0.1 Introduction

0.1.1 Sample

A **sample** is a *subset* of a *population* on which statistical studies are made in order to draw conclusions relative to the *population*. w is a "sample point".
S is a "sample space" if

w is a "sample point and $w \in S$;
and $\forall_{i,j}(w_i \cap w_j = \emptyset$, if $i \neq j)$;
and $w_1 \cup w_2 \cup ... \cup w_n = S$.

A is a "family of events" if is a set of "sample points"

$A = \{w\}$ where w is a "sample point"
$A^{\complement} = \{w : w \notin A\}$
$A \cup B = \{w : w \in A \vee w \in B\}$
$A \cap B = \{w : w \in A \wedge w \in B\}$
$S^{\complement} = \emptyset$
$A \cup A^{\complement} = S$
$A \cap A^{\complement} = \emptyset$
$A \cap S = A$
$A \cup S = S$
$A \cup \emptyset = A$
$\cup$ is commutative, associative, distributive
$\cap$ is commutative, associative, distributive

P is a "probability measure" if is a mapping between S and the "real numbers" with the following properties:

$P = f : S \mapsto \mathbb{R}$
$P(A) = f(A)$
$f(S) = \sum_{\forall i} f(A_i) = 1$
$0 <= f(A) <= 1$
if $A \cap B = \emptyset$ then $f(A \cup B) = f(A) + f(B)$.

The triplet (S,A,P) defines a "probability system", a consistent axiomatic theory of probability of finite "sample spaces".
"Conditional probability" is the probability of "family of events" A, given that the "family of events B" occurred.

$$P(A|B) \triangleq \frac{P(A \cap B)}{P(B)}$$

A and B are "statistical independent" if:

$$P(A \cap B) = P(A) * P(B)$$

wich can be extended to:

$$P(\bigcap_{\forall i} A_i) = \prod_{\forall i} P(A_i)$$

Given the last two properties we have that the "conditional property" of two "statistical independent" "family of events" is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B)}{P(B)} = P(A)$$

. The "Theorem of total probability" states that:

$$P(B) = \sum_{\forall i} P(A_i|B)$$

0.2 Reference

1. The Concise Encyclopedia of Statistics - Yadolah Dodge