1 Karatsuba Multiplication

Two numbers l and r, can be written in the following way:

$$l = a * 10^N + b$$
$$r = c * 10^N + d$$

In this case their multiplication can be written

$$\begin{aligned} l * r &= (a * 10^N + b) * (c * 10^N + d) \\ &= (a * c * 10^{2N}) + (a * d * 10^N) + (b * c * 10^N) + (b * d) \\ &= (a * c * 10^{2N}) + ([(a * d) + (b * c)] * 10^N) +] + (b * d) \end{aligned}$$

but $(a * d) + (b * c)$ is equal to

$$\begin{aligned} &= (a * d) + (b * c) \\ &= (a + b) * (c + d) - ac - bd \\ &= a * c + a * d + b * c + b * d - a * c - b * d \\ &= a * d + b * c \end{aligned}$$

now we can change two multiplications $(a*d)$ and $(b*c)$ to just one: $(a+b)*(c+d)$, given that we have already calculated $a * c$ and $b * d$.

This gives us the final form:

$$\begin{aligned} l * r = &(a * c * 10^{2N}) \\ &+ ([(a + b) * (c + d) - a * c - b * d] * 10^N) \\ &+ (b * d) \end{aligned}$$

See:
`https://academic.microsoft.com/#/detail/204623740`

`https://scholar.google.co.uk/scholar?q=Multiplication+of+Many-Digital+Numbers+by+Automatic+Computers&btnG=&hl=en&as_sdt=0%2C5`

`http://cstheory.stackexchange.com/questions/21564/why-did-kolmogorov-publish-karatsubas-al`

The Art of Computer Programming, Volume II, page 294, section 4.3.3