

# eisenberg-noe-2001-debt-model-with-default-costs

November 13, 2021

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## DEBT MODEL WITH DEFAULT COSTS

ALPHA = 0.5 and BETA = 0.5

Running Debt Model in MANUAL mode with 3 nodes...

Scenario 4 - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3, MODE == 'MANUAL', NUM\_AGENTS = 3, NOMINAL\_LIABILITY\_MATRIX = np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING\_CASH\_FLOW\_VECTOR = [3, 0, 4], ALPHA = 0.5, BETA = 0.5

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## AGENT LABELS

Agent labels ['A', 'B', 'C']

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NOMINAL LIABILITY MATRIX Data Frame  
i.e. what node i expects to pay node j...

	A	B	C
A	0	2	9
B	7	0	9
C	3	1	0

Nominal liabilities for each node:

Liability of Node A to Node B is 2

Liability of Node A to Node C is 9

Liability of Node B to Node A is 7

Liability of Node B to Node C is 9

Liability of Node C to Node A is 3

Liability of Node C to Node B is 1

NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame

i.e. what node j expects to receive from i...

	A	B	C
A	0	7	3
B	2	0	1
C	9	9	0

Node A expects to receive 7 from Node B  
Node A expects to receive 3 from Node C  
Node B expects to receive 2 from Node A  
Node B expects to receive 1 from Node C  
Node C expects to receive 9 from Node A  
Node C expects to receive 9 from Node B

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#### OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 3  
Exogenous cash flow for Node B: 0  
Exogenous cash flow for Node C: 4  
[3, 0, 4]

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#### CALCULATING RELATIVE LIABILITIES FOR EACH NODE

##### \*\*\*Node A\*\*\*

###### - Liabilities for Node A

Liability of Node A to Node B (i.e. P\_01) is 2.0  
Liability of Node A to Node C (i.e. P\_02) is 9.0

###### - Total obligations for Node A

Total Obligation Vector updated in round 1 for Node A with value 11.0  
Total nominal liabilities for Node A (i.e. p\_bar\_1) is 11.0

###### - Relative Liabilities of Node A

Relative Liability of Node A to Node B is 0.181818181818182  
Relative Liability of Node A to Node C is 0.8181818181818182  
Sum of Relative Liabilities for Node A is 1.0

##### \*\*\*Node B\*\*\*

###### - Liabilities for Node B

Liability of Node B to Node A (i.e. P\_10) is 7.0  
Liability of Node B to Node C (i.e. P\_12) is 9.0

- Total obligations for Node B  
Total Obligation Vector updated in round 1 for Node B with value 16.0  
Total nominal liabilities for Node B (i.e.  $p_{\text{bar}_2}$ ) is 16.0

- Relative Liabilities of Node B  
Relative Liability of Node B to Node A is 0.4375  
Relative Liability of Node B to Node C is 0.5625  
Sum of Relative Liabilities for Node B is 1.0

\*\*\*Node C\*\*\*

- Liabilities for Node C  
Liability of Node C to Node A (i.e.  $P_{20}$ ) is 3.0  
Liability of Node C to Node B (i.e.  $P_{21}$ ) is 1.0

- Total obligations for Node C  
Total Obligation Vector updated in round 1 for Node C with value 4.0  
Total nominal liabilities for Node C (i.e.  $p_{\text{bar}_3}$ ) is 4.0

- Relative Liabilities of Node C  
Relative Liability of Node C to Node A is 0.75  
Relative Liability of Node C to Node B is 0.25  
Sum of Relative Liabilities for Node C is 1.0

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RELATIVE LIABILITY MATRIX Data Frame

	A	B	C
A	0.0000	0.181818	0.818182
B	0.4375	0.000000	0.562500
C	0.7500	0.250000	0.000000

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RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

	A	B	C	Relative Liability Total	CORRECT VALUE?
A	0.0000	0.181818	0.818182	1.0	True
B	0.4375	0.000000	0.562500	1.0	True
C	0.7500	0.250000	0.000000	1.0	True

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RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame

i.e. what node i expects to receive from j in relative terms...

	A	B	C
A	0.000000	0.4375	0.75
B	0.181818	0.0000	0.25
C	0.818182	0.5625	0.00

\*\*\*Expected nominal payments in for Node A - both proportion and total amount\*\*\*

Node A expects to receive proportion 0.4375 from Node B

Node A expects to receive proportion 0.75 from Node C

Total payments in to Node A is 10.0

\*\*\*Expected nominal payments in for Node B - both proportion and total amount\*\*\*

Node B expects to receive proportion 0.18181818181818182 from Node A

Node B expects to receive proportion 0.25 from Node C

Total payments in to Node B is 3.0

\*\*\*Expected nominal payments in for Node C - both proportion and total amount\*\*\*

Node C expects to receive proportion 0.8181818181818182 from Node A

Node C expects to receive proportion 0.5625 from Node B

Total payments in to Node C is 18.0

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START OF ROUND 1

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TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e.  $p_{\text{bar}_i}$

Total nominal obligation for Node A (i.e.  $p_{\text{bar}_1}$ ): 11.0

Total nominal obligation for Node B (i.e.  $p_{\text{bar}_2}$ ): 16.0

Total nominal obligation for Node C (i.e.  $p_{\text{bar}_3}$ ): 4.0

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TOTAL PAYMENT MADE PER NODE

i.e.  $\min[\text{nominal obligations, cashflow (payments in + exogenous cash flow)}]$  for each node...

