eisenberg-noe-2001-debt-model-with-default-costs

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Data Privacy and Provenance
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Description
We model 3 scenarios for which we hold the number of agents (n=3), nominal liabilities (see below) and exogenous assets (operating cash flow) constant whilst varying alpha and beta parameters.

Alpha and Beta are the fraction of exogenous assets (outside financial network) and endogenous assets (inside financial network i.e., interbank obligations) that are realised on liquidation in the event of default. The two fractions may be different; but we generally expect that Alpha would be low, because a defaulting bank would be having to sell off its loan portfolio, probably at a knock-down price or fire sale. In contrast, Beta might be much closer to 1, because an obligation from a solvent bank would probably be paid in full (though perhaps with some negotiated discount to compensate for the inconvenience of early repayment).

Parameters
Constant params
NUM_AGENTS = 3

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NOMINAL LIABILITY MATRIX Data Frame
i.e. what node i expects to pay node j…

   A  B  C
A  0  2  9
B  7  0  9
C  3  1  0


Nominal liabilities for each node:
Liability of Node A to Node B is 2
Liability of Node A to Node C is 9
Liability of Node B to Node A is 7
Liability of Node B to Node C is 9
Liability of Node C to Node A is 3
Liability of Node C to Node B is 1
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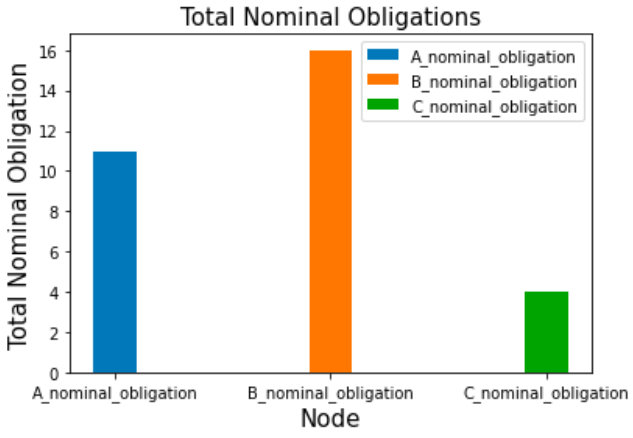


OPERATING_CASH_FLOW_VECTOR = [3, 0, 4]

Alpha and Beta params

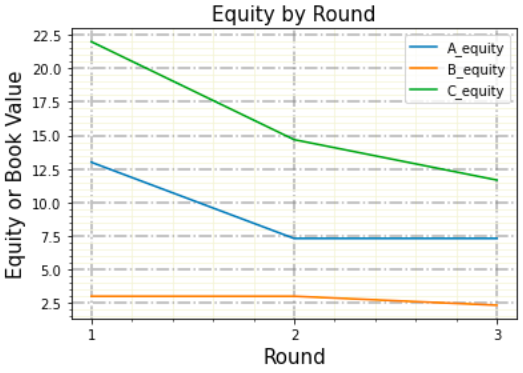
We test Alpha & Beta values of (1,1), (0.5, 0.5) and (0.1, 0.9) respectively. When Alpha=Beta=1 we have the equivalent of Eisenberg and Noe with no default costs.

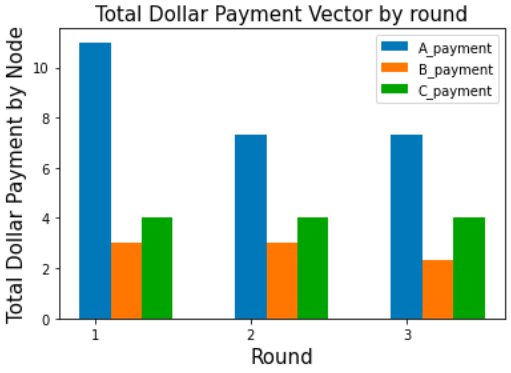
Scenarios

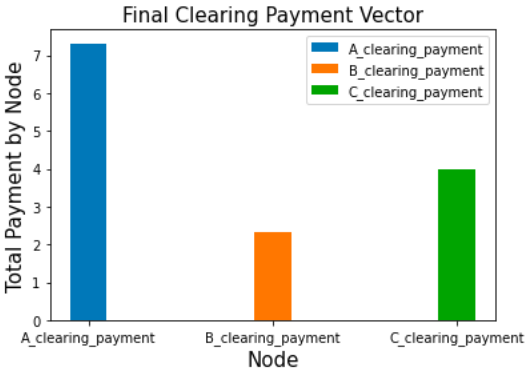
SCENARIO 3 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 1 and BETA = 1

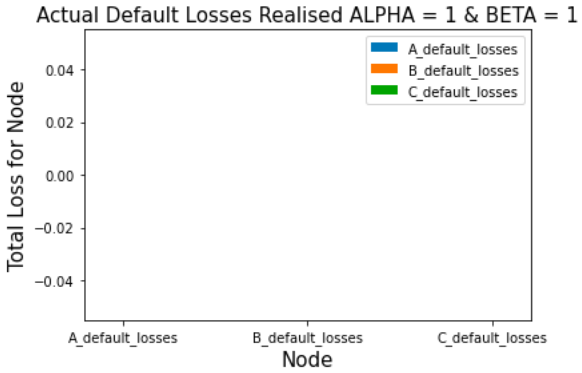
Description - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3.

