Probability and Statistics

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Contents

Chapter 1

Discrite Distributions

1.1 Generic Formulas

1.1.1 Expected Value

X is discrete random variable
$g : \mathbf{R} \to \mathbf{R}$
$\Omega X = Im(X)$

$$E[g(X)] = \sum_{x \in \Omega X} P(X = x) * g(x) \qquad (1.1)$$

1.1.2   Variance

$$var(g(X)) = E([g(X) - E(g(X))]^2) \tag{1.2}$$

$$= \sum_{x \in \Omega X} [g(X) - E(g(X))]^2 * P(X = x)] \tag{1.3}$$

$$= \sum_{x \in \Omega X} [g(X)^2 - 2 * g(X) * E(g(X)) + E(g(X))^2] * P(X = x) \tag{1.4}$$

$$= \sum_{x \in \Omega X} g(X)^2 * P(X = x) \tag{1.5}$$

$$- \sum_{x \in \Omega X} 2 * g(X) * E(g(X)) * P(X = x) \tag{1.6}$$

$$+ \sum_{x \in \Omega X} E(g(X))^2 * P(X = x) \tag{1.7}$$

$$= \sum_{x \in \Omega X} g(X)^2 * P(X = x) \tag{1.8}$$

$$- 2 * E(g(X)) * \sum_{x \in \Omega X} g(X) * P(X = x) \tag{1.9}$$

$$+ E(g(X))^2 * \sum_{x \in \Omega X} P(X = x) \tag{1.10}$$

$$= E(g(X)^2) - 2 * E(g(X)) * E(g(X)) + E(g(X))^2 * 1 \tag{1.11}$$

$$= E(g(X)^2) - 2 * E(g(X))^2 + E(g(X))^2 \tag{1.12}$$

$$= E(g(X)^2) - E(g(X))^2 \tag{1.13}$$

$$\square \tag{1.14}$$

1.1.3 Covariance Matrix

$$cov(X) = E[(X - E(X)]^2 \tag{1.15}$$

$$= \sum_{x \in \Omega X} (X - E(X))^2 * P(X = x) \tag{1.16}$$

$$= \sum_{x \in \Omega X} [X^2 - 2XE(X) + E(X)^2] * P(X = x) \tag{1.17}$$

$$= \sum_{x \in \Omega X} X^2 P(X = x) \tag{1.18}$$

$$- \sum_{x \in \Omega X} 2XE(X) * P(X = x) \tag{1.19}$$

$$+ \sum_{x \in \Omega X} E(X)^2 P(X = x) \tag{1.20}$$

$$= \sum_{x \in \Omega X} X^2 P(X = x) \tag{1.21}$$

$$- 2E(X) * \sum_{x \in \Omega X} X P(X = x) \tag{1.22}$$

$$+ E(X)^2 * \sum_{x \in \Omega X} P(X = x) \tag{1.23}$$

$$= E(X^2) - 2E(X)E(X) + E(X)^2 * 1 \tag{1.24}$$

$$= E(X^2) - 2 * E(X)^2 + E(X)^2 \tag{1.25}$$

$$= E(X^2) - E(X)^2 \tag{1.26}$$

$$= E(X^t X) - \mu^t \mu \tag{1.27}$$

$$\square \tag{1.28}$$

1.1.4   Variance of the Sample Mean

$$Var\left(\overline{X}\right) = Var\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right) \tag{1.29}$$

$$= \frac{1}{n^2}Var\left(\sum_{i=1}^{n}X_i\right) \tag{1.30}$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}Var(X_i), \text{ by independence} \tag{1.31}$$

$$= \frac{1}{n^2}\left[Var(X_1) + Var(X_2) + \ldots + Var(X_n)\right] \tag{1.32}$$

$$= \frac{1}{n^2}\left[\sigma^2 + \sigma^2 + \ldots + \sigma^2\right], \text{ since the } X_i \text{ are identically distributed} \tag{1.33}$$

