eisenberg-noe-2001-debt-model-with-default-costs

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Data Privacy and Provenance
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SECTION A

Description
We model 7 scenarios for which we hold the number of agents (n=3) and nominal liabilities (see below) constant whilst varying shock size (which determines exogenous assets i.e. operating cash flow) and the alpha and beta parameters (which determine default costs and losses).

The scenarios are described below:

- SCENARIO 3 - Debt Model with 3 nodes (A, B, C), ALPHA = 1, BETA = 1
    - SHOCK = 2 with GREATER CLEARING VECTOR as output
    - SHOCK = 4 with GREATER CLEARING VECTOR as output
    - SHOCK = 8 with GREATER CLEARING VECTOR as output
- SCENARIO 5 - Debt Model with 3 nodes (A, B, C), ALPHA = 0.1, BETA = 0.9
    - SHOCK = 2 with GREATER CLEARING VECTOR as output
    - SHOCK = 4 with GREATER CLEARING VECTOR as output
    - SHOCK = 8 with GREATER CLEARING VECTOR as output
    - SHOCK = 8 with LOWER CLEARING VECTOR as output

Alpha and Beta are the fraction of exogenous assets (outside financial network) and endogenous assets (inside financial network i.e., interbank obligations) that are realised on liquidation in the event of default. The two fractions may be different; but we generally expect that Alpha would be low, because a defaulting bank would be having to sell off its loan portfolio, probably at a knock-down price or fire sale. In contrast, Beta might be much closer to 1, because an obligation from a solvent bank would probably be paid in full (though perhaps with some negotiated discount to compensate for the inconvenience of early repayment).

The algorithm to determine the greater clearing vector initially assumes that interbank payments received by each node are the same as what they expect to receive (i.e. endogenous claims), which is then added to the exogenous assets, after which the outward interbank payments are calculated. In contrast, the algorithm to determine the lower clearing vector initially assumes that only exogenous assets are on hand and no interbank payments are received, after which outward interbank payments are calculated.

In general, we find that the larger the shock value, the less the equity and clearing payments and the less the default loss.

Parameters
Constant params

NUM_AGENTS = 3
OPERATING_CASHFLOW_BEFORE_SHOCK = [11, 8, 12]
e.g. Resulting exogenous assets when SHOCK = 8 is
OPERATING_CASH_FLOW_VECTOR = [3, 0, 4]

```
NOMINAL LIABILITY MATRIX Data Frame
i.e. what node i expects to pay node j…

   A  B  C
A  0  2  9
B  7  0  9
C  3  1  0


Nominal liabilities for each node:
Liability of Node A to Node B is 2
Liability of Node A to Node C is 9
Liability of Node B to Node A is 7
Liability of Node B to Node C is 9
Liability of Node C to Node A is 3
Liability of Node C to Node B is 1
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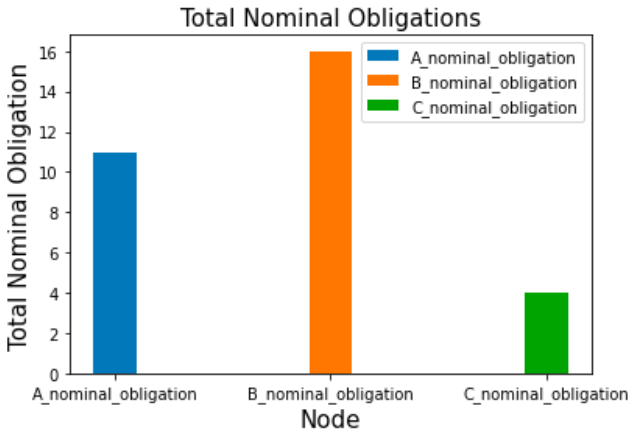
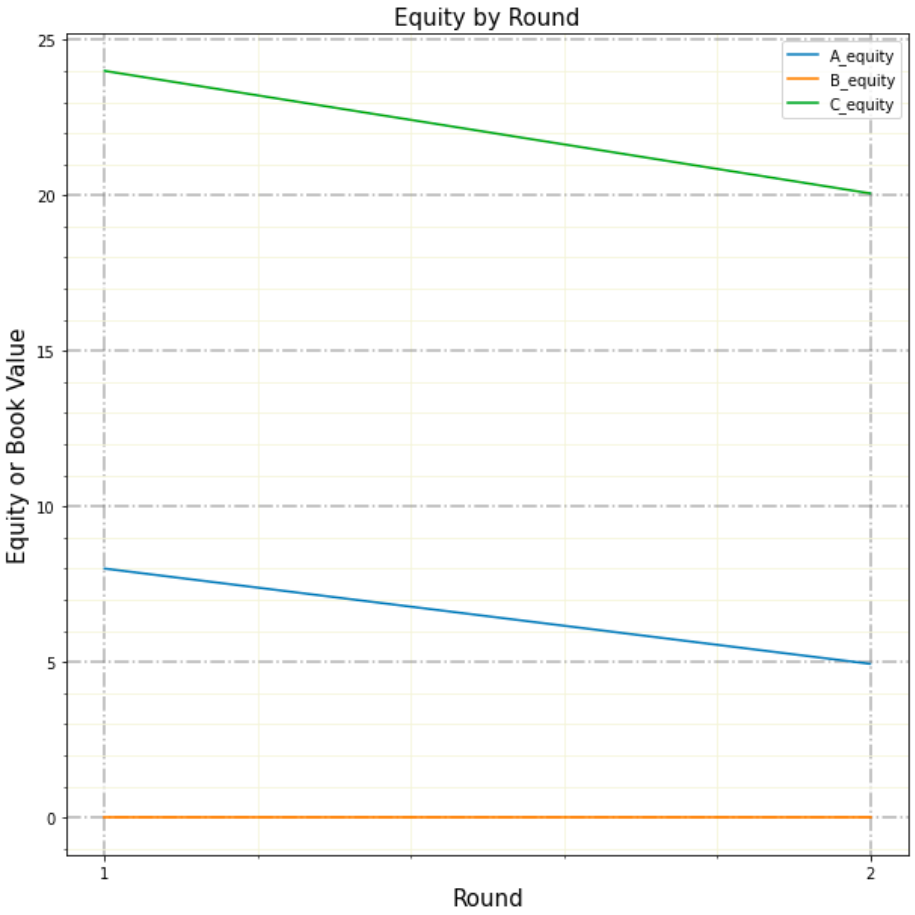
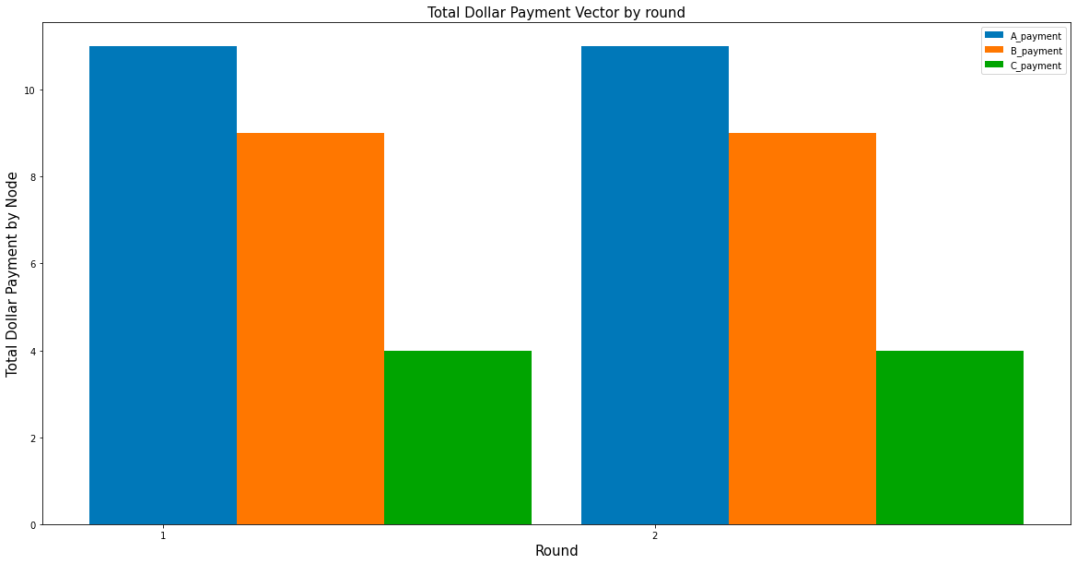


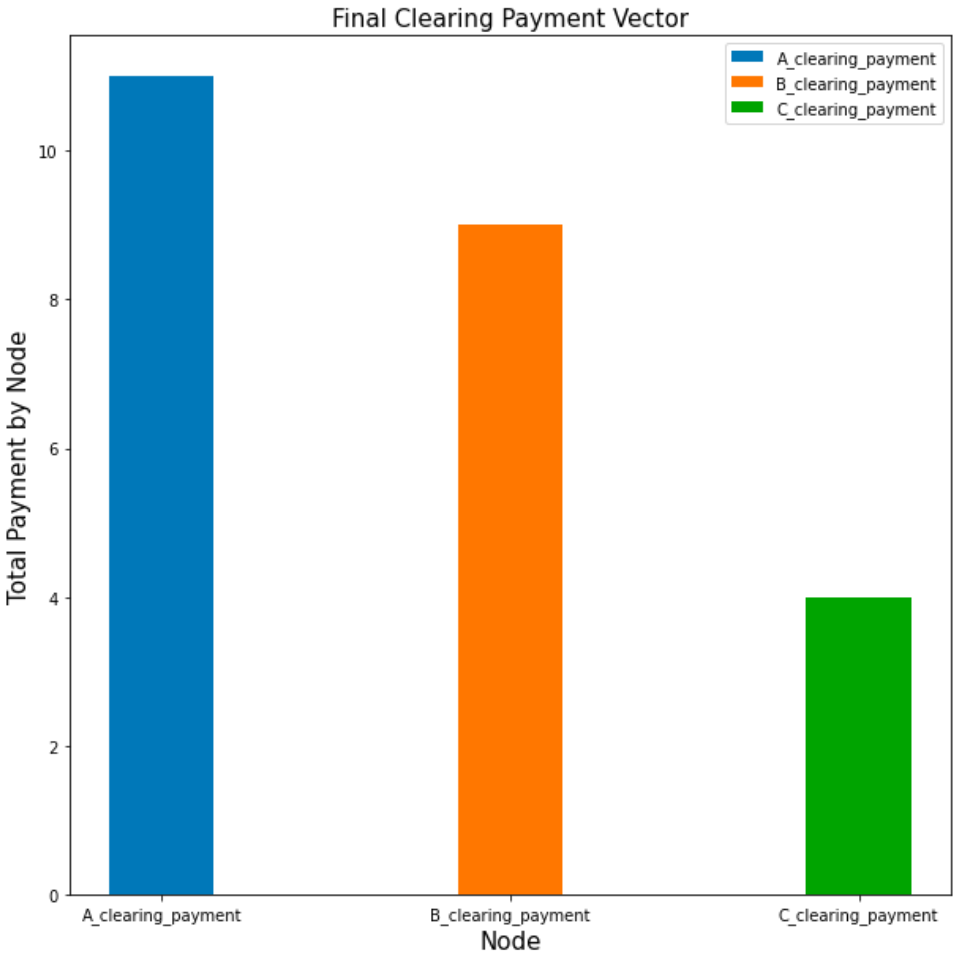
Alpha and Beta params
We test Alpha & Beta values of (1,1) and (0.1, 0.9) respectively. When Alpha=Beta=1 we have the equivalent of Eisenberg and Noe with no default costs.

