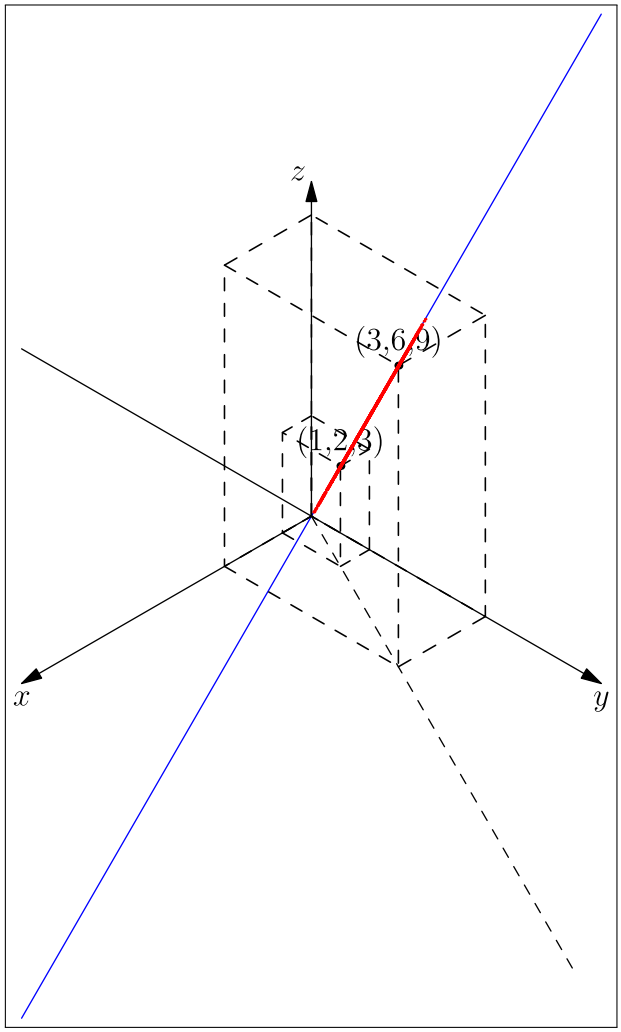
Describe geometrically (line, plane, or all of R^3) all linear combinations of:

Linear combination is $a * A + b * B$ where $a \in R$ and $b \in R$.

(a) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

Both points are on the same line, so their linear combination can only express a line. The blue line is the line that is formed by all linear combinations of A and B. The red points are random linear combinations of A and B that we confirm that fall all in the line.

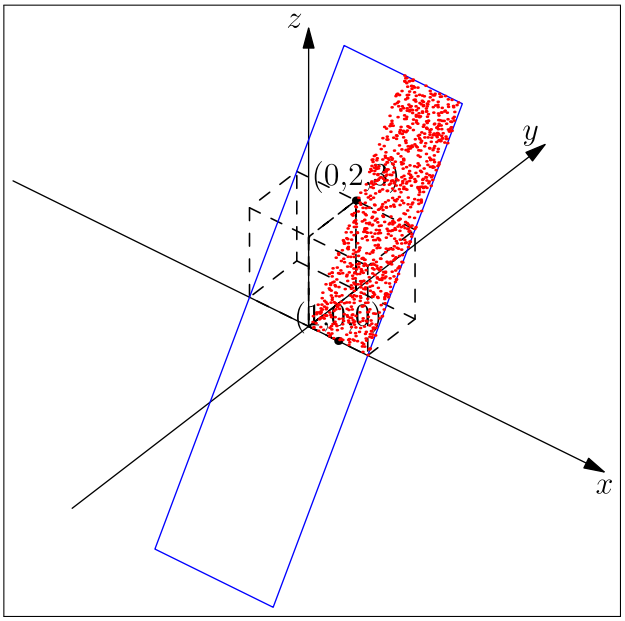

z
(3,6,9)
(1,2,3)
x
y

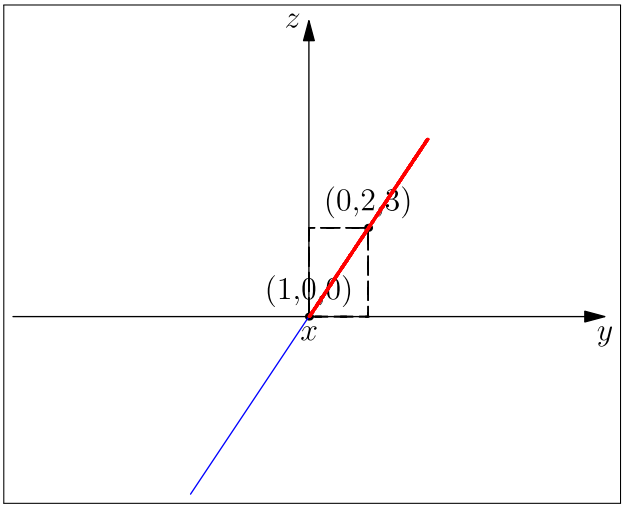
(b) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

It is obvious that we cannot have one line passing throught $(0, 0, 0)$ that also passes throught $(1, 0, 0)$ and $(0, 2, 3)$. So, the linear combinations of theses points forms a plane.

The blue plane represents all linear combinations of A and B.

The red poins are, again, random linear combinations of A and B. We can see in the second figure how all points fall perfectly in the plane.




z
(0,2,3)
(1,0,0)
x
y

(c) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$