$$= \frac{1}{n^2}(n\sigma^2) \tag{1.34}$$

$$= \frac{\sigma^2}{n} \tag{1.35}$$

1.1.5   Law of Iterated Expectation

$$E[X] = E[E[X|Y]] \tag{1.36}$$

1.1.6   Law of Total Variance

$$var(X) = E[var(X|Y)] + var(E[X|Y]) \tag{1.37}$$

1.1.7 MSE

$$\hat{\theta} = \hat{\theta}(X) \text{ Random Variable} \tag{1.38}$$
$$E[\hat{\theta}] = \text{constant} \tag{1.39}$$
$$\theta = \text{true value, constant} \tag{1.40}$$
$$\tag{1.41}$$
$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] \tag{1.42}$$
$$= E[(\hat{\theta} - E[\hat{\theta}] + E[\hat{\theta}]) - \theta)^2] \tag{1.43}$$
$$= E[([\hat{\theta} - E[\hat{\theta}]] + [E[\hat{\theta}] - \theta])^2] \tag{1.44}$$
$$= E[(A + B)^2] \tag{1.45}$$
$$\tag{1.46}$$
$$A = \hat{\theta} - E[\hat{\theta}] \tag{1.47}$$
$$B = E[\hat{\theta}] - \theta \tag{1.48}$$
$$\tag{1.49}$$
$$= E[A^2 + 2AB + B^2] \tag{1.50}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2 + 2(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta) + (E[\hat{\theta}] - \theta)^2] \tag{1.51}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + E[2(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta)] + E[(E[\hat{\theta}] - \theta)^2] \tag{1.52}$$
$$\tag{1.53}$$
$$C = E[\hat{\theta}] - \theta \text{ is a constant} \tag{1.54}$$
$$\tag{1.55}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + E[2(\hat{\theta} - E[\hat{\theta}])(C)] + E[C^2] \tag{1.56}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2CE[\hat{\theta} - E[\hat{\theta}]] + C^2 \tag{1.57}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(E[\hat{\theta}] - E[E[\hat{\theta}]]) + C^2 \tag{1.58}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(E[\hat{\theta}] - E[\hat{\theta}]) + C^2 \tag{1.59}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + 2C(0) + C^2 \tag{1.60}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + C^2 \tag{1.61}$$
$$= E[(\hat{\theta} - E[\hat{\theta}])^2] + (E[\hat{\theta}] - \theta)^2 \tag{1.62}$$
$$\tag{1.63}$$
$$var(\hat{\theta}) = E[(\hat{\theta} - E[\hat{\theta}])^2] \tag{1.64}$$
$$bias(\hat{\theta}, \theta) = E[\hat{\theta}] - \theta \tag{1.65}$$
$$\tag{1.66}$$
$$= var(\hat{\theta}) + (bias(\hat{\theta}, \theta))^2 \tag{1.67}$$
$$\tag{1.68}$$

1.2   Uniform Random Variable

1.3   Bernoulli Distribution

The Bernoulli Distribution is a special case of the Binomial Distribution, where

$$n = 1$$

.

1.3.1   PMF

$$P(X = k) = \binom{1}{k} p^k (1-p)^{1-k} \tag{1.69}$$

$$= p^k (1-p)^{n-k} \tag{1.70}$$

1.3.2   Expected Value

$$E(x) = \sum_{k \geqslant 1} [\binom{n}{k} p^k (1-p)^{n-k}] * k \tag{1.71}$$

$$= np \qquad\qquad \text{see Binomial Distribution E[X]} \tag{1.72}$$

$$= 1 * p \tag{1.73}$$

$$= p \tag{1.74}$$

1.3.3   Variance

$$Var(X) = np * (1-p) \tag{1.75}$$

$$= p * (1-p) \tag{1.76}$$

$$\tag{1.77}$$

1.3.4   Likelihood of IID Bernoulli

$$x_i \stackrel{iid}{\sim} Bernoulli(p) \tag{1.78}$$

$$L(x_i | p) = p(x_1, x_2, ..., x_n | p) \tag{1.79}$$

$$= \prod_{n=1}^{n} p(x_i | p) \tag{1.80}$$

$$= p^S * (1-p)^{n-S} \tag{1.81}$$

1.3.5 Maximun Likelihood

$$\frac{d[L(x_i|p)]}{dp} = \frac{d[p^S * (1-p)^{n-S}]}{dp} \tag{1.82}$$

$$\frac{d[log(L(x_i|p))]}{dp} = \frac{d[log(p^S * (1-p)^{n-S})]}{dp} \tag{1.83}$$

