eisenberg-noe-2001-debt-model-with-default-costs

November 15, 2021

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DEBT MODEL WITH DEFAULT COSTS

ALPHA = 0.1 and BETA = 0.9
Running Debt Model in MANUAL mode with 3 nodes…

Scenario 5 - Firm B defaults in first round, Firm A in second round, algorithm
terminates round 3, MODE == 'MANUAL', NUM_AGENTS = 3, NOMINAL_LIABILITY_MATRIX =
np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING_CASH_FLOW_VECTOR = [3, 0, 4],
ALPHA = 0.1, BETA = 0.9

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AGENT LABELS

Agent labels ['A', 'B', 'C']

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NOMINAL LIABILITY MATRIX Data Frame
i.e. what node i expects (row) to pay node j (column)…

   A  B  C
A  0  2  9
B  7  0  9
C  3  1  0



Nominal liabilities for each node:
Liability of Node A to Node B is 2
Liability of Node A to Node C is 9
Liability of Node B to Node A is 7
Liability of Node B to Node C is 9
Liability of Node C to Node A is 3
Liability of Node C to Node B is 1

NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame
```

i.e. what node j expects to receive from i…

```
    A  B  C
A   0  7  3
B   2  0  1
C   9  9  0
```

Node A expects to receive 7 from Node B
Node A expects to receive 3 from Node C
Node B expects to receive 2 from Node A
Node B expects to receive 1 from Node C
Node C expects to receive 9 from Node A
Node C expects to receive 9 from Node B

OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 3
Exogenous cash flow for Node B: 0
Exogenous cash flow for Node C: 4
[3, 0, 4]

CALCULATING RELATIVE LIABILITIES FOR EACH NODE

Node A

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 9.0

- Total obligations for Node A
Total Obligation Vector updated in round 1 for Node A with value 11.0
Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0

- Relative Liabilities of Node A
Relative Liability of Node A to Node B is 0.18181818181818182
Relative Liability of Node A to Node C is 0.8181818181818182
Sum of Relative Liabilities for Node A is 1.0

Node B

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 7.0
Liability of Node B to Node C (i.e. P_12) is 9.0

- Total obligations for Node B
Total Obligation Vector updated in round 1 for Node B with value 16.0
Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0

- Relative Liabilities of Node B
Relative Liability of Node B to Node A is 0.4375
Relative Liability of Node B to Node C is 0.5625
Sum of Relative Liabilities for Node B is 1.0

Node C

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 3.0
Liability of Node C to Node B (i.e. P_21) is 1.0

- Total obligations for Node C
Total Obligation Vector updated in round 1 for Node C with value 4.0
Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0

- Relative Liabilities of Node C
Relative Liability of Node C to Node A is 0.75
Relative Liability of Node C to Node B is 0.25
Sum of Relative Liabilities for Node C is 1.0

RELATIVE LIABILITY MATRIX Data Frame

	A	B	C
A	0.0000	0.181818	0.818182
B	0.4375	0.000000	0.562500
C	0.7500	0.250000	0.000000

RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

	A	B	C	Relative Liability Total	CORRECT VALUE?
A	0.0000	0.181818	0.818182	1.0	True
B	0.4375	0.000000	0.562500	1.0	True
C	0.7500	0.250000	0.000000	1.0	True

RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame
i.e. what node i (row) expects to receive from node j (column) in relative

terms…

