Probability and Statistics

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Contents

Chapter 1

Discrite Distributions

1.1 Generic Formulas

1.1.1 Expected Value

X is discrete random variable
$g : \mathbf{R} \to \mathbf{R}$
$\Omega X = Im(X)$

$$E[g(X)] = \sum_{x \in \Omega X} P(X = x) * g(x) \tag{1.1}$$

1.1.2 Variance

$$
\begin{aligned}
var(g(X)) &= E([g(X) - E(g(X))]^2) && (1.2)\\
&= \sum_{x \in \Omega X} [g(X) - E(g(X))]^2 * P(X = x)] && (1.3)\\
&= \sum_{x \in \Omega X} [g(X)^2 - 2 * g(X) * E(g(X)) + E(g(X))^2] * P(X = x) && (1.4)\\
&= \sum_{x \in \Omega X} g(X)^2 * P(X = x) && (1.5)\\
&- \sum_{x \in \Omega X} 2 * g(X) * E(g(X)) * P(X = x) && (1.6)\\
&+ \sum_{x \in \Omega X} E(g(X))^2 * P(X = x) && (1.7)\\
&= \sum_{x \in \Omega X} g(X)^2 * P(X = x) && (1.8)\\
&- 2 * E(g(X)) * \sum_{x \in \Omega X} g(X) * P(X = x) && (1.9)\\
&+ E(g(X))^2 * \sum_{x \in \Omega X} P(X = x) && (1.10)\\
&= E(g(X)^2) - 2 * E(g(X)) * E(g(X)) + E(g(X))^2 * 1 && (1.11)\\
&= E(g(X)^2) - 2 * E(g(X))^2 + E(g(X))^2 && (1.12)\\
&= E(g(X)^2) - E(g(X))^2 && (1.13)\\
\square & && (1.14)
\end{aligned}
$$

1.1.3 Covariance Matrix

$$
\begin{aligned}
cov(X) &= E[(X - E(X)]^2 && (1.15)\\
&= \sum_{x \in \Omega X} (X - E(X))^2 * P(X = x) && (1.16)\\
&= \sum_{x \in \Omega X} [X^2 - 2XE(X) + E(X)^2] * P(X = x) && (1.17)\\
&= \sum_{x \in \Omega X} X^2 P(X = x) && (1.18)\\
&- \sum_{x \in \Omega X} 2XE(X) * P(X = x) && (1.19)\\
&+ \sum_{x \in \Omega X} E(X)^2 P(X = x) && (1.20)\\
&= \sum_{x \in \Omega X} X^2 P(X = x) && (1.21)\\
&- 2E(X) * \sum_{x \in \Omega X} XP(X = x) && (1.22)\\
&+ E(X)^2 * \sum_{x \in \Omega X} P(X = x) && (1.23)\\
&= E(X^2) - 2E(X)E(X) + E(X)^2 * 1 && (1.24)\\
&= E(X^2) - 2 * E(X)^2 + E(X)^2 && (1.25)\\
&= E(X^2) - E(X)^2 && (1.26)\\
&= E(X^t X) - \mu^t \mu && (1.27)\\
\square & && (1.28)
\end{aligned}
$$

1.1.4 Variance of the Sample Mean

$$
\begin{aligned}
Var\left(\overline{X}\right) &= Var\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) && (1.29)\\
&= \frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right) && (1.30)\\
&= \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i), \text{ by independence} && (1.31)\\
&= \frac{1}{n^2}\left[Var(X_1) + Var(X_2) + \ldots + Var(X_n)\right] && (1.32)\\
&= \frac{1}{n^2}\left[\sigma^2 + \sigma^2 + \ldots + \sigma^2\right], \text{ since the } X_i \text{ are identically distributed} && (1.33)\\
&= \frac{1}{n^2}(n\sigma^2) && (1.34)\\
&= \frac{\sigma^2}{n} && (1.35)
\end{aligned}
$$

