eisenberg-noe-2001-debt-model

October 23, 2021

DEBT MODEL MODE Running Debt Model in RANDOM mode with 3 nodes... Scenario 4 - Random Initialisation with 3 nodes, MODE == 'RANDOM', NUM_AGENTS = AGENT LABELS Agent labels ['A', 'B', 'C'] NOMINAL LIABILITY MATRIX Data Frame i.e. what node i expects to pay node j... A B C A 0 4 4 B 4 0 1 C 4 4 0 Nominal liabilities for each node: Liability of Node A to Node B is 4 Liability of Node A to Node C is 4 Liability of Node B to Node A is 4 Liability of Node B to Node C is 1 Liability of Node C to Node A is 4 Liability of Node C to Node B is 4 NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame i.e. what node j expects to receive from i...

A B C

A 0 4 4 B 4 0 4 C 4 1 0

Node A expects to receive 4 from Node B Node A expects to receive 4 from Node C Node B expects to receive 4 from Node A Node B expects to receive 4 from Node C Node C expects to receive 4 from Node A Node C expects to receive 1 from Node B

OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 0 Exogenous cash flow for Node B: 6 Exogenous cash flow for Node C: 0 [0, 6, 0]

CALCULATING RELATIVE LIABILITIES FOR EACH NODE

Node A

- Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 4.0 Liability of Node A to Node C (i.e. P_02) is 4.0
- Total obligations for Node A Total Obligation Vector updated in round 1 for Node A with value 8.0 Total nominal liabilities for Node A (i.e. p_bar_1) is 8.0
- Relative Liabilities of Node A
 Relative Liability of Node A to Node B is 0.5
 Relative Liability of Node A to Node C is 0.5
 Sum of Relative Liabilities for Node A is 1.0

Node B

- Liabilities for Node B Liability of Node B to Node A (i.e. P_10) is 4.0 Liability of Node B to Node C (i.e. P_12) is 1.0
- Total obligations for Node B Total Obligation Vector updated in round 1 for Node B with value 5.0

Total nominal liabilities for Node B (i.e. p_bar_2) is 5.0

- Relative Liabilities of Node B
Relative Liability of Node B to Node A is 0.8
Relative Liability of Node B to Node C is 0.2
Sum of Relative Liabilities for Node B is 1.0

Node C

- Liabilities for Node C Liability of Node C to Node A (i.e. P_20) is 4.0 Liability of Node C to Node B (i.e. P_21) is 4.0
- Total obligations for Node C
 Total Obligation Vector updated in round 1 for Node C with value 8.0
 Total nominal liabilities for Node C (i.e. p_bar_3) is 8.0
- Relative Liabilities of Node C
 Relative Liability of Node C to Node A is 0.5
 Relative Liability of Node C to Node B is 0.5
 Sum of Relative Liabilities for Node C is 1.0

RELATIVE LIABILITY MATRIX Data Frame

A B C A 0.0 0.5 0.5 B 0.8 0.0 0.2 C 0.5 0.5 0.0

RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

	Α	В	С	Relative Liability Total	CORRECT VALUE?
Α	0.0	0.5		1.0	True
В	0.8	0.0	0.2	1.0	True
С	0.5	0.5	0.0	1.0	True

RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame

i.e. what node i expects to receive from j in relative terms...

A B C

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B 0.5 0.0 0.5
C 0.5 0.2 0.0
***Expected nominal payments in for Node A - both proportion and total amount***
Node A expects to receive proportion 0.8 from Node B
Node A expects to receive proportion 0.5 from Node C
Total payments in to Node A is 8.0
***Expected nominal payments in for Node B - both proportion and total amount***
Node B expects to receive proportion 0.5 from Node A
Node B expects to receive proportion 0.5 from Node C
Total payments in to Node B is 8.0
***Expected nominal payments in for Node C - both proportion and total amount***
Node C expects to receive proportion 0.5 from Node A
Node C expects to receive proportion 0.2 from Node B
Total payments in to Node C is 5.0
______
START OF ROUND 1
_____
TOTAL OBLIGATION VECTOR
i.e. total nominal obligations for each node i.e. p_bar_i...
Total nominal obligation for Node A (i.e. p_bar_1): 8.0
Total nominal obligation for Node B (i.e. p_bar_2): 5.0
Total nominal obligation for Node C (i.e. p_bar_3): 8.0
TOTAL PAYMENT MADE PER NODE
i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for
each node...
***Node A***
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A 0.0 0.8 0.5

