Exercise 1.4.21

Let

$$A = \begin{Bmatrix} 16 & 4 & 8 & 4 \\ 4 & 10 & 8 & 4 \\ 8 & 8 & 12 & 10 \\ 4 & 4 & 10 & 12 \end{Bmatrix}$$

and

$$b = \begin{Bmatrix} 32 \\ 26 \\ 38 \\ 30 \end{Bmatrix}$$

Notice that A is symmetric,
(a) Use the inner-product formulation of Cholesky's method to show that A is positive definite and compute its Cholesky factor,
(b) Use forward and back substitution to solve the linear system $Ax = b$.
Answers:
(a)
Using octave:
octave:1¿ A =[16,4,8,4;4,10,8,4;8,8,12,10;4,4,10,12]
A =

16 4 8 4
4 10 8 4
8 8 12 10
4 4 10 12

octave:2¿ chol(A)
ans =

4 1 2 1
0 3 2 1
0 0 2 3
0 0 0 1

(b)

$$\begin{aligned} A &= R^T R \\ Ax &= b \\ R^T Rx &= b \\ y &= Rx \\ R^T y &= b \end{aligned}$$

Using octave

octave:1¿ A = [16,4,8,4;4,10,8,4;8,8,12,10;4,4,10,12]
A =

16 4 8 4
4 10 8 4
8 8 12 10
4 4 10 12

octave:2¿ R = chol(A)
R =

4 1 2 1
0 3 2 1
0 0 2 3
0 0 0 1

octave:3¿ b = [32;26;38;30]
b =

32
26
38
30

octave:4¿ y = R'\b
y =

8
6
5
1

$$R^t y = b$$

$$y = \begin{Bmatrix} 8 \\ 6 \\ 5 \\ 1 \end{Bmatrix}$$

$$Rx = y$$

using octave
octave:6¿ x = R\y
x =

1
1
1
1

$$x = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$