1.1.5 Law of Iterated Expectation

$$E[X] = E[E[X|Y]] \tag{1.36}$$

1.1.6 Law of Total Variance

$$var(X) = E[var(X|Y)] + var(E[X|Y]) \tag{1.37}$$

1.1.7 MSE

$$
\begin{aligned}
\hat{\theta} &= \hat{\theta}(X) \text{ Random Variable} && (1.38)\\
E[\hat{\theta}] &= \text{constant} && (1.39)\\
\theta &= \text{true value, constant} && (1.40)\\
& && (1.41)\\
MSE(\hat{\theta}) &= E[(\hat{\theta}-\theta)^2] && (1.42)\\
&= E[(\hat{\theta}-E[\hat{\theta}]+E[\hat{\theta}])-\theta)^2] && (1.43)\\
&= E[([\hat{\theta}-E[\hat{\theta}]]+[E[\hat{\theta}]-\theta])^2] && (1.44)\\
&= E[(A+B)^2] && (1.45)\\
& && (1.46)\\
& A = \hat{\theta}-E[\hat{\theta}] && (1.47)\\
& B = E[\hat{\theta}]-\theta && (1.48)\\
& && (1.49)\\
&= E[A^2+2AB+B^2] && (1.50)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2+2(\hat{\theta}-E[\hat{\theta}])(E[\hat{\theta}]-\theta)+(E[\hat{\theta}]-\theta)^2] && (1.51)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+E[2(\hat{\theta}-E[\hat{\theta}])(E[\hat{\theta}]-\theta)]+E[(E[\hat{\theta}]-\theta)^2] && (1.52)\\
& && (1.53)\\
& C = E[\hat{\theta}]-\theta \text{ is a constant} && (1.54)\\
& && (1.55)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+E[2(\hat{\theta}-E[\hat{\theta}])(C)]+E[C^2] && (1.56)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+2CE[\hat{\theta}-E[\hat{\theta}]]+C^2 && (1.57)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+2C(E[\hat{\theta}]-E[E[\hat{\theta}]])+C^2 && (1.58)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+2C(E[\hat{\theta}]-E[\hat{\theta}])+C^2 && (1.59)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+2C(0)+C^2 && (1.60)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+C^2 && (1.61)\\
&= E[(\hat{\theta}-E[\hat{\theta}])^2]+(E[\hat{\theta}]-\theta)^2 && (1.62)\\
& && (1.63)\\
& var(\hat{\theta}) = E[(\hat{\theta}-E[\hat{\theta}])^2] && (1.64)\\
& bias(\hat{\theta},\theta) = E[\hat{\theta}]-\theta && (1.65)\\
& && (1.66)\\
&= var(\hat{\theta})+(bias(\hat{\theta},\theta))^2 && (1.67)\\
& && (1.68)
\end{aligned}
$$

1.2 Bernoulli Distribution

The Bernoulli Distribution is a special case of the Binomial Distribution, where

$$n = 1$$

.

1.2.1 PMF

$$\begin{aligned} P(X = k) &= \binom{1}{k} p^k (1-p)^{1-k} && (1.69) \\ &= p^k (1-p)^{n-k} && (1.70) \end{aligned}$$

1.2.2 Expected Value

$$\begin{aligned} E(x) &= \sum_{k \geqslant 1} [\binom{n}{k} p^k (1-p)^{n-k}] * k && (1.71) \\ &= np && \text{see Binomial Distribution E[X]} \quad (1.72) \\ &= 1 * p && (1.73) \\ &= p && (1.74) \end{aligned}$$

1.2.3 Variance

$$\begin{aligned} Var(X) &= np * (1-p) && (1.75) \\ &= p * (1-p) && (1.76) \\ & && (1.77) \end{aligned}$$

1.2.4 Likelihood of IID Bernoulli

$$\begin{aligned} x_i \overset{iid}{\sim} Bernoulli(p) & && (1.78) \\ L(x_i|p) &= p(x_1, x_2, ..., x_n|p) && (1.79) \\ &= \prod_{n=1}^{n} p(x_i|p) && (1.80) \\ &= p^S * (1-p)^{n-S} && (1.81) \end{aligned}$$

