

Probability and Statistics

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Contents

1 Generic Formulas

1.1 Expected Value

X is discrete random variable

$$g : \mathbf{R} \rightarrow \mathbf{R}$$

$$\Omega X = \text{Im}(X)$$

$$E[g(X)] = \sum_{x \in \Omega X} P(X = x) * g(x) \quad (1)$$

1.2 Variance

$$\text{var}(g(X)) = E([g(X) - E(g(X))]^2) \quad (2)$$

$$= \sum_{x \in \Omega X} [g(X) - E(g(X))]^2 * P(X = x) \quad (3)$$

$$= \sum_{x \in \Omega X} [g(X)^2 - 2 * g(X) * E(g(X)) + E(g(X))^2] * P(X = x) \quad (4)$$

$$= \sum_{x \in \Omega X} g(X)^2 * P(X = x) \quad (5)$$

$$- \sum_{x \in \Omega X} 2 * g(X) * E(g(X)) * P(X = x) \quad (6)$$

$$+ \sum_{x \in \Omega X} E(g(X))^2 * P(X = x) \quad (7)$$

$$= \sum_{x \in \Omega X} g(X)^2 * P(X = x) \quad (8)$$

$$- 2 * E(g(X)) * \sum_{x \in \Omega X} g(X) * P(X = x) \quad (9)$$

$$+ E(g(X))^2 * \sum_{x \in \Omega X} P(X = x) \quad (10)$$

$$= E(g(X)^2) - 2 * E(g(X)) * E(g(X)) + E(g(X))^2 * 1 \quad (11)$$

$$= E(g(X)^2) - 2 * E(g(X))^2 + E(g(X))^2 \quad (12)$$

$$= E(g(X)^2) - E(g(X))^2 \quad (13)$$

$$\square \quad (14)$$

1.3 Covariance Matrix

$$\text{cov}(X) = E[(X - E(X))^2] \quad (15)$$

$$= \sum_{x \in \Omega X} (X - E(X))^2 * P(X = x) \quad (16)$$

$$= \sum_{x \in \Omega X} [X^2 - 2XE(X) + E(X)^2] * P(X = x) \quad (17)$$

$$= \sum_{x \in \Omega X} X^2 P(X = x) \quad (18)$$

$$- \sum_{x \in \Omega X} 2XE(X) * P(X = x) \quad (19)$$

$$+ \sum_{x \in \Omega X} E(X)^2 P(X = x) \quad (20)$$

$$= \sum_{x \in \Omega X} X^2 P(X = x) \quad (21)$$

$$- 2E(X) * \sum_{x \in \Omega X} XP(X = x) \quad (22)$$

$$+ E(X)^2 * \sum_{x \in \Omega X} P(X = x) \quad (23)$$

$$= E(X^2) - 2E(X)E(X) + E(X)^2 * 1 \quad (24)$$

$$= E(X^2) - 2 * E(X)^2 + E(X)^2 \quad (25)$$

$$= E(X^2) - E(X)^2 \quad (26)$$

$$= E(X^t X) - \mu^t \mu \quad (27)$$

$$\square \quad (28)$$

1.4 Variance of the Sample Mean

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{n} \sum_{i=1}^n X_i\right) \quad (29)$$

$$= \frac{1}{n^2} \text{Var}\left(\sum_{i=1}^n X_i\right) \quad (30)$$

$$= \frac{1}{n^2} \sum_{i=1}^n \text{Var}(X_i), \text{ by independence} \quad (31)$$

$$= \frac{1}{n^2} [\text{Var}(X_1) + \text{Var}(X_2) + \dots + \text{Var}(X_n)] \quad (32)$$

$$= \frac{1}{n^2} [\sigma^2 + \sigma^2 + \dots + \sigma^2], \text{ since the } X_i \text{ are identically distributed} \quad (33)$$

$$= \frac{1}{n^2} (n\sigma^2) \quad (34)$$

$$= \frac{\sigma^2}{n} \quad (35)$$

1.5 Law of Iterated Expectation

$$E[X] = E[E[X|Y]] \quad (36)$$

1.6 Law of Total Variance

$$\text{var}(X) = E[\text{var}(X|Y)] + \text{var}(E[X|Y]) \quad (37)$$

1.7 MSE

$$\hat{\theta} = \hat{\theta}(X) \text{ Random Variable} \quad (38)$$

$$E[\hat{\theta}] = \text{constant} \quad (39)$$

$$\theta = \text{true value, constant} \quad (40)$$

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] \quad (41)$$

$$= E[(\hat{\theta} - E[\hat{\theta}] + E[\hat{\theta}] - \theta)^2] \quad (42)$$

