eisenberg-noe-2001-debt-model

October 23, 2021

```
DEBT MODEL MODE
Running Debt Model in MANUAL mode with 3 nodes...
Scenario 3 - No firm defaults during first round, MODE == 'MANUAL', NUM_AGENTS =
3, NOMINAL_LIABILITY_MATRIX = np.array([[0,2,9],[7,0,9],[3,1,0]]),
OPERATING_CASH_FLOW_VECTOR = [3, 0, 4]
AGENT LABELS
Agent labels ['A', 'B', 'C']
NOMINAL LIABILITY MATRIX Data Frame
i.e. what node i expects to pay node j...
   A B C
A 0 2 9
B 7 0 9
C 3 1 0
Nominal liabilities for each node:
Liability of Node A to Node B is 2
Liability of Node A to Node C is 9
Liability of Node B to Node A is 7
Liability of Node B to Node C is 9
Liability of Node C to Node A is 3
Liability of Node C to Node B is 1
```

NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame i.e. what node j expects to receive from i...

```
A B C
A 0 7 3
B 2 0 1
C 9 9 0
```

```
Node A expects to receive 7 from Node B Node A expects to receive 3 from Node C Node B expects to receive 2 from Node A Node B expects to receive 1 from Node C Node C expects to receive 9 from Node A Node C expects to receive 9 from Node B
```

OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 3 Exogenous cash flow for Node B: 0 Exogenous cash flow for Node C: 4 [3, 0, 4]

CALCULATING RELATIVE LIABILITIES FOR EACH NODE

Node A

- Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 2.0 Liability of Node A to Node C (i.e. P_02) is 9.0
- Total obligations for Node A Total Obligation Vector updated in round 1 for Node A with value 11.0 Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0
- Relative Liabilities of Node A
 Relative Liability of Node A to Node B is 0.181818181818182
 Relative Liability of Node A to Node C is 0.81818181818182
 Sum of Relative Liabilities for Node A is 1.0

Node B

- Liabilities for Node B Liability of Node B to Node A (i.e. P_10) is 7.0 Liability of Node B to Node C (i.e. P_12) is 9.0
- Total obligations for Node B

Total Obligation Vector updated in round 1 for Node B with value 16.0 Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0

- Relative Liabilities of Node B
Relative Liability of Node B to Node A is 0.4375
Relative Liability of Node B to Node C is 0.5625
Sum of Relative Liabilities for Node B is 1.0

Node C

- Liabilities for Node C Liability of Node C to Node A (i.e. P_20) is 3.0 Liability of Node C to Node B (i.e. P_21) is 1.0
- Total obligations for Node C Total Obligation Vector updated in round 1 for Node C with value 4.0 Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0
- Relative Liabilities of Node C
 Relative Liability of Node C to Node A is 0.75
 Relative Liability of Node C to Node B is 0.25
 Sum of Relative Liabilities for Node C is 1.0

RELATIVE LIABILITY MATRIX Data Frame

A B C
A 0.0000 0.181818 0.818182
B 0.4375 0.000000 0.562500
C 0.7500 0.250000 0.000000

RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

	Α	В	C	Relative Liability Total	CORRECT VALUE?
Α	0.0000	0.181818	0.818182	1.0	True
В	0.4375	0.000000	0.562500	1.0	True
C	0.7500	0.250000	0.000000	1.0	True

RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame i.e. what node i expects to receive from j in relative terms...

```
В
A 0.000000 0.4375 0.75
B 0.181818 0.0000 0.25
C 0.818182 0.5625 0.00
***Expected nominal payments in for Node A - both proportion and total amount***
Node A expects to receive proportion 0.4375 from Node B
Node A expects to receive proportion 0.75 from Node C
Total payments in to Node A is 10.0
***Expected nominal payments in for Node B - both proportion and total amount***
Node B expects to receive proportion 0.181818181818182 from Node A
Node B expects to receive proportion 0.25 from Node C
Total payments in to Node B is 3.0
***Expected nominal payments in for Node C - both proportion and total amount***
Node C expects to receive proportion 0.8181818181818182 from Node A
Node C expects to receive proportion 0.5625 from Node B
Total payments in to Node C is 18.0
_____
START OF ROUND 1
TOTAL OBLIGATION VECTOR
i.e. total nominal obligations for each node i.e. p_bar_i...
Total nominal obligation for Node A (i.e. p_bar_1): 11.0
Total nominal obligation for Node B (i.e. p_bar_2): 16.0
Total nominal obligation for Node C (i.e. p_bar_3): 4.0
```

TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node...