1.2.5 Maximun Likelihood

$$
\begin{aligned}
\frac{d[L(x_i|p)]}{dp} &= \frac{d[p^S * (1-p)^{n-S}]}{dp} && (1.82)\\
\frac{d[log(L(x_i|p))]}{dp} &= \frac{d[log(p^S * (1-p)^{n-S})]}{dp} && (1.83)\\
&= \frac{d}{dp} * [log(p^S * (1-p)^{n-S})] && (1.84)\\
&= \frac{d}{dp}[log(p^S) + log((1-p)^{n-S})] && (1.85)\\
&= \frac{d}{dp}[S * log(p) + (n-S) * log(1-p)] && (1.86)\\
&= S * \frac{d}{dp}[log(p)] + (n-S) * \frac{d}{dp}[log(1-p)] && (1.87)\\
&= S * [\frac{1}{p}] + (n-S) * \frac{d}{dp}[log(1-p)] && \text{chain rule} \quad (1.88)\\
&= S * \frac{1}{p} + (n-S) * \frac{1}{p-1} && (1.89)\\
&= \frac{S}{p} + \frac{n-S}{p-1} && (1.90)\\
&= \frac{S * (p-1)}{p * (p-1)} + \frac{p * (n-S)}{p * (p-1)} && (1.91)\\
&= \frac{S * (p-1) + p * (n-S)}{p * (p-1)} && (1.92)\\
&= \frac{S * p - S + p * n - p * S}{p * (p-1)} && (1.93)\\
&= \frac{-S + p * n}{p * (p-1)} && (1.94)\\
0 &= \frac{-S + p * n}{p * (p-1)} && (1.95)\\
0 * (p * (p-1)) &= -S + p * n && (1.96)\\
0 &= -S + p * n && (1.97)\\
S &= p * n && (1.98)\\
\frac{S}{n} &= p && (1.99)\\
p &= \frac{S}{n} && (1.100)\\
& && (1.101)
\end{aligned}
$$

1.2.6 MGF

$$
\begin{aligned}
M(t) &= E[e^{tX}] && (1.102)\\
&= (1-p)e^{t*0} + pe^{t*1} && (1.103)\\
&= (1-p) + pe^{t} && (1.104)\\
& && (1.105)
\end{aligned}
$$

E[X] using MGF

$$
\begin{aligned}
E[X] &= \frac{d^1}{dt^1}[M(t)](0) && (1.106)\\
& && (1.107)\\
M(t) &= (1-p) + pe^{t} && (1.108)\\
& && (1.109)\\
E[X] &= \frac{d^1}{dt^1}[(1-p) + pe^{t}](0) && (1.110)\\
&= [\frac{d^1}{dt^1}[(1-p)] + \frac{d^1}{dt^1}[pe^{t}]](0) && (1.111)\\
&= [0 + \frac{d^1}{dt^1}[pe^{t}]](0) && (1.112)\\
&= [\frac{d^1}{dt^1}[pe^{t}]](0) && (1.113)\\
&= [pe^{t}](0) && (1.114)\\
&= [pe^{0}] && (1.115)\\
&= p * 1 && (1.116)\\
&= p && (1.117)\\
& && (1.118)
\end{aligned}
$$

$E[X^2]$ **using MGF**

$$E[X^2] = \frac{d^2}{dt^2}[M(t)](0) \tag{1.119}$$
$$\tag{1.120}$$
$$M(t) = (1-p) + pe^t \tag{1.121}$$
$$\tag{1.122}$$
$$E[X^2] = \frac{d^2}{dt^2}[(1-p) + pe^t](0) \tag{1.123}$$
$$= \frac{d^1}{dt^1}[pe^t](0) \qquad \text{see E[X] using MGF} \tag{1.124}$$
$$= [pe^t](0) \tag{1.125}$$
$$= [pe^0] \tag{1.126}$$
$$= p * 1 \tag{1.127}$$
$$= p \tag{1.128}$$
$$\tag{1.129}$$

And these values can be checked calculating the $var[X]$.

$$var[X] = E[X^2] - E[X]^2 \tag{1.130}$$
$$= p - p^2 \tag{1.131}$$
$$= p * (1-p) \tag{1.132}$$
$$\square \tag{1.133}$$

1.3 Binomial Distribution

PMF

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{1.134}$$

1.3.1 Expected Value

$E[g(X)]$ when $g(X) = X$.

