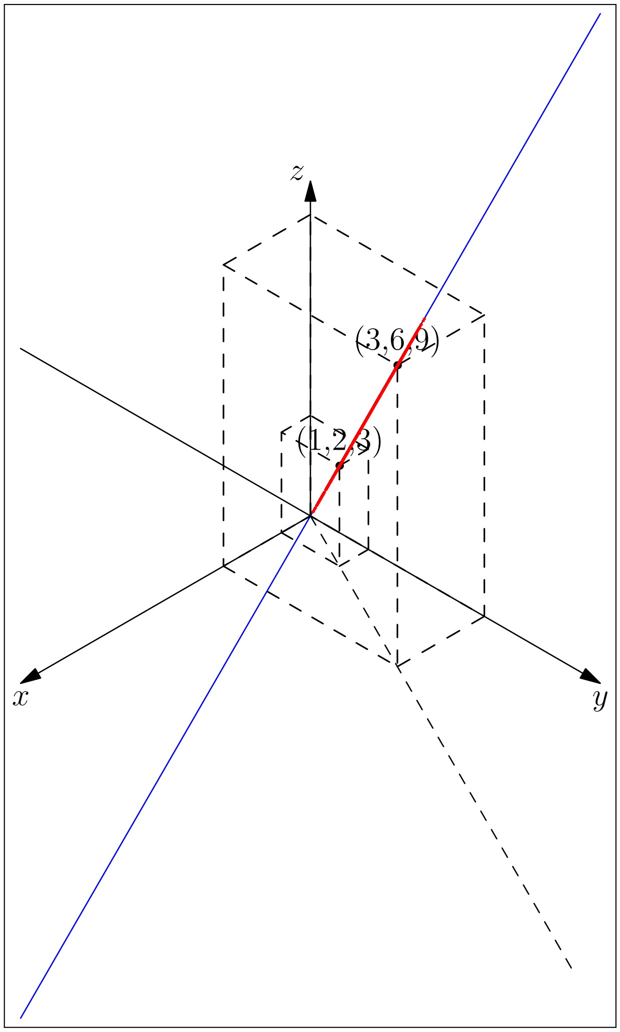
Exercise 1:

Describe geometrically (line, plane, or all of R^3) all linear combinations of:

Linear combination is $a * A + b * B$ where $a \in R$ and $b \in R$.

(a) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Both points are on the same line, so their linear combination can only express a line. The blue line is the line that is formed by all linear combinations of A and B. The red points are random linear combinations of A and B that we confirm that fall all in the line.