Node A

- Total payments in for Node A Total payments in to Node A is 10.0
- Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 2.0 Liability of Node A to Node C (i.e. P_02) is 9.0
- Total obligations for Node A Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0 Payment out is min[payment out, total cash flow] i.e. min[11.0, 13.0]
- Total Dollar Payment Vector for round 1 and Node A
 Total Dollar Payment Vector for round 1 and Node A updated with value 11.0

Node B

- Total payments in for Node B Total payments in to Node B is 3.0
- Liabilities for Node B Liability of Node B to Node A (i.e. P_10) is 7.0 Liability of Node B to Node C (i.e. P_12) is 9.0
- Total obligations for Node B
 Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0
 Payment out is min[payment out, total cash flow] i.e. min[16.0, 3.0]
 Round 1 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 3.0
- Total Dollar Payment Vector for round 1 and Node B Total Dollar Payment Vector for round 1 and Node B updated with value 3.0

Node C

- Total payments in for Node C Total payments in to Node C is 18.0
- Liabilities for Node C Liability of Node C to Node A (i.e. P_20) is 3.0 Liability of Node C to Node B (i.e. P_21) is 1.0
- Total obligations for Node C Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0 Payment out is min[payment out, total cash flow] i.e. min[4.0, 22.0]
- Total Dollar Payment Vector for round 1 and Node C

Total Dollar Payment Vector for round 1 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 11.0 Total payment by Node B (i.e. p_2): 3.0 Total payment by Node C (i.e. p_3): 4.0 [11.0, 3.0, 4.0]

UPDATE EQUITY FOR EACH NODE

Total payments in to Node A is 10.0

Total dollar payment by Node A (i.e. p_1) is 11.0

- Equity Vector for round 1 and Node A
 Equity Vector for round 1 and Node A updated with value 13.0
 Total payments in to Node B is 3.0
 Total dollar payment by Node B (i.e. p_2) is 3.0
- Equity Vector for round 1 and Node B
 Equity Vector for round 1 and Node B updated with value 3.0
 Total payments in to Node C is 18.0
 Total dollar payment by Node C (i.e. p_3) is 4.0
- Equity Vector for round 1 and Node C Equity Vector for round 1 and Node C updated with value 22.0

EQUITY FOR EACH NODE

Equity for Node A: 13.0 Equity for Node B: 3.0 Equity for Node C: 22.0 [13.0, 3.0, 22.0]

ROUND 1 DEFAULTERS

Node B has defaulted in round 1 {'A': False, 'B': True, 'C': False}
There are defaulters in this round (i.e. round 1), algorithm will proceed for another round.

END OF ROUND 1 START OF ROUND 2 TOTAL OBLIGATION VECTOR i.e. total nominal obligations for each node i.e. p_bar_i... Total nominal obligation for Node A (i.e. p_bar_1): 11.0 Total nominal obligation for Node B (i.e. p_bar_2): 16.0 Total nominal obligation for Node C (i.e. p_bar_3): 4.0 TOTAL PAYMENT MADE PER NODE i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node... ***Node A*** - Total payments in for Node A Relative Payment in to Node A from Node B is 0.4375 Total dollar payment by Node B (i.e. p_2) is 3.0 Relative Payment in to Node A from Node C is 0.75 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node A is 4.3125 - Liabilities for Node A Liability of Node A to Node B (i.e. P_01) is 2.0 Liability of Node A to Node C (i.e. P_02) is 9.0 Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0 Payment out is min[payment out, total cash flow] i.e. min[11.0, 7.3125] Round 2 and Node A has defaulted due to nominal obligations 11.0 being greater than cash flow 7.3125 - Total Dollar Payment Vector for round 2 and Node A Total Dollar Payment Vector for round 2 and Node A updated with value 7.3125 ***Node B***