Scenarios
SCENARIO 3 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 1 and BETA = 1, SHOCK = 2, GREATER CLEARING VECTOR
Description - Firm B defaults in first round, algorithm terminates round 2.

- ALPHA=BETA=1 (no default costs / losses).
- Clearing payment vector found in round 2 = [11.0, 9.0, 4.0]
  - Node A pays: 11.0
  - Node B pays: 9.0
  - Node C pays: 4.0
  - No default losses as Alpha and Beta are set to 1 which assumes full recovery i.e. equivalent of Eisenberg and Noe with no default costs.
- Systemic Risk
  - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
  - Node A has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
  - Node C has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).


Equity by Round
A_equity
B_equity
C_equity
Equity or Book Value
Round
25
20
15
10
5
0
1
2


Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round
10
8
6
4
2
0
1
2

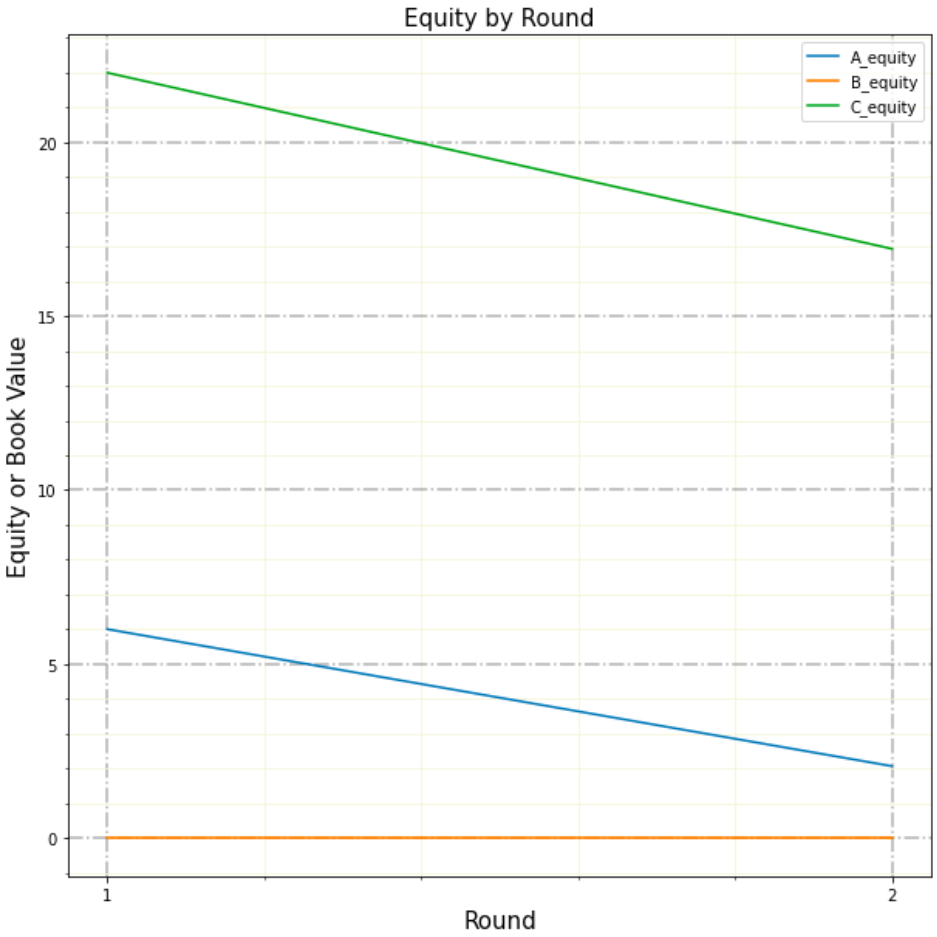


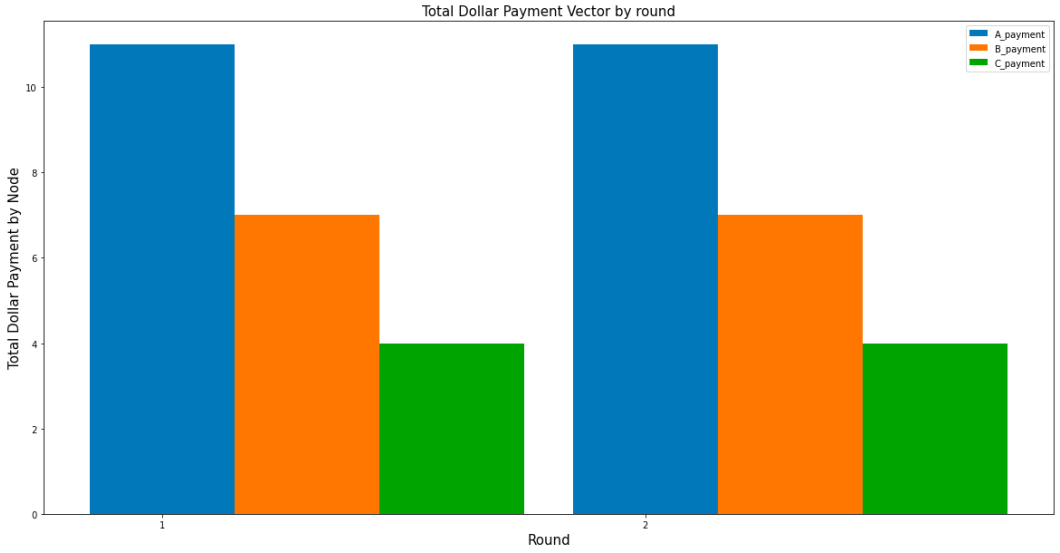
Comment – Equity (assets – liabilities) decreases over time due to default by Firm B.

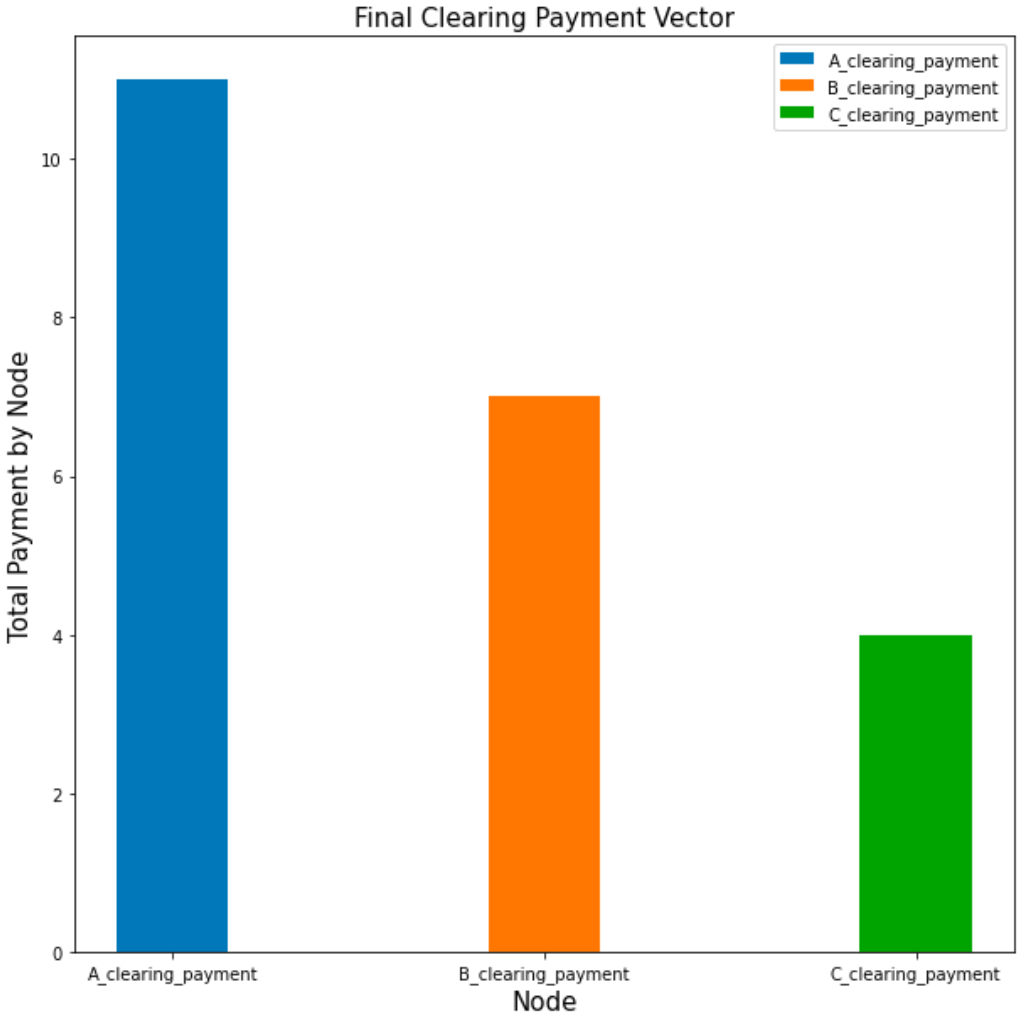
SCENARIO 3 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 1 and BETA = 1, SHOCK = 4, GREATER CLEARING VECTOR

Description - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3.

- ALPHA=BETA=1 (no default costs / losses).
- Clearing payment vector found in round 2 = [11, 7, 4]
    - Node A pays: 11
    - Node B pays: 7
    - Node C pays: 4
- No default losses as Alpha and Beta are set to 1 which assumes full recovery i.e. equivalent of Eisenberg and Noe with no default costs.
- Systemic Risk
    - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node A has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
    - Node C has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).