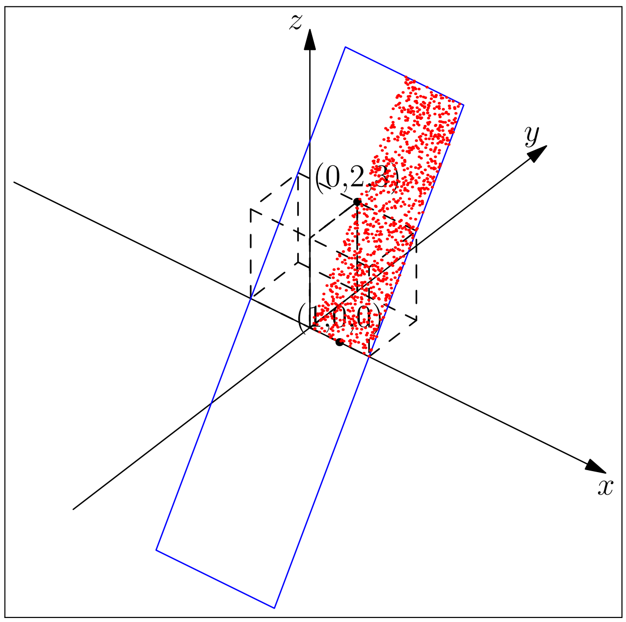
z
(3,6,9)
(1,2,3)
x
y

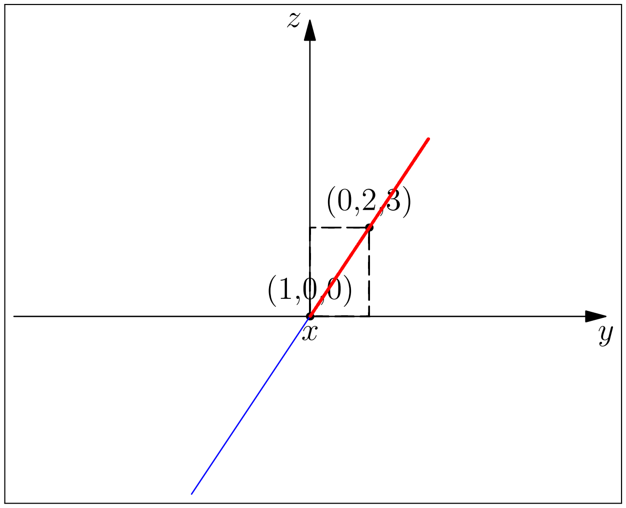
(b) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

It is obvious that we cannot have one line passing throught $(0, 0, 0)$ that also passes throught $(1, 0, 0)$ and $(0, 2, 3)$. So, the linear combinations of theses points forms a plane.

The blue plane represents all linear combinations of A and B.

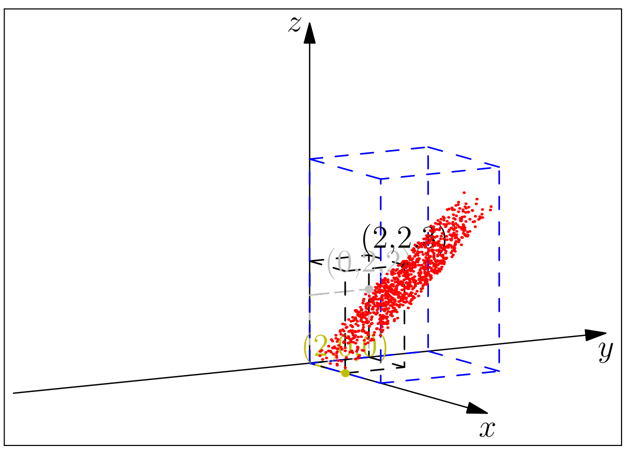
The red poins are, again, random linear combinations of A and B. We can see in the second figure how all points fall perfectly in the plane.

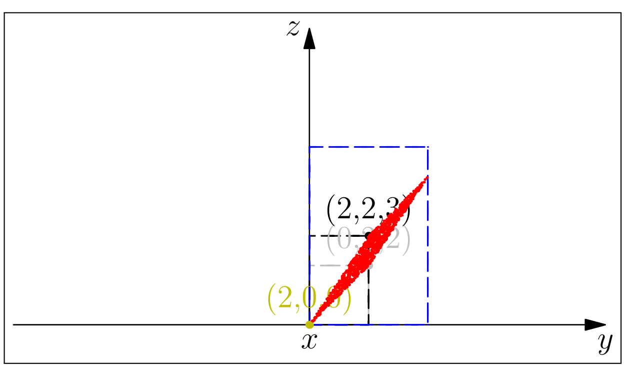


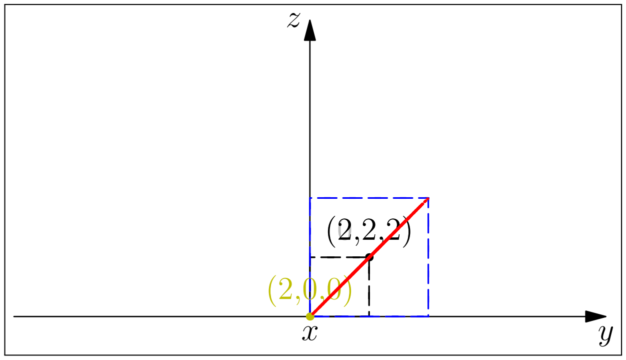

z
(0,2,3)
(1,0,0)
x
y

(c) $A = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

In this case it is also impossible to have a line passing through $(0, 0, 0)$, $(2, 0, 0)$ and $(0, 2, 2)$, but it is also impossible to have a plane passing through $(2, 2, 3)$. So, all linear combinations of A, B, C span all possible points of R^3.
Again, we show the blue dashed box as a representation of the space, but the linear combinations are not limited inside this blue box, they will span all R^3.
Deceivingly, the random points appear to span a plane. That happens because C is almost the sum of A and B. The second picture show us that the points are not perfectedly in the plane. The third picture helps us to see the difference because it shows what happens when $C = (2, 2, 2)$ and the linear combination fall all in the plane.






z
(2,2,2)
(2,0,0)
x
y