$$E(X) = \sum_{k \geqslant 0} P(x = k) * k \tag{1.135}$$
$$= \sum_{k \geqslant 0} [\binom{n}{k} p^k (1-p)^{n-k}] * k \tag{1.136}$$
$$\tag{1.137}$$

when

$$k = 0$$

, the formula

$$[\binom{n}{k} p^k (1-p)^{n-k}] * k = [\binom{n}{0} p^k (1-p)^n] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$E(X) = \sum_{k \geqslant 1} \binom{n}{k} p^k (1-p)^{n-k} * k \tag{1.138}$$

$$= \sum_{k \geqslant 1} \frac{n}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} * k \qquad \text{see BinomialCoefficient} \tag{1.139}$$

$$= \sum_{k \geqslant 1} \frac{n * k}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} \tag{1.140}$$

$$= \sum_{k \geqslant 1} n * \binom{n-1}{k-1} p^k (1-p)^{n-k} \tag{1.141}$$

$$= \sum_{k \geqslant 1} n * p * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \tag{1.142}$$

$$= np * \sum_{k \geqslant 1} \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \tag{1.143}$$

$$u = n - 1 \tag{1.144}$$

$$z = k - 1 \tag{1.145}$$

$$u - z = (n-1) - (k-1) \tag{1.146}$$

$$= n - 1 - k + 1 \tag{1.147}$$

$$= n - k \tag{1.148}$$

$$k > 1 = (z+1) > 1 \tag{1.149}$$

$$= z > 0 \tag{1.150}$$

$$= np * \sum_{z > 0} \binom{u}{z} p^z (1-p)^{u-z} \tag{1.151}$$

$$= np * 1 \qquad see Binomial Distribution Proof Equals 1 \tag{1.152}$$

$$= np \tag{1.153}$$

$$\square \tag{1.154}$$

1.3.2 Variance

$$Var(X) = E(X^2) - E(X)^2 \quad \text{see Variance} \tag{1.155}$$
$$= \sum_{k \geqslant 0} [\binom{n}{k} p^k (1-p)^{n-k}] * k^2 - np \quad \text{see Binomial Expected Value} \tag{1.156}$$

when

$$k = 0$$

, the formula

$$[\binom{n}{k} p^k (1-p)^{n-k}] * k = [\binom{n}{0} p^k (1-p)^{n}] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$= \sum_{k \geqslant 1} [\binom{n}{k} p^k (1-p)^{n-k}] * k^2 - (np)^2 \tag{1.157}$$

$$= \sum_{k \geqslant 1} \frac{n}{k} [\binom{n-1}{k-1} p^k (1-p)^{n-k}] * k^2 - (np)^2 \tag{1.158}$$

$$= \sum_{k \geqslant 1} \frac{n * k^2}{k} [\binom{n-1}{k-1} p^k (1-p)^{n-k}] - (np)^2 \tag{1.159}$$

$$= \sum_{k \geqslant 1} [nk * \binom{n-1}{k-1} p^k (1-p)^{n-k}] - (np)^2 \tag{1.160}$$

$$= \sum_{k \geqslant 1} [nkp * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \tag{1.161}$$

$$= np * \sum_{k \geqslant 1} [k * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \tag{1.162}$$

$$u = n - 1 \tag{1.163}$$

$$z = k - 1 \tag{1.164}$$

$$u - z = (n-1) - (k-1) \tag{1.165}$$

$$= n - 1 - k + 1 \tag{1.166}$$

$$= n - k \tag{1.167}$$

$$k >= 1 = (z+1) >= 1 \tag{1.168}$$

$$= z >= 0 \tag{1.169}$$

$$= np * \sum_{z \geqslant 0} [(z+1) * \binom{u}{z} p^z (1-p)^{u-z}] - (np)^2 \tag{1.170}$$

$$= np * [\sum_{z \geqslant 0} [z * \binom{u}{z} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.171}$$

$$= np * [\sum_{z \geqslant 0} [z * \frac{u}{z} * \binom{u-1}{z-1} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.172}$$

$$= np * [u * \sum_{z \geqslant 0} [\binom{u-1}{z-1} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.173}$$

