Recursive Call Count Operator
if f is a function, #f returns how much recursive call f does.
Fibonnaci Numbers

```
int fibonnaci(int n){
        if(n <= 2) return 1;
        return fibonnaci(n-1) + fibonnaci(n-2);
}
```

Call tree

```
fibonnaci(5)
├─fibonnaci(4)
│ ├─fibonnaci(3)
│ │ ├─fibonnaci(2) = 1
│ │ └─fibonnaci(1) = 1
│ └─fibonnaci(2) = 1
└─fibonnaci(3)
  ├─fibonnaci(2) = 1
  └─fibonnaci(1) = 1
```

All leafs always have value 1. So
fibonnaci(n) = 1 + 1 + ... + 1 + 1
In this case the call tree have:
9 nodes =
5 leafs
4 aggregators

Every aggregator comes from a recursive call. So
fibonnaci(5) makes 4 recursives calls.

So #fibonnaci(n) = #fibonnaci(n-1) + #fibonnaci(n-2) + 1

Thesis:
#fibonnaci(n) = fibonnaci(n) - 1

Proof by Induction
Base
#fibonnaci(1) = fibonnaci(1) - 1 = 1 - 1 = 0 (OK)
#fibonnaci(2) = fibonnaci(2) - 1 = 1 - 1 = 0 (OK)

Step
#fibonnaci(n) = fibonnaci(n) - 1
#fibonnaci(n) + fibonnaci(n-1) = fibonnaci(n) - 1 + fibonnaci(n-1)
#fibonnaci(n) + fibonnaci(n-1) = fibonnaci(n) + fibonnaci(n-1) - 1
#fibonnaci(n) + #fibonnaci(n-1) + 1 = fibonnaci(n+1) - 1
#fibonnaci(n+1) = fibonnaci(n+1) - 1

QED

Binomial Numbers

```
int binomial(int n, int k){
        if(k > n) return 0;
        if(k == 0 || n == k) return 1;
        return binomial(n-1,k) + binomial(n-1,k-1)
}
```

Call Tree:

```
binomial(4,2)
 ├─binomial(3,2)
 │  ├─binomial(2,2) = 1
 │  └─binomial(2,1)
 │     ├─binomial(1,1) = 1
 │     └─binomial(1,0) = 1
 └─binomial(3,1)
    ├─binomial(2,1)
    │  ├─binomial(1,1) = 1
    │  └─binomial(1,0) = 1
    └─binomial(2,0) = 1
```

All leafs always have value 1. So
binomial(n,k) = 1 + 1 + ... + 1 + 1
In this case the call tree have:
11 nodes =
6 leafs
5 aggregators

Every aggregator comes from a recursive call. So binomial(4,2) makes 5 recursives calls.

#binomial(n,k) = #binomial(n-1,k) + #binomial(n-1,k-1) + 1

Thesis:
#binomial(n,k) = binomial(n,k) - 1

Bases:
#binomial(1,0) = binomial(1,0) - 1 = 1 - 1 = 0 (OK)

Step:
#binomial(n,k) = binomial(n,k) - 1
#binomial(n,k) + binomial(n,k+1) = binomial(n,k) - 1 + binomial(n,k+1)
#binomial(n,k) + #binomial(n,k+1) + 1 = binomial(n+1,k+1) - 1
#binomial(n+1,k+1) = binomial(n+1,k+1) - 1
QED