\*\*\*Node A\*\*\*

- Total payments in for Node A

Total payments in to Node A is 10.0

- Liabilities for Node A

Liability of Node A to Node B (i.e. P\_01) is 2.0

Liability of Node A to Node C (i.e. P\_02) is 9.0

- Total obligations for Node A

Total nominal liabilities for Node A (i.e. p\_bar\_1) is 11.0

Payment out is min[payment out, total cash flow] i.e. min[11.0, 13.0]

- Total Dollar Payment Vector for round 1 and Node A

Total Dollar Payment Vector for round 1 and Node A updated with value 11.0

\*\*\*Node B\*\*\*

- Total payments in for Node B

Total payments in to Node B is 3.0

- Liabilities for Node B

Liability of Node B to Node A (i.e. P\_10) is 7.0

Liability of Node B to Node C (i.e. P\_12) is 9.0

- Total obligations for Node B

Total nominal liabilities for Node B (i.e. p\_bar\_2) is 16.0

Payment out is min[payment out, total cash flow] i.e. min[16.0, 3.0]

Total payments in to Node B is 3.0

Total payments in to Node B is 3.0

Total payments in to Node B is 3.0

Round 1 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 3.0. Default loss for Node B is 1.5

- Total Dollar Payment Vector for round 1 and Node B

Total Dollar Payment Vector for round 1 and Node B updated with value 1.5

\*\*\*Node C\*\*\*

- Total payments in for Node C

Total payments in to Node C is 18.0

- Liabilities for Node C

Liability of Node C to Node A (i.e. P\_20) is 3.0

Liability of Node C to Node B (i.e. P\_21) is 1.0

- Total obligations for Node C

Total nominal liabilities for Node C (i.e. p\_bar\_3) is 4.0

Payment out is min[payment out, total cash flow] i.e. min[4.0, 22.0]

- Total Dollar Payment Vector for round 1 and Node C

Total Dollar Payment Vector for round 1 and Node C updated with value 4.0

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#### TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p\_1): 11.0

Total payment by Node B (i.e. p\_2): 1.5

Total payment by Node C (i.e. p\_3): 4.0

[11.0, 1.5, 4.0]

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#### UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE

Total payments in to Node A is 10.0

Total dollar payment by Node A (i.e. p\_1) is 11.0

- Equity Vector for round 1 and Node A

Equity Vector for round 1 and Node A updated with value 13.0

Total payments in to Node B is 3.0

Total dollar payment by Node B (i.e. p\_2) is 1.5

- Equity Vector for round 1 and Node B

Equity Vector for round 1 and Node B updated with value 3.0

Total payments in to Node C is 18.0

Total dollar payment by Node C (i.e. p\_3) is 4.0

- Equity Vector for round 1 and Node C

Equity Vector for round 1 and Node C updated with value 22.0

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#### EQUITY FOR EACH NODE

Equity for Node A: 13.0

Equity for Node B: 3.0

Equity for Node C: 22.0

[13.0, 3.0, 22.0]

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#### ROUND 1 DEFAULTERS

Node B has defaulted in round 1

{'A': False, 'B': True, 'C': False}

There are defaulters in this round (i.e. round 1), algorithm will proceed for another round.

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END OF ROUND 1  
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START OF ROUND 2  
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TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e.  $p_{bar_i}$   
Total nominal obligation for Node A (i.e.  $p_{bar_1}$ ): 11.0  
Total nominal obligation for Node B (i.e.  $p_{bar_2}$ ): 16.0  
Total nominal obligation for Node C (i.e.  $p_{bar_3}$ ): 4.0  
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TOTAL PAYMENT MADE PER NODE

i.e.  $\min[\text{nominal obligations, cashflow (payments in + exogenous cash flow)}]$  for each node...

\*\*\*Node A\*\*\*

- Total payments in for Node A

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

- Liabilities for Node A

Liability of Node A to Node B (i.e.  $P_{01}$ ) is 2.0

Liability of Node A to Node C (i.e.  $P_{02}$ ) is 9.0

Total nominal liabilities for Node A (i.e.  $p_{bar_1}$ ) is 11.0

Payment out is  $\min[\text{payment out, total cash flow}]$  i.e.  $\min[11.0, 6.65625]$

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5  
 Relative Payment in to Node A from Node C is 0.75  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node A is 3.65625  
 Relative Payment in to Node A from Node B is 0.4375  
 Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5  
 Relative Payment in to Node A from Node C is 0.75  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node A is 3.65625  
 Round 2 and Node A has defaulted due to nominal obligations 11.0 being greater than cash flow 6.65625. Default loss for Node A is 3.328125

- Total Dollar Payment Vector for round 2 and Node A  
 Total Dollar Payment Vector for round 2 and Node A updated with value 3.328125