$$= \frac{d}{dp} * [log(p^S * (1-p)^{n-S})] \tag{1.84}$$

$$= \frac{d}{dp}[log(p^S) + log((1-p)^{n-S})] \tag{1.85}$$

$$= \frac{d}{dp}[S * log(p) + (n-S) * log(1-p)] \tag{1.86}$$

$$= S * \frac{d}{dp}[log(p)] + (n-S) * \frac{d}{dp}[log(1-p)] \tag{1.87}$$

$$= S * [\frac{1}{p}] + (n-S) * \frac{d}{dp}[log(1-p)] \qquad \text{chain rule} \tag{1.88}$$

$$= S * \frac{1}{p} + (n-S) * \frac{1}{p-1} \tag{1.89}$$

$$= \frac{S}{p} + \frac{n-S}{p-1} \tag{1.90}$$

$$= \frac{S * (p-1)}{p * (p-1)} + \frac{p * (n-S)}{p * (p-1)} \tag{1.91}$$

$$= \frac{S * (p-1) + p * (n-S)}{p * (p-1)} \tag{1.92}$$

$$= \frac{S * p - S + p * n - p * S}{p * (p-1)} \tag{1.93}$$

$$= \frac{-S + p * n}{p * (p-1)} \tag{1.94}$$

$$0 = \frac{-S + p * n}{p * (p-1)} \tag{1.95}$$

$$0 * (p * (p-1)) = -S + p * n \tag{1.96}$$

$$0 = -S + p * n \tag{1.97}$$

$$S = p * n \tag{1.98}$$

$$\frac{S}{n} = p \tag{1.99}$$

$$p = \frac{S}{n} \tag{1.100}$$

$$\tag{1.101}$$

1.3.6 MGF

$$M(t) = E[e^{tX}] \tag{1.102}$$
$$= (1-p)e^{t*0} + pe^{t*1} \tag{1.103}$$
$$= (1-p) + pe^t \tag{1.104}$$
$$\tag{1.105}$$

E[X] using MGF

$$E[X] = \frac{d^1}{dt^1}[M(t)](0) \tag{1.106}$$
$$\tag{1.107}$$
$$M(t) = (1-p) + pe^t \tag{1.108}$$
$$\tag{1.109}$$

$$E[X] = \frac{d^1}{dt^1}[(1-p) + pe^t](0) \tag{1.110}$$
$$= [\frac{d^1}{dt^1}[(1-p)] + \frac{d^1}{dt^1}[pe^t]](0) \tag{1.111}$$
$$= [0 + \frac{d^1}{dt^1}[pe^t]](0) \tag{1.112}$$
$$= [\frac{d^1}{dt^1}[pe^t]](0) \tag{1.113}$$
$$= [pe^t](0) \tag{1.114}$$
$$= [pe^0] \tag{1.115}$$
$$= p * 1 \tag{1.116}$$
$$= p \tag{1.117}$$
$$\tag{1.118}$$

$E[X^2]$ **using MGF**

$$E[X^2] = \frac{d^2}{dt^2}[M(t)](0) \tag{1.119}$$

$$\tag{1.120}$$

$$M(t) = (1-p) + pe^t \tag{1.121}$$

$$\tag{1.122}$$

$$E[X^2] = \frac{d^2}{dt^2}[(1-p) + pe^t](0) \tag{1.123}$$

$$= \frac{d^1}{dt^1}[pe^t](0) \qquad \text{see E[X] using MGF} \tag{1.124}$$

$$= [pe^t](0) \tag{1.125}$$

$$= [pe^0] \tag{1.126}$$

$$= p * 1 \tag{1.127}$$

$$= p \tag{1.128}$$

$$\tag{1.129}$$

And these values can be checked calculating the $var[X]$.

$$var[X] = E[X^2] - E[X]^2 \tag{1.130}$$

$$= p - p^2 \tag{1.131}$$

$$= p * (1-p) \tag{1.132}$$

$$\square \tag{1.133}$$

$E[X^n]$ **using MGF**

$$E[X^2] = \frac{d^2}{dt^2}[M(t)](0) \tag{1.134}$$

$$\tag{1.135}$$

$$M(t) = (1-p) + pe^t \tag{1.136}$$

$$\tag{1.137}$$

$$E[X^2] = \frac{d^2}{dt^2}[(1-p) + pe^t](0) \tag{1.138}$$

$$\tag{1.139}$$

1.4   Binomial Distribution

1.4.1   PMF

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \tag{1.140}$$

1.4.2   Expected Value

$E[g(X)]$ when $g(X) = X$.

