Week2_Lab_MM1

March 18, 2018

MOOC: Understanding queues
Python Lab
Week II: M/M/1 queue simulation

In this lab, we are going to simulate the evolution of the number of customers in a M/M/1 queue. Let λ and μ represent the arrival and departure rates. We simulate the following events: arrival of a new client in the system, or departure of a client from the system. Additionally, we record the value of the number of customers in the system at these instants.

1) We assume that the system is not empty. For $\lambda = 4$ and $\mu = 5$, what is the probability Pa that the next event is an arrival?

In [29]:
```
%matplotlib inline
from pylab import *

lambda_ = 4
mu      = 5

###################
# Write a function that computes the probability Pa that the next event
# is an arrival (when the system is not empty)
def Pa(lambda_,mu):
    return lambda_/(mu+lambda_)
###################
V1 = Pa(lambda_,mu)
```

2) Assume that the system is not empty. The time before the next event (departure or arrival) follows an exponential distribution. What is the rate of this exponential distribution?

In [30]:
```
###################
# Supply the rate of the exponential distribution
# that represents the time until the next event (departure or arrival)
# if the system is not empty
def Rate(lambda_,mu):
    return lambda_+mu
###################
V2 = Rate(lambda_,mu)
```

3) The implementation of the function **generate_MM1(lambda_=4, mu=5, N0 = 5, Tmax=200)** with entries

- lambda, mu: arrival and departure rates
- N0: initial number of customers in the system
- Tmax: time interval over which the evolution of the queue is simulated

and outputs

- T: vector of instants of events (arrivals or departures) over [0,Tmax]
- N: vector of the number of customers in the system at instants in T

is given below. Execute this code to plot the evolution the number of clients in the system against time.

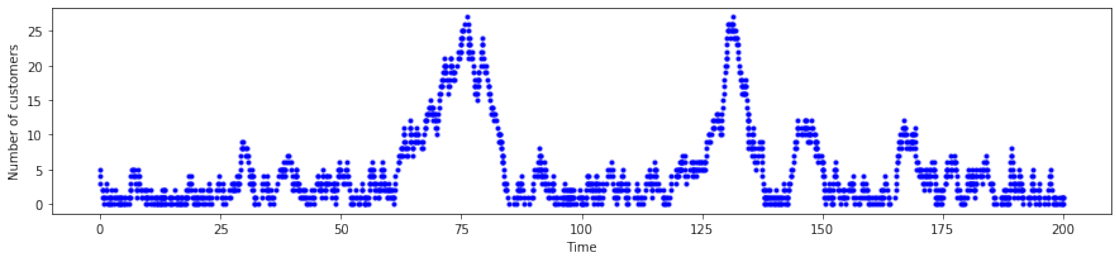
In [31]:
```python
def generate_MM1(lambda_=4,mu=5,N0=5,Tmax=200):
    """
    function generate_MM1(N0 = 5,Tmax=200)
    generates an MM1 file
    INPUTS
    ------
    lambda, mu: arrival and departure rates
    N0:         initial state of the system (default = 5)
    Tmax:       duration of the observation (default = 200)
    OUTPUTS
    -------
    T:          list of time of events (arrivals or departures) over [0,T]
    N:          list of system states (at T(t): N->N+1 or N->N-1)
    """
    seed(20)
    tau   = 0       # initial instant
    T     = [0]     # list of instants of events
    N     = [N0]    # initial state of the system, list of state evolutions

    while T[-1]<Tmax:
        if N[-1]==0:
            tau   = -1./lambda_*log(rand()) # inter-event time when N(t)=0
            event = 1    # arrival
        else:
            tau   = -1./Rate(lambda_,mu)*log(rand()) # inter-event time when N(t)>0
            event = 2*(rand()<Pa(lambda_,mu))-1
            # +1 for an arrival (with probability Pa), -1 for a departure
        N = N + [N[-1]+event]
        T = T + [T[-1]+tau]

    T = T[:-1] # event after Tmax is discarded
    N = N[:-1]
    return T,N
```

```
In [32]: # Plotting the number of clients in the system
         T,N = generate_MM1()
         rcParams['figure.figsize'] = [15,3]
         plot(T,N,'.b')
         xlabel('Time')
         ylabel('Number of customers')
```

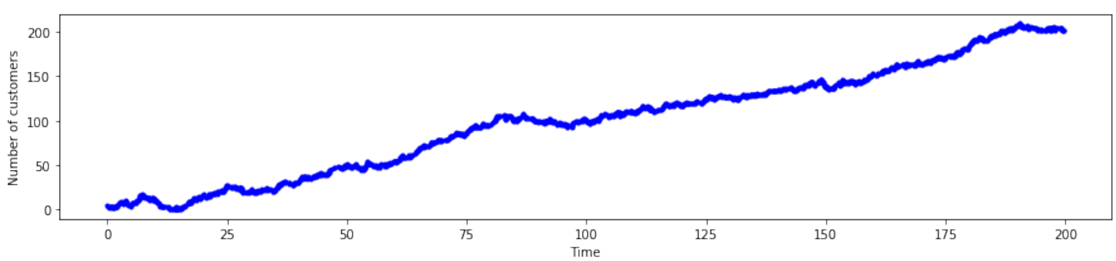
```
Out[32]: <matplotlib.text.Text at 0x7f831460f048>
```



4) Letting now $\lambda = 4$ and $\mu = 3$, what do you notice when running the function generate_MM1? What is the value of the number of customers at $Tmax = 200$?

```
In [33]: T,N = generate_MM1(lambda_=4,mu=3)
         rcParams['figure.figsize'] = [15,3]
         plot(T,N,'.b')
         xlabel('Time')
         ylabel('Number of customers')
```

```
Out[33]: <matplotlib.text.Text at 0x7f83145057b8>
```




```
In [34]: #####################
         # Supply the number of customers at Tmax
         n = N[-1]
         print('At Tmax, N={}'.format(n))
         #####################
         V3 = n
```

```
At Tmax, N=201
```

1 Your answers for the exercise

```
In [35]: print("---------------------------\n"
               +"RESULTS SUPPLIED FOR LAB 2:\n"
               +"---------------------------")
         results = ("V"+str(k) for k in range(1,4))
         for x in results:
             try:
                 print(x+" = {0:.2f}".format(eval(x)))
             except:
                 print(x+": variable is undefined")
```

```
---------------------------
RESULTS SUPPLIED FOR LAB 2:
---------------------------
V1 = 0.44
V2 = 9.00
V3 = 201.00
```