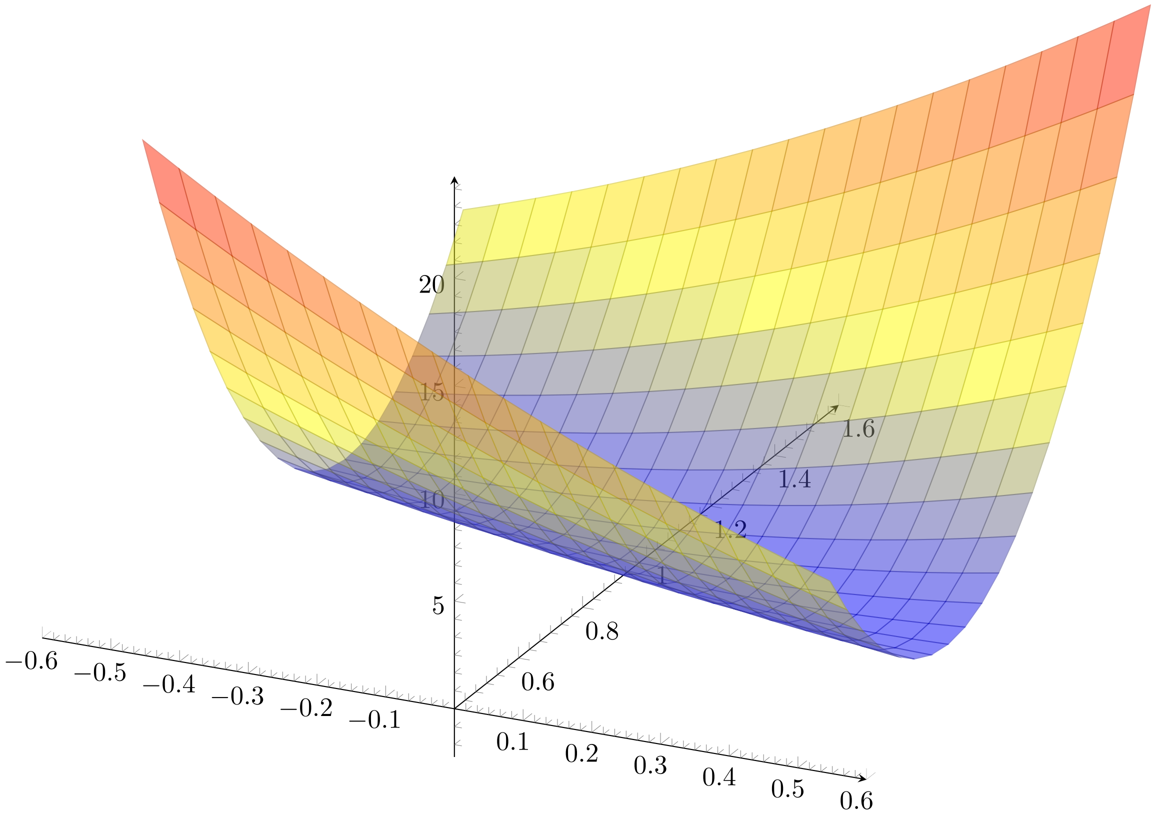
Squared Error Line

$$
\begin{aligned}
SE_{line} &= \sum_{i=0}^{N} [y_i - (w_1 * x_i + w_0)]^2 \\
&= \sum_{i=0}^{N} [y_i^2 - 2 * y_i * (w_1 * x_i + w_0) + (w_1 * x_i + w_0)^2] \\
&= \sum_{i=0}^{N} [y_i^2 - 2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i + w_0)^2] \\
&= \sum_{i=0}^{N} [y_i^2 - 2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - \sum_{i=0}^{N} [2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - \sum_{i=0}^{N} [2 * y_i * w_1 * x_i] - \sum_{i=0}^{N} [2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2 * w_1 * \sum_{i=0}^{N} [y_i * x_i] - \sum_{i=0}^{N} [2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2 * w_1 * \sum_{i=0}^{N} [y_i * x_i] - \sum_{i=0}^{N} [2 * y_i * w_0] + \sum_{i=0}^{N} [(w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2 * w_1 * \sum_{i=0}^{N} [y_i * x_i] - 2 * w_0 * \sum_{i=0}^{N} [y_i] + \sum_{i=0}^{N} [(w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2 * w_1 * \sum_{i=0}^{N} [y_i * x_i] - 2 * w_0 * \sum_{i=0}^{N} [y_i] + \sum_{i=0}^{N} [(w_1 * x_i)^2] + \sum_{i=0}^{N} [2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2 * w_1 * \sum_{i=0}^{N} [y_i * x_i] - 2 * w_0 * \sum_{i=0}^{N} [y_i] + w_1^2 * \sum_{i=0}^{N} (x_i)^2 + \sum_{i=0}^{N} [2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2w_1 \sum_{i=0}^{N} [y_i x_i] - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + \sum_{i=0}^{N} [2w_1 x_i w_0 + w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2w_1 \sum_{i=0}^{N} [y_i x_i] - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + \sum_{i=0}^{N} [2w_1 x_i w_0] + \sum_{i=0}^{N} [w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2w_1 \sum_{i=0}^{N} [y_i x_i] - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + \sum_{i=0}^{N} [w_0^2] \\
&= \sum_{i=0}^{N} [y_i^2] - 2w_1 \sum_{i=0}^{N} [y_i x_i] - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + w_0^2 N
\end{aligned}
$$