$$E(X) = \sum_{k \geqslant 0} P(x = k) * k \tag{1.141}$$

$$= \sum_{k \geqslant 0} [\binom{n}{k} p^k (1 - p)^{n-k}] * k \tag{1.142}$$

$$\tag{1.143}$$

when

$$k = 0$$

, the formula

$$[\binom{n}{k} p^k (1 - p)^{n-k}] * k = [\binom{n}{0} p^k (1 - p)^n] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$E(X) = \sum_{k \geqslant 1} \binom{n}{k} p^k (1-p)^{n-k} * k \tag{1.144}$$

$$= \sum_{k \geqslant 1} \frac{n}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} * k \quad \text{see BinomialCoefficient}$$
$$\tag{1.145}$$

$$= \sum_{k \geqslant 1} \frac{n*k}{k} * \binom{n-1}{k-1} p^k (1-p)^{n-k} \tag{1.146}$$

$$= \sum_{k \geqslant 1} n * \binom{n-1}{k-1} p^k (1-p)^{n-k} \tag{1.147}$$

$$= \sum_{k \geqslant 1} n * p * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \tag{1.148}$$

$$= np * \sum_{k \geqslant 1} \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k} \tag{1.149}$$

$$u = n - 1 \tag{1.150}$$
$$z = k - 1 \tag{1.151}$$
$$u - z = (n-1) - (k-1) \tag{1.152}$$
$$= n - 1 - k + 1 \tag{1.153}$$
$$= n - k \tag{1.154}$$
$$k > 1 = (z + 1) > 1 \tag{1.155}$$
$$= z > 0 \tag{1.156}$$

$$= np * \sum_{z > 0} \binom{u}{z} p^z (1-p)^{u-z} \tag{1.157}$$

$$= np * 1 \qquad\qquad see\,Binomial\,Distribution\,Proof\,Equals\,1$$
$$\tag{1.158}$$

$$= np \tag{1.159}$$
$$\square \tag{1.160}$$

1.4.3 Variance

$$Var(X) = E(X^2) - E(X)^2 \qquad \text{see Variance} \tag{1.161}$$

$$= \sum_{k \geqslant 0} [\binom{n}{k} p^k (1-p)^{n-k}] * k^2 - np \quad \text{see Binomial Expected Value}$$
$$\tag{1.162}$$

when

$$k = 0$$

, the formula

$$[\binom{n}{k}p^k(1-p)^{n-k}] * k = [\binom{n}{0}p^k(1-p)^n] * 0 = 0$$

, so the index of the summation can be increased by 1.

$$= \sum_{k \geqslant 1} [\binom{n}{k} p^k (1-p)^{n-k}] * k^2 - (np)^2 \tag{1.163}$$

$$= \sum_{k \geqslant 1} \frac{n}{k} [\binom{n-1}{k-1} p^k (1-p)^{n-k}] * k^2 - (np)^2 \tag{1.164}$$

$$= \sum_{k \geqslant 1} \frac{n * k^2}{k} [\binom{n-1}{k-1} p^k (1-p)^{n-k}] - (np)^2 \tag{1.165}$$

$$= \sum_{k \geqslant 1} [nk * \binom{n-1}{k-1} p^k (1-p)^{n-k}] - (np)^2 \tag{1.166}$$

$$= \sum_{k \geqslant 1} [nkp * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \tag{1.167}$$

$$= np * \sum_{k \geqslant 1} [k * \binom{n-1}{k-1} p^{k-1} (1-p)^{n-k}] - (np)^2 \tag{1.168}$$

$$u = n - 1 \tag{1.169}$$

$$z = k - 1 \tag{1.170}$$

$$u - z = (n-1) - (k-1) \tag{1.171}$$

$$= n - 1 - k + 1 \tag{1.172}$$

$$= n - k \tag{1.173}$$

$$k >= 1 = (z+1) >= 1 \tag{1.174}$$

$$= z >= 0 \tag{1.175}$$

$$= np * \sum_{z \geqslant 0} [(z+1) * \binom{u}{z} p^z (1-p)^{u-z}] - (np)^2 \tag{1.176}$$

$$= np * [\sum_{z \geqslant 0} [z * \binom{u}{z} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.177}$$