Equity by Round
A_equity
B_equity
C_equity
Equity or Book Value
Round
0
5
10
15
20
1
2


Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round
0
2
4
6
8
10
1
2



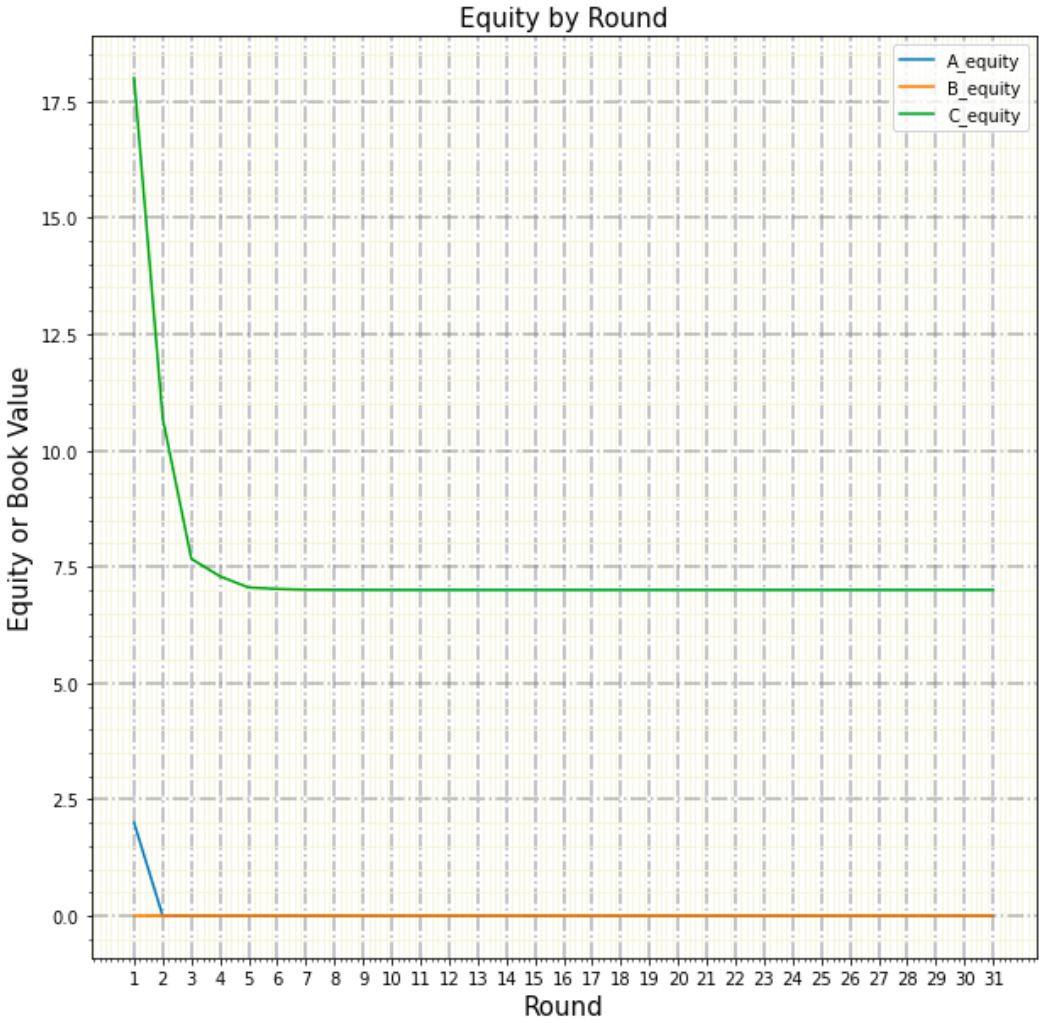
Comment – Equity (assets – liabilities) decreases over time due to default by Firm B. Overall equity for non-defaulting firms is reduced due to the larger shock value (4).The larger shock value also sees the clearing payment by defaulting Firm B reducing in value compared to when the shock = 2.

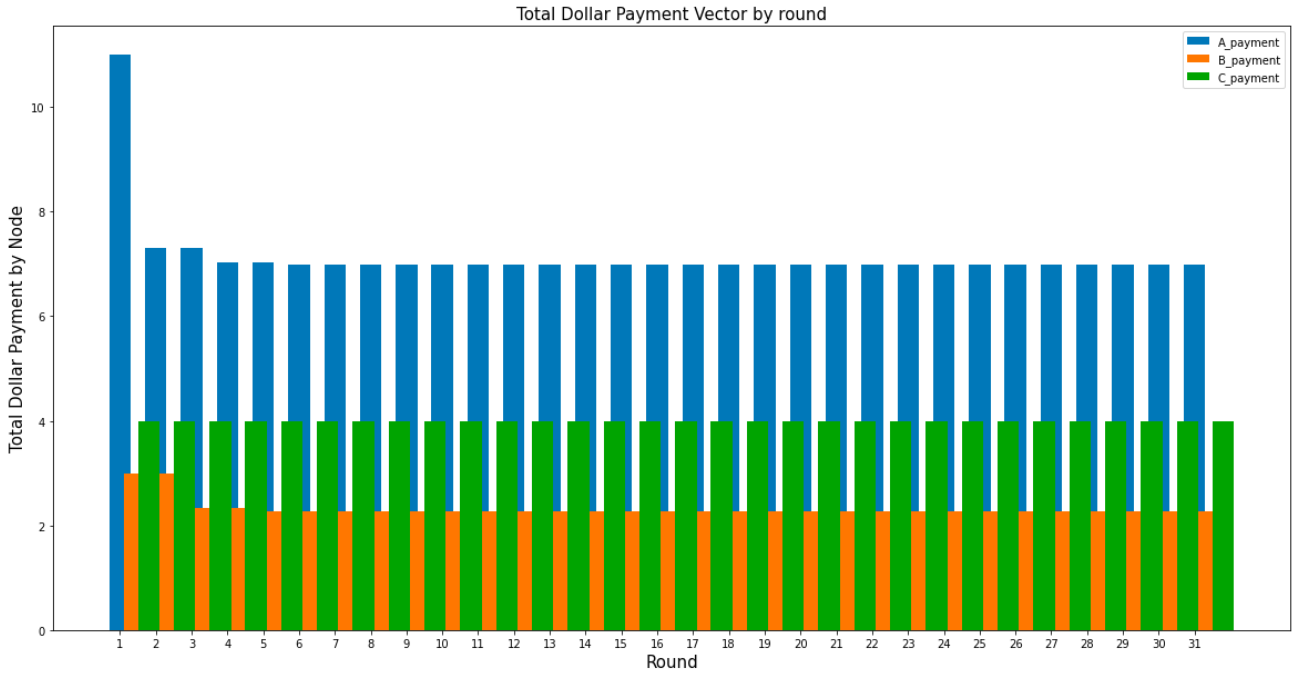
SCENARIO 3 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 1 and BETA = 1, SHOCK = 8, GREATER CLEARING VECTOR

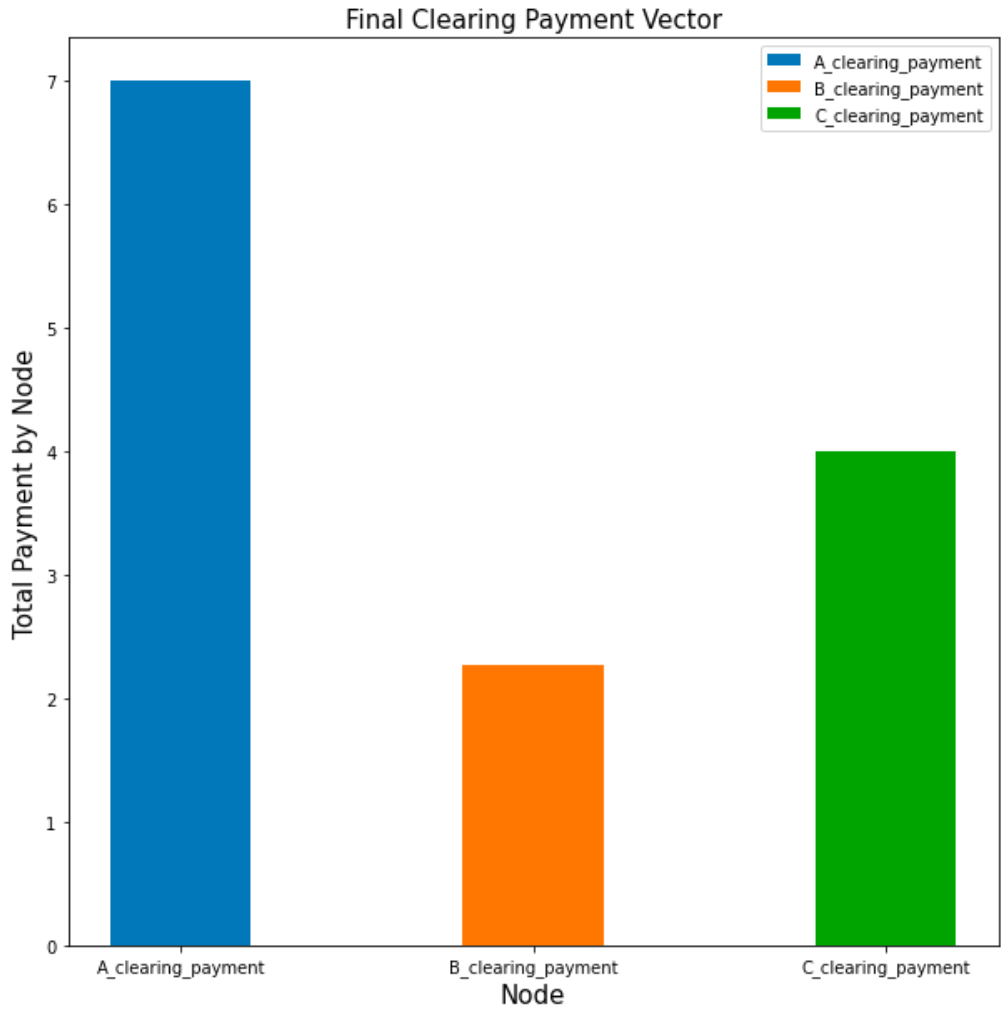
Description - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3.

- ALPHA=BETA=1 (no default costs / losses).
- Clearing payment vector found in round 31 = [6.993827160493828, 2.2716049382716053, 4.0]
    - Node A pays: 6.993827160493828
    - Node B pays: 2.2716049382716053
    - Node C pays: 4.0
    - No default losses as Alpha and Beta are set to 1 which assumes full recovery i.e. equivalent of Eisenberg and Noe with no default costs.
- Systemic Risk
    - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).

- Node A has defaulted in round 2. The number of prior default waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
- Node C has not defaulted after 31 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).




Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round

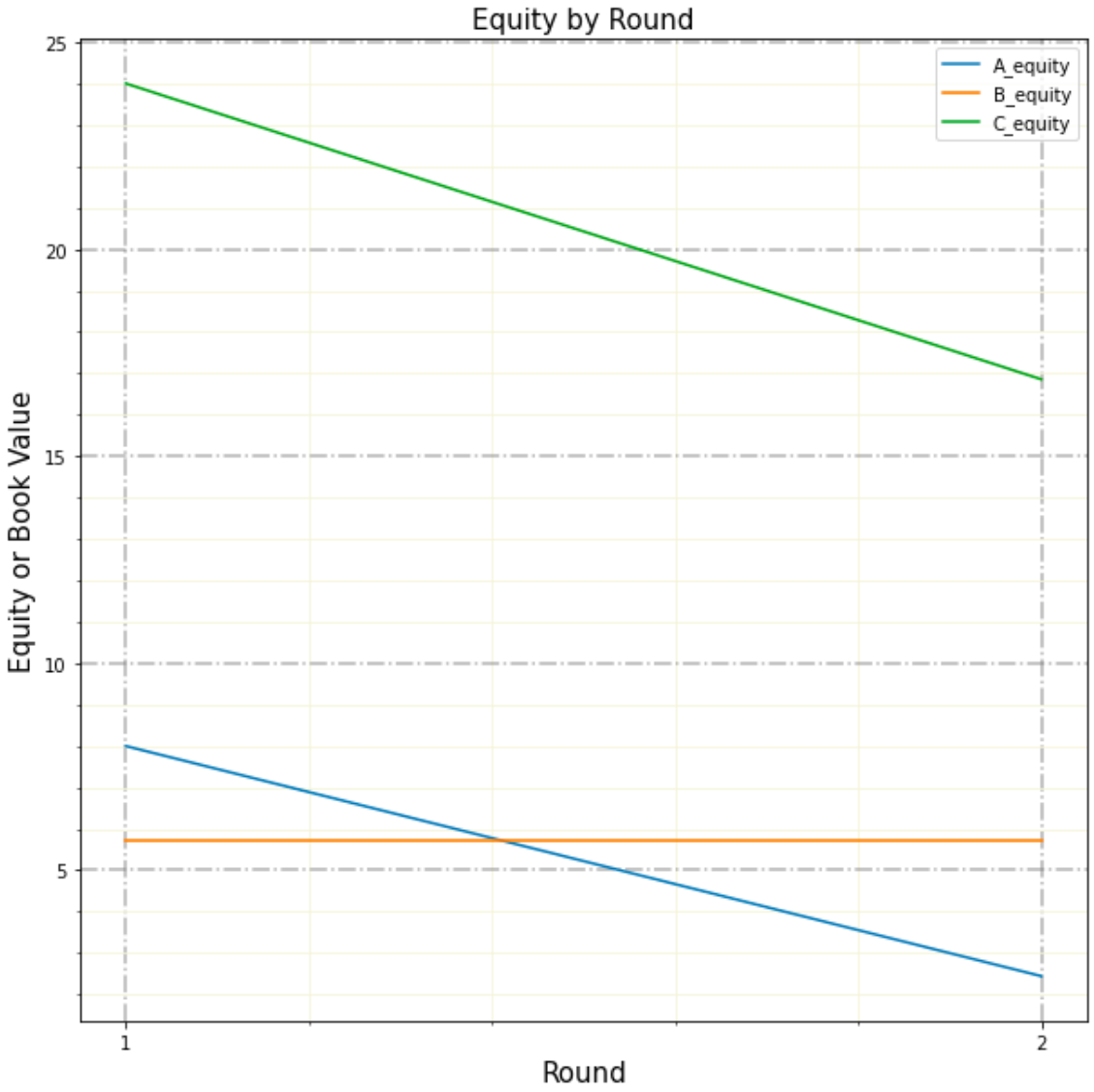

Final Clearing Payment Vector
A_clearing_payment
B_clearing_payment
C_clearing_payment
Total Payment by Node
Node

Comment – The larger shock value (8) drastically reduces equity in the system and causes more defaults i.e. Firm A which did not default previously (shock = 4) has now defaulted.

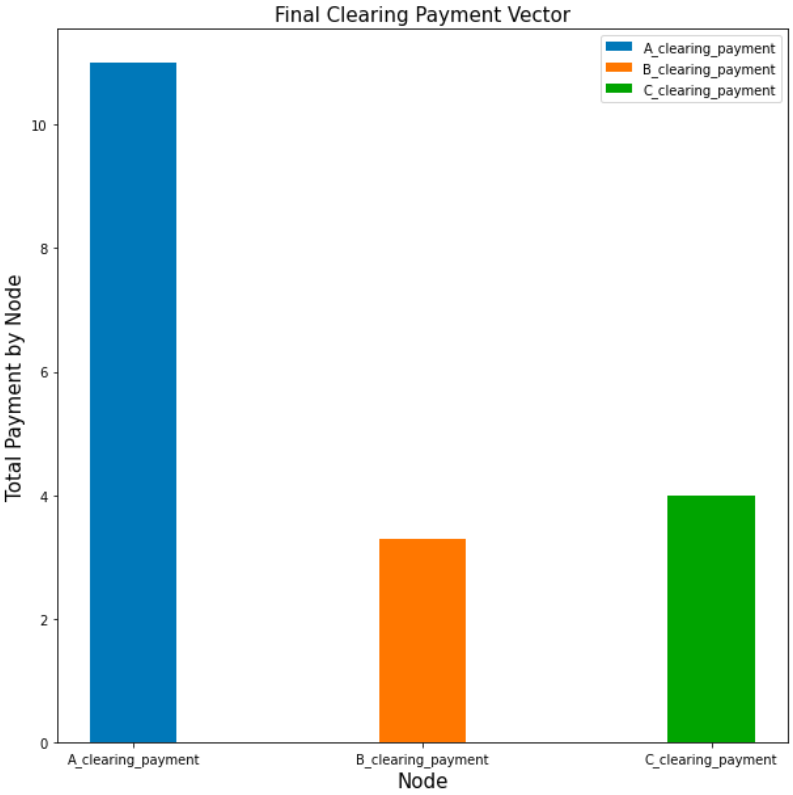
SCENARIO 5 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.1 and BETA = 0.9, SHOCK = 2, GREATER CLEARING VECTOR

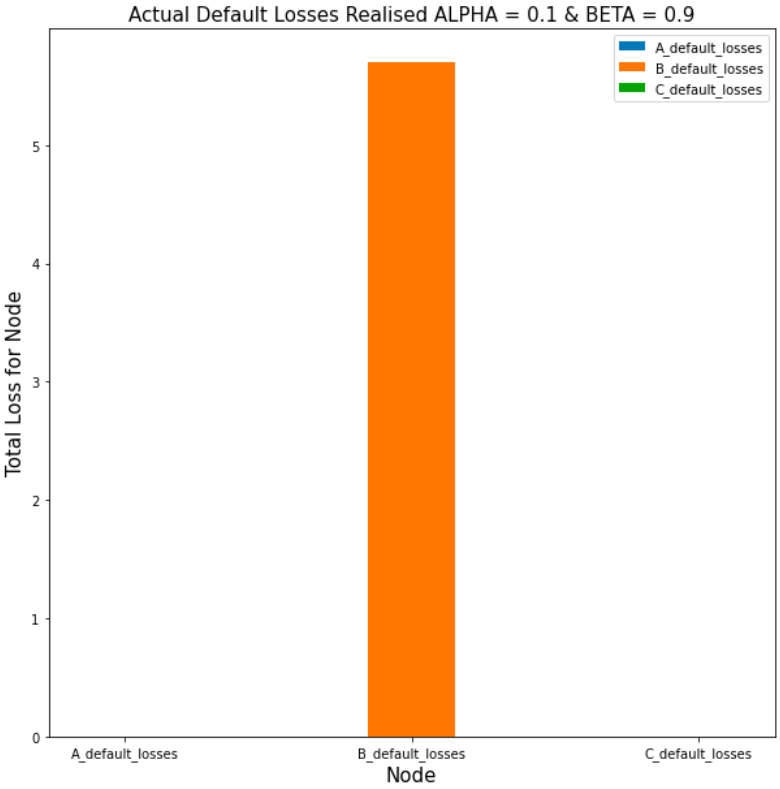
Scenario 5 - Firm B defaults in first round, algorithm terminates round 2.

- ALPHA = 0.1, BETA = 0.9
- Clearing payment vector found in round 2 = [11.0, 3.3000000000000003, 4.0]
    - Node A pays: 11.0
    - Node B pays: 3.3000000000000003
    - Node C pays: 4.0
    - Default loss incurred by Node A is: 0
    - Default loss incurred by Node B is: 5.699999999999999
    - Default loss incurred by Node C is: 0
- Systemic Risk:
    - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node A has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
    - Node C has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).


Equity by Round
A_equity
B_equity
C_equity
Equity or Book Value
Round
25
20
15
10
5
1
2


Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round
0
2
4
6
8
10
1
2



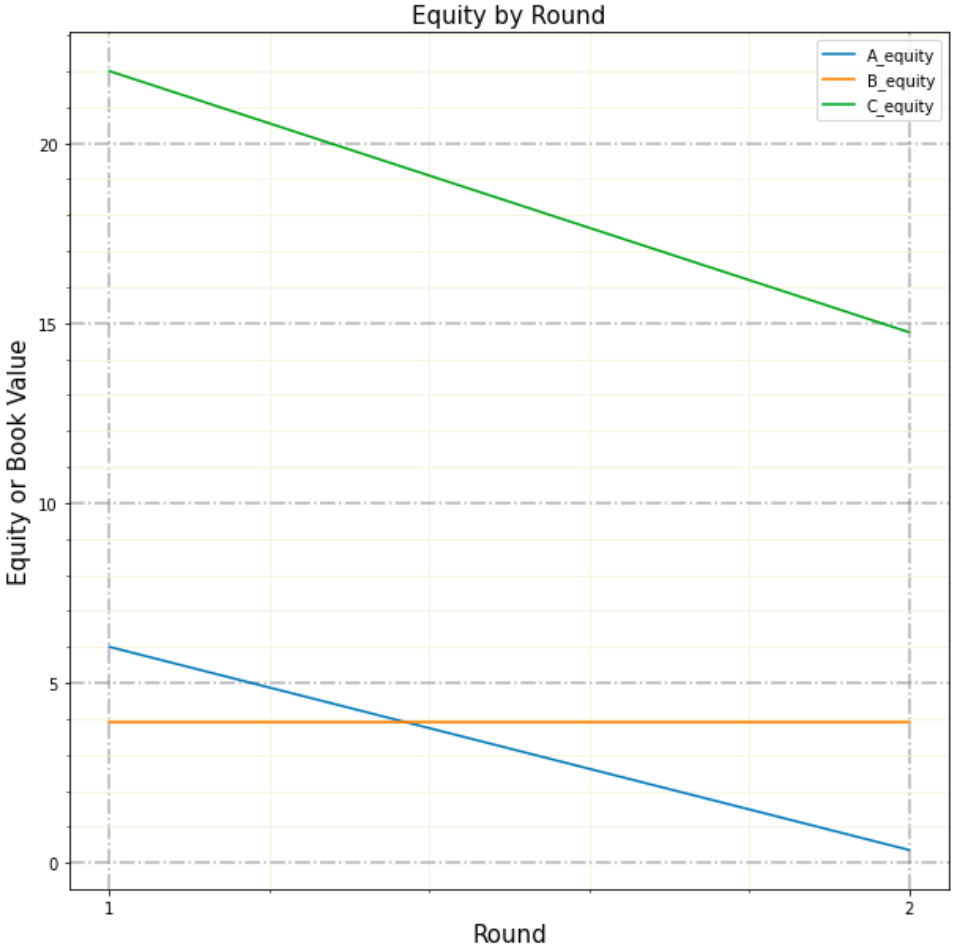


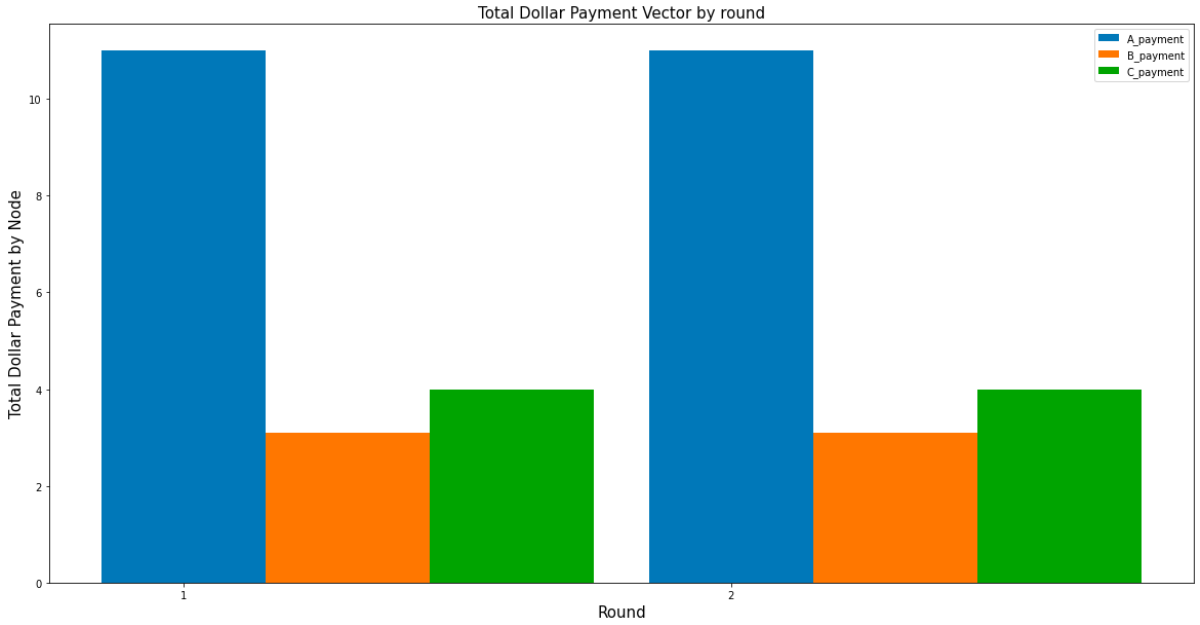
Comment – Equity graphs for models with default costs do not seem to be correct for defaulting firms.