$$= E[(\hat{\theta} - E[\hat{\theta}]) + (E[\hat{\theta}] - \theta)]^2 \quad (43)$$

$$= E[(A + B)^2] \quad (44)$$

$$= E[A^2 + 2AB + B^2] \quad (45)$$

$$A = \hat{\theta} - E[\hat{\theta}] \quad (46)$$

$$B = E[\hat{\theta}] - \theta \quad (47)$$

$$= E[A^2 + 2AB + B^2] \quad (48)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2 + 2(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta) + (E[\hat{\theta}] - \theta)^2] \quad (49)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + E[2(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta)] + E[(E[\hat{\theta}] - \theta)^2] \quad (50)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + E[2(\hat{\theta} - E[\hat{\theta}])(C)] + E[C^2] \quad (51)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2CE[\hat{\theta} - E[\hat{\theta}]] + C^2 \quad (52)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(E[\hat{\theta}] - E[E[\hat{\theta}]]) + C^2 \quad (53)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(E[\hat{\theta}] - E[\hat{\theta}]) + C^2 \quad (54)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(0) + C^2 \quad (55)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + C^2 \quad (56)$$

$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + (E[\hat{\theta}] - \theta)^2 \quad (57)$$

$$= \text{var}(\hat{\theta}) + (bias(\hat{\theta}, \theta))^2 \quad (58)$$

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2 Binomial Distribution

PMF

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (69)$$

2.1 Expected Value

$E[g(X)]$ when $g(X) = X$.

$$E(X) = \sum_{k \geq 0} P(x = k) * k \quad (70)$$

$$= \sum_{k \geq 0} \left[\binom{n}{k} p^k (1 - p)^{n-k} \right] * k \quad (71)$$

$$(72)$$

when

$$k = 0$$

, the formula

$$\left[\binom{n}{k} p^k (1 - p)^{n-k} \right] * k = \left[\binom{n}{0} p^k (1 - p)^n \right] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$E(X) = \sum_{k \geq 1} \binom{n}{k} p^k (1-p)^{n-k} * k \quad (73)$$

$$= \sum_{k \geq 1} \frac{n}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} * k \quad \text{see BinomialCoefficient} \quad (74)$$

$$= \sum_{k \geq 1} \frac{n * k}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} \quad (75)$$

$$= \sum_{k \geq 1} n * \binom{n-1}{k-1} p^k (1-p)^{n-k} \quad (76)$$

$$= \sum_{k \geq 1} n * p * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \quad (77)$$

$$= np * \sum_{k \geq 1} \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \quad (78)$$

$$u = n - 1 \quad (79)$$

$$z = k - 1 \quad (80)$$

$$u - z = (n - 1) - (k - 1) \quad (81)$$

$$= n - 1 - k + 1 \quad (82)$$

$$= n - k \quad (83)$$

$$k > 1 = (z + 1) > 1 \quad (84)$$

$$= z > 0 \quad (85)$$

$$= np * \sum_{z > 0} \binom{u}{z} p^z (1-p)^{u-z} \quad (86)$$

$$= np * 1 \quad \text{see BinomialDistributionProof Equals 1} \quad (87)$$

$$= np \quad (88)$$

$$\square \quad (89)$$

2.2 Variance

$$Var(X) = E(X^2) - E(X)^2 \quad \text{see Variance} \quad (90)$$

$$= \sum_{k \geq 0} \left[\binom{n}{k} p^k (1-p)^{n-k} \right] * k^2 - np \quad \text{see Binomial Expected Value} \quad (91)$$

when

$$k = 0$$

, the formula

$$[\binom{n}{k} p^k (1-p)^{n-k}] * k = [\binom{n}{0} p^k (1-p)^n] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$= \sum_{k \geq 1} \left[\binom{n}{k} p^k (1-p)^{n-k} \right] * k^2 - (np)^2 \quad (92)$$

$$= \sum_{k \geq 1} \frac{n}{k} \left[\binom{n-1}{k-1} p^k (1-p)^{n-k} \right] * k^2 - (np)^2 \quad (93)$$

