

1 Introduction

Definition 1.0.1 (Log Definition).

$$\begin{aligned} \log_a b &= c \\ a^{\log_a b} &= a^c \\ b &= a^c \\ \log_a b &= \log_a a^c \\ \log_a b &= c \end{aligned}$$

Theorem 1.1 (Logarithm Product Rule).

$$\log_x(A * B) = \log_x A + \log_x B$$

Proof.

$$x^l = A \tag{1}$$

$$\log_x x^l = \log_x A \tag{2}$$

$$l = \log_x A \tag{3}$$

$$x^m = B \tag{4}$$

$$\log_x x^m = \log_x B \tag{5}$$

$$m = \log_x B \tag{6}$$

$$x^n = A * B \tag{7}$$

$$\log_x x^n = \log_x(A * B) \tag{8}$$

$$n = \log_x(A * B) \tag{9}$$

$$\log_x(A * B) = n \tag{10}$$

$$x^n = A * B \tag{11}$$

$$x^n = x^l * x^m \tag{12}$$

$$x^n = x^{l+m} \tag{13}$$

$$n = l + m \tag{14}$$

use (3) (6) (9)

$$\log_x(A * B) = \log_x A + \log_x B \tag{15}$$

□

Theorem 1.2 (Logarithm Power Rule).

$$\log_x A^B = B * \log_x(A)$$

Proof.

$$\log_x(A^B) \tag{16}$$

$$\log_x(\prod_{n=1}^B A) \tag{17}$$

$$\sum_{n=1}^B (\log_x A) \tag{18}$$

use (1.1)

$$\log_x A * \sum_{n=1}^B 1 \tag{19}$$

$$B * \log_x A \tag{20}$$

□