\*\*\*Node B\*\*\*

- Total payments in for Node B  
 Relative Payment in to Node B from Node A is 0.18181818181818182  
 Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0  
 Relative Payment in to Node B from Node C is 0.25  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node B is 3.0  
  
 - Liabilities for Node B  
 Liability of Node B to Node A (i.e.  $P_{10}$ ) is 7.0  
 Liability of Node B to Node C (i.e.  $P_{12}$ ) is 9.0  
 Total nominal liabilities for Node B (i.e.  $p_{\text{bar}_2}$ ) is 16.0  
 Payment out is  $\min[\text{payment out}, \text{total cash flow}]$  i.e.  $\min[16.0, 3.0]$   
 Relative Payment in to Node B from Node A is 0.18181818181818182  
 Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0  
 Relative Payment in to Node B from Node C is 0.25  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node B is 3.0  
 Relative Payment in to Node B from Node A is 0.18181818181818182  
 Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0  
 Relative Payment in to Node B from Node C is 0.25  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node B is 3.0  
 Relative Payment in to Node B from Node A is 0.18181818181818182  
 Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0  
 Relative Payment in to Node B from Node C is 0.25  
 Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
 Total payments in to Node B is 3.0  
 Round 2 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 3.0. Default loss for Node B is 1.5

- Total Dollar Payment Vector for round 2 and Node B



Total Dollar Payment Vector for round 2 and Node B updated with value 1.5

\*\*\*Node C\*\*\*

- Total payments in for Node C

Relative Payment in to Node C from Node A is 0.8181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0

Relative Payment in to Node C from Node B is 0.5625

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Total payments in to Node C is 9.84375

- Liabilities for Node C

Liability of Node C to Node A (i.e.  $P_{20}$ ) is 3.0

Liability of Node C to Node B (i.e.  $P_{21}$ ) is 1.0

Total nominal liabilities for Node C (i.e.  $p_{\text{bar}_3}$ ) is 4.0

Payment out is  $\min[\text{payment out, total cash flow}]$  i.e.  $\min[4.0, 13.84375]$

- Total Dollar Payment Vector for round 2 and Node C

Total Dollar Payment Vector for round 2 and Node C updated with value 4.0

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TOTAL PAYMENT VECTOR

Total payment by Node A (i.e.  $p_1$ ): 3.328125

Total payment by Node B (i.e.  $p_2$ ): 1.5

Total payment by Node C (i.e.  $p_3$ ): 4.0

[3.328125, 1.5, 4.0]  
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UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

- Equity Vector for round 2 and Node A

Equity Vector for round 2 and Node A updated with value 6.65625

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 11.0

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node B is 3.0

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

- Equity Vector for round 2 and Node B  
Equity Vector for round 2 and Node B updated with value 3.0  
Relative Payment in to Node C from Node A is 0.8181818181818182  
Total dollar payment by Node A (i.e. p\_1) is 11.0  
Relative Payment in to Node C from Node B is 0.5625  
Total dollar payment by Node B (i.e. p\_2) is 1.5  
Total payments in to Node C is 9.84375  
Total dollar payment by Node C (i.e. p\_3) is 4.0

- Equity Vector for round 2 and Node C  
Equity Vector for round 2 and Node C updated with value 13.84375

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#### EQUITY FOR EACH NODE

Equity for Node A: 6.65625  
Equity for Node B: 3.0  
Equity for Node C: 13.84375  
[6.65625, 3.0, 13.84375]

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#### ROUND 2 DEFAULTERS

Node A has defaulted in round 2  
{'A': True, 'B': True, 'C': False}  
There are defaulters in this round (i.e. round 2), algorithm will proceed for another round.

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#### END OF ROUND 2

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#### START OF ROUND 3

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#### TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p\_bar\_i...  
Total nominal obligation for Node A (i.e. p\_bar\_1): 11.0  
Total nominal obligation for Node B (i.e. p\_bar\_2): 16.0  
Total nominal obligation for Node C (i.e. p\_bar\_3): 4.0

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TOTAL PAYMENT MADE PER NODE

i.e.  $\min[\text{nominal obligations, cashflow (payments in + exogenous cash flow)}]$  for each node...

\*\*\*Node A\*\*\*

- Total payments in for Node A

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

- Liabilities for Node A

Liability of Node A to Node B (i.e.  $P_{01}$ ) is 2.0

Liability of Node A to Node C (i.e.  $P_{02}$ ) is 9.0

Total nominal liabilities for Node A (i.e.  $p_{\text{bar}_1}$ ) is 11.0

Payment out is  $\min[\text{payment out, total cash flow}]$  i.e.  $\min[11.0, 6.65625]$

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node A is 3.65625

Round 3 and Node A has defaulted due to nominal obligations 11.0 being greater than cash flow 6.65625. Default loss for Node A is 3.328125

- Total Dollar Payment Vector for round 3 and Node A

Total Dollar Payment Vector for round 3 and Node A updated with value 3.328125

\*\*\*Node B\*\*\*

- Total payments in for Node B

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Total payments in to Node B is 1.6051136363636362