```
          A        B      C
A  0.000000  0.4375  0.75
B  0.181818  0.0000  0.25
C  0.818182  0.5625  0.00
```

Expected nominal payments in for Node A - both proportion and total amount

Node A expects to receive proportion 0.4375 from Node B
Node A expects to receive proportion 0.75 from Node C
Total payments in to Node A is 3.0 in round 1.

Expected nominal payments in for Node B - both proportion and total amount

Node B expects to receive proportion 0.18181818181818182 from Node A
Node B expects to receive proportion 0.25 from Node C
Total payments in to Node B is 1.5454545454545454 in round 1.

Expected nominal payments in for Node C - both proportion and total amount

Node C expects to receive proportion 0.8181818181818182 from Node A
Node C expects to receive proportion 0.5625 from Node B
Total payments in to Node C is 2.4545454545454546 in round 1.

START OF ROUND 1

TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p_bar_i…
Total nominal obligation for Node A (i.e. p_bar_1): 11.0
Total nominal obligation for Node B (i.e. p_bar_2): 16.0
Total nominal obligation for Node C (i.e. p_bar_3): 4.0

TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for

each node…

Node A

- Total payments in for Node A
Total payments in to Node A is 3.0 in round 1.

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 9.0

- Total obligations for Node A
Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0
Payment out is min[payment out, total cash flow] i.e. min[11.0, 6.0]
Total payments in to Node A is 3.0 in round 1.
Total payments in to Node A is 3.0 in round 1.
Total payments in to Node A is 3.0 in round 1.
Round 1 and Node A has defaulted due to nominal obligations 11.0 being greater
than cash flow 6.0. Default loss for Node A is 3.0

 - Total Dollar Payment Vector for round 1 and Node A
Total Dollar Payment Vector for round 1 and Node A updated with value 3.0

Node B

- Total payments in for Node B
Total payments in to Node B is 1.5454545454545454 in round 1.

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 7.0
Liability of Node B to Node C (i.e. P_12) is 9.0

- Total obligations for Node B
Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0
Payment out is min[payment out, total cash flow] i.e. min[16.0,
1.5454545454545454]
Total payments in to Node B is 1.5454545454545454 in round 1.