- ALPHA=BETA=1 (no default costs / losses).
- Clearing payment vector found in round 3 = [7.3125, 2.3295454545454546, 4.0]
  - Node A pays: 7.3125
  - Node B pays: 2.3295454545454546
  - Node C pays: 4.0
  - Default loss incurred by Node A is: 0.0
  - Default loss incurred by Node B is: 0.0
  - Default loss incurred by Node C is: 0
- Systemic Risk
  - Node B has defaulted in round 1. The number of prior default waves is 0 - there are 3 nodes in the system (0 of which have defaulted i.e. []).
  - Node A has defaulted in round 2. The number of prior default waves is 1 - there are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
  - Node C has not defaulted after 3 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).







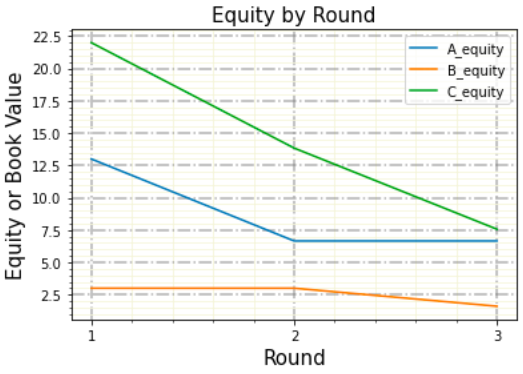


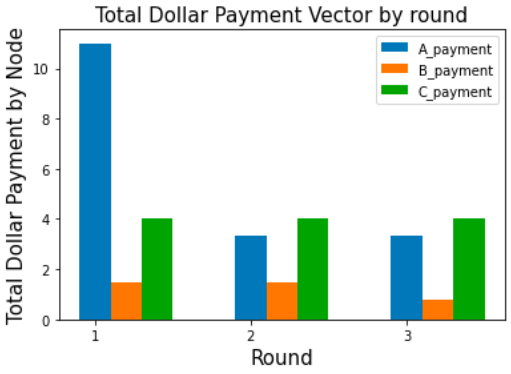
Comment – No default losses as Alpha and Beta are set to 1 which assumes full recovery.

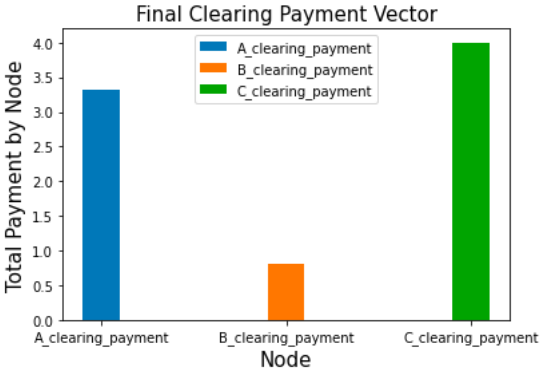
SCENARIO 4 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.5 and BETA = 0.5

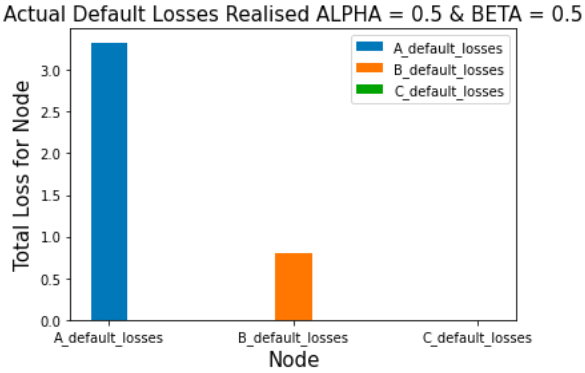
Description - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3.

- ALPHA = 0.5, BETA = 0.5
- Clearing payment vector found in round 3 = [3.328125, 0.8025568181818181, 4.0]
    - Node A pays: 3.328125
    - Node B pays: 0.8025568181818181
    - Node C pays: 4.0
    - Default loss incurred by Node A is: 3.328125
    - Default loss incurred by Node B is: 0.8025568181818181
    - Default loss incurred by Node C is: 0
- Systemic Risk:
    - Node B has defaulted in round 1. The number of prior default waves is 0 - there are 3 nodes in the system (0 of which have defaulted i.e. []).
    - Node A has defaulted in round 2. The number of prior default waves is 1 - there are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
    - Node C has not defaulted after 3 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).