- Liabilities for Node B

Liability of Node B to Node A (i.e.  $P_{10}$ ) is 7.0

Liability of Node B to Node C (i.e.  $P_{12}$ ) is 9.0

Total nominal liabilities for Node B (i.e.  $p_{\text{bar}_2}$ ) is 16.0

Payment out is  $\min[\text{payment out}, \text{total cash flow}]$  i.e.  $\min[16.0, 1.6051136363636362]$

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node B is 1.6051136363636362

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node B is 1.6051136363636362

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0

Total payments in to Node B is 1.6051136363636362

Round 3 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 1.6051136363636362. Default loss for Node B is 0.8025568181818181

- Total Dollar Payment Vector for round 3 and Node B

Total Dollar Payment Vector for round 3 and Node B updated with value 0.8025568181818181

\*\*\*Node C\*\*\*

- Total payments in for Node C

Relative Payment in to Node C from Node A is 0.8181818181818182

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125

Relative Payment in to Node C from Node B is 0.5625

Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5

Total payments in to Node C is 3.5667613636363638

- Liabilities for Node C

Liability of Node C to Node A (i.e.  $P_{20}$ ) is 3.0

Liability of Node C to Node B (i.e.  $P_{21}$ ) is 1.0

Total nominal liabilities for Node C (i.e.  $p_{\text{bar}_3}$ ) is 4.0

Payment out is  $\min[\text{payment out}, \text{total cash flow}]$  i.e.  $\min[4.0, 7.566761363636363]$

- Total Dollar Payment Vector for round 3 and Node C

Total Dollar Payment Vector for round 3 and Node C updated with value 4.0

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TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p\_1): 3.328125  
Total payment by Node B (i.e. p\_2): 0.8025568181818181  
Total payment by Node C (i.e. p\_3): 4.0  
[3.328125, 0.8025568181818181, 4.0]

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UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE

Relative Payment in to Node A from Node B is 0.4375  
Total dollar payment by Node B (i.e. p\_2) is 1.5  
Relative Payment in to Node A from Node C is 0.75  
Total dollar payment by Node C (i.e. p\_3) is 4.0  
Total payments in to Node A is 3.65625  
Total dollar payment by Node A (i.e. p\_1) is 3.328125

- Equity Vector for round 3 and Node A

Equity Vector for round 3 and Node A updated with value 6.65625  
Relative Payment in to Node B from Node A is 0.18181818181818182  
Total dollar payment by Node A (i.e. p\_1) is 3.328125  
Relative Payment in to Node B from Node C is 0.25  
Total dollar payment by Node C (i.e. p\_3) is 4.0  
Total payments in to Node B is 1.6051136363636362  
Total dollar payment by Node B (i.e. p\_2) is 0.8025568181818181

- Equity Vector for round 3 and Node B

Equity Vector for round 3 and Node B updated with value 1.6051136363636362  
Relative Payment in to Node C from Node A is 0.8181818181818182  
Total dollar payment by Node A (i.e. p\_1) is 3.328125  
Relative Payment in to Node C from Node B is 0.5625  
Total dollar payment by Node B (i.e. p\_2) is 1.5  
Total payments in to Node C is 3.5667613636363638  
Total dollar payment by Node C (i.e. p\_3) is 4.0

- Equity Vector for round 3 and Node C

Equity Vector for round 3 and Node C updated with value 7.566761363636363

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EQUITY FOR EACH NODE

Equity for Node A: 6.65625

Equity for Node B: 1.6051136363636362  
Equity for Node C: 7.566761363636363  
[6.65625, 1.6051136363636362, 7.566761363636363]

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### ROUND 3 DEFAULTERS

Node A has defaulted in round 3  
{'A': True, 'B': True, 'C': False}

There are defaulters from earlier rounds but no new defaulters in the current round, algorithm will not proceed for another round.

\*\*\*Checking limited liability and absolute priority for Node A\*\*\*

Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125  
Relative Payment in to Node A from Node B is 0.4375  
Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5  
Relative Payment in to Node A from Node C is 0.75  
Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Total payments in to Node A is 3.65625

- Limited liability is met. Node A made a payment of 3.328125 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 6.65625.

Total nominal obligation for Node A (i.e.  $p_{\text{bar}_1}$ ): 11.0  
Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125  
Relative Payment in to Node A from Node B is 0.4375  
Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5  
Relative Payment in to Node A from Node C is 0.75  
Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Total payments in to Node A is 3.65625

-Checking absolute priority for Node A in round 3. Nominal obligations is 11.0 and Dollar payments is 3.328125  
Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node A  
All value i.e. cash flow available to Node A is 6.65625  
-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 11.0, Dollar payment was 3.328125 and Total cash flow was 6.65625

Node A in round 3 passes candidate clearing vector payment entry checks.

\*\*\*Checking limited liability and absolute priority for Node B\*\*\*

Total dollar payment by Node B (i.e.  $p_2$ ) is 0.8025568181818181  
Relative Payment in to Node B from Node A is 0.18181818181818182  
Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125  
Relative Payment in to Node B from Node C is 0.25  
Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Total payments in to Node B is 1.6051136363636362

- Limited liability is met. Node B made a payment of 0.8025568181818181 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 1.6051136363636362.