- Total payments in for Node B

```
Relative Payment in to Node B from Node A is 0.18181818181818182 Total dollar payment by Node A (i.e. p_1) is 11.0 Relative Payment in to Node B from Node C is 0.25 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node B is 3.0
```

- Liabilities for Node B
 Liability of Node B to Node A (i.e. P_10) is 7.0
 Liability of Node B to Node C (i.e. P_12) is 9.0
 Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0
 Payment out is min[payment out, total cash flow] i.e. min[16.0, 3.0]
 Round 2 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 3.0
- Total Dollar Payment Vector for round 2 and Node B
 Total Dollar Payment Vector for round 2 and Node B updated with value 3.0

Node C

- Total payments in for Node C
 Relative Payment in to Node C from Node A is 0.81818181818182
 Total dollar payment by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node C from Node B is 0.5625
 Total dollar payment by Node B (i.e. p_2) is 3.0
 Total payments in to Node C is 10.6875
- Liabilities for Node C
 Liability of Node C to Node A (i.e. P_20) is 3.0
 Liability of Node C to Node B (i.e. P_21) is 1.0
 Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0
 Payment out is min[payment out, total cash flow] i.e. min[4.0, 14.6875]
- Total Dollar Payment Vector for round 2 and Node C Total Dollar Payment Vector for round 2 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 7.3125 Total payment by Node B (i.e. p_2): 3.0 Total payment by Node C (i.e. p_3): 4.0 [7.3125, 3.0, 4.0]

UPDATE EQUITY FOR EACH NODE

Relative Payment in to Node A from Node B is 0.4375 Total dollar payment by Node B (i.e. p_2) is 3.0 Relative Payment in to Node A from Node C is 0.75 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node A is 4.3125 Total dollar payment by Node A (i.e. p_1) is 7.3125

- Equity Vector for round 2 and Node A

Equity Vector for round 2 and Node A updated with value 7.3125

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e. p_1) is 11.0

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e. p_3) is 4.0

Total payments in to Node B is 3.0

Total dollar payment by Node B (i.e. p_2) is 3.0

- Equity Vector for round 2 and Node B
Equity Vector for round 2 and Node B updated with value 3.0
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 11.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 3.0
Total payments in to Node C is 10.6875
Total dollar payment by Node C (i.e. p_3) is 4.0

- Equity Vector for round 2 and Node C Equity Vector for round 2 and Node C updated with value 14.6875

EQUITY FOR EACH NODE

Equity for Node A: 7.3125 Equity for Node B: 3.0 Equity for Node C: 14.6875 [7.3125, 3.0, 14.6875]

ROUND 2 DEFAULTERS

Node A has defaulted in round 2 {'A': True, 'B': True, 'C': False}
There are defaulters in this round (i.e. round 2), algorithm will proceed for another round.

END OF ROUND 2

START OF ROUND 3

TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p_bar_i...

Total nominal obligation for Node A (i.e. p_bar_1): 11.0

Total nominal obligation for Node B (i.e. p_bar_2): 16.0

Total nominal obligation for Node C (i.e. p_bar_3): 4.0

TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node...

Node A

- Total payments in for Node A
 Relative Payment in to Node A from Node B is 0.4375
 Total dollar payment by Node B (i.e. p_2) is 3.0
 Relative Payment in to Node A from Node C is 0.75
 Total dollar payment by Node C (i.e. p_3) is 4.0
 Total payments in to Node A is 4.3125
- Liabilities for Node A
 Liability of Node A to Node B (i.e. P_01) is 2.0
 Liability of Node A to Node C (i.e. P_02) is 9.0
 Total nominal liabilities for Node A (i.e. p_bar_1) is 11.0
 Payment out is min[payment out, total cash flow] i.e. min[11.0, 7.3125]
 Round 3 and Node A has defaulted due to nominal obligations 11.0 being greater than cash flow 7.3125
- Total Dollar Payment Vector for round 3 and Node A Total Dollar Payment Vector for round 3 and Node A updated with value 7.3125