$$= np * [\sum_{z \geqslant 0} [z * \frac{u}{z} * \binom{u-1}{z-1} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.178}$$

$$= np * [u * \sum_{z \geqslant 0} [\binom{u-1}{z-1} p^z (1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z (1-p)^{u-z}]] - (np)^2 \tag{1.179}$$

$$\tag{1.180}$$

$$= np * [up * \sum_{z \geqslant 0} [\binom{u-1}{z-1} p^{z-1}(1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z(1-p)^{u-z}]] - (np)^2$$

$$(1.181)$$

$$= np * [up * \sum_{z \geqslant 1} [\binom{u-1}{z-1} p^{z-1}(1-p)^{(u-1)-(z-1)}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z(1-p)^{u-z}]] - (np)^2$$

$$(1.182)$$

$$= np * [up * \sum_{z \geqslant 0} [\binom{u}{z} p^z(1-p)^{u-z}] + \sum_{z \geqslant 0} [\binom{u}{z} p^z(1-p)^{u-z}]] - (np)^2$$

$$(1.183)$$

$$= np * [up * (p+q)^{u-1} + (p+q)^u] - (np)^2 \tag{1.184}$$

$$= np * [(n-1) * p * (p+q)^{n-1-1} + (p+q)^{(n-1)}] - (np)^2 \tag{1.185}$$

$$= np * [(n-1) * p * (p+q)^{n-2} + (p+q)^{(n-1)}] - (np)^2 \tag{1.186}$$

$$= np * ([(n-1) * p * (p+q)^{n-2} + (p+q)^{(n-1)}] - np) \tag{1.187}$$

$$= np * ([(n-1) * p + 1] - np) \qquad\qquad\qquad\qquad \text{p+q=1}$$

$$(1.188)$$

$$= np * ([(n-1) * p + 1] - np) \tag{1.189}$$

$$= np * ([np - p + 1] - np) \tag{1.190}$$

$$= np * (np - p + 1 - np) \tag{1.191}$$

$$= np * (-p + 1) \tag{1.192}$$

$$= np * (1 - p) \tag{1.193}$$

$$\square \tag{1.194}$$

1.5 Geometric Distribution

1.5.1 CDF

$$P(X \le x) =$$

$$CDF(X = x) = \sum_{i=0}^{x} (1-p)^i p \qquad \text{by geometric summation}$$

$$= p \frac{1 - (1-p)^x}{1 - (1-p)}$$

$$= p \frac{1 - (1-p)^x}{1 - 1 + p}$$

$$= p \frac{1 - (1-p)^x}{p}$$

$$= 1 - (1-p)^x$$

$$\square$$

$$P(X > x) = 1 - CDF(X = x)$$

$$= 1 - (1 - (1-p)^x)$$

$$= 1 - 1 + (1-p)^x$$

$$= (1-p)^x$$

$$\square$$

1.6 Normal Distribution

1.6.1 Definition

$$pdf(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \qquad (1.195)$$

Chapter 2

Continuous Distributions

2.1   Uniform Distribution

2.1.1   PDF

$$\int_a^b k\,dx = 1$$
$$= k\int_a^b dx$$
$$= k[x|_a^b]$$
$$= k[b - a]$$

$$k[b - a] = 1$$
$$k = \frac{1}{b - a}$$

2.1.2   Expected Value

$$\int_a^b x\Big(\frac{1}{b-a}\Big)dx =$$

$$= \int_a^b x\Big(\frac{1}{b-a}\Big)dx$$

$$= \frac{1}{b-a}\int_a^b xdx$$

$$= \frac{1}{b-a}\Big[\frac{x^2}{2}\big|_a^b\Big]$$

$$= \frac{1}{b-a}\Big[\frac{b^2}{2}-\frac{a^2}{2}\Big]$$

$$= \frac{1}{b-a}\Big[\frac{b^2-a^2}{2}\Big]$$

$$= \frac{1}{b-a}\Big[\frac{(b+a)(b-a)}{2}\Big]$$

$$= \frac{(b+a)(b-a)}{2(b-a)}$$

$$= \frac{b+a}{2}$$

2.1.3 Variance

$$var(X) = E[X^2] - E[X]^2$$

$$= [\int_a^b x^2 \left(\frac{1}{b-a}\right) dx] - \left(\frac{b+a}{2}\right)^2$$