Total payments in to Node B is 1.5454545454545454 in round 1.
Total payments in to Node B is 1.5454545454545454 in round 1.
Round 1 and Node B has defaulted due to nominal obligations 16.0 being greater
than cash flow 1.5454545454545454. Default loss for Node B is
0.15454545454545454

 - Total Dollar Payment Vector for round 1 and Node B
Total Dollar Payment Vector for round 1 and Node B updated with value
1.3909090909090909

Node C

- Total payments in for Node C
Total payments in to Node C is 2.4545454545454546 in round 1.

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 3.0
Liability of Node C to Node B (i.e. P_21) is 1.0

- Total obligations for Node C
Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0
Payment out is min[payment out, total cash flow] i.e. min[4.0,
6.454545454545455]

 - Total Dollar Payment Vector for round 1 and Node C
Total Dollar Payment Vector for round 1 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 3.0
Total payment by Node B (i.e. p_2): 1.3909090909090909
Total payment by Node C (i.e. p_3): 4.0
[3.0, 1.3909090909090909, 4.0]

UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE

Total payments in to Node A is 3.0 in round 1.
Total dollar payment by Node A (i.e. p_1) is 3.0

 - Equity Vector for round 1 and Node A
Equity Vector for round 1 and Node A updated with value 6.0
Total payments in to Node B is 1.5454545454545454 in round 1.
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909

 - Equity Vector for round 1 and Node B
Equity Vector for round 1 and Node B updated with value 1.5454545454545454
Total payments in to Node C is 2.4545454545454546 in round 1.
Total dollar payment by Node C (i.e. p_3) is 4.0

 - Equity Vector for round 1 and Node C
Equity Vector for round 1 and Node C updated with value 6.454545454545455

EQUITY FOR EACH NODE

Equity for Node A: 6.0
Equity for Node B: 1.5454545454545454
Equity for Node C: 6.454545454545455
[6.0, 1.5454545454545454, 6.454545454545455]

ROUND 1 DEFAULTERS

Node A has defaulted in round 1
{'A': True, 'B': True, 'C': False}
There are defaulters in this round (i.e. round 1), algorithm will proceed for
another round.

END OF ROUND 1

START OF ROUND 2

TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p_bar_i…
Total nominal obligation for Node A (i.e. p_bar_1): 11.0