$$\tag{1.174}$$

$$= np * [up * \sum_{z \geqslant 0} [\binom{u-1}{z-1} p^{z-1}(1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (1.175)$$

$$= np * [up * \sum_{z \geqslant 1} [\binom{u-1}{z-1} p^{z-1}(1-p)^{(u-1)-(z-1)}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (1.176)$$

$$= np * [up * \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (1.177)$$

$$= np * [up * (p+q)^{u-1} + (p+q)^u] - (np)^2 \quad (1.178)$$

$$= np * [(n-1) * p * (p+q)^{n-1-1} + (p+q)^(n-1)] - (np)^2 \quad (1.179)$$

$$= np * [(n-1) * p * (p+q)^{n-2} + (p+q)^(n-1)] - (np)^2 \quad (1.180)$$

$$= np * ([(n-1) * p * (p+q)^{n-2} + (p+q)^(n-1)] - np) \quad (1.181)$$

$$= np * ([(n-1) * p + 1] - np) \quad \text{p+q=1} \quad (1.182)$$

$$= np * ([(n-1) * p + 1] - np) \quad (1.183)$$

$$= np * ([np - p + 1] - np) \quad (1.184)$$

$$= np * (np - p + 1 - np) \quad (1.185)$$

$$= np * (-p + 1) \quad (1.186)$$

$$= np * (1 - p) \quad (1.187)$$

$$\square \quad (1.188)$$

1.4 Geometric Distribution

1.4.1 CDF

$$
\begin{aligned}
P(X \leq x) &= \\
CDF(X = x) &= \sum_{i=0}^{x} (1-p)^i p && \text{by geometric summation} \\
&= p\frac{1-(1-p)^x}{1-(1-p)} \\
&= p\frac{1-(1-p)^x}{1-1+p} \\
&= p\frac{1-(1-p)^x}{p} \\
&= 1-(1-p)^x
\end{aligned}
$$

$\square$

$$
\begin{aligned}
P(X > x) &= 1 - CDF(X = x) \\
&= 1-(1-(1-p)^x) \\
&= 1-1+(1-p)^x \\
&= (1-p)^x
\end{aligned}
$$

$\square$

1.5 Normal Distribution

1.5.1 Definition

$$
pdf(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{1.189}
$$

Chapter 2

Continuous Distributions

2.1 Uniform Distribution

2.1.1 PDF

$$\begin{aligned}
\int_a^b k dx &= 1 \\
&= k \int_a^b dx \\
&= k[x|_a^b] \\
&= k[b-a]
\end{aligned}$$

$$\begin{aligned}
k[b-a] &= 1 \\
k &= \frac{1}{b-a}
\end{aligned}$$

2.1.2 Expected Value

$$
\begin{aligned}
\int_a^b x(\frac{1}{b-a})dx = & \\
&= \int_a^b x(\frac{1}{b-a})dx \\
&= \frac{1}{b-a}\int_a^b xdx \\
&= \frac{1}{b-a}[\frac{x^2}{2}|_a^b] \\
&= \frac{1}{b-a}[\frac{b^2}{2} - \frac{a^2}{2}] \\
&= \frac{1}{b-a}[\frac{b^2-a^2}{2}] \\
&= \frac{1}{b-a}[\frac{(b+a)(b-a)}{2}] \\
&= \frac{(b+a)(b-a)}{2(b-a)} \\
&= \frac{b+a}{2}
\end{aligned}
$$