$$= [\frac{1}{b-a} \int_a^b x^2 dx] - \left(\frac{b+a}{2}\right)^2$$

$$= [\frac{1}{b-a} \frac{x^3}{3}|_a^b] - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{1}{b-a} \left(\frac{b^3}{3} - \frac{a^3}{3}\right) - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{b^3 - a^3}{3(b-a)} - \frac{(b+a)^2}{4}$$

$$= \frac{(b-a)(b^2 - ab + a^2)}{3(b-a)} - \frac{(b+a)^2}{4}$$

$$= \frac{(b^2 - ab + a^2)}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{4(b^2 - ab + a^2)}{12} - \frac{3(b+a)^2}{12}$$

$$= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3(b^2 + 2ab + a^2)}{12}$$

$$= \frac{4b^2 - 4ab + 4a^2}{12} - \frac{3b^2 + 6ab + 3a^2}{12}$$

$$= \frac{4b^2 - 3b^2 - 4ab - 6ab + 4a^2 - 3a^2}{12}$$

$$= \frac{b^2 - 2ab + a^2}{12}$$

$$= \frac{(b-a)^2}{12}$$

2.2   Exponential Distribution

2.2.1   PDF

$$f_x(x) = \lambda e^{-\lambda x} \qquad\qquad x >= 0$$

2.2.2   CDF

$$CDF(x) = \int_{-\infty}^{x} f_x dx$$

$$CDF(x) = \int_{-\infty}^{x} \lambda e^{-\lambda x} dx$$

$$u = e^{-\lambda x}$$

$$\frac{du}{dx} = d[e^{-\lambda x}]$$

$$du = d[e^{-\lambda x}] * dx$$

$$du = [-\lambda * e^{-\lambda x}] * dx$$

$$du = -\lambda e^{-\lambda x} dx$$

$$CDF(x) = \int_{-\infty}^{x} \lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^{x} -1 * \lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^{x} -\lambda e^{-\lambda x} dx$$

$$CDF(x) = -1 * \int_{-\infty}^{x} du$$

$$CDF(x) = -1 * \int_{0}^{x} du \qquad\qquad \text{by x bounds}$$

$$CDF(x) = -1 * [u]_0^x$$

$$CDF(x) = -1 * [e^{-\lambda x}]_0^x$$

$$CDF(x) = -1 * [e^{-\lambda x} - e^{-\lambda 0}]$$

$$CDF(x) = -1 * [e^{-\lambda x} - e^0]$$

$$CDF(x) = -1 * [e^{-\lambda x} - 1]$$

$$CDF(x) = [1 - e^{-\lambda x}]$$

$$CDF(x) = 1 - e^{-\lambda x}$$

2.2.3 Expected Value

$$f_x(x) = \lambda e^{-\lambda x}$$

$$E[x] = \int x * f_x(x) dx$$

$$E[x] = \int x * \lambda e^{-\lambda x} dx$$

$$u = x$$
$$du = 1 * dx$$
$$v = e^{-\lambda x}$$
$$dv = e^{-\lambda x} * -\lambda * dx$$
$$dv = -\lambda e^{-\lambda x} dx$$

$$E[x] = \int [x][\lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int [x][-1 * \lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int [x][-\lambda e^{-\lambda x} dx]$$

$$E[x] = -1 * \int u dv$$

$$E[x] = -1 * [uv|_{-\infty}^{\infty} - \int v du]$$

$$E[x] = -1 * [uv|_{0}^{\infty} - \int v du] \qquad \text{because x bounds}$$

$$E[x] = -1 * [[xe^{-\lambda x}]_{0}^{\infty} - \int e^{-\lambda x} dx]$$

$$[xe^{-\lambda x}]_0^\infty = \lim_{x->\infty} [xe^{-\lambda x} - (0)e^{-\lambda(0)}]$$

$$= \lim_{x->\infty} [xe^{-\lambda x} + 0e^0]$$

$$= \lim_{x->\infty} [xe^{-\lambda x}]$$

$$= 0$$

$$E[x] = -1 * - \int e^{-\lambda x} dx$$

$$E[x] = \frac{1}{\lambda} \int \lambda e^{-\lambda x} dx$$

$$E[x] = \frac{1}{\lambda}$$

Variables Relationships.png