

Theorem 1.4.4

Let M be any $n \times n$ nonsingular matrix, and let $A = M^T M$. Then A is positive definite.

Proposition 1.4.55

If A and X are $n \times n$, A is "Positive Definite", and X is nonsingular then the matrix $B = X^T A X$ is also "Positive Definite".

Considering the special case $A = I$ (which is clearly "Positive Definite"), we see that this proposition is a generalization of Theorem 1.4.4.

Exercise 1.4.56

Prove Proposition 1.4.55.

Proof:

$$z^T A z > 0 \text{ because } A \text{ is "PD"}$$

$$\begin{aligned} Xy &= z \\ (Xy)^T A (Xy) &> 0 \\ y^T X^T A X y &> 0 \\ y^T B y &> 0 \text{ if } y > 0 \end{aligned}$$

We already know that $z > 0$ because of the "Positive Definiteness" of A . So we need:

$$\begin{aligned} Xy &= z \\ z &> 0 \\ y &> 0 \\ Xy &> 0 \end{aligned}$$

But if X is "Singular" it is possible that a $y > 0$ that $Xy = 0$. So to guarantee that $y > 0$ and $Xy > 0$ we need that X be "NonSingular".