2.1.3 Variance

$$
\begin{aligned}
var(X) &= E[X^2] - E[X]^2 \\
&= [\int_a^b x^2 (\frac{1}{b-a}) dx] - (\frac{b+a}{2})^2 \\
&= [\frac{1}{b-a} \int_a^b x^2 dx] - (\frac{b+a}{2})^2 \\
&= [\frac{1}{b-a} \frac{x^3}{3} |_a^b] - (\frac{b+a}{2})^2 \\
&= \frac{1}{b-a} (\frac{b^3}{3} - \frac{a^3}{3}) - (\frac{b+a}{2})^2 \\
&= \frac{b^3 - a^3}{3(b-a)} - \frac{(b+a)^2}{4} \\
&= \frac{(b-a)(b^2 - ab + a^2)}{3(b-a)} - \frac{(b+a)^2}{4} \\
&= \frac{(b^2 - ab + a^2)}{3} - \frac{(b+a)^2}{4} \\
&= \frac{4(b^2 - ab + a^2)}{12} - \frac{3(b+a)^2}{12} \\
&= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3(b^2 + 2ab + a^2)}{12} \\
&= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3b^2 + 6ab + 3a^2}{12} \\
&= \frac{4b^2 - 3b^2 - 4ab - 6ab + 4a^2 - 3a^2}{12} \\
&= \frac{b^2 - 2ab + a^2}{12} \\
&= \frac{(b-a)^2}{12}
\end{aligned}
$$

2.2 Exponential Distribution

2.2.1 PDF

$$f_x(x) = \lambda e^{-\lambda x} \qquad x >= 0$$

2.2.2 CDF

$$CDF(x) = \int_{-\infty}^{x} f_x dx$$
$$CDF(x) = \int_{-\infty}^{x} \lambda e^{-\lambda x} dx$$

$$u = e^{-\lambda x}$$
$$\frac{du}{dx} = d[e^{-\lambda x}]$$
$$du = d[e^{-\lambda x}] * dx$$
$$du = [-\lambda * e^{-\lambda x}] * dx$$
$$du = -\lambda e^{-\lambda x} dx$$

$$CDF(x) = \int_{-\infty}^{x} \lambda e^{-\lambda x} dx$$
$$CDF(x) = -1 * \int_{-\infty}^{x} -1 * \lambda e^{-\lambda x} dx$$
$$CDF(x) = -1 * \int_{-\infty}^{x} -\lambda e^{-\lambda x} dx$$
$$CDF(x) = -1 * \int_{-\infty}^{x} du$$
$$CDF(x) = -1 * \int_{0}^{x} du \qquad \text{by x bounds}$$
$$CDF(x) = -1 * [u]_0^x$$
$$CDF(x) = -1 * [e^{-\lambda x}]_0^x$$
$$CDF(x) = -1 * [e^{-\lambda x} - e^{-\lambda 0}]$$
$$CDF(x) = -1 * [e^{-\lambda x} - e^{0}]$$
$$CDF(x) = -1 * [e^{-\lambda x} - 1]$$
$$CDF(x) = [1 - e^{-\lambda x}]$$
$$CDF(x) = 1 - e^{-\lambda x}$$

2.2.3 Expected Value

$$f_x(x) = \lambda e^{-\lambda x}$$

$$E[x] = \int x * f_x(x) dx$$
$$E[x] = \int x * \lambda e^{-\lambda x} dx$$

$$u = x$$
$$du = 1 * dx$$
$$v = e^{-\lambda x}$$
$$dv = e^{-\lambda x} * -\lambda * dx$$
$$dv = -\lambda e^{-\lambda x} dx$$

$$E[x] = \int [x][\lambda e^{-\lambda x} dx]$$
$$E[x] = -1 * \int [x][-1 * \lambda e^{-\lambda x} dx]$$
$$E[x] = -1 * \int [x][-\lambda e^{-\lambda x} dx]$$
$$E[x] = -1 * \int u dv$$
$$E[x] = -1 * [uv|_{-\infty}^{\infty} - \int v du]$$
$$E[x] = -1 * [uv|_{0}^{\infty} - \int v du] \qquad \text{because x bounds}$$
$$E[x] = -1 * [[x e^{-\lambda x}]_{0}^{\infty} - \int e^{-\lambda x} dx]$$

$$\begin{aligned}
[xe^{-\lambda x}]_0^\infty &= \lim_{x->\infty} [xe^{-\lambda x} - (0)e^{-\lambda(0)}] \\
&= \lim_{x->\infty} [xe^{-\lambda x} + 0e^0] \\
&= \lim_{x->\infty} [xe^{-\lambda x}] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
E[x] &= -1 * - \int e^{-\lambda x} dx \\
E[x] &= \frac{1}{\lambda} \int \lambda e^{-\lambda x} dx \\
E[x] &= \frac{1}{\lambda}
\end{aligned}$$