$$\frac{\sum_{i=0}^{N} y_i^2}{N} = \overline{y^2}$$

$$\sum_{i=0}^{N} y_i^2 = \overline{y^2} * N$$

$$\begin{aligned}
&= \overline{y^2}N - 2w_1 \sum_{i=0}^{N} [y_i x_i] - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{yx} N - 2w_0 \sum_{i=0}^{N} [y_i] + w_1^2 \sum_{i=0}^{N} (x_i)^2 + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{yx} N - 2w_0 \overline{y} N + w_1^2 \sum_{i=0}^{N} (x_i)^2 + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{yx} N - 2w_0 \overline{y} N + w_1^2 \overline{x^2} N + 2w_1 w_0 \sum_{i=0}^{N} [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{yx} N - 2w_0 \overline{y} N + w_1^2 \overline{x^2} N + 2w_1 w_0 \overline{x} N + w_0^2 N \\
SE_{line} &= N\overline{y^2} - 2Nw_1 \overline{yx} - 2Nw_0 \overline{y} + Nw_1^2 \overline{x^2} + 2Nw_1 w_0 \overline{x} + Nw_0^2
\end{aligned}$$



Now to find the line that minimizes the squared errors we can take the partial directives in relation to w_0 and w_1

First partial derivative in relation of w_0

$$
\begin{aligned}
MSE_{line} &= \frac{d}{dw_0}[N\overline{y^2} - 2Nw_1\overline{yx} - 2Nw_0\overline{y} + Nw_1^2\overline{x^2} + 2Nw_1w_0\overline{x} + Nw_0^2] \\
&= \frac{d}{dw_0}[N\overline{y^2}] - \frac{d}{dw_0}[2Nw_1\overline{yx}] - \frac{d}{dw_0}[2Nw_0\overline{y}] + \frac{d}{dw_0}[Nw_1^2\overline{x^2}] + \frac{d}{dw_0}[2Nw_1w_0\overline{x}] + \frac{d}{dw_0}[Nw_0^2] \\
&= -\frac{d}{dw_0}[2Nw_1\overline{yx}] - \frac{d}{dw_0}[2Nw_0\overline{y}] + \frac{d}{dw_0}[Nw_1^2\overline{x^2}] + \frac{d}{dw_0}[2Nw_1w_0\overline{x}] + \frac{d}{dw_0}[Nw_0^2] \\
&= -\frac{d}{dw_0}[2Nw_0\overline{y}] + \frac{d}{dw_0}[Nw_1^2\overline{x^2}] + \frac{d}{dw_0}[2Nw_1w_0\overline{x}] + \frac{d}{dw_0}[Nw_0^2] \\
&= -2N\overline{y} + \frac{d}{dw_0}[Nw_1^2\overline{x^2}] + \frac{d}{dw_0}[2Nw_1w_0\overline{x}] + \frac{d}{dw_0}[Nw_0^2] \\
&= -2N\overline{y} + \frac{d}{dw_0}[2Nw_1w_0\overline{x}] + \frac{d}{dw_0}[Nw_0^2] \\
&= -2N\overline{y} + 2Nw_1\overline{x} + \frac{d}{dw_0}[Nw_0^2] \\
&= -2N\overline{y} + 2Nw_1\overline{x} + 2Nw_0
\end{aligned}
$$

