If A^{-1} exists then $det(A) \neq 0$.
To solve this we need.

$$det(A * B) = det(A) * det(B)$$
$$det(A^{-1}) = \frac{1}{det(A)}$$

$$\begin{aligned} Ax &= b \\ A^{-1}Ax &= A^{-1}b \\ Ix &= A^{-1}b \\ x &= A^{-1}b \\ det(x) &= det(A^{-1}b) \\ det(x) &= det(A^{-1})det(b) \\ det(x) &= \frac{det(b)}{det(A)} \end{aligned}$$

And this last step is definided in all points that $det(A) \neq 0$.