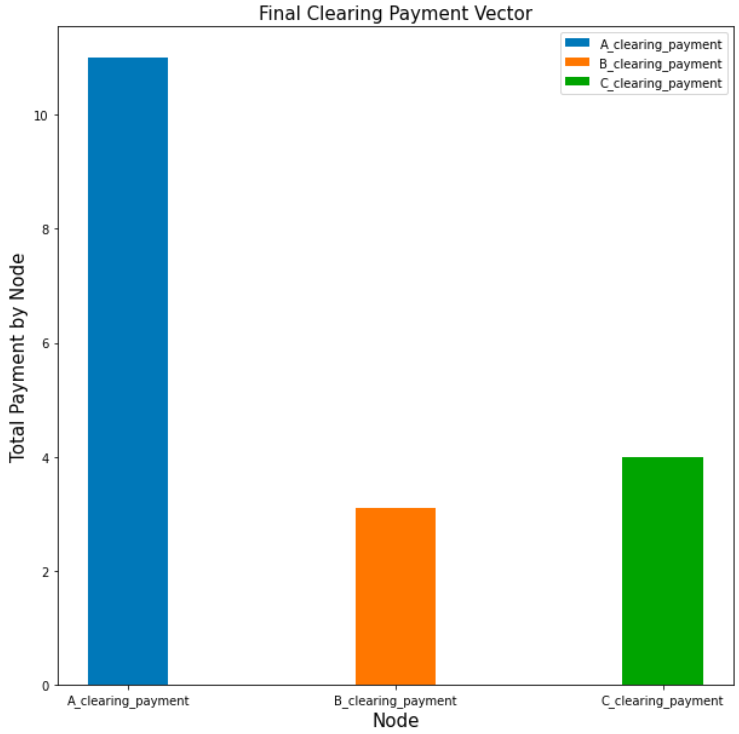
SCENARIO 5 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.1 and BETA = 0.9, SHOCK = 4, GREATER CLEARING VECTOR

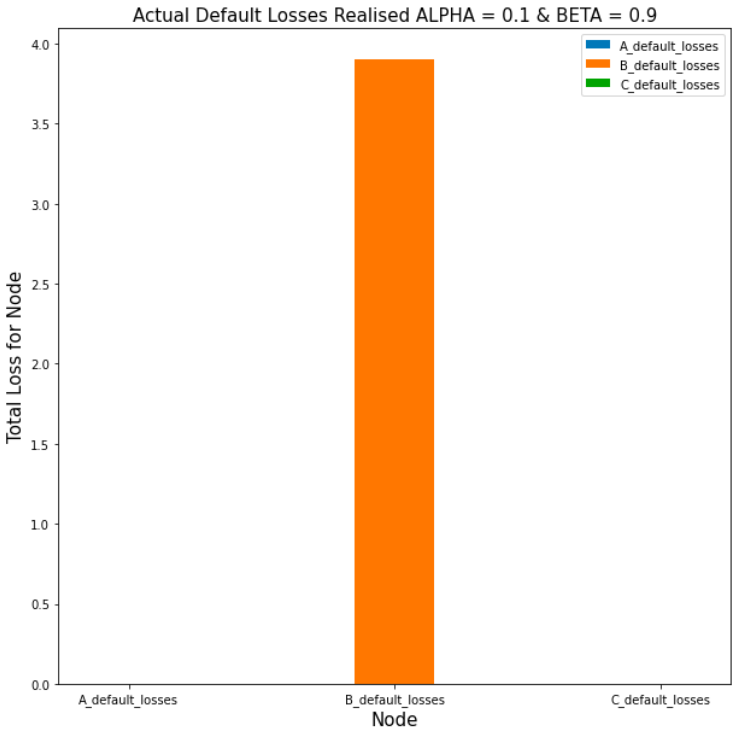
Scenario 5 - Firm B defaults in first round, algorithm terminates round 2.

- ALPHA = 0.1, BETA = 0.9
- Clearing payment vector found in round 2 = [11.0, 3.1, 4.0]
    - Node A pays: 11.0
    - Node B pays: 3.1
    - Node C pays: 4.0
    - 12
    - Default loss incurred by Node A is: 0
    - Default loss incurred by Node B is: 3.9
    - Default loss incurred by Node C is: 0
- Systemic Risk:
    - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node A has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
    - Node C has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).




Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
0
2
4
6
8
10
1
2
Round


Final Clearing Payment Vector
A_clearing_payment
B_clearing_payment
C_clearing_payment
Total Payment by Node
0
2
4
6
8
10
A_clearing_payment
B_clearing_payment
C_clearing_payment
Node

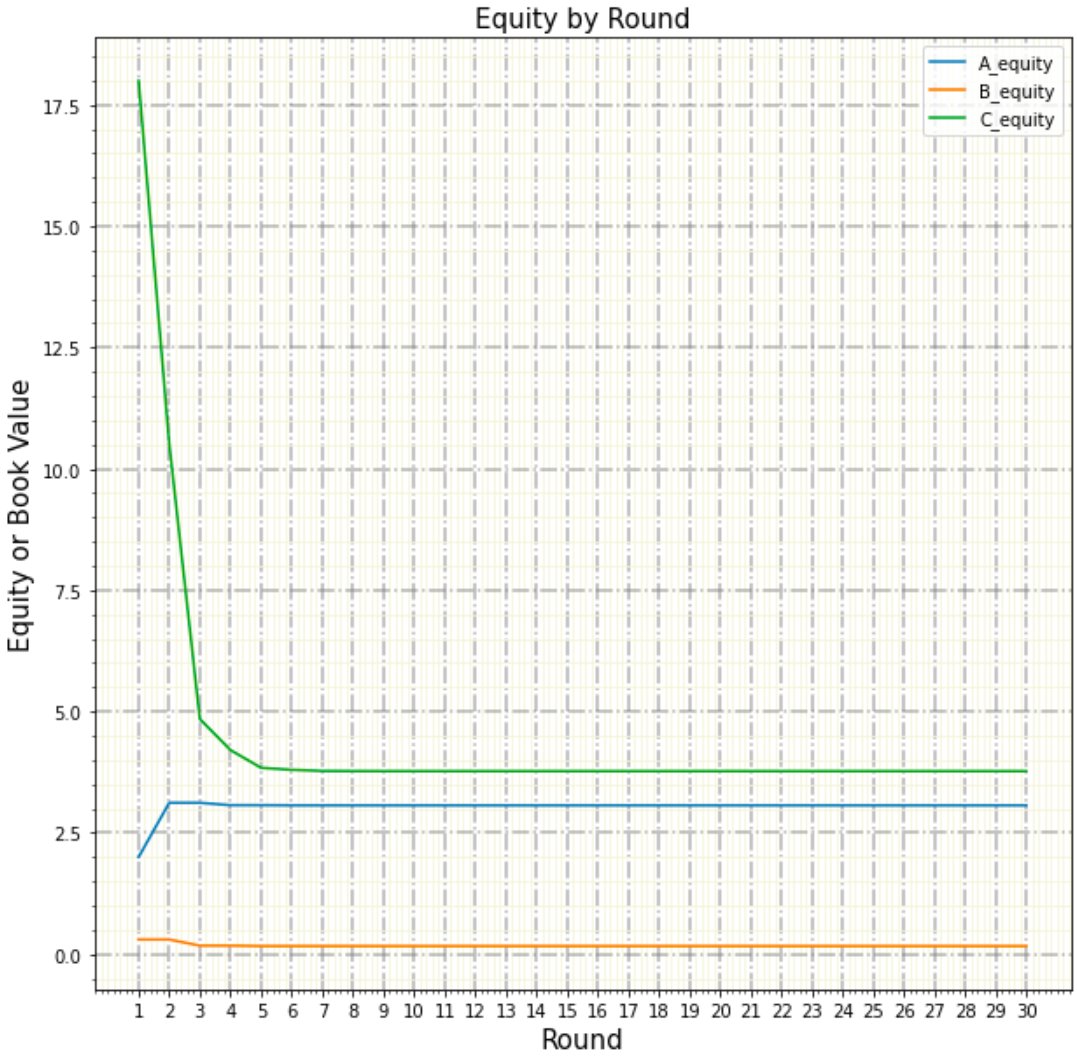


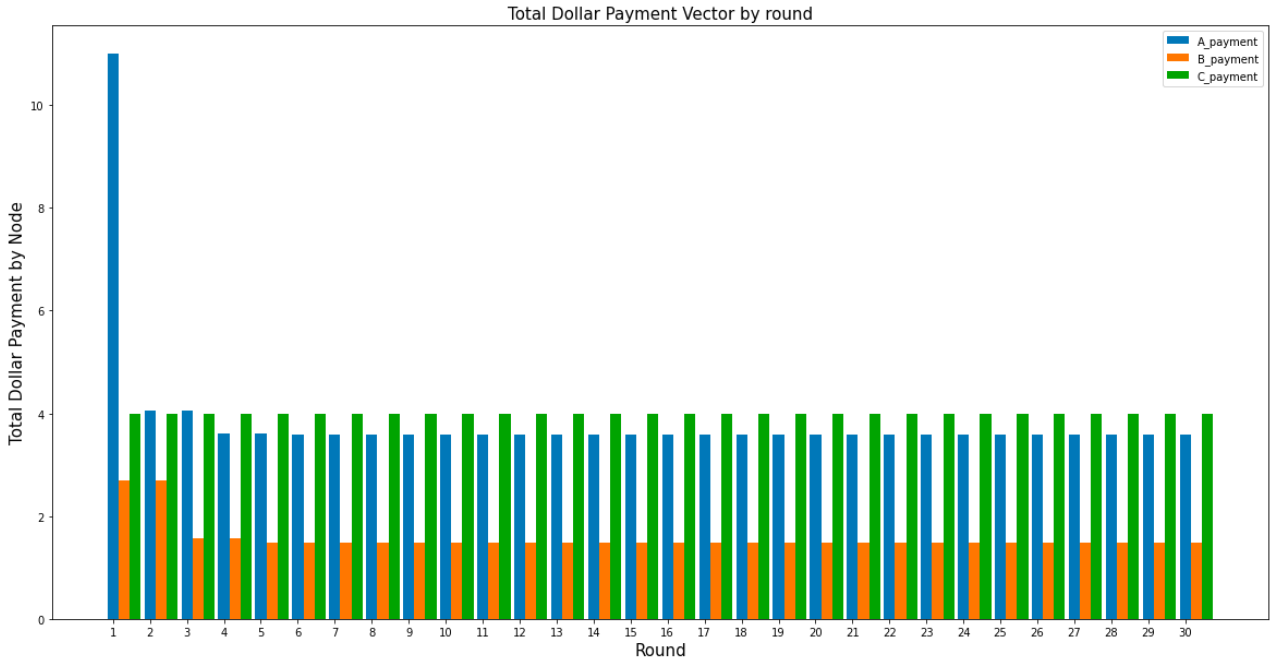
Comment – The larger shock reduces equity in system and payment by defaulting firm.

