Gaussian Maximum Likelihood

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1 Gaussian Maximum Likelihood

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\begin{aligned}
X &\overset{iid}{\sim} N(\mu, \sigma^2) \\
L(theta|X) &= p(X|theta) \\
&= p(X|\mu, \sigma^2) \\
&= \prod p(x_i|\mu, \sigma^2)
\end{aligned}
$$

$$
\begin{aligned}
\max_{\mu, sd} L(theta|X) &= \max_{\mu, \sigma^2} \prod p(x_i|\mu, \sigma^2) \\
&= \max_{\mu, \sigma^2} ln(\prod p(x_i|\mu, \sigma^2))) \\
&= \max_{\mu, \sigma^2} \sum ln(p(x_i|\mu, \sigma^2))
\end{aligned}
$$

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p(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
$$

$$
\begin{aligned}
&= \max_{\mu, \sigma^2} \sum ln(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}) \\
&= \max_{\mu, \sigma^2} \sum \left[ln(\frac{1}{\sqrt{2\pi\sigma^2}}) + ln(e^{-\frac{(x_i-\mu)^2}{2\sigma^2}})\right] \\
&= \max_{\mu, \sigma^2} \sum \left[ln(\frac{1}{\sqrt{2\pi\sigma^2}}) + -\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&= \max_{\mu, \sigma^2} \sum \left[ln(\frac{1}{\sqrt{2\pi\sigma^2}})\right] - \sum \left[\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&= \max_{\mu, \sigma^2} \sum \left[ln((2\pi\sigma^2)^{\frac{1}{2}})\right] - \sum \left[\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&= \max_{\mu, \sigma^2} \left[ln((2\pi\sigma^2)^{\frac{1}{2}})\right] \sum 1 - \sum \left[\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&= \max_{\mu, \sigma^2} \left[\frac{1}{2} ln(2\pi\sigma^2)\right] \sum 1 - \sum \left[\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&= \max_{\mu, \sigma^2} \left[\frac{1}{2} ln(2\pi\sigma^2)\right] N - \sum \left[\frac{(x_i-\mu)^2}{2\sigma^2}\right]
\end{aligned}
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TODO

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\begin{aligned}
\overset{*}{\mu} &= \frac{1}{N} \sum x_i \\
\overset{*}{\sigma}{}^2 &= \frac{1}{N} \sum (x_n - \overset{*}{\mu})^2
\end{aligned}
$$