- Total payments in for Node A Total payments in to Node A is 8.0
- Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 4.0 Liability of Node A to Node C (i.e. P_02) is 4.0
- Total obligations for Node A
 Total nominal liabilities for Node A (i.e. p_bar_1) is 8.0
 Payment out is min[payment out, total cash flow] i.e. min[8.0, 8.0]
- Total Dollar Payment Vector for round 1 and Node A Total Dollar Payment Vector for round 1 and Node A updated with value 8.0

Node B

- Total payments in for Node B Total payments in to Node B is 8.0
- Liabilities for Node B Liability of Node B to Node A (i.e. P_10) is 4.0 Liability of Node B to Node C (i.e. P_12) is 1.0
- Total obligations for Node B Total nominal liabilities for Node B (i.e. p_bar_2) is 5.0 Payment out is min[payment out, total cash flow] i.e. min[5.0, 14.0]
- Total Dollar Payment Vector for round 1 and Node B Total Dollar Payment Vector for round 1 and Node B updated with value 5.0

Node C

- Total payments in for Node C Total payments in to Node C is 5.0
- Liabilities for Node C Liability of Node C to Node A (i.e. P_20) is 4.0 Liability of Node C to Node B (i.e. P_21) is 4.0
- Total obligations for Node C Total nominal liabilities for Node C (i.e. p_bar_3) is 8.0 Payment out is min[payment out, total cash flow] i.e. min[8.0, 5.0] Round 1 and Node C has defaulted due to nominal obligations 8.0 being greater than cash flow 5.0
- Total Dollar Payment Vector for round 1 and Node C Total Dollar Payment Vector for round 1 and Node C updated with value 5.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 8.0 Total payment by Node B (i.e. p_2): 5.0 Total payment by Node C (i.e. p_3): 5.0 [8.0, 5.0, 5.0]

UPDATE EQUITY FOR EACH NODE

Total payments in to Node A is 8.0 Total dollar payment by Node A (i.e. p_1) is 8.0

- Equity Vector for round 1 and Node A
 Equity Vector for round 1 and Node A updated with value 8.0
 Total payments in to Node B is 8.0
 Total dollar payment by Node B (i.e. p_2) is 5.0
- Equity Vector for round 1 and Node B
 Equity Vector for round 1 and Node B updated with value 14.0
 Total payments in to Node C is 5.0
 Total dollar payment by Node C (i.e. p_3) is 5.0
- Equity Vector for round 1 and Node C Equity Vector for round 1 and Node C updated with value 5.0

EQUITY FOR EACH NODE

Equity for Node A: 8.0 Equity for Node B: 14.0 Equity for Node C: 5.0 [8.0, 14.0, 5.0]

ROUND 1 DEFAULTERS

Node C has defaulted in round 1 {'A': False, 'B': False, 'C': True}
There are defaulters in this round (i.e. round 1), algorithm will proceed for another round.