Total nominal obligation for Node B (i.e. p_bar_2): 16.0
Total nominal obligation for Node C (i.e. p_bar_3): 4.0

TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for
each node…

Node A

- Total payments in for Node A
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 9.0
Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0
Payment out is min[payment out, total cash flow] i.e. min[11.0,
6.608522727272727]
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.
Round 2 and Node A has defaulted due to nominal obligations 11.0 being greater
than cash flow 6.608522727272727. Default loss for Node A is 3.0608522727272724

 - Total Dollar Payment Vector for round 2 and Node A
Total Dollar Payment Vector for round 2 and Node A updated with value
3.5476704545454547

Node B

- Total payments in for Node B
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 7.0
Liability of Node B to Node C (i.e. P_12) is 9.0
Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0
Payment out is min[payment out, total cash flow] i.e. min[16.0,
1.5454545454545454]
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.

Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.
Round 2 and Node B has defaulted due to nominal obligations 16.0 being greater
than cash flow 1.5454545454545454. Default loss for Node B is
0.15454545454545454

 - Total Dollar Payment Vector for round 2 and Node B
Total Dollar Payment Vector for round 2 and Node B updated with value
1.3909090909090909

Node C

- Total payments in for Node C
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Total payments in to Node C is 3.236931818181818 in round 2.

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 3.0
Liability of Node C to Node B (i.e. P_21) is 1.0
Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0
Payment out is min[payment out, total cash flow] i.e. min[4.0,
7.236931818181818]

 - Total Dollar Payment Vector for round 2 and Node C
Total Dollar Payment Vector for round 2 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 3.5476704545454547
Total payment by Node B (i.e. p_2): 1.3909090909090909
Total payment by Node C (i.e. p_3): 4.0
[3.5476704545454547, 1.3909090909090909, 4.0]

UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE

Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.
