

Question 03: Which piece(s) of information guarantee that the graph of P is always increasing?

Answer:

$$\frac{d}{d\theta}(1 - c) = 0$$

$$\begin{aligned} 1 + e^{-a(\theta-b)} &= 1 + e^{-a\theta+ab} \\ &= 1 + e^{-a\theta}e^{ab} \end{aligned}$$

$$\begin{aligned} \frac{d}{d\theta}[1 + e^{-a\theta}e^{ab}] &= e^{ab} * -a * e^{-a\theta} \\ &= -ae^{ab}e^{-a\theta} \\ &= -ae^{a(b-\theta)} \\ &= -ae^{-a(\theta-b)} \end{aligned}$$

$$\begin{aligned} (1 + e^{-a(\theta-b)})^2 &= (1^2 + 2 * 1 * e^{-a\theta}e^{ab} + (e^{-a\theta}e^{ab})^2) \\ &= (1 + 2e^{-a\theta}e^{ab} + e^{-2a\theta}e^{ab}) \end{aligned}$$

$$\begin{aligned} \frac{d}{d\theta}\left[c + \frac{1-c}{1 + e^{-a(\theta-b)}}\right] &= \\ &= 0 + \frac{d}{d\theta}\left[\frac{1-c}{1 + e^{-a(\theta-b)}}\right] = \\ &= 0 + \frac{d}{d\theta}\left[\frac{1-c}{1 + e^{-a(\theta-b)}}\right] = \\ &= \frac{-(1-c)(ae^{-a(\theta-b)}) + 0 * (...)}{(1 + e^{-a(\theta-b)})^2} \\ &= \frac{-(1-c)(-ae^{-a(\theta-b)})}{(1 + e^{-a(\theta-b)})^2} \\ &= \frac{a(1-c)e^{-a(\theta-b)}}{(1 + e^{-a(\theta-b)})^2} \end{aligned}$$

$$\begin{aligned} P'(b) &= \frac{a(1-c)e^{-a(b-b)}}{(1 + e^{-a(b-b)})^2} \\ &= \frac{a(1-c)e^{-a(0)}}{(1 + e^{-a(0)})^2} \\ &= \frac{a(1-c)e^0}{(1 + e^0)^2} \\ &= \frac{a(1-c)}{(1 + 1)^2} \\ &= \frac{a(1-c)}{4} \end{aligned}$$