END OF ROUND 1 START OF ROUND 2 TOTAL OBLIGATION VECTOR i.e. total nominal obligations for each node i.e. p_bar_i... Total nominal obligation for Node A (i.e. p_bar_1): 8.0 Total nominal obligation for Node B (i.e. p_bar_2): 5.0 Total nominal obligation for Node C (i.e. p_bar_3): 8.0 TOTAL PAYMENT MADE PER NODE i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node... ***Node A*** - Total payments in for Node A Relative Payment in to Node A from Node B is 0.8 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node A is 6.5 - Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 4.0 Liability of Node A to Node C (i.e. P_02) is 4.0 Total nominal liabilities for Node A (i.e. p_bar_1) is 8.0 Payment out is min[payment out, total cash flow] i.e. min[8.0, 6.5] Round 2 and Node A has defaulted due to nominal obligations 8.0 being greater than cash flow 6.5 - Total Dollar Payment Vector for round 2 and Node A Total Dollar Payment Vector for round 2 and Node A updated with value 6.5 ***Node B*** - Total payments in for Node B

Relative Payment in to Node B from Node A is 0.5

Total dollar payment by Node A (i.e. p_1) is 8.0 Relative Payment in to Node B from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node B is 6.5

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 4.0
Liability of Node B to Node C (i.e. P_12) is 1.0
Total nominal liabilities for Node B (i.e. p_bar_2) is 5.0
Payment out is min[payment out, total cash flow] i.e. min[5.0, 12.5]

- Total Dollar Payment Vector for round 2 and Node B
Total Dollar Payment Vector for round 2 and Node B updated with value 5.0

Node C

- Total payments in for Node C
 Relative Payment in to Node C from Node A is 0.5
 Total dollar payment by Node A (i.e. p_1) is 8.0
 Relative Payment in to Node C from Node B is 0.2
 Total dollar payment by Node B (i.e. p_2) is 5.0
 Total payments in to Node C is 5.0
- Liabilities for Node C
 Liability of Node C to Node A (i.e. P_20) is 4.0
 Liability of Node C to Node B (i.e. P_21) is 4.0
 Total nominal liabilities for Node C (i.e. p_bar_3) is 8.0
 Payment out is min[payment out, total cash flow] i.e. min[8.0, 5.0]
 Round 2 and Node C has defaulted due to nominal obligations 8.0 being greater than cash flow 5.0
- Total Dollar Payment Vector for round 2 and Node C Total Dollar Payment Vector for round 2 and Node C updated with value 5.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 6.5 Total payment by Node B (i.e. p_2): 5.0 Total payment by Node C (i.e. p_3): 5.0 [6.5, 5.0, 5.0]

UPDATE EQUITY FOR EACH NODE

Relative Payment in to Node A from Node B is 0.8

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Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node A is 6.5 Total dollar payment by Node A (i.e. p_1) is 6.5
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- Equity Vector for round 2 and Node A
Equity Vector for round 2 and Node A updated with value 6.5
Relative Payment in to Node B from Node A is 0.5
Total dollar payment by Node A (i.e. p_1) is 8.0
Relative Payment in to Node B from Node C is 0.5
Total dollar payment by Node C (i.e. p_3) is 5.0
Total payments in to Node B is 6.5
Total dollar payment by Node B (i.e. p_2) is 5.0

- Equity Vector for round 2 and Node B
Equity Vector for round 2 and Node B updated with value 12.5
Relative Payment in to Node C from Node A is 0.5
Total dollar payment by Node A (i.e. p_1) is 8.0
Relative Payment in to Node C from Node B is 0.2
Total dollar payment by Node B (i.e. p_2) is 5.0
Total payments in to Node C is 5.0
Total dollar payment by Node C (i.e. p_3) is 5.0

- Equity Vector for round 2 and Node C Equity Vector for round 2 and Node C updated with value 5.0

EQUITY FOR EACH NODE

Equity for Node A: 6.5 Equity for Node B: 12.5 Equity for Node C: 5.0 [6.5, 12.5, 5.0]

ROUND 2 DEFAULTERS

Node A has defaulted in round 2 {'A': True, 'B': False, 'C': True} There are defaulters in this round (i.e. round 2), algorithm will proceed for another round.