Node B

- Total payments in for Node B
Relative Payment in to Node B from Node A is 0.181818181818182
Total dollar payment by Node A (i.e. p_1) is 7.3125
Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node B is 2.3295454545454546

- Liabilities for Node B

Liability of Node B to Node A (i.e. P_10) is 7.0

Liability of Node B to Node C (i.e. P_12) is 9.0

Total nominal liabilities for Node B (i.e. p_bar_2) is 16.0

Payment out is min[payment out, total cash flow] i.e. min[16.0,

2.32954545454545461

Round 3 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 2.3295454545454546

- Total Dollar Payment Vector for round 3 and Node B Total Dollar Payment Vector for round 3 and Node B updated with value 2.32954545454546

Node C

- Total payments in for Node C
 Relative Payment in to Node C from Node A is 0.81818181818182
 Total dollar payment by Node A (i.e. p_1) is 7.3125
 Relative Payment in to Node C from Node B is 0.5625
 Total dollar payment by Node B (i.e. p_2) is 3.0
 Total payments in to Node C is 7.6704545454546
- Liabilities for Node C
 Liability of Node C to Node A (i.e. P_20) is 3.0
 Liability of Node C to Node B (i.e. P_21) is 1.0
 Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0
 Payment out is min[payment out, total cash flow] i.e. min[4.0, 11.6704545454547]
- Total Dollar Payment Vector for round 3 and Node C Total Dollar Payment Vector for round 3 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 7.3125

Total payment by Node B (i.e. p_2): 2.3295454545454546

Total payment by Node C (i.e. p_3): 4.0

[7.3125, 2.3295454545454546, 4.0]

UPDATE EQUITY FOR EACH NODE

Relative Payment in to Node A from Node B is 0.4375 Total dollar payment by Node B (i.e. p_2) is 3.0 Relative Payment in to Node A from Node C is 0.75 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node A is 4.3125 Total dollar payment by Node A (i.e. p_1) is 7.3125

- Equity Vector for round 3 and Node A

Equity Vector for round 3 and Node A updated with value 7.3125

Relative Payment in to Node B from Node A is 0.18181818181818182

Total dollar payment by Node A (i.e. p_1) is 7.3125

Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e. p_3) is 4.0

Total payments in to Node B is 2.32954545454546

Total dollar payment by Node B (i.e. p_2) is 2.3295454545454546

- Equity Vector for round 3 and Node B
Equity Vector for round 3 and Node B updated with value 2.3295454545454646
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment by Node A (i.e. p_1) is 7.3125
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment by Node B (i.e. p_2) is 3.0
Total payments in to Node C is 7.6704545454546
Total dollar payment by Node C (i.e. p_3) is 4.0

EQUITY FOR EACH NODE

Equity for Node A: 7.3125

Equity for Node B: 2.3295454545454546 Equity for Node C: 11.670454545454547

[7.3125, 2.3295454545454546, 11.670454545454547]

ROUND 3 DEFAULTERS

Node A has defaulted in round 3 {'A': True, 'B': True, 'C': False}

There are defaulters from earlier rounds but no new defaulters in the current round, algorithm will not proceed for another round.

Checking limited liability and absolute priority for Node A

Total dollar payment by Node A (i.e. p_1) is 7.3125 Relative Payment in to Node A from Node B is 0.4375 Total dollar payment by Node B (i.e. p_2) is 3.0 Relative Payment in to Node A from Node C is 0.75 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node A is 4.3125

- Limited liability is met. Node A made a payment of 7.3125 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 7.3125. Total nominal obligation for Node A (i.e. p_bar_1): 11.0

Total dollar payment by Node A (i.e. p_1) is 7.3125

Relative Payment in to Node A from Node B is 0.4375

Total dollar payment by Node B (i.e. p_2) is 3.0

Relative Payment in to Node A from Node C is 0.75

Total dollar payment by Node C (i.e. p_3) is 4.0

Total payments in to Node A is 4.3125

-Checking absolute priority for Node A in round 3. Nominal obligations is 11.0 and Dollar payments is 7.3125
Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node A
All value i.e. cash flow available to Node A is 7.3125
-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 11.0, Dollar payment was 7.3125 and Total cash flow was 7.3125