Total dollar payment by Node A (i.e. p_1) is 3.5476704545454547

 - Equity Vector for round 2 and Node A
Equity Vector for round 2 and Node A updated with value 6.608522727272727
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909

 - Equity Vector for round 2 and Node B
Equity Vector for round 2 and Node B updated with value 1.5454545454545454
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Total payments in to Node C is 3.236931818181818 in round 2.
Total dollar payment by Node C (i.e. p_3) is 4.0

 - Equity Vector for round 2 and Node C
Equity Vector for round 2 and Node C updated with value 7.236931818181818

EQUITY FOR EACH NODE

Equity for Node A: 6.608522727272727
Equity for Node B: 1.5454545454545454
Equity for Node C: 7.236931818181818
[6.608522727272727, 1.5454545454545454, 7.236931818181818]

ROUND 2 DEFAULTERS

Node A has defaulted in round 2
{'A': True, 'B': True, 'C': False}
There are defaulters from earlier rounds but no new defaulters in the current
round, algorithm will not proceed for another round.

Checking limited liability and absolute priority for Node A

Total dollar payment by Node A (i.e. p_1) is 3.5476704545454547
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.

- Limited liability is met. Node A made a payment of 3.5476704545454547 in round 2 which is less than or equal to the cash flow (payments in + exogenous cash) of 6.608522727272727.
Total nominal obligation for Node A (i.e. p_bar_1): 11.0
Total dollar payment by Node A (i.e. p_1) is 3.5476704545454547
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node A from Node C is 0.75
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 3.608522727272727 in round 2.

 -Checking absolute priority for Node A in round 2. Nominal obligations is 11.0 and Dollar payments is 3.5476704545454547
Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node A
All value i.e. cash flow available to Node A is 6.608522727272727
-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 2 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 11.0, Dollar payment was 3.5476704545454547 and Total cash flow was 6.608522727272727

Node A in round 2 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node B

Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.