END OF ROUND 2

START OF ROUND 3 TOTAL OBLIGATION VECTOR i.e. total nominal obligations for each node i.e. p_bar_i... Total nominal obligation for Node A (i.e. p_bar_1): 8.0 Total nominal obligation for Node B (i.e. p_bar_2): 5.0 Total nominal obligation for Node C (i.e. p_bar_3): 8.0 TOTAL PAYMENT MADE PER NODE i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node... ***Node A*** - Total payments in for Node A Relative Payment in to Node A from Node B is 0.8 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node A is 6.5 - Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 4.0 Liability of Node A to Node C (i.e. P_02) is 4.0 Total nominal liabilities for Node A (i.e. p_bar_1) is 8.0 Payment out is min[payment out, total cash flow] i.e. min[8.0, 6.5] Round 3 and Node A has defaulted due to nominal obligations 8.0 being greater than cash flow 6.5 - Total Dollar Payment Vector for round 3 and Node A Total Dollar Payment Vector for round 3 and Node A updated with value 6.5 ***Node B*** - Total payments in for Node B Relative Payment in to Node B from Node A is 0.5 Total dollar payment by Node A (i.e. p_1) is 6.5

Relative Payment in to Node B from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0

Total payments in to Node B is 5.75

- Liabilities for Node B
 Liability of Node B to Node A (i.e. P_10) is 4.0
 Liability of Node B to Node C (i.e. P_12) is 1.0
 Total nominal liabilities for Node B (i.e. p_bar_2) is 5.0
 Payment out is min[payment out, total cash flow] i.e. min[5.0, 11.75]
- Total Dollar Payment Vector for round 3 and Node B
 Total Dollar Payment Vector for round 3 and Node B updated with value 5.0

Node C

- Total payments in for Node C
 Relative Payment in to Node C from Node A is 0.5
 Total dollar payment by Node A (i.e. p_1) is 6.5
 Relative Payment in to Node C from Node B is 0.2
 Total dollar payment by Node B (i.e. p_2) is 5.0
 Total payments in to Node C is 4.25
- Liabilities for Node C
 Liability of Node C to Node A (i.e. P_20) is 4.0
 Liability of Node C to Node B (i.e. P_21) is 4.0
 Total nominal liabilities for Node C (i.e. p_bar_3) is 8.0
 Payment out is min[payment out, total cash flow] i.e. min[8.0, 4.25]
 Round 3 and Node C has defaulted due to nominal obligations 8.0 being greater than cash flow 4.25
- Total Dollar Payment Vector for round 3 and Node C Total Dollar Payment Vector for round 3 and Node C updated with value 4.25

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 6.5 Total payment by Node B (i.e. p_2): 5.0 Total payment by Node C (i.e. p_3): 4.25 [6.5, 5.0, 4.25]

UPDATE EQUITY FOR EACH NODE

Relative Payment in to Node A from Node B is 0.8 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0

Total payments in to Node A is 6.5 Total dollar payment by Node A (i.e. p_1) is 6.5

- Equity Vector for round 3 and Node A

 Equity Vector for round 3 and Node A updated with value 6.5

 Relative Payment in to Node B from Node A is 0.5

 Total dollar payment by Node A (i.e. p_1) is 6.5

 Relative Payment in to Node B from Node C is 0.5

 Total dollar payment by Node C (i.e. p_3) is 5.0

 Total payments in to Node B is 5.75

 Total dollar payment by Node B (i.e. p_2) is 5.0
- Equity Vector for round 3 and Node B
 Equity Vector for round 3 and Node B updated with value 11.75
 Relative Payment in to Node C from Node A is 0.5
 Total dollar payment by Node A (i.e. p_1) is 6.5
 Relative Payment in to Node C from Node B is 0.2
 Total dollar payment by Node B (i.e. p_2) is 5.0
 Total payments in to Node C is 4.25
 Total dollar payment by Node C (i.e. p_3) is 4.25
- Equity Vector for round 3 and Node C Equity Vector for round 3 and Node C updated with value 4.25

EQUITY FOR EACH NODE

Equity for Node A: 6.5 Equity for Node B: 11.75 Equity for Node C: 4.25 [6.5, 11.75, 4.25]

ROUND 3 DEFAULTERS

Node A has defaulted in round 3 {'A': True, 'B': False, 'C': True}

There are defaulters from earlier rounds but no new defaulters in the current round, algorithm will not proceed for another round.