Node A in round 3 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node B

Total dollar payment by Node B (i.e. p_2) is 2.32954545454546 Relative Payment in to Node B from Node A is 0.181818181818182 Total dollar payment by Node A (i.e. p_1) is 7.3125 Relative Payment in to Node B from Node C is 0.25 Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node B is 2.32954545454546

- Limited liability is met. Node B made a payment of 2.3295454545454546 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 2.32954545454546.

Total nominal obligation for Node B (i.e. p_bar_2): 16.0 Total dollar payment by Node B (i.e. p_2) is 2.32954545454546 Relative Payment in to Node B from Node A is 0.181818181818182 Total dollar payment by Node A (i.e. p_1) is 7.3125 Relative Payment in to Node B from Node C is 0.25

Total dollar payment by Node C (i.e. p_3) is 4.0 Total payments in to Node B is 2.3295454545454546

-Checking absolute priority for Node B in round 3. Nominal obligations is 16.0 and Dollar payments is 2.32954545454546

Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node B

All value i.e. cash flow available to Node B is 2.32954545454546

-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 16.0, Dollar payment was 2.32954545454546 and Total cash flow was 2.3295454545454546

Node B in round 3 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node C

Total dollar payment by Node C (i.e. p_3) is 4.0 Relative Payment in to Node C from Node A is 0.81818181818182 Total dollar payment by Node A (i.e. p_1) is 7.3125 Relative Payment in to Node C from Node B is 0.5625 Total dollar payment by Node B (i.e. p_2) is 3.0 Total payments in to Node C is 7.6704545454546

- Limited liability is met. Node C made a payment of 4.0 in round 3 which is less than or equal to the cash flow (payments in + exogenous cash) of 11.6704545454547.

Total nominal obligation for Node C (i.e. p_bar_3): 4.0 Total dollar payment by Node C (i.e. p_3) is 4.0 Relative Payment in to Node C from Node A is 0.81818181818182 Total dollar payment by Node A (i.e. p_1) is 7.3125 Relative Payment in to Node C from Node B is 0.5625 Total dollar payment by Node B (i.e. p_2) is 3.0 Total payments in to Node C is 7.6704545454546

- -Checking absolute priority for Node C in round 3. Nominal obligations is 4.0 and Dollar payments is 4.0
- -Absolute priority is satisfied for Node C
- Absolute priority is met by Node C in round 3 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 4.0, Dollar payment was 4.0 and Total cash flow was 11.6704545454547

Node C in round 3 passes candidate clearing vector payment entry checks.

CLEARING_PAYMENT_VECTOR

Clearing payment vector found in round 3.

[7.3125, 2.3295454545454546, 4.0]