$$= \sum_{k \geq 1} \frac{n * k^2}{k} \left[\binom{n-1}{k-1} p^k (1-p)^{n-k} \right] - (np)^2 \quad (94)$$

$$= \sum_{k \geq 1} [nk * \binom{n-1}{k-1} p^k (1-p)^{n-k}] - (np)^2 \quad (95)$$

$$= \sum_{k \geq 1} [nkp * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \quad (96)$$

$$= np * \sum_{k \geq 1} [k * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \quad (97)$$

$$u = n - 1 \quad (98)$$

$$z = k - 1 \quad (99)$$

$$u - z = (n - 1) - (k - 1) \quad (100)$$

$$= n - 1 - k + 1 \quad (101)$$

$$= n - k \quad (102)$$

$$k \geq 1 = (z + 1) \geq 1 \quad (103)$$

$$= z \geq 0 \quad (104)$$

$$= np * \sum_{z \geq 0} [(z + 1) * \binom{u}{z} p^z (1-p)^{u-z}] - (np)^2 \quad (105)$$

$$= np * \left[\sum_{z \geq 0} [z * \binom{u}{z} p^z (1-p)^{u-z}] + \sum_{z \geq 0} \left[\binom{u}{z} p^z (1-p)^{u-z} \right] \right] - (np)^2 \quad (106)$$

$$= np * \left[\sum_{z \geq 0} \left[z * \frac{u}{z} * \binom{u-1}{z-1} p^z (1-p)^{u-z} \right] + \sum_{z \geq 0} \left[\binom{u}{z} p^z (1-p)^{u-z} \right] \right] - (np)^2 \quad (107)$$

$$= np * \left[u * \sum_{z \geq 0} \left[\binom{u-1}{z-1} p^z (1-p)^{u-z} \right] + \sum_{z \geq 0} \left[\binom{u}{z} p^z (1-p)^{u-z} \right] \right] - (np)^2 \quad (108)$$

$$(109)$$

$$= np * [up * \sum_{z \geq 0} [\binom{u-1}{z-1} p^{z-1} (1-p)^{u-z}] + \sum_{z \geq 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (110)$$

$$= np * [up * \sum_{z \geq 1} [\binom{u-1}{z-1} p^{z-1} (1-p)^{(u-1)-(z-1)}] + \sum_{z \geq 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (111)$$

$$= np * [up * \sum_{z \geq 0} [\binom{u}{z} p^z (1-p)^{u-z}] + \sum_{z \geq 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \quad (112)$$

$$= np * [up * (p+q)^{u-1} + (p+q)^u] - (np)^2 \quad (113)$$

$$= np * [(n-1) * p * (p+q)^{n-1-1} + (p+q)^{(n-1)}] - (np)^2 \quad (114)$$

$$= np * [(n-1) * p * (p+q)^{n-2} + (p+q)^{(n-1)}] - (np)^2 \quad (115)$$

$$= np * [(n-1) * p * (p+q)^{n-2} + (p+q)^{(n-1)}] - np \quad (116)$$

$$= np * [(n-1) * p + 1] - np \quad (117)$$

$$= np * [(n-1) * p + 1] - np \quad (118)$$

$$= np * [np - p + 1] - np \quad (119)$$

$$= np * (np - p + 1 - np) \quad (120)$$

$$= np * (-p + 1) \quad (121)$$

$$= np * (1 - p) \quad (122)$$

$$\square \quad (123)$$

p+q=1

3 Bernoulli Distribution

The Bernoulli Distribution is a special case of the Binomial Distribution, where

$$n = 1$$

PMF

$$P(X = k) = \binom{1}{k} p^k (1-p)^{1-k} \quad (124)$$

$$= p^k (1-p)^{n-k} \quad (125)$$

3.1 Expected Value

$$E(x) = \sum_{k \geq 1} \left[\binom{n}{k} p^k (1-p)^{n-k} \right] * k \quad (126)$$