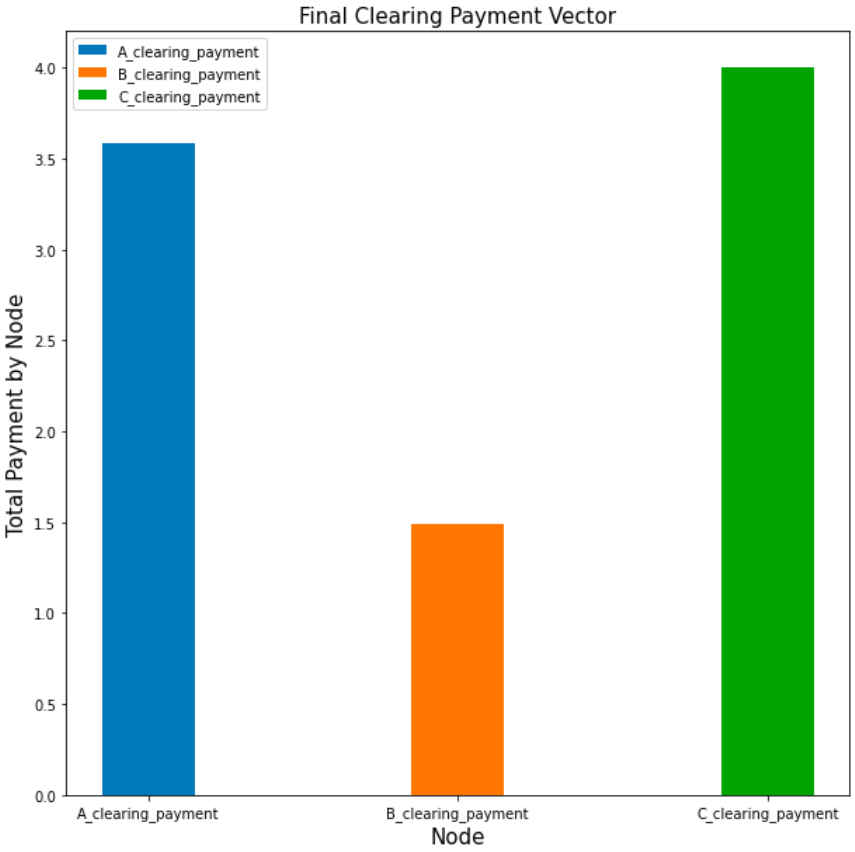
SCENARIO 5 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.1 and BETA = 0.9, SHOCK = 8, GREATER CLEARING VECTOR

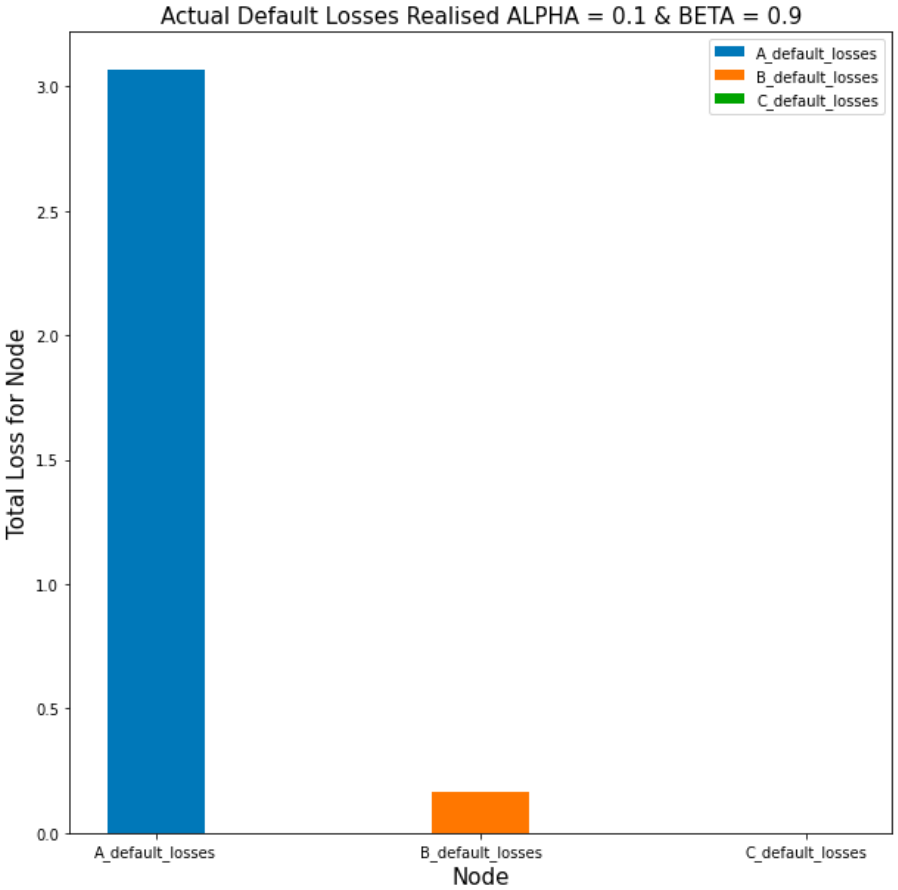
Scenario 5 - Firm B defaults in first round, Firm A in second round, algorithm terminates round 30.

- ALPHA = 0.1, BETA = 0.9
- Clearing payment vector found in round 30 = [3.585388072391595, 1.486699866391352, 4.0]
    - Node A pays: 3.585388072391595
    - Node B pays: 1.486699866391352
    - Node C pays: 4.0
    - Default loss incurred by Node A is: 3.0650431191546215
    - Default loss incurred by Node B is: 0.16518887404348348
    - Default loss incurred by Node C is: 0
- Systemic Risk:
    - Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node A has defaulted in round 2. The number of prior default waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
    - Node C has not defaulted after 30 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).