Total nominal obligation for Node B (i.e.  $p_{\text{bar}_2}$ ): 16.0  
Total dollar payment by Node B (i.e.  $p_2$ ) is 0.8025568181818181  
Relative Payment in to Node B from Node A is 0.18181818181818182  
Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125  
Relative Payment in to Node B from Node C is 0.25  
Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Total payments in to Node B is 1.6051136363636362

-Checking absolute priority for Node B in round 3. Nominal obligations is 16.0 and Dollar payments is 0.8025568181818181  
Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node B  
All value i.e. cash flow available to Node B is 1.6051136363636362  
-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 16.0, Dollar payment was 0.8025568181818181 and Total cash flow was 1.6051136363636362

Node B in round 3 passes candidate clearing vector payment entry checks.

\*\*\*Checking limited liability and absolute priority for Node C\*\*\*

Total dollar payment by Node C (i.e.  $p_3$ ) is 4.0  
Relative Payment in to Node C from Node A is 0.8181818181818182  
Total dollar payment by Node A (i.e.  $p_1$ ) is 3.328125  
Relative Payment in to Node C from Node B is 0.5625  
Total dollar payment by Node B (i.e.  $p_2$ ) is 1.5  
Total payments in to Node C is 3.5667613636363638

- Limited liability is met. Node C made a payment of 4.0 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 7.566761363636363.  
Total nominal obligation for Node C (i.e.  $p_{\text{bar}_3}$ ): 4.0

Total dollar payment by Node C (i.e. p\_3) is 4.0  
Relative Payment in to Node C from Node A is 0.8181818181818182  
Total dollar payment by Node A (i.e. p\_1) is 3.328125  
Relative Payment in to Node C from Node B is 0.5625  
Total dollar payment by Node B (i.e. p\_2) is 1.5  
Total payments in to Node C is 3.5667613636363638

-Checking absolute priority for Node C in round 3. Nominal obligations is 4.0  
and Dollar payments is 4.0

-Absolute priority is satisfied for Node C

- Absolute priority is met by Node C in round 3 i.e. either obligations are paid  
in full or all available cash flow (i.e. sum of the payments received by the  
node plus the exogenous operating cash flow) is paid to creditors. Nominal  
obligations were 4.0, Dollar payment was 4.0 and Total cash flow was  
7.566761363636363

Node C in round 3 passes candidate clearing vector payment entry checks.

---

#### CLEARING\_PAYMENT\_VECTOR

Clearing payment vector found in round 3.  
[3.328125, 0.8025568181818181, 4.0]  
Node A pays: 3.328125  
Node B pays: 0.8025568181818181  
Node C pays: 4.0  
Default loss incurred by Node A is: 3.328125  
Default loss incurred by Node B is: 0.8025568181818181  
Default loss incurred by Node C is: 0

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#### END OF ROUND 3

- Systemic Risk: Node B has defaulted in round 1. The number of prior default  
waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).  
- Systemic Risk: Node A has defaulted in round 2. The number of prior default  
waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e.  
['B']).  
- Systemic Risk: Node C has not defaulted after 3 rounds. There are 3 nodes in  
the system (2 of which have defaulted i.e. ['B', 'A']).