$$= np \quad \text{see Binomial Distribution} \quad (127)$$

$$= 1 * p \quad (128)$$

$$= p \quad (129)$$

3.2 Variance

$$Var(X) = np * (1-p) \quad (130)$$

$$= p * (1-p) \quad (131)$$

$$(132)$$

3.3 Likelihood of IID Bernoulli

$$x_i \stackrel{iid}{\sim} \text{Bernoulli}(p) \quad (133)$$

$$L(x_i|p) = p(x_1, x_2, \dots, x_n|p) \quad (134)$$

$$= \prod_{n=1}^n p(x_i|p) \quad (135)$$

$$= p^S * (1-p)^{n-S} \quad (136)$$

3.4 Maximun Likelihood

$$\frac{d[L(x_i|p)]}{dp} = \frac{d[p^S * (1-p)^{n-S}]}{dp} \quad (137)$$

$$\frac{d[\log(L(x_i|p))]}{dp} = \frac{d[\log(p^S * (1-p)^{n-S})]}{dp} \quad (138)$$

$$= \frac{d}{dp} * [\log(p^S * (1-p)^{n-S})] \quad (139)$$

$$= \frac{d}{dp} [\log(p^S) + \log((1-p)^{n-S})] \quad (140)$$

$$= \frac{d}{dp} [S * \log(p) + (n-S) * \log(1-p)] \quad (141)$$

$$= S * \frac{d}{dp} [\log(p)] + (n-S) * \frac{d}{dp} [\log(1-p)] \quad (142)$$

$$= S * \left[\frac{1}{p}\right] + (n-S) * \frac{d}{dp} [\log(1-p)] \quad \text{chain rule} \quad (143)$$

$$= S * \frac{1}{p} + (n-S) * \frac{1}{p-1} \quad (144)$$

$$= \frac{S}{p} + \frac{n-S}{p-1} \quad (145)$$

$$= \frac{S * (p-1)}{p * (p-1)} + \frac{p * (n-S)}{p * (p-1)} \quad (146)$$

$$= \frac{S * (p-1) + p * (n-S)}{p * (p-1)} \quad (147)$$

$$= \frac{S * p - S + p * n - p * S}{p * (p-1)} \quad (148)$$

$$= \frac{-S + p * n}{p * (p-1)} \quad (149)$$

$$0 = \frac{-S + p * n}{p * (p-1)} \quad (150)$$

$$0 * (p * (p-1)) = -S + p * n \quad (151)$$

$$0 = -S + p * n \quad (152)$$

$$S = p * n \quad (153)$$

$$\frac{S}{n} = p \quad (154)$$

$$p = \frac{S}{n} \quad (155)$$

$$(156)$$

4 Geometric Distribution

4.1 CDF

$$\begin{aligned}P(X \leq x) &= \\CDF(X = x) &= \sum_{i=0}^x (1-p)^i p && \text{by geometric summation} \\&= p \frac{1 - (1-p)^{x+1}}{1 - (1-p)} \\&= p \frac{1 - (1-p)^{x+1}}{1 - 1 + p} \\&= p \frac{1 - (1-p)^{x+1}}{p} \\&= 1 - (1-p)^{x+1} \\&\square\end{aligned}$$

$$\begin{aligned}P(X > x) &= 1 - CDF(X = x) \\&= 1 - (1 - (1-p)^{x+1}) \\&= 1 - 1 + (1-p)^{x+1} \\&= (1-p)^{x+1} \\&\square\end{aligned}$$

5 Normal Distribution

5.1 Definition

$$pdf(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (157)$$

Continuous Distributions

6 Uniform Distribution

6.1 PDF

$$\begin{aligned}\int_a^b k dx &= 1 \\ &= k \int_a^b dx \\ &= k[x]_a^b \\ &= k[b - a]\end{aligned}$$

$$\begin{aligned}k[b - a] &= 1 \\ k &= \frac{1}{b - a}\end{aligned}$$

6.2 Expected Value

$$\begin{aligned}\int_a^b x \left(\frac{1}{b - a} \right) dx &= \\ &= \int_a^b x \left(\frac{1}{b - a} \right) dx \\ &= \frac{1}{b - a} \int_a^b x dx \\ &= \frac{1}{b - a} \left[\frac{x^2}{2} \right]_a^b \\ &= \frac{1}{b - a} \left[\frac{b^2}{2} - \frac{a^2}{2} \right] \\ &= \frac{1}{b - a} \left[\frac{b^2 - a^2}{2} \right] \\ &= \frac{1}{b - a} \left[\frac{(b + a)(b - a)}{2} \right] \\ &= \frac{(b + a)(b - a)}{2(b - a)} \\ &= \frac{b + a}{2}\end{aligned}$$

