Proposition 1.4.51
If A is positive definite, then $a_{ii} > 0$ for $i = 1, ..., n$.
Exercise 1.4.52
Prove Proposition 1.4.51. Do not use the Cholesky decomposition in your proof; we want to use this result to prove that the Cholesky decomposition exists. (Hint: Find a nonzero vector x such that $x^T A x = a_{ii}$.)

Proof:
If A is Positive Definite, then $x^T Ax > 0$ for all x.

$$x^T Ax = \begin{Bmatrix} x_1 & x_2 & ... & x_n \end{Bmatrix} * \begin{Bmatrix} a_{11} & ... & a1n \\ ... & & ... \\ a_{n1} & ... & ann \end{Bmatrix} * \begin{Bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{Bmatrix}$$

But we know that "line vector and matrix multipliction" when the "line vector" is e_i just selects one of the rows of the matrix.
For example

$$e_1 = \begin{Bmatrix} 1 \\ 0 \\ ... \\ 0 \end{Bmatrix}$$

$$e_1^T * \begin{Bmatrix} a_{11} & ... & a_{1n} \\ ... & & ... \\ a_{n1} & ... & a_{nn} \end{Bmatrix} = \begin{Bmatrix} a_{11} & ... & a_{1n} \end{Bmatrix}$$

On the other hand, "matrix and column vector multiplication", when the "column vector" is e_i we choose a column of A;

$$e_1 = \begin{Bmatrix} 1 \\ 0 \\ ... \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} a_{11} & ... & a_{1n} \\ ... & & ... \\ a_{n1} & ... & a_{nn} \end{Bmatrix} * e_i = \begin{Bmatrix} a_{11} \\ ... \\ a_{n1} \end{Bmatrix}$$

So we can "choose" a element of the matrix by doing:

$$e_i^T Ae_j = a_{ij}$$

So we can easily "choose" the diagonal elements by doing:

$$e_i^T Ae_i = a_{ii}$$

But, given that A is "Positive Definite" this expression is always positive. So we have:

$$e_i^T A e_i > 0$$
$$a_{ii} > 0$$
$$\square$$