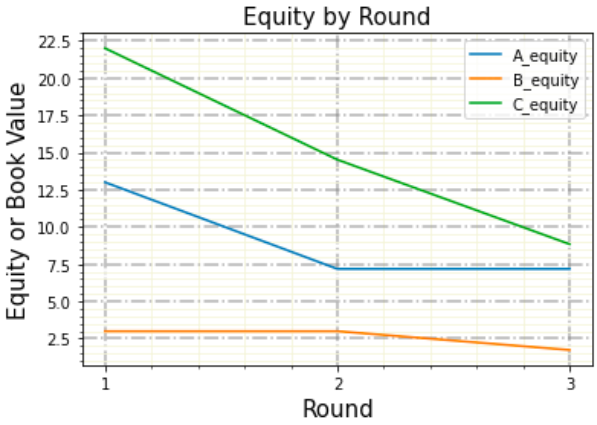


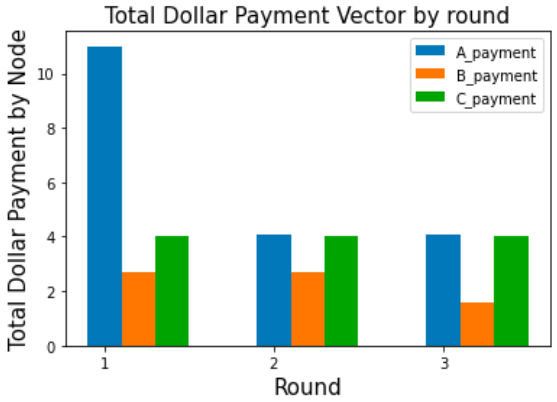
Comment – Default losses are incurred as 100% percent recoveries of assets no longer holds. This also drops the overall equity at each node as well as the final clearing payments, as compared to Scenario 3.

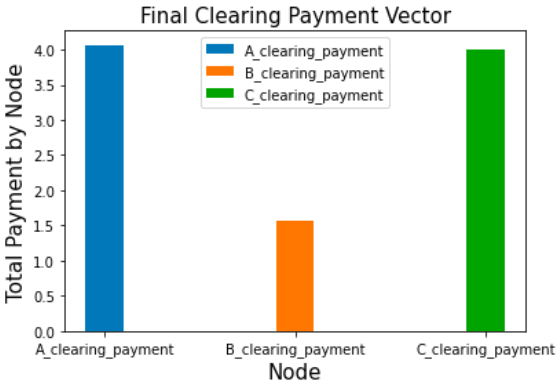
SCENARIO 5 - Running Debt Model in MANUAL mode with 3 nodes and ALPHA = 0.1 and BETA = 0.9

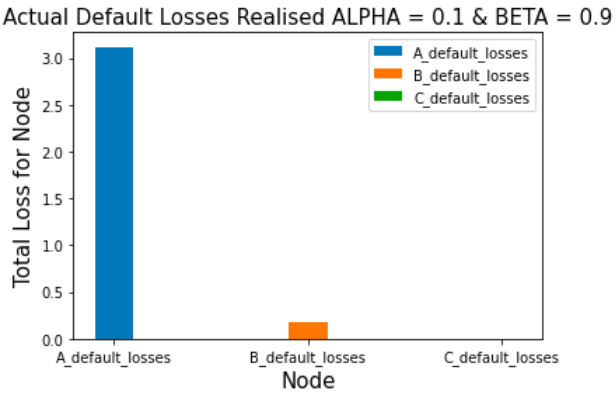
Scenario 5 - Firm B defaults in first round, Firm A in second round, algorithm terminates round 3.

- ALPHA = 0.1, BETA = 0.9
- Clearing payment vector found in round 3 = [4.063125, 1.564875, 4.0]
  - Node A pays: 4.063125
  - Node B pays: 1.564875
  - Node C pays: 4.0
  - Default loss incurred by Node A is: 3.118125
  - Default loss incurred by Node B is: 0.173875
  - Default loss incurred by Node C is: 0
- Systemic Risk:
  - Node B has defaulted in round 1. The number of prior default waves is 0 -there are 3 nodes in the system (0 of which have defaulted i.e. []).
  - Node A has defaulted in round 2. The number of prior default waves is 1 -there are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
  - Node C has not defaulted after 3 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).









Comment - Lower default costs due to larger Beta which recovers more from endogenous interbank assets. Although Equity and Clearing payment vector are lower than Scenario 3, they are greater than Scenario 4 due to the larger Beta. Solvent banks have an incentive to pay obligations to lower risk in overall system.