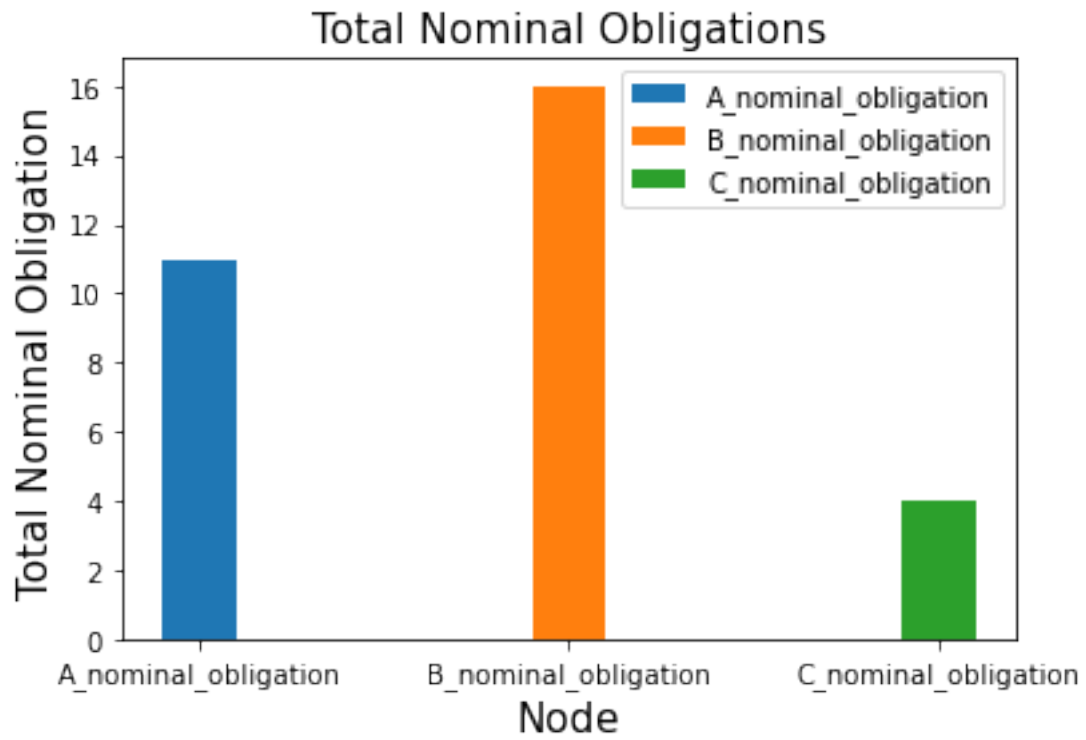
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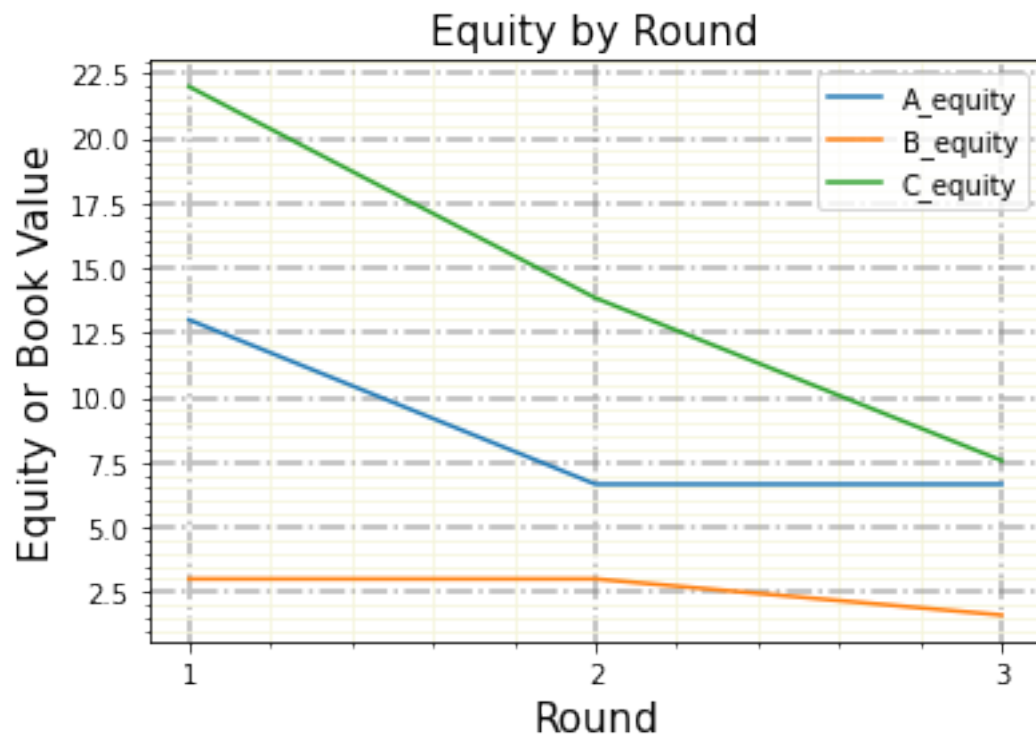
Scenario 4 - Firm B defaults in first round, Firm A in second round, algorithm

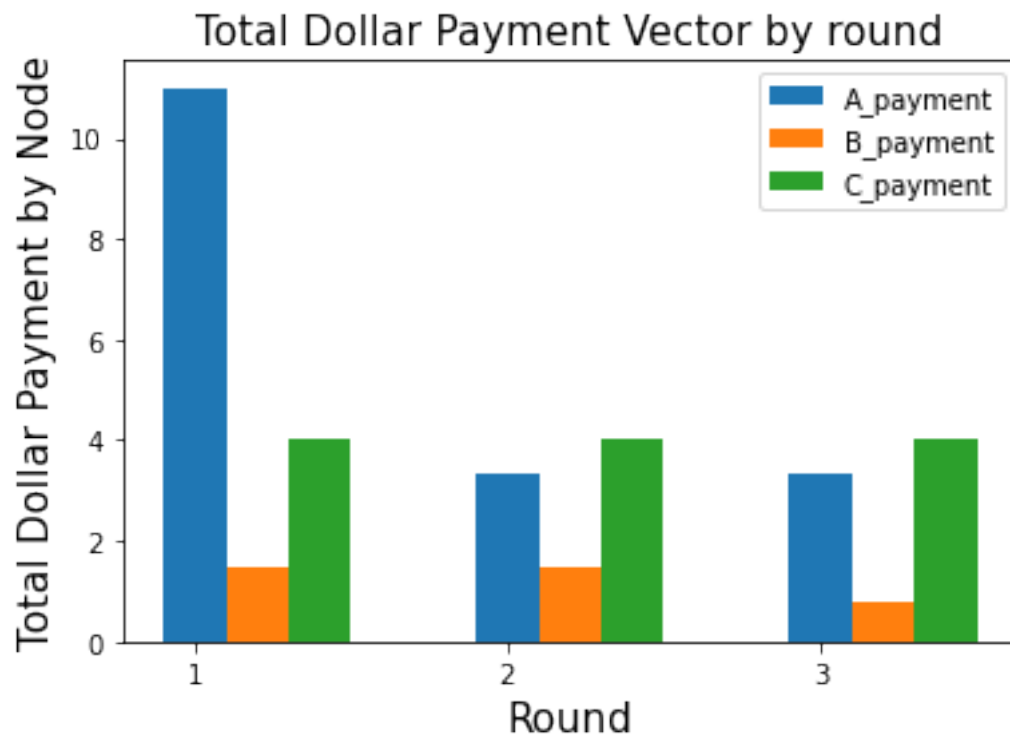


```
terminates round 3, MODE == 'MANUAL', NUM_AGENTS = 3, NOMINAL_LIABILITY_MATRIX =  
np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING_CASH_FLOW_VECTOR = [3, 0, 4],  
ALPHA = 0.5, BETA = 0.5
```