Checking limited liability and absolute priority for Node A

Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node A from Node B is 0.8 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5

Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node A is 6.5

- Limited liability is met. Node A made a payment of 6.5 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 6.5. Total nominal obligation for Node A (i.e. p_bar_1): 8.0 Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node A from Node B is 0.8 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node A from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node A is 6.5

-Checking absolute priority for Node A in round 3. Nominal obligations is 8.0 and Dollar payments is 6.5
Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node A
All value i.e. cash flow available to Node A is 6.5
-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 8.0, Dollar payment was 6.5 and Total cash flow was 6.5

Node A in round 3 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node B

Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node B from Node A is 0.5 Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node B from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node B is 5.75

- Limited liability is met. Node B made a payment of 5.0 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 11.75. Total nominal obligation for Node B (i.e. p_bar_2): 5.0 Total dollar payment by Node B (i.e. p_2) is 5.0 Relative Payment in to Node B from Node A is 0.5 Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node B from Node C is 0.5 Total dollar payment by Node C (i.e. p_3) is 5.0 Total payments in to Node B is 5.75

-Checking absolute priority for Node B in round 3. Nominal obligations is 5.0 and Dollar payments is 5.0

-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 5.0, Dollar payment was 5.0 and Total cash flow was 11.75

Node B in round 3 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node C

Total dollar payment by Node C (i.e. p_3) is 4.25 Relative Payment in to Node C from Node A is 0.5 Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node C from Node B is 0.2 Total dollar payment by Node B (i.e. p_2) is 5.0 Total payments in to Node C is 4.25

- Limited liability is met. Node C made a payment of 4.25 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 4.25. Total nominal obligation for Node C (i.e. p_bar_3): 8.0 Total dollar payment by Node C (i.e. p_3) is 4.25 Relative Payment in to Node C from Node A is 0.5 Total dollar payment by Node A (i.e. p_1) is 6.5 Relative Payment in to Node C from Node B is 0.2 Total dollar payment by Node B (i.e. p_2) is 5.0 Total payments in to Node C is 4.25

-Checking absolute priority for Node C in round 3. Nominal obligations is 8.0 and Dollar payments is 4.25 Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node C All value i.e. cash flow available to Node C is 4.25 -Absolute priority is satisfied for Node C

- Absolute priority is met by Node C in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 8.0, Dollar payment was 4.25 and Total cash flow was 4.25

Node C in round 3 passes candidate clearing vector payment entry checks.

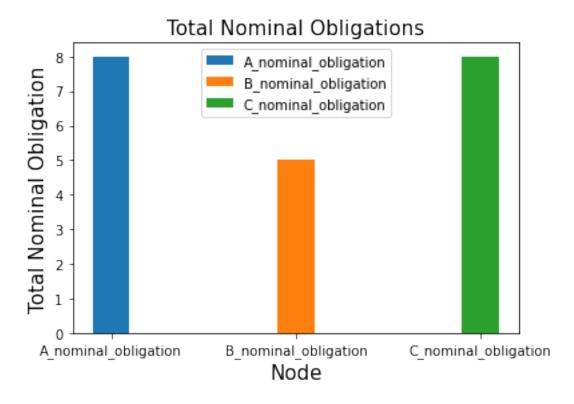
CLEARING_PAYMENT_VECTOR

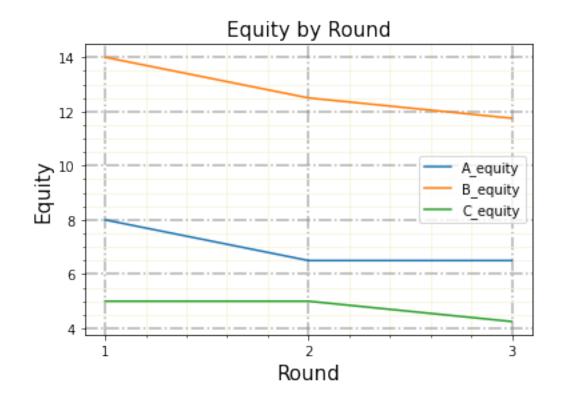
Clearing payment vector found in round 3. [6.5, 5.0, 4.25]