6.3 Variance

$$\begin{aligned}
\text{var}(X) &= E[X^2] - E[X]^2 \\
&= \left[\int_a^b x^2 \left(\frac{1}{b-a} \right) dx \right] - \left(\frac{b+a}{2} \right)^2 \\
&= \left[\frac{1}{b-a} \int_a^b x^2 dx \right] - \left(\frac{b+a}{2} \right)^2 \\
&= \left[\frac{1}{b-a} \frac{x^3}{3} \Big|_a^b \right] - \left(\frac{b+a}{2} \right)^2 \\
&= \frac{1}{b-a} \left(\frac{b^3}{3} - \frac{a^3}{3} \right) - \left(\frac{b+a}{2} \right)^2 \\
&= \frac{b^3 - a^3}{3(b-a)} - \frac{(b+a)^2}{4} \\
&= \frac{(b-a)(b^2 - ab + a^2)}{3(b-a)} - \frac{(b+a)^2}{4} \\
&= \frac{(b^2 - ab + a^2)}{3} - \frac{(b+a)^2}{4} \\
&= \frac{4(b^2 - ab + a^2)}{12} - \frac{3(b+a)^2}{12} \\
&= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3(b^2 + 2ab + a^2)}{12} \\
&= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3b^2 + 6ab + 3a^2}{12} \\
&= \frac{4b^2 - 3b^2 - 4ab - 6ab + 4a^2 - 3a^2}{12} \\
&= \frac{b^2 - 2ab + a^2}{12} \\
&= \frac{(b-a)^2}{12}
\end{aligned}$$

7 Exponential Distribution

7.1 PDF

$$f_x(x) = \lambda e^{-\lambda x} \quad x \geq 0$$

7.2 CDF

$$CDF(x) = \int_{-\infty}^x f_x dx$$

$$CDF(x) = \int_{-\infty}^x \lambda e^{-\lambda x} dx$$

$$u = e^{-\lambda x}$$

$$\frac{du}{dx} = d[e^{-\lambda x}]$$

$$du = d[e^{-\lambda x}] * dx$$

$$du = [-\lambda * e^{-\lambda x}] * dx$$

$$du = -\lambda e^{-\lambda x} dx$$

$$CDF(x) = \int_{-\infty}^x \lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^x -1 * \lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^x -\lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^x du$$

$$CDF(x) = -1 * \int_0^x du$$

by x bounds

$$CDF(x) = -1 * [u]_0^x$$

$$CDF(x) = -1 * [e^{-\lambda x}]_0^x$$

$$CDF(x) = -1 * [e^{-\lambda x} - e^{-\lambda 0}]$$

$$CDF(x) = -1 * [e^{-\lambda x} - e^0]$$

$$CDF(x) = -1 * [e^{-\lambda x} - 1]$$

$$CDF(x) = [1 - e^{-\lambda x}]$$

$$CDF(x) = 1 - e^{-\lambda x}$$

7.3 Expected Value

$$f_x(x) = \lambda e^{-\lambda x}$$

$$E[x] = \int x * f_x(x) dx$$

$$E[x] = \int x * \lambda e^{-\lambda x} dx$$

$$u = x$$

$$du = 1 * dx$$

$$v = e^{-\lambda x}$$

$$dv = e^{-\lambda x} * -\lambda * dx$$

$$dv = -\lambda e^{-\lambda x} dx$$

$$E[x] = \int [x] [\lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int [x] [-1 * \lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int [x] [-\lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int u dv$$

$$E[x] = -1 * [uv|_{-\infty}^{\infty} - \int v du]$$

$$E[x] = -1 * [uv|_0^{\infty} - \int v du] \quad \text{because x bounds}$$

$$E[x] = -1 * [[xe^{-\lambda x}]_0^{\infty} - \int e^{-\lambda x} dx]$$

$$\begin{aligned}
[xe^{-\lambda x}]_0^\infty &= \lim_{x \rightarrow \infty} [xe^{-\lambda x} - (0)e^{-\lambda(0)}] \\
&= \lim_{x \rightarrow \infty} [xe^{-\lambda x} + 0e^0] \\
&= \lim_{x \rightarrow \infty} [xe^{-\lambda x}] \\
&= 0
\end{aligned}$$

$$E[x] = -1 * - \int e^{-\lambda x} dx$$

$$E[x] = \frac{1}{\lambda} \int \lambda e^{-\lambda x} dx$$

$$E[x] = \frac{1}{\lambda}$$