- Limited liability is met. Node B made a payment of 1.3909090909090909 in round 2 which is less than or equal to the cash flow (payments in + exogenous cash) of 1.5454545454545454.
Total nominal obligation for Node B (i.e. p_bar_2): 16.0

Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Relative Payment in to Node B from Node A is 0.18181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 1.5454545454545454 in round 2.

 -Checking absolute priority for Node B in round 2. Nominal obligations is 16.0
and Dollar payments is 1.3909090909090909
Dollar payments less than nominal obligations. Now checking if all value is paid
to creditors, i.e. Total cash flow for Node B
All value i.e. cash flow available to Node B is 1.5454545454545454
-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 2 i.e. either obligations are paid
in full or all available cash flow (i.e. sum of the payments received by the
node plus the exogenous operating cash flow) is paid to creditors. Nominal
obligations were 16.0, Dollar payment was 1.3909090909090909 and Total cash
flow was 1.5454545454545454

Node B in round 2 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node C

Total dollar payment by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Total payments in to Node C is 3.236931818181818 in round 2.

- Limited liability is met. Node C made a payment of 4.0 in round 2 which is
less than or equal to the cash flow (payments in + exogenous cash) of
7.236931818181818.
Total nominal obligation for Node C (i.e. p_bar_3): 4.0
Total dollar payment by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 1.3909090909090909
Total payments in to Node C is 3.236931818181818 in round 2.

 -Checking absolute priority for Node C in round 2. Nominal obligations is 4.0
and Dollar payments is 4.0
-Absolute priority is satisfied for Node C

- Absolute priority is met by Node C in round 2 i.e. either obligations are paid
in full or all available cash flow (i.e. sum of the payments received by the

12

node plus the exogenous operating cash flow) is paid to creditors. Nominal
obligations were 4.0, Dollar payment was 4.0 and Total cash flow was
7.236931818181818

Node C in round 2 passes candidate clearing vector payment entry checks.

CLEARING_PAYMENT_VECTOR

Clearing payment vector found in round 2.
[3.5476704545454547, 1.3909090909090909, 4.0]