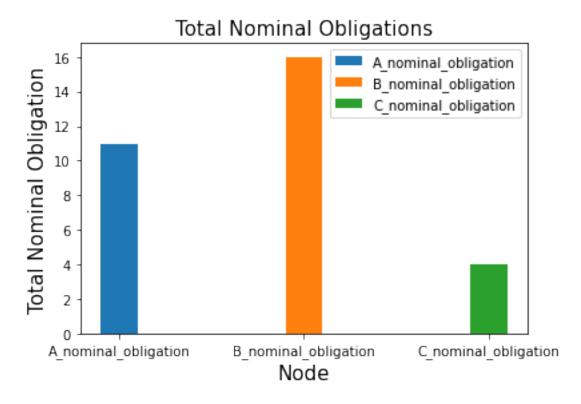
Node A pays: 7.3125

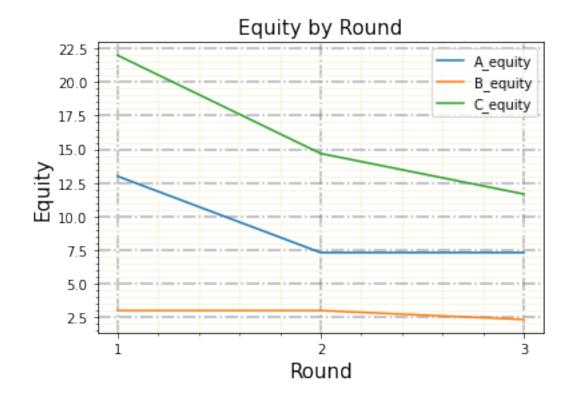
Node B pays: 2.3295454545454546

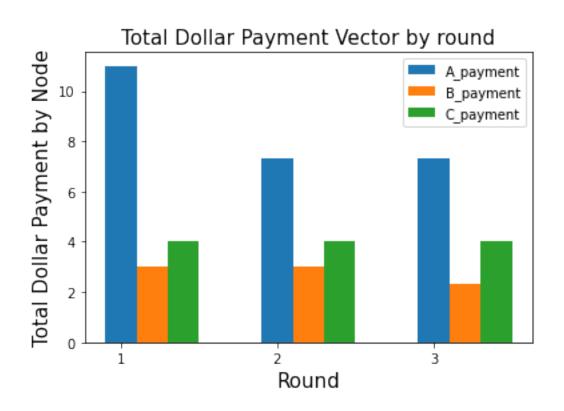
Node C pays: 4.0

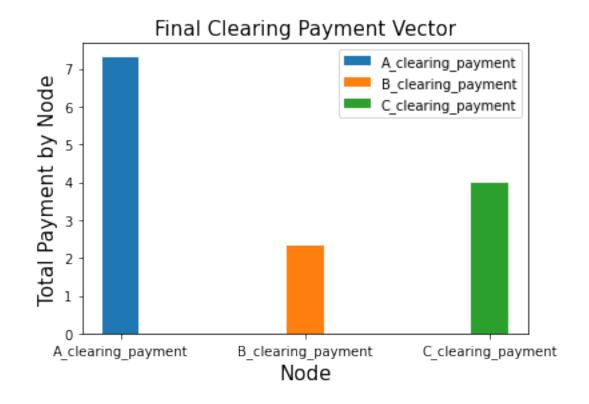
END OF ROUND 3

- Systemic Risk: Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
- Systemic Risk: Node A has defaulted in round 2. The number of prior default waves is 1. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
- Systemic Risk: Node C has not defaulted after 3 rounds. There are 3 nodes in the system (2 of which have defaulted i.e. ['B', 'A']).

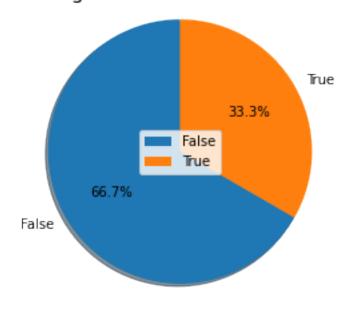








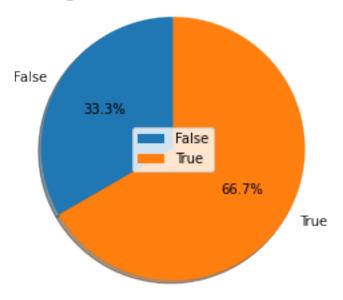
Percentage of Defaulters After Round 1



Node A has NOT defaulted in round 1 Node B has defaulted in round 1

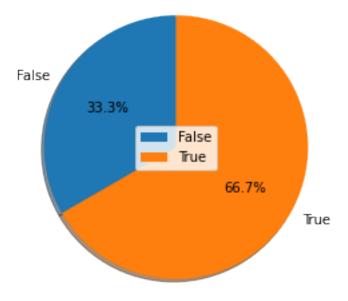
Node C has NOT defaulted in round 1

Percentage of Defaulters After Round 2



Node A has defaulted in round 2 Node B has defaulted in round 2 Node C has NOT defaulted in round 2

Percentage of Defaulters After Round 3



Node A has defaulted in round 3 Node B has defaulted in round 3 Node C has NOT defaulted in round 3