

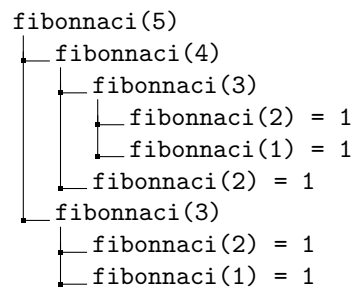
Recursive Call Count Operator

if f is a function, #f returns how much recursive call f does.

Fibonacci Numbers

```
int fibonnaci(int n){
    if(n <= 2) return 1;
    return fibonnaci(n-1) + fibonnaci(n-2);
}
```

Call tree



All leafs always have value 1. So

$\text{fibonnaci}(n) = 1 + 1 + \dots + 1 + 1$

In this case the call tree have:

9 nodes =

5 leafs

4 aggregators

Every aggregator comes from a recursive call. So
 $\text{fibonnaci}(5)$ makes 4 recursive calls.

So $\# \text{fibonnaci}(n) = \# \text{fibonnaci}(n-1) + \# \text{fibonnaci}(n-2) + 1$

Thesis:

$\# \text{fibonnaci}(n) = \text{fibonnaci}(n) - 1$

Proof by Induction

Base

$\# \text{fibonnaci}(1) = \text{fibonnaci}(1) - 1 = 1 - 1 = 0$ (OK)

$\# \text{fibonnaci}(2) = \text{fibonnaci}(2) - 1 = 1 - 1 = 0$ (OK)

Step

$\# \text{fibonnaci}(n) = \text{fibonnaci}(n) - 1$

$\# \text{fibonnaci}(n) + \text{fibonnaci}(n-1) = \text{fibonnaci}(n) - 1 + \text{fibonnaci}(n-1)$

$\# \text{fibonnaci}(n) + \text{fibonnaci}(n-1) = \text{fibonnaci}(n) + \text{fibonnaci}(n-1) - 1$

$\# \text{fibonnaci}(n) + \# \text{fibonnaci}(n-1) + 1 = \text{fibonnaci}(n+1) - 1$

$\# \text{fibonnaci}(n+1) = \text{fibonnaci}(n+1) - 1$

QED

Binomial Numbers

```
int binomial(int n, int k){
    if(k > n) return 0;
    if(k == 0 || n == k) return 1;
    return binomial(n-1,k) + binomial(n-1,k-1)
}
```

Call Tree:

```
binomial(4,2)
├── binomial(3,2)
│   ├── binomial(2,2) = 1
│   └── binomial(2,1)
│       ├── binomial(1,1) = 1
│       └── binomial(1,0) = 1
└── binomial(3,1)
    ├── binomial(2,1)
    │   ├── binomial(1,1) = 1
    │   └── binomial(1,0) = 1
    └── binomial(2,0) = 1
```

All leafs always have value 1. So

$\text{binomial}(n,k) = 1 + 1 + \dots + 1 + 1$

In this case the call tree have:

11 nodes =

6 leafs

5 aggregators

Every aggregator comes from a recursive call. So

$\text{binomial}(4,2)$ makes 5 recursive calls.

$$\# \text{binomial}(n,k) = \# \text{binomial}(n-1,k) + \# \text{binomial}(n-1,k-1) + 1$$

Thesis:

$$\# \text{binomial}(n,k) = \text{binomial}(n,k) - 1$$

Bases:

$$\# \text{binomial}(1,0) = \text{binomial}(1,0) - 1 = 1 - 1 = 0 \text{ (OK)}$$

Step:

$$\# \text{binomial}(n,k) = \text{binomial}(n,k) - 1$$

$$\# \text{binomial}(n,k) + \text{binomial}(n,k+1) = \text{binomial}(n,k) - 1 + \text{binomial}(n,k+1)$$

$$\# \text{binomial}(n,k) + \# \text{binomial}(n,k+1) + 1 = \text{binomial}(n+1,k+1) - 1$$

$$\# \text{binomial}(n+1,k+1) = \text{binomial}(n+1,k+1) - 1$$

QED