Node A pays: 3.5476704545454547
Node B pays: 1.3909090909090909
Node C pays: 4.0
Default loss incurred by Node A is: 3.0608522727272724
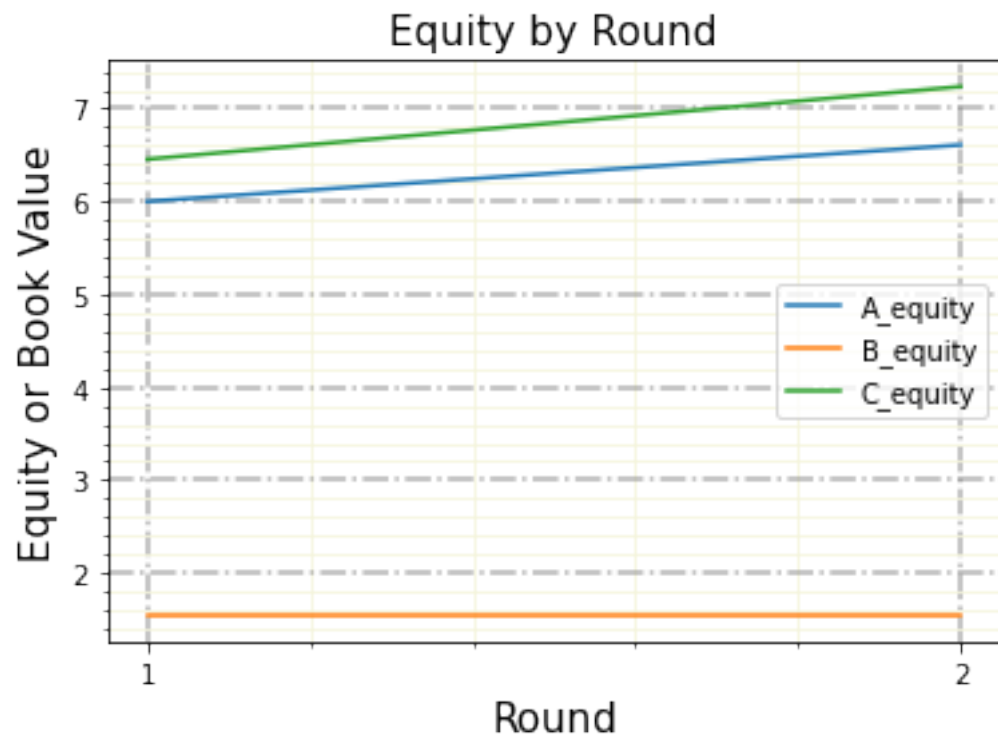Default loss incurred by Node B is: 0.15454545454545454
Default loss incurred by Node C is: 0

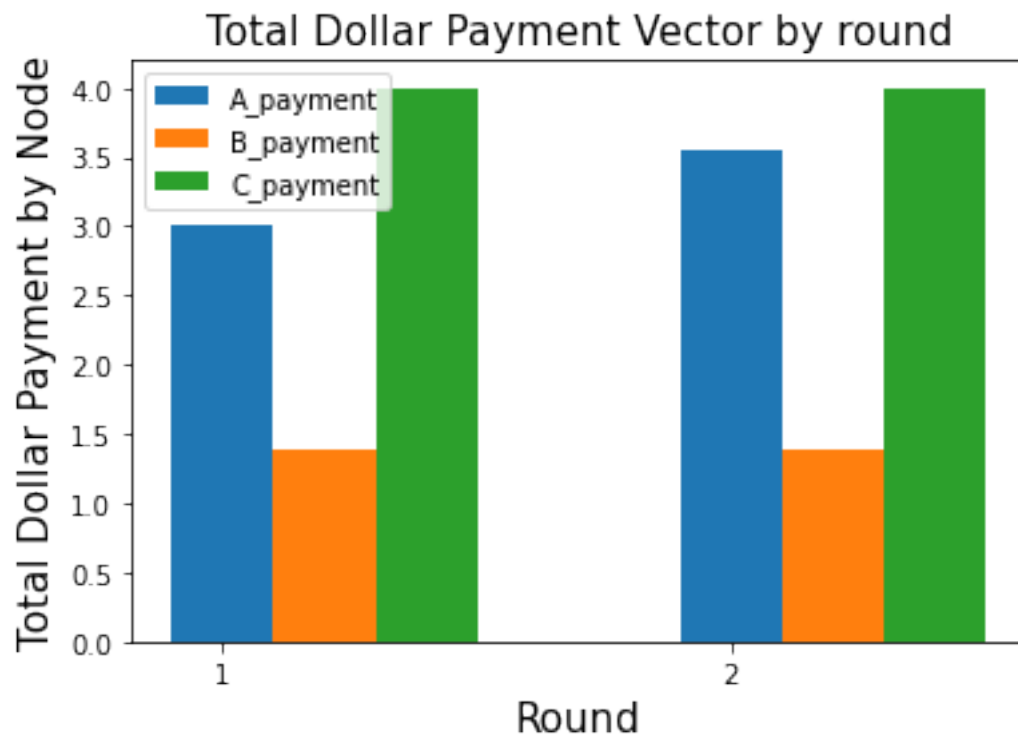---
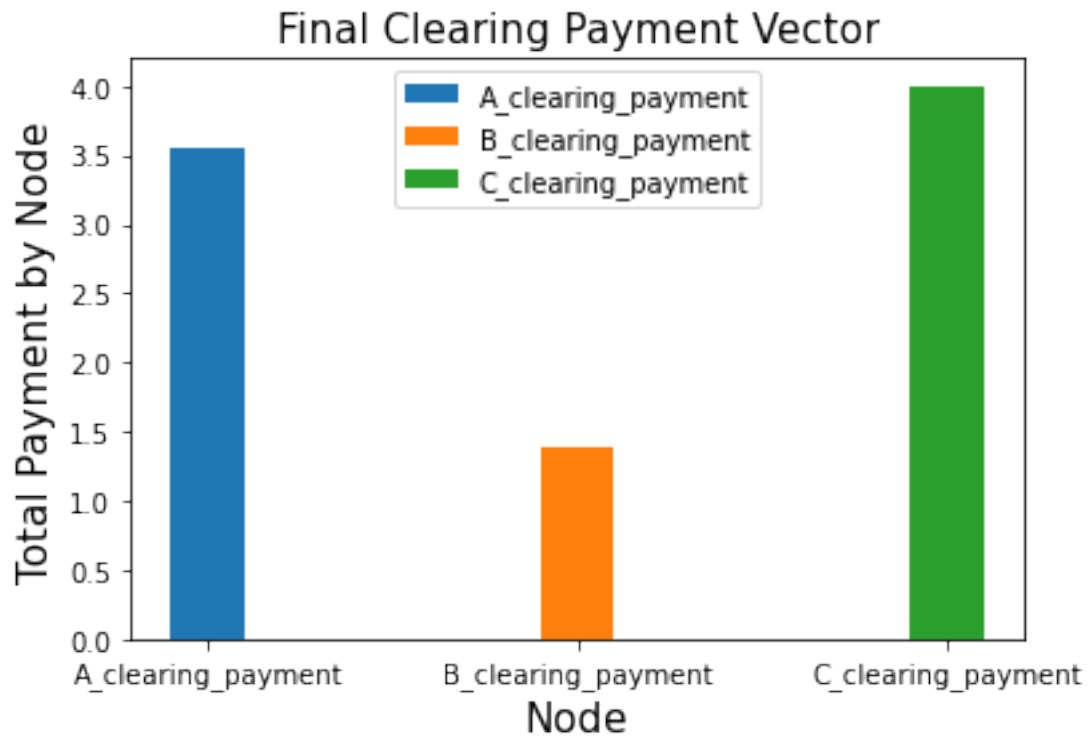
END OF ROUND 2
- Systemic Risk: Node A has defaulted in round 1. The number of prior default
waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
- Systemic Risk: Node B has defaulted in round 1. The number of prior default
waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e.
['A']).
- Systemic Risk: Node C has not defaulted after 2 rounds. There are 3 nodes in
the system (2 of which have defaulted i.e. ['A', 'B']).

Scenario 5 - Firm B defaults in first round, Firm A in second round, algorithm
terminates round 3, MODE == 'MANUAL', NUM_AGENTS = 3, NOMINAL_LIABILITY_MATRIX =
np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING_CASH_FLOW_VECTOR = [3, 0, 4],
ALPHA = 0.1, BETA = 0.9

Total Nominal Obligations

Equity by Round

Total Dollar Payment Vector by round

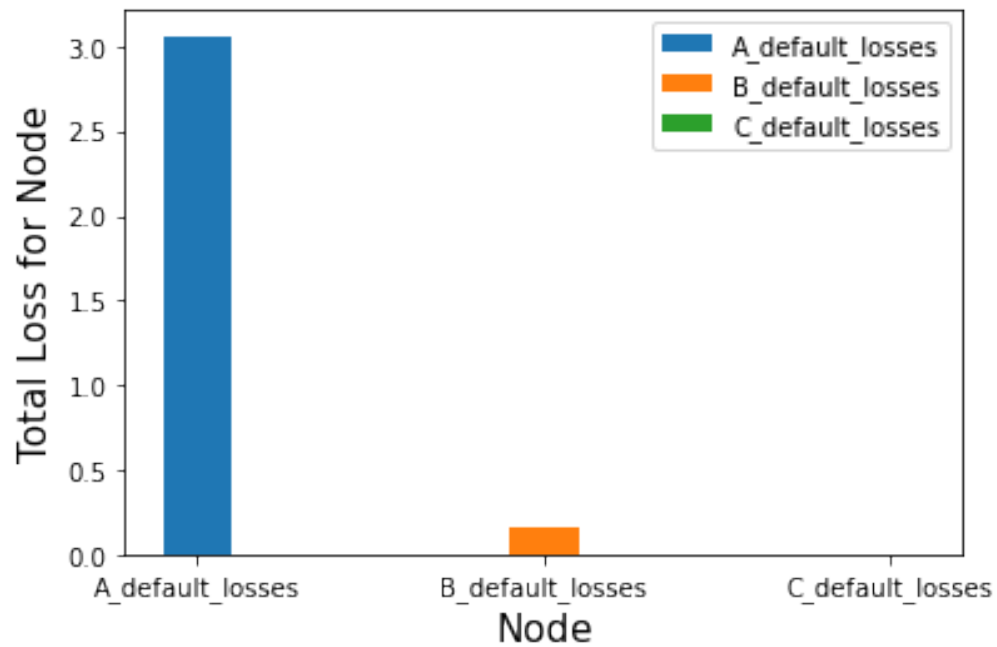
Final Clearing Payment Vector
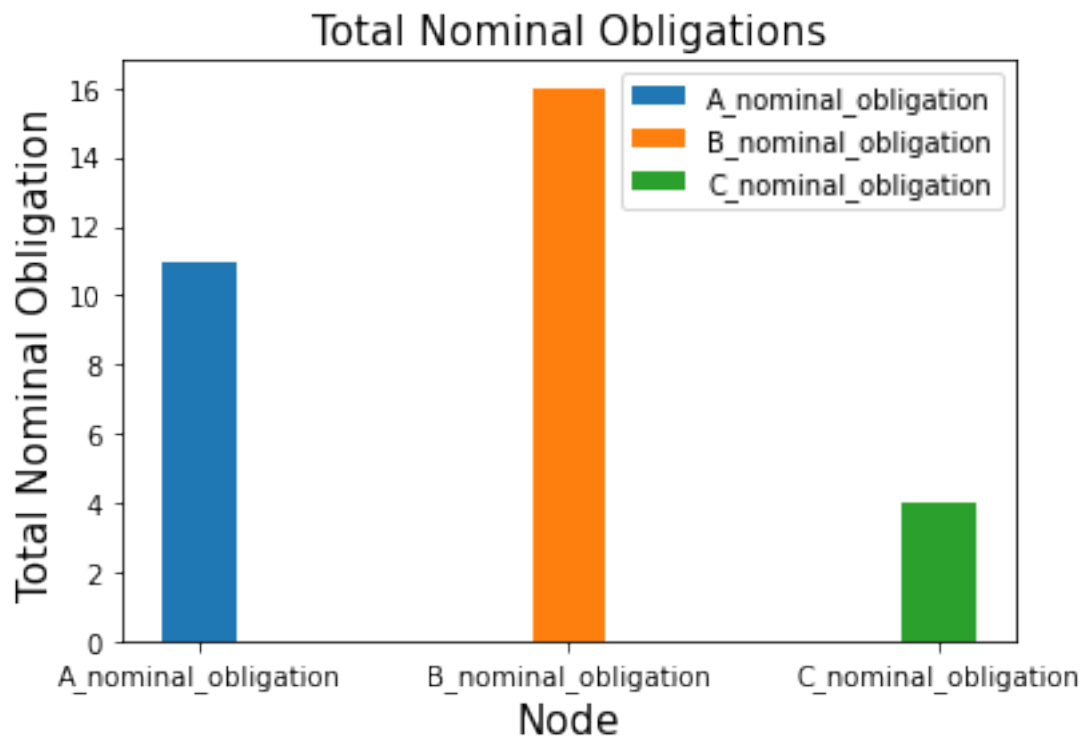
Actual Default Losses Realised ALPHA = 0.1 & BETA = 0.9

ALPHA and BETA are the fraction of exogenous assets (outside financial network) and endogenous assets (inside financial network i.e. interbank obligations) that are realised on liquidation in the event of default. The two fractions may conceivably be different; we would typically expect that ALPHA would be low,because the bank would be having to sell off its loan portfolio, probably at a knock-down price or fire sale. In contrast, BETA might be much closer to 1, because an obligation from a solvent bank would probably be paid in full (though perhaps with some negotiated discount to compensate for the inconvenience of early repayment).

Percentage of Defaulters After Round 1
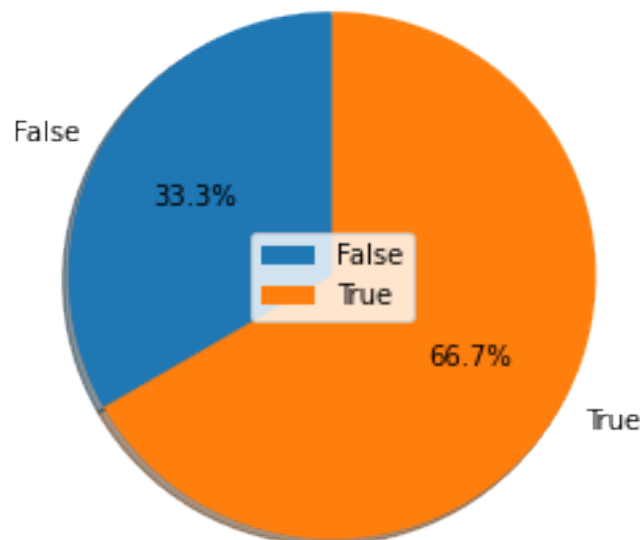
Node A has defaulted in round 1
Node B has defaulted in round 1
Node C has NOT defaulted in round 1

Percentage of Defaulters After Round 2
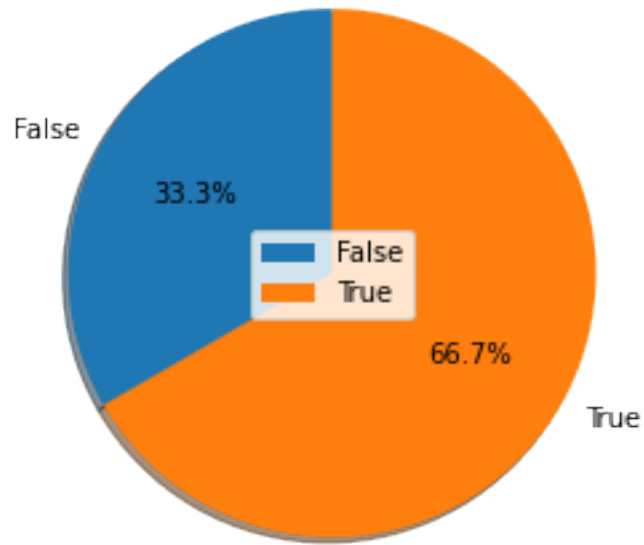

```
Node A has defaulted in round 2
Node B has defaulted in round 2
Node C has NOT defaulted in round 2
```