Equity by Round
A_equity
B_equity
C_equity
Equity or Book Value
Round


Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round


Final Clearing Payment Vector
A_clearing_payment
B_clearing_payment
C_clearing_payment
Total Payment by Node
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
A_clearing_payment
B_clearing_payment
C_clearing_payment
Node

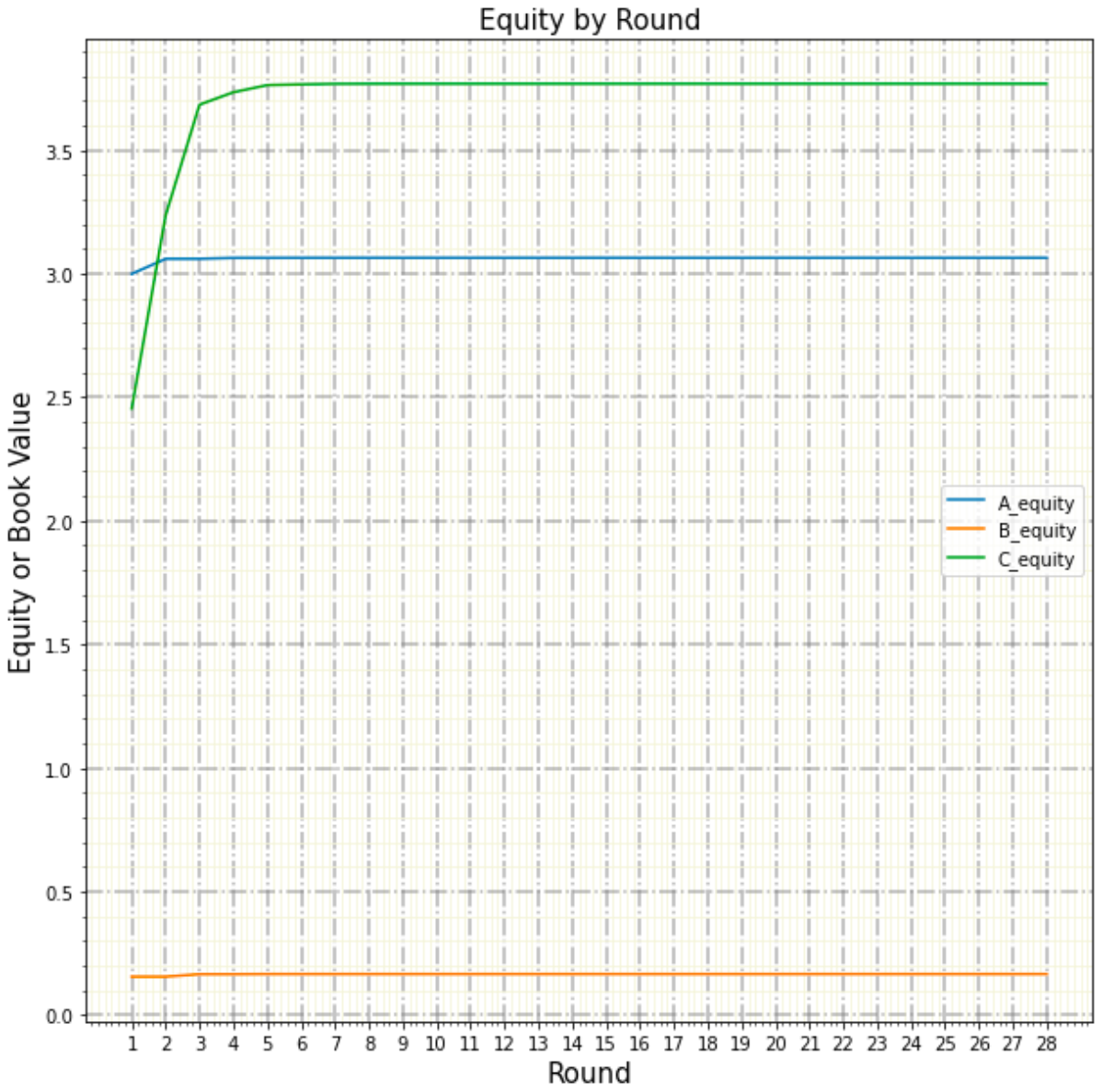

Actual Default Losses Realised ALPHA = 0.1 & BETA = 0.9
A_default_losses
B_default_losses
C_default_losses
Total Loss for Node
0.0
0.5
1.0
1.5
2.0
2.5
3.0
A_default_losses
B_default_losses
C_default_losses
Node

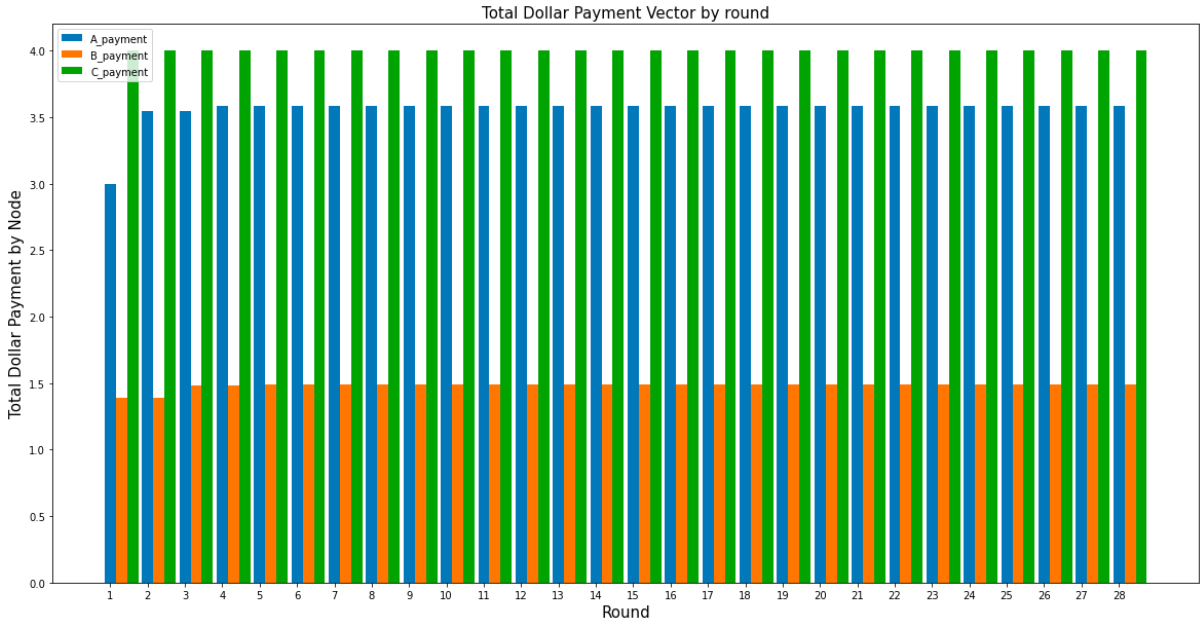
Comment – Larger shock value (4) means lower equity and more defaulters (there are now 2 defaulters unlike previous scenario with shock = 2 and 1 defaulting firm).

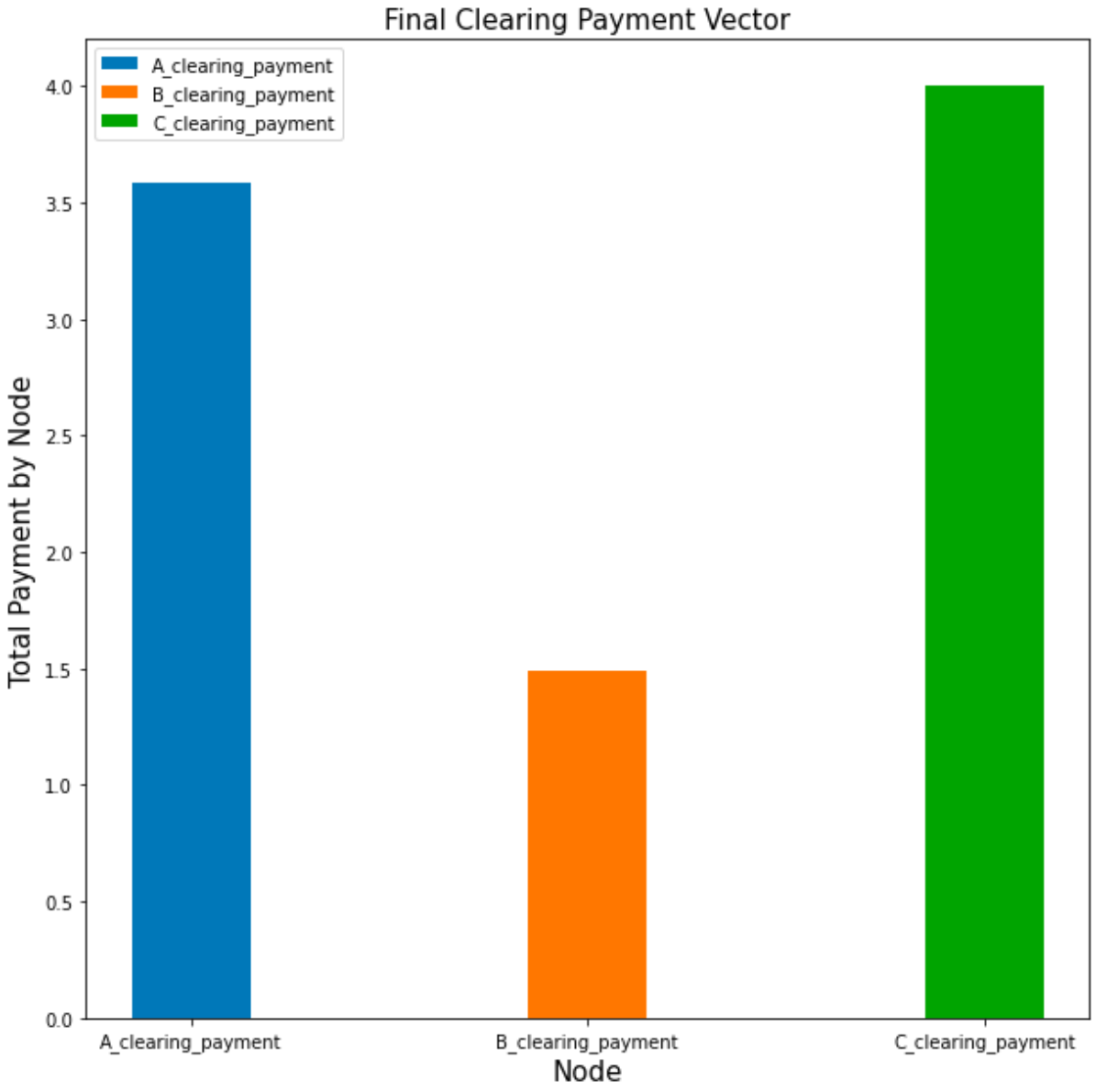
SCENARIO 5 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.1 and BETA = 0.9, SHOCK = 8, LOWER CLEARING VECTOR

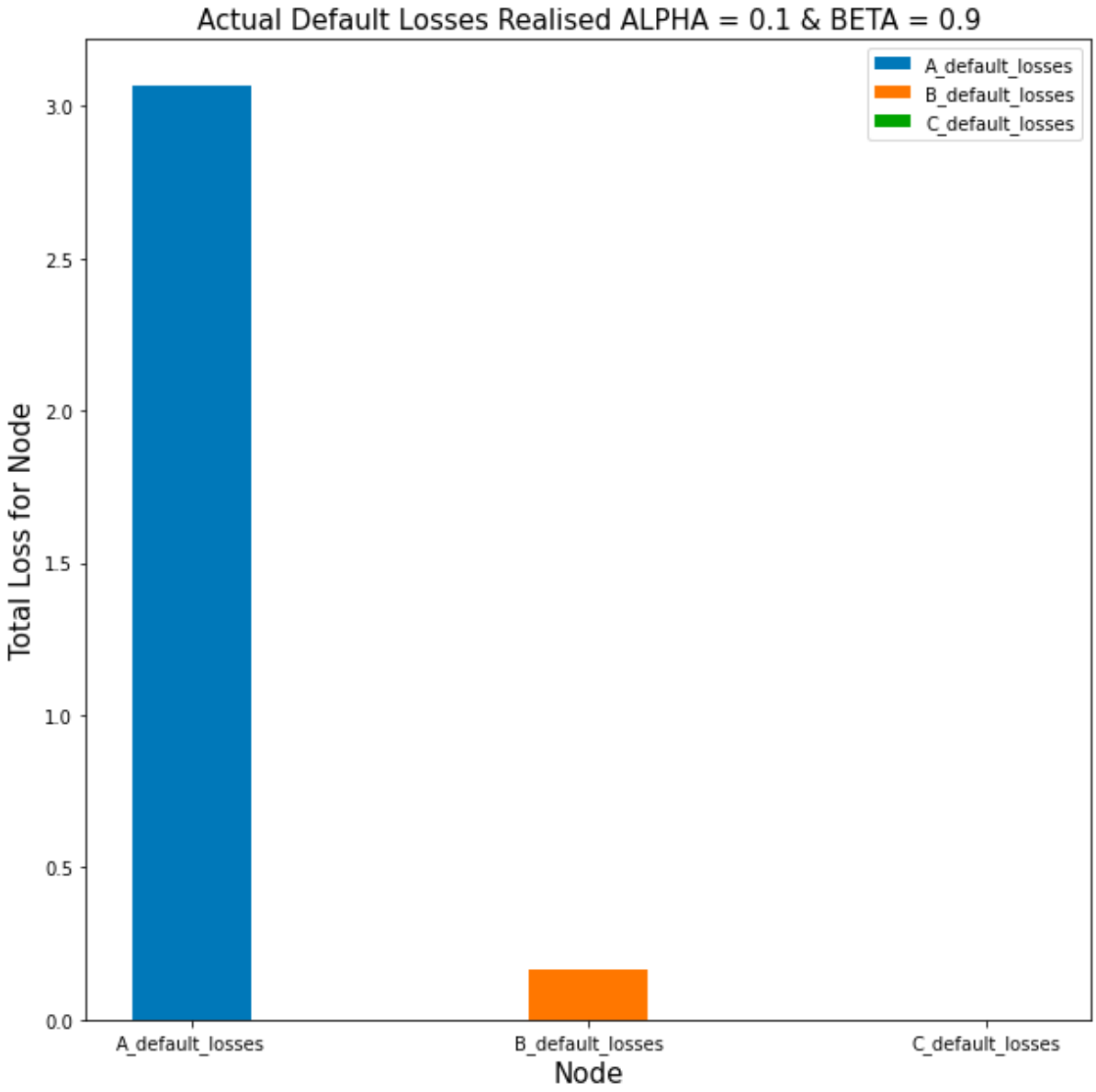
Scenario 5 - Firm A defaults in first round, Firm B in first round, algorithm terminates round 28.