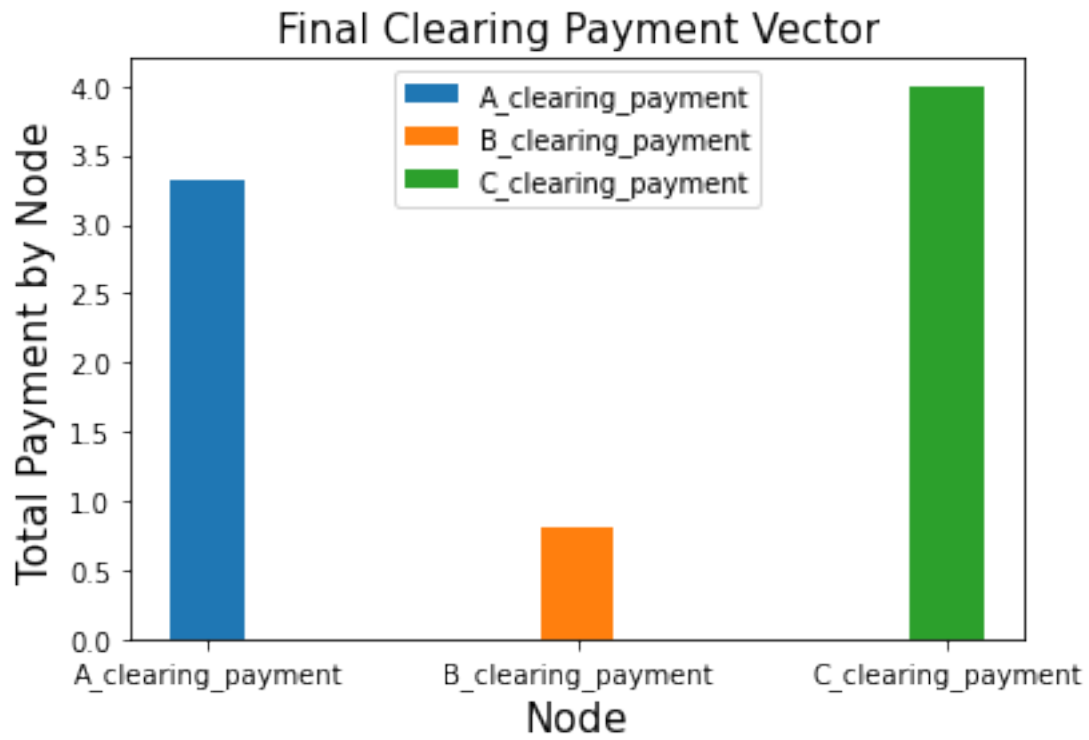
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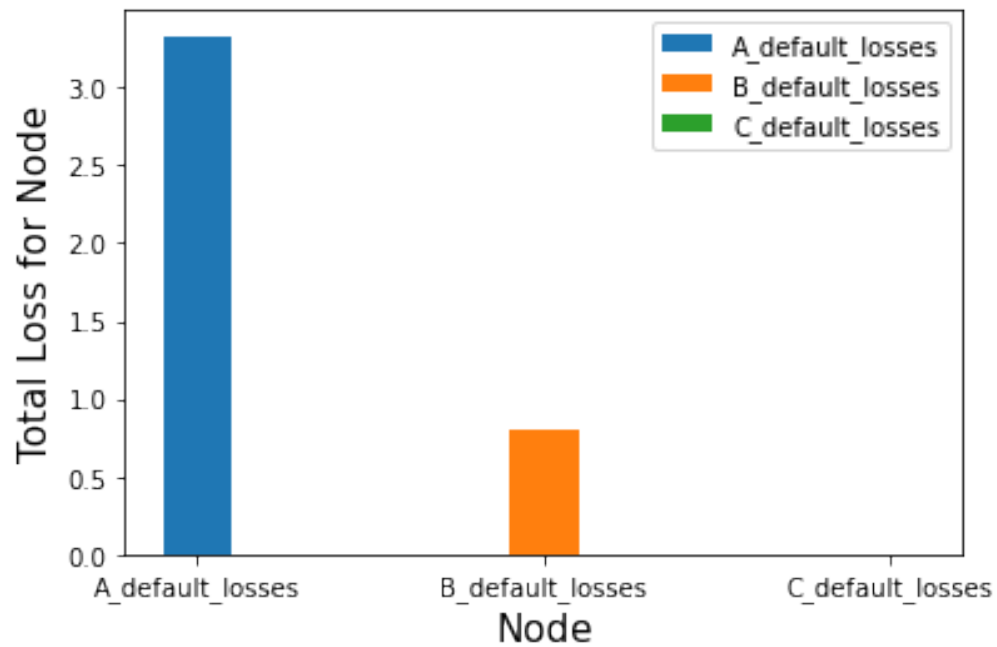


