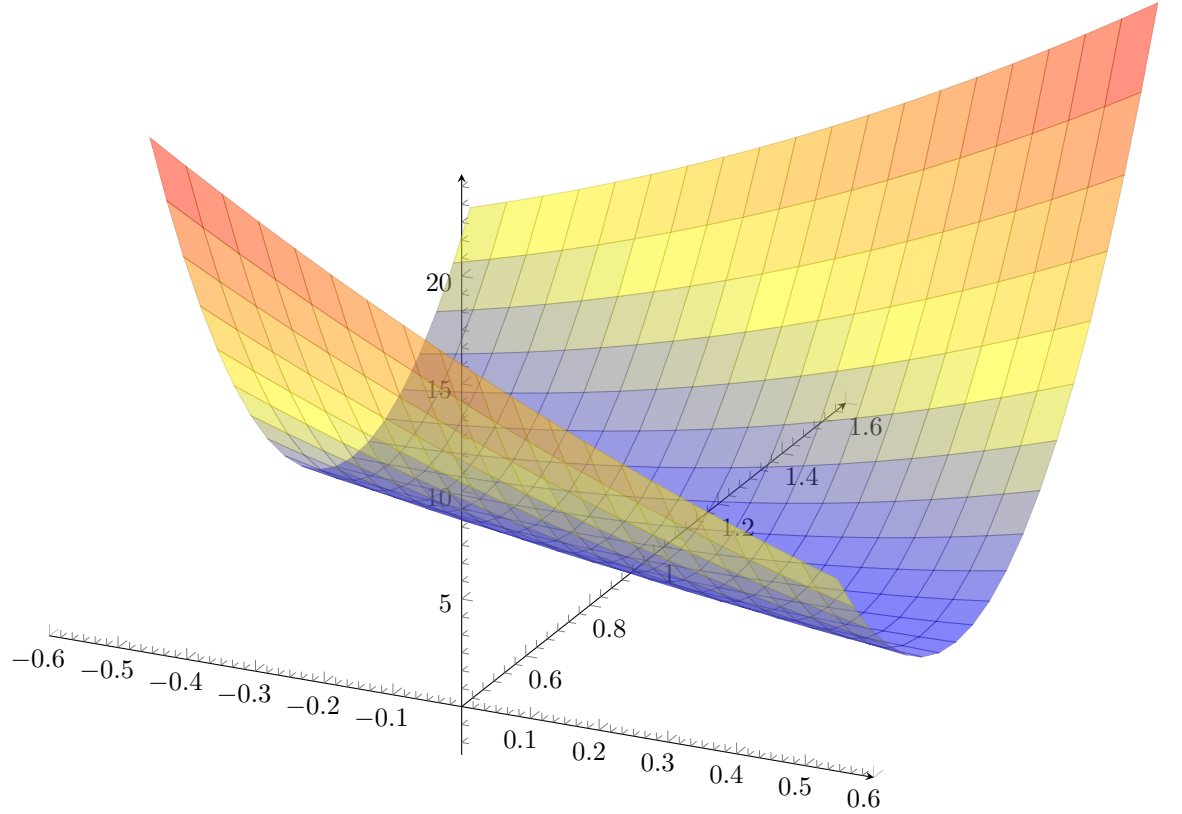


Squared Error Line

$$\begin{aligned}
SE_{line} &= \sum_{i=0}^N [y_i - (w_1 * x_i + w_0)]^2 \\
&= \sum_{i=0}^N [y_i^2 - 2 * y_i * (w_1 * x_i + w_0) + (w_1 * x_i + w_0)^2] \\
&= \sum_{i=0}^N [y_i^2 - 2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i + w_0)^2] \\
&= \sum_{i=0}^N [y_i^2 - 2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - \sum_{i=0}^N [2 * y_i * w_1 * x_i - 2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - \sum_{i=0}^N [2 * y_i * w_1 * x_i] - \sum_{i=0}^N [2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2 * w_1 * \sum_{i=0}^N [y_i * x_i] - \sum_{i=0}^N [2 * y_i * w_0 + (w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2 * w_1 * \sum_{i=0}^N [y_i * x_i] - \sum_{i=0}^N [2 * y_i * w_0] + \sum_{i=0}^N [(w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2 * w_1 * \sum_{i=0}^N [y_i * x_i] - 2 * w_0 * \sum_{i=0}^N [y_i] + \sum_{i=0}^N [(w_1 * x_i)^2 + 2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2 * w_1 * \sum_{i=0}^N [y_i * x_i] - 2 * w_0 * \sum_{i=0}^N [y_i] + \sum_{i=0}^N [(w_1 * x_i)^2] + \sum_{i=0}^N [2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2 * w_1 * \sum_{i=0}^N [y_i * x_i] - 2 * w_0 * \sum_{i=0}^N [y_i] + w_1^2 * \sum_{i=0}^N (x_i)^2 + \sum_{i=0}^N [2 * w_1 * x_i * w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2w_1 \sum_{i=0}^N [y_i x_i] - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + \sum_{i=0}^N [2w_1 x_i w_0 + w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2w_1 \sum_{i=0}^N [y_i x_i] - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + \sum_{i=0}^N [2w_1 x_i w_0] + \sum_{i=0}^N [w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2w_1 \sum_{i=0}^N [y_i x_i] - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + 2w_1 w_0 \sum_{i=0}^N [x_i] + \sum_{i=0}^N [w_0^2] \\
&= \sum_{i=0}^N [y_i^2] - 2w_1 \sum_{i=0}^N [y_i x_i] - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + 2w_1 w_0 \sum_{i=0}^N [x_i] + w_0^2 N
\end{aligned}$$

$$\begin{aligned}