- ALPHA = 0.1, BETA = 0.9
- Clearing payment vector found in round 28 = [3.585388072391595, 1.486699866391352, 4.0]
    - Node A pays: 3.585388072391595
    - Node B pays: 1.486699866391352
    - Node C pays: 4.0
    - Default loss incurred by Node A is: 3.0650431191546215
    - Default loss incurred by Node B is: 0.16518887404348348
    - Default loss incurred by Node C is: 0
- Systemic Risk:
    - Node A has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node B has defaulted in round 1. The number of prior default waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e. ['A']).
    - Node C has not defaulted after 28 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['A', 'B']).


Equity by Round
Equity or Book Value
Round
A_equity
B_equity
C_equity


Total Dollar Payment Vector by round
Total Dollar Payment by Node
Round
A_payment
B_payment
C_payment


Final Clearing Payment Vector
A_clearing_payment
B_clearing_payment
C_clearing_payment
Total Payment by Node
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
A_clearing_payment
B_clearing_payment
C_clearing_payment
Node



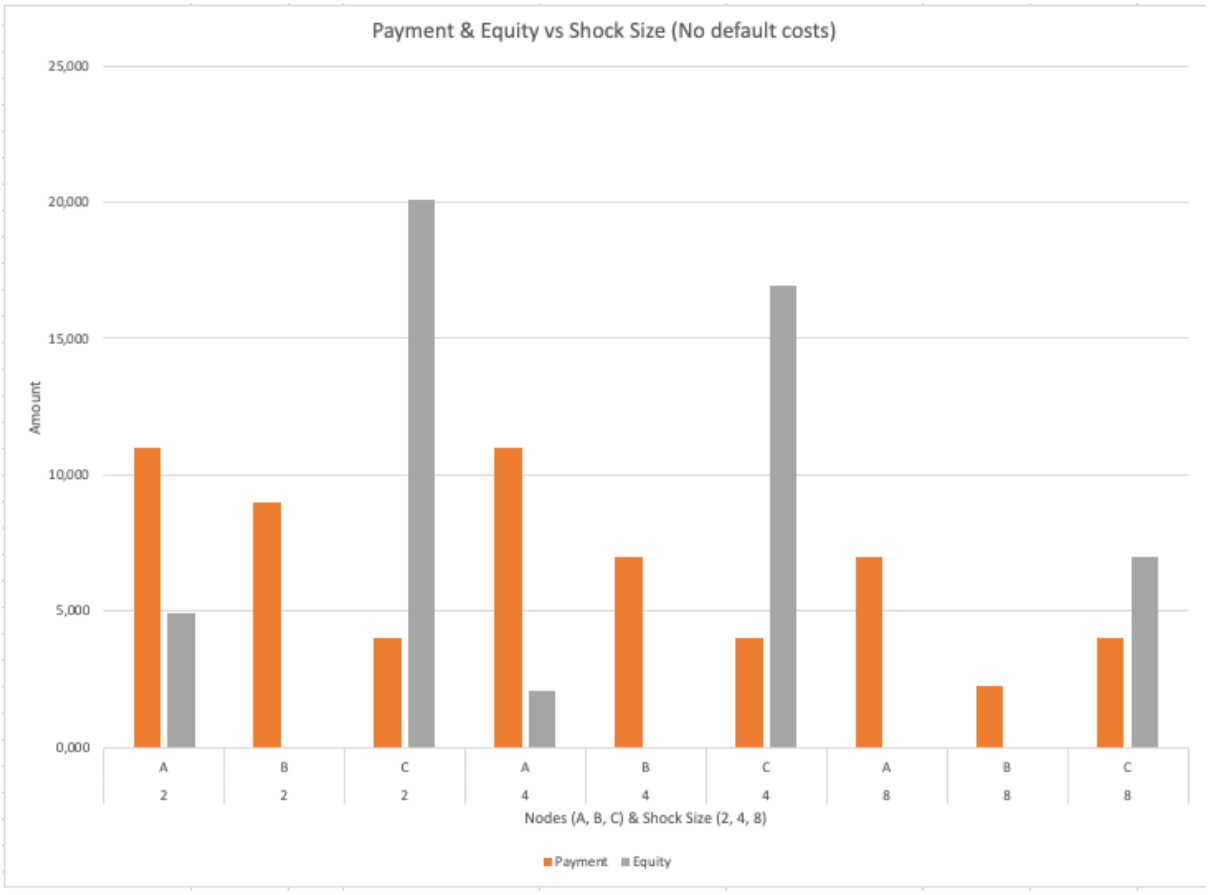
Comment – This is calculating the lowest clearing vector (and therefore not really considering the shock size (8) but rather a comparison with the previous scenario which is the greatest clearing vector). Interestingly, the lowest clearing vector is identical to the highest clearing vector and notably converges to the clearing payment vector faster (in 28 rounds) than the algorithm for the greatest clearing vector (in 30 rounds).

SECTION B – Effects of Shock Size

Effect of shock size with no default costs

The graph below shows the effect of shock size on clearing payments and equity for each firm. This corresponds to Eisenberg and Noe i.e. with no default costs. These scenario's are recapped below:

- SCENARIO 3 - Debt Model with 3 nodes (A, B, C), ALPHA = 1, BETA = 1
  - SHOCK = 2 with GREATER CLEARING VECTOR as output
  - SHOCK = 4 with GREATER CLEARING VECTOR as output
  - SHOCK = 8 with GREATER CLEARING VECTOR as output

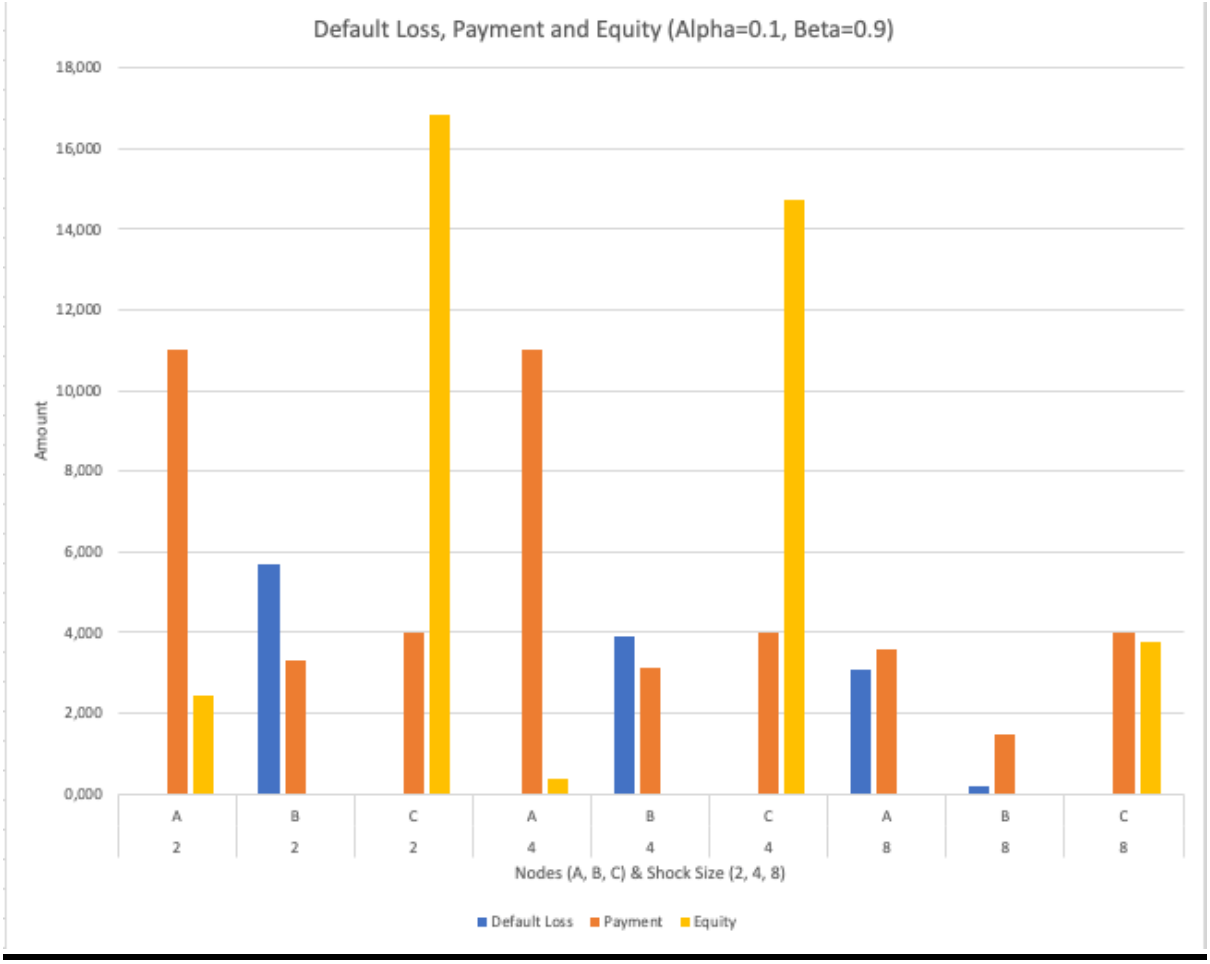


Comment – This relates to Scenario 3 which is the debt model with no default costs (i.e. classic Eisenberg and Noe). Firms with no equity have defaulted (e.g. B). The larger the shock size, the less the equity, the more the number of defaulters (e.g. shock size 8, A and B default) and the smaller the clearing payment vector.

Effect of shock size with default costs

The graph below shows the effect of shock size on default loss, clearing payments and equity for each firm. This corresponds to Rogers and Veraart i.e. with default costs. These scenario's are recapped below:

- SCENARIO 5 - Debt Model with 3 nodes (A, B, C), ALPHA = 0.1, BETA = 0.9
    - SHOCK = 2 with GREATER CLEARING VECTOR as output
    - SHOCK = 4 with GREATER CLEARING VECTOR as output
    - SHOCK = 8 with GREATER CLEARING VECTOR as output



Comment – This relates to Scenario 5 which is the debt model with default costs (i.e. Rogers and Veraart). Firms with no equity have defaulted (e.g. B). The larger the shock size, the less the equity, the more the number of defaulters (e.g. shock size 8, A and B default) and the smaller the clearing payment vector. Interestingly, the larger the shock size, the smaller the default loss.