1 Introduction

Mathematics Handout - Logaritmics

Daniel Frederico Lins Leite

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$$x_n$$

is a sequence of real numbers Th sequence converge if:

$$\exists a \in Re$$

$$\exists \epsilon \in Rsuchthat \epsilon > 0$$

$$\exists N_a \in IsuchthatN_a > 0$$

$$\forall n > N_a, |a - x_n| < \epsilon$$

if for example exists another

$$\exists b \in Re$$

such that

$$\exists N_b \in IsuchthatN_b > 0$$

$$\forall n > N_b, |b - x_n| < \epsilon$$

then

$$|a - b| = |a - x_n + x_n - b|$$

Given that the

$$R^n$$

, the space of the elements of the sequence is a normed vector space, we can apply the Triangle Inequality. So

$$|a - x_n + x_n - b| <= |a - x_n| + |x_n - b|$$

given that

$$|x_n - b|$$

is a even function

$$f(x) = f(-x)$$

we have that

$$|x_n - b| = -1 * |x_n - b| = |-x_n + b| = |b - x_x|$$

so we have

$$|a - x_n| + |x_n - b| < |a - x_n| + |b - x_n| <= \epsilon + \epsilon <= 2\epsilon$$

So

$$0 <= |a - b| <= 2\epsilon$$

So $|a - b|$ is squeezed between this two function. Given that this is true for every ϵ, if $\epsilon -> 0$ we will have that:

$$|a - b| = 0$$

and

$$a = b$$

The value a is called the limit of the sequence.

The

$$S = \sum_{i=0}^{\mathrm{inf}} x_i$$

is called the series associated with se sequence.

The partial sum is

$$S_n = \sum_{i=0}^{n} x_i$$

So

S_n is also a sequence. In the same way it can converge and have a limit. If the sequence converge, we say that the series converge and that the limit is

$$limitofS_n = \sum_{i=0}^{\mathrm{inf}} x_i$$

If we have two convergent series:

$$S_1 = \sum_{i=0}^{\mathrm{inf}} a_i$$

$$S_2 = \sum_{i=0}^{\mathrm{inf}} b_i$$

then

$$\sum_{i=0}^{inf} (a_i + b_i) = \sum_{i=0}^{\mathrm{inf}} a_i + \sum_{i=0}^{\mathrm{inf}} b_i$$