\frac{\sum_{i=0}^N y_i^2}{N} &= \overline{y^2} \\
\sum_{i=0}^N y_i^2 &= \overline{y^2} * N \\
&= \overline{y^2}N - 2w_1 \sum_{i=0}^N [y_i x_i] - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + 2w_1 w_0 \sum_{i=0}^N [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{y\bar{x}}N - 2w_0 \sum_{i=0}^N [y_i] + w_1^2 \sum_{i=0}^N (x_i)^2 + 2w_1 w_0 \sum_{i=0}^N [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{y\bar{x}}N - 2w_0 \overline{y}N + w_1^2 \sum_{i=0}^N (x_i)^2 + 2w_1 w_0 \sum_{i=0}^N [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{y\bar{x}}N - 2w_0 \overline{y}N + w_1^2 \overline{x^2}N + 2w_1 w_0 \sum_{i=0}^N [x_i] + w_0^2 N \\
&= \overline{y^2}N - 2w_1 \overline{y\bar{x}}N - 2w_0 \overline{y}N + w_1^2 \overline{x^2}N + 2w_1 w_0 \overline{x}N + w_0^2 N \\
SE_{line} &= N\overline{y^2} - 2Nw_1 \overline{y\bar{x}} - 2Nw_0 \overline{y} + Nw_1^2 \overline{x^2} + 2Nw_1 w_0 \overline{x} + Nw_0^2
\end{aligned}$$



Now to find the line that minimizes the squared errors we can take the partial derivatives in relation to w_0 and w_1

First partial derivative in relation of w_0

$$\begin{aligned}
MSE_{line} &= \frac{d}{dw_0} [N\bar{y}^2 - 2Nw_1\bar{y}\bar{x} - 2Nw_0\bar{y} + Nw_1^2\bar{x}^2 + 2Nw_1w_0\bar{x} + Nw_0^2] \\
&= \frac{d}{dw_0} [N\bar{y}^2] - \frac{d}{dw_0} [2Nw_1\bar{y}\bar{x}] - \frac{d}{dw_0} [2Nw_0\bar{y}] + \frac{d}{dw_0} [Nw_1^2\bar{x}^2] + \frac{d}{dw_0} [2Nw_1w_0\bar{x}] + \frac{d}{dw_0} [Nw_0^2] \\
&= -\frac{d}{dw_0} [2Nw_1\bar{y}\bar{x}] - \frac{d}{dw_0} [2Nw_0\bar{y}] + \frac{d}{dw_0} [Nw_1^2\bar{x}^2] + \frac{d}{dw_0} [2Nw_1w_0\bar{x}] + \frac{d}{dw_0} [Nw_0^2] \\
&= -\frac{d}{dw_0} [2Nw_0\bar{y}] + \frac{d}{dw_0} [Nw_1^2\bar{x}^2] + \frac{d}{dw_0} [2Nw_1w_0\bar{x}] + \frac{d}{dw_0} [Nw_0^2] \\
&= -2N\bar{y} + \frac{d}{dw_0} [Nw_1^2\bar{x}^2] + \frac{d}{dw_0} [2Nw_1w_0\bar{x}] + \frac{d}{dw_0} [Nw_0^2] \\
&= -2N\bar{y} + \frac{d}{dw_0} [2Nw_1w_0\bar{x}] + \frac{d}{dw_0} [Nw_0^2] \\
&= -2N\bar{y} + 2Nw_1\bar{x} + \frac{d}{dw_0} [Nw_0^2] \\
&= -2N\bar{y} + 2Nw_1\bar{x} + 2Nw_0
\end{aligned}$$

