If A^{-1} exists then the only possible solution for $Ay = 0$ is $y = 0$.

$$Ay = 0$$
$$A^{-1}Ay = A^{-1}0$$
$$Iy = 0$$
$$y = 0$$