Now the derivative in relation to w_1

$$
\begin{aligned}
MSE_{line} &= \frac{d}{dw_1}[N\overline{y^2} - 2Nw_1\overline{yx} - 2Nw_0\overline{y} + Nw_1^2\overline{x^2} + 2Nw_1w_0\overline{x} + Nw_0^2] \\
&= \frac{d}{dw_1}[N\overline{y^2}] - \frac{d}{dw_1}[2Nw_1\overline{yx}] - \frac{d}{dw_1}[2Nw_0\overline{y}] + \frac{d}{dw_1}[Nw_1^2\overline{x^2}] + \frac{d}{dw_1}[2Nw_1w_0\overline{x}] + \frac{d}{dw_1}[Nw_0^2] \\
&= -\frac{d}{dw_1}[2Nw_1\overline{yx}] - \frac{d}{dw_1}[2Nw_0\overline{y}] + \frac{d}{dw_1}[Nw_1^2\overline{x^2}] + \frac{d}{dw_1}[2Nw_1w_0\overline{x}] + \frac{d}{dw_1}[Nw_0^2] \\
&= -2N\overline{yx} - \frac{d}{dw_1}[2Nw_0\overline{y}] + \frac{d}{dw_1}[Nw_1^2\overline{x^2}] + \frac{d}{dw_1}[2Nw_1w_0\overline{x}] + \frac{d}{dw_1}[Nw_0^2] \\
&= -2N\overline{yx} + \frac{d}{dw_1}[Nw_1^2\overline{x^2}] + \frac{d}{dw_1}[2Nw_1w_0\overline{x}] + \frac{d}{dw_1}[Nw_0^2] \\
&= -2N\overline{yx} + 2Nw_1\overline{x^2} + \frac{d}{dw_1}[2Nw_1w_0\overline{x}] + \frac{d}{dw_1}[Nw_0^2] \\
&= -2N\overline{yx} + 2Nw_1\overline{x^2} + 2Nw_0\overline{x} + \frac{d}{dw_1}[Nw_0^2] \\
&= -2N\overline{yx} + 2Nw_1\overline{x^2} + 2Nw_0\overline{x}
\end{aligned}
$$

Now we must solve the lienar system

$$\begin{cases} -2N\overline{y} + 2Nw_1\overline{x} + 2Nw_0 = 0 \\ -2N\overline{yx} + 2Nw_1\overline{x^2} + 2Nw_0\overline{x} = 0 \end{cases}$$

divide first 2N

$$\begin{cases} -\overline{y} + w_1\overline{x} + w_0 = 0 \\ -2N\overline{yx} + 2Nw_1\overline{x^2} + 2Nw_0\overline{x} = 0 \end{cases}$$

divide second 2N

$$\begin{cases} -\overline{y} + w_1\overline{x} + w_0 = 0 \\ -\overline{yx} + w_1\overline{x^2} + w_0\overline{x} = 0 \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1\overline{x^2} + w_0\overline{x} = \overline{yx} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1\frac{\overline{x^2}}{\overline{x}} + w_0\frac{\overline{x}}{\overline{x}} = \frac{\overline{yx}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1\frac{\overline{x^2}}{\overline{x}} + w_0 = \frac{\overline{yx}}{\overline{x}} \end{cases}$$

This means that the line that minimizes the squared error pass through the points

$$(\overline{x}, \overline{y})$$

$$(\frac{\overline{x^2}}{\overline{x}}, \frac{\overline{yx}}{\overline{x}})$$

Solving for w_1

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1\frac{\overline{x^2}}{\overline{x}} + w_0 = \frac{\overline{yx}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ -w_1\frac{\overline{x^2}}{\overline{x}} - w_0 = -\frac{\overline{yx}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1\overline{x} - w_1\frac{\overline{x^2}}{\overline{x}} = \overline{y} - \frac{\overline{yx}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1(\overline{x} - \frac{\overline{x^2}}{\overline{x}}) = \overline{y} - \frac{\overline{yx}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1 = \frac{\overline{y} - \frac{\overline{yx}}{\overline{x}}}{\overline{x} - \frac{\overline{x^2}}{\overline{x}}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1 = \frac{\overline{y} - \frac{\overline{yx}}{\overline{x}}}{\overline{x} - \frac{\overline{x^2}}{\overline{x}}} * \frac{\overline{x}}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{\overline{x} - \frac{\overline{x^2}}{\overline{x}}} * \frac{1}{\overline{x}} \end{cases}$$

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}} \end{cases}$$

and now for w_0

$$\begin{cases} w_1\overline{x} + w_0 = \overline{y} \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}} \end{cases}$$

$$\begin{cases} w_0 = \overline{y} - w_1\overline{x} \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}} \end{cases}$$

$$\begin{cases} w_0 = \overline{y} - \left(\frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}}\right)\overline{x} \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}} \end{cases}$$

$$\begin{cases} w_0 = \overline{y} - \left(\frac{(\overline{x})^2\overline{y} - \overline{x} * \overline{yx}}{(\overline{x})^2 - \overline{x^2}}\right) \\ w_1 = \frac{\overline{x} * \overline{y} - \overline{yx}}{(\overline{x})^2 - \overline{x^2}} \end{cases}$$