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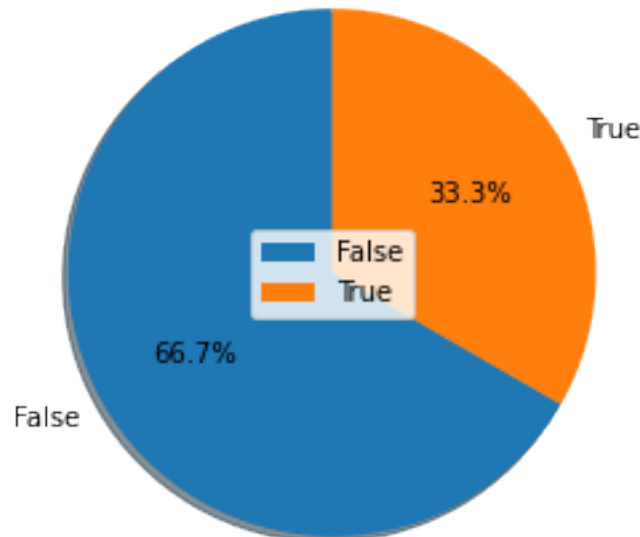
Actual Default Losses Realised ALPHA = 0.5 & BETA = 0.5



ALPHA and BETA are the fraction of exogenous assets (outside financial network) and endogenous assets (inside financial network i.e. interbank obligations) that are realised on liquidation in the event of default. The two fractions may conceivably be different; we would typically expect that ALPHA would be low, because the bank would be having to sell off its loan portfolio, probably at a knock-down price or fire sale. In contrast, BETA might be much closer to 1, because an obligation from a solvent bank would probably be paid in full (though perhaps with some negotiated discount to compensate for the inconvenience of early repayment).

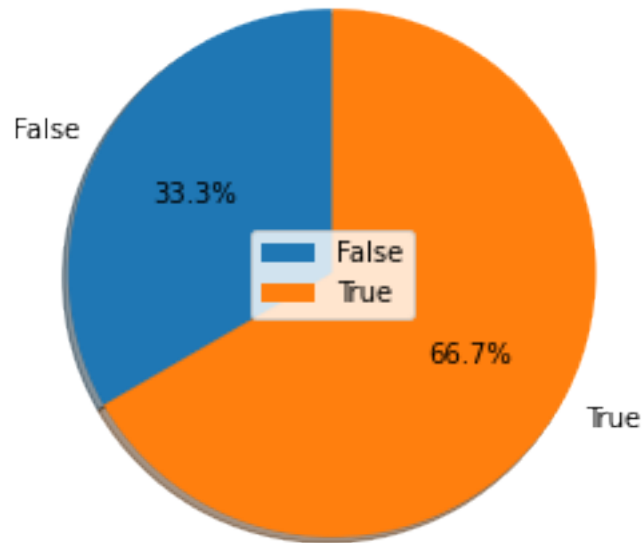
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Percentage of Defaulters After Round 1



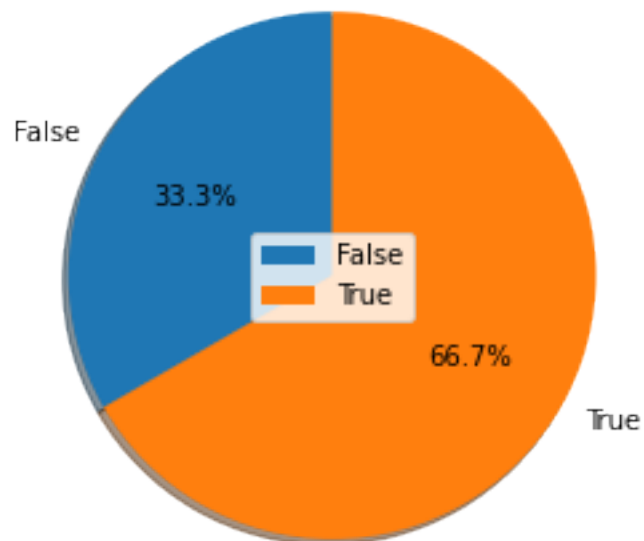
Node A has NOT defaulted in round 1  
Node B has defaulted in round 1  
Node C has NOT defaulted in round 1

## Percentage of Defaulters After Round 2



Node A has defaulted in round 2  
Node B has defaulted in round 2  
Node C has NOT defaulted in round 2

## Percentage of Defaulters After Round 3



Node A has defaulted in round 3

Node B has defaulted in round 3  
Node C has NOT defaulted in round 3

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