Now the derivative in relation to w_1

$$\begin{aligned}
MSE_{line} &= \frac{d}{dw_1} [N\bar{y}^2 - 2Nw_1\bar{y}\bar{x} - 2Nw_0\bar{y} + Nw_1^2\bar{x}^2 + 2Nw_1w_0\bar{x} + Nw_0^2] \\
&= \frac{d}{dw_1} [N\bar{y}^2] - \frac{d}{dw_1} [2Nw_1\bar{y}\bar{x}] - \frac{d}{dw_1} [2Nw_0\bar{y}] + \frac{d}{dw_1} [Nw_1^2\bar{x}^2] + \frac{d}{dw_1} [2Nw_1w_0\bar{x}] + \frac{d}{dw_1} [Nw_0^2] \\
&= -\frac{d}{dw_1} [2Nw_1\bar{y}\bar{x}] - \frac{d}{dw_1} [2Nw_0\bar{y}] + \frac{d}{dw_1} [Nw_1^2\bar{x}^2] + \frac{d}{dw_1} [2Nw_1w_0\bar{x}] + \frac{d}{dw_1} [Nw_0^2] \\
&= -2N\bar{y}\bar{x} - \frac{d}{dw_1} [2Nw_0\bar{y}] + \frac{d}{dw_1} [Nw_1^2\bar{x}^2] + \frac{d}{dw_1} [2Nw_1w_0\bar{x}] + \frac{d}{dw_1} [Nw_0^2] \\
&= -2N\bar{y}\bar{x} + \frac{d}{dw_1} [Nw_1^2\bar{x}^2] + \frac{d}{dw_1} [2Nw_1w_0\bar{x}] + \frac{d}{dw_1} [Nw_0^2] \\
&= -2N\bar{y}\bar{x} + 2Nw_1\bar{x}^2 + \frac{d}{dw_1} [2Nw_1w_0\bar{x}] + \frac{d}{dw_1} [Nw_0^2] \\
&= -2N\bar{y}\bar{x} + 2Nw_1\bar{x}^2 + 2Nw_0\bar{x} + \frac{d}{dw_1} [Nw_0^2] \\
&= -2N\bar{y}\bar{x} + 2Nw_1\bar{x}^2 + 2Nw_0\bar{x}
\end{aligned}$$

Now we must solve the lienar system

$$\begin{cases} -2N\bar{y} + 2Nw_1\bar{x} + 2Nw_0 = 0 \\ -2N\bar{y}\bar{x} + 2Nw_1\bar{x}^2 + 2Nw_0\bar{x} = 0 \end{cases}$$

divide first 2N

$$\begin{cases} -\bar{y} + w_1\bar{x} + w_0 = 0 \\ -2N\bar{y}\bar{x} + 2Nw_1\bar{x}^2 + 2Nw_0\bar{x} = 0 \end{cases}$$

divide second 2N

$$\begin{cases} -\bar{y} + w_1\bar{x} + w_0 = 0 \\ -\bar{y}\bar{x} + w_1\bar{x}^2 + w_0\bar{x} = 0 \end{cases}$$

$$\begin{cases} w_1\bar{x} + w_0 = \bar{y} \\ w_1\bar{x}^2 + w_0\bar{x} = \bar{y}\bar{x} \end{cases}$$

$$\begin{cases} w_1\bar{x} + w_0 = \bar{y} \\ w_1\frac{\bar{x}^2}{\bar{x}} + w_0\frac{\bar{x}}{\bar{x}} = \frac{\bar{y}\bar{x}}{\bar{x}} \end{cases}$$

$$\begin{cases} w_1\bar{x} + w_0 = \bar{y} \\ w_1\frac{\bar{x}^2}{\bar{x}} + w_0 = \frac{\bar{y}\bar{x}}{\bar{x}} \end{cases}$$

This means that the line that minimizes the squared error pass through the points

$$\begin{pmatrix} \bar{x}, \bar{y} \\ \frac{\bar{x}^2}{\bar{x}}, \frac{\bar{y}\bar{x}}{\bar{x}} \end{pmatrix}$$