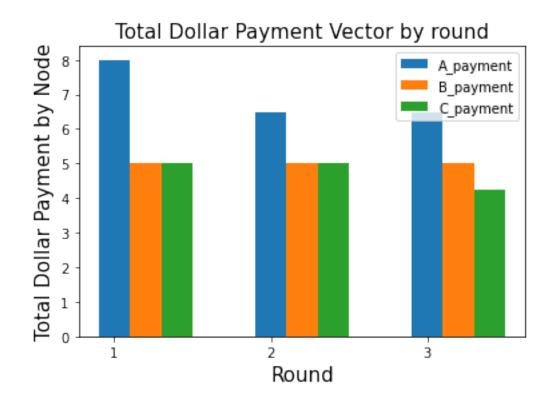
Node A pays: 6.5 Node B pays: 5.0 Node C pays: 4.25

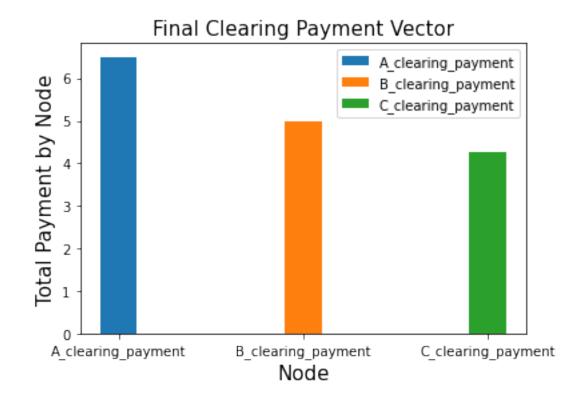
END OF ROUND 3

- Systemic Risk: Node C has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
- Systemic Risk: Node A has defaulted in round 2. The number of prior default waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e. ['C']).
- Systemic Risk: Node B has not defaulted after 3 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['C', 'A']).

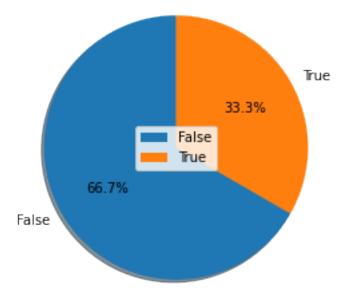








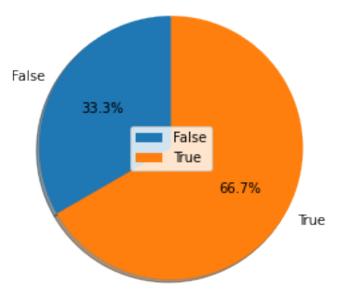
Percentage of Defaulters After Round 1



Node A has NOT defaulted in round 1 Node B has NOT defaulted in round 1

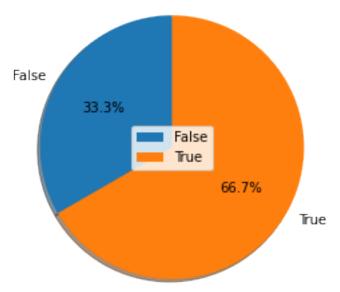
Node C has defaulted in round 1

Percentage of Defaulters After Round 2



Node A has defaulted in round 2 Node B has NOT defaulted in round 2 Node C has defaulted in round 2

Percentage of Defaulters After Round 3



Node A has defaulted in round 3 Node B has NOT defaulted in round 3 Node C has defaulted in round 3