eisenberg-noe-2001-debt-model

October 23, 2021

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DEBT MODEL MODE

Running Debt Model in MANUAL mode with 3 nodes…

Scenario 2 - No firm defaults during first round, MODE == 'MANUAL', NUM_AGENTS =
3, NOMINAL_LIABILITY_MATRIX = np.array([[0,2,1],[3,0,1],[2,5,0]]),
OPERATING_CASH_FLOW_VECTOR = [5, 4, 2]

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AGENT LABELS

Agent labels ['A', 'B', 'C']

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NOMINAL LIABILITY MATRIX Data Frame
i.e. what node i expects to pay node j…

   A  B  C
A  0  2  1
B  3  0  1
C  2  5  0



Nominal liabilities for each node:
Liability of Node A to Node B is 2
Liability of Node A to Node C is 1
Liability of Node B to Node A is 3
Liability of Node B to Node C is 1
Liability of Node C to Node A is 2
Liability of Node C to Node B is 5

NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame
i.e. what node j expects to receive from i…
```

```
   A  B  C
A  0  3  2
B  2  0  5
C  1  1  0


Node A expects to receive 3 from Node B
Node A expects to receive 2 from Node C
Node B expects to receive 2 from Node A
Node B expects to receive 5 from Node C
Node C expects to receive 1 from Node A
Node C expects to receive 1 from Node B

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OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 5
Exogenous cash flow for Node B: 4
Exogenous cash flow for Node C: 2
[5, 4, 2]

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CALCULATING RELATIVE LIABILITIES FOR EACH NODE

***Node A***

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 1.0

- Total obligations for Node A
Total Obligation Vector updated in round 1 for Node A with value 3.0
Total nominal liabilities for Node A (i.e. p_bar_1) is 3.0

- Relative Liabilities of Node A
Relative Liability of Node A to Node B is 0.6666666666666666
Relative Liability of Node A to Node C is 0.3333333333333333
Sum of Relative Liabilities for Node A is 1.0

***Node B***

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 3.0
Liability of Node B to Node C (i.e. P_12) is 1.0

- Total obligations for Node B
```

```
Total Obligation Vector updated in round 1 for Node B with value 4.0
Total nominal liabilities for Node B (i.e. p_bar_2) is 4.0

- Relative Liabilities of Node B
Relative Liability of Node B to Node A is 0.75
Relative Liability of Node B to Node C is 0.25
Sum of Relative Liabilities for Node B is 1.0

***Node C***

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 2.0
Liability of Node C to Node B (i.e. P_21) is 5.0

- Total obligations for Node C
Total Obligation Vector updated in round 1 for Node C with value 7.0
Total nominal liabilities for Node C (i.e. p_bar_3) is 7.0

- Relative Liabilities of Node C
Relative Liability of Node C to Node A is 0.2857142857142857
Relative Liability of Node C to Node B is 0.7142857142857143
Sum of Relative Liabilities for Node C is 1.0

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RELATIVE LIABILITY MATRIX Data Frame

          A         B         C
A  0.000000  0.666667  0.333333
B  0.750000  0.000000  0.250000
C  0.285714  0.714286  0.000000



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RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

          A         B         C  Relative Liability Total  CORRECT VALUE?
A  0.000000  0.666667  0.333333                       1.0            True
B  0.750000  0.000000  0.250000                       1.0            True
C  0.285714  0.714286  0.000000                       1.0            True



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RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame
i.e. what node i expects to receive from j in relative terms…
```

```
          A     B         C
A  0.000000  0.75  0.285714
B  0.666667  0.00  0.714286
C  0.333333  0.25  0.000000


***Expected nominal payments in for Node A - both proportion and total amount***

Node A expects to receive proportion 0.75 from Node B
Node A expects to receive proportion 0.2857142857142857 from Node C
Total payments in to Node A is 5.0

***Expected nominal payments in for Node B - both proportion and total amount***

Node B expects to receive proportion 0.6666666666666666 from Node A
Node B expects to receive proportion 0.7142857142857143 from Node C
Total payments in to Node B is 7.0

***Expected nominal payments in for Node C - both proportion and total amount***

Node C expects to receive proportion 0.3333333333333333 from Node A
Node C expects to receive proportion 0.25 from Node B
Total payments in to Node C is 2.0

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START OF ROUND 1

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TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p_bar_i…
Total nominal obligation for Node A (i.e. p_bar_1): 3.0
Total nominal obligation for Node B (i.e. p_bar_2): 4.0
Total nominal obligation for Node C (i.e. p_bar_3): 7.0

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TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for
each node…
```

```
***Node A***

- Total payments in for Node A
Total payments in to Node A is 5.0

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 1.0

- Total obligations for Node A
Total nominal liabilities for Node A (i.e. p_bar_1) is 3.0
Payment out is min[payment out, total cash flow] i.e. min[3.0, 10.0]

 - Total Dollar Payment Vector for round 1 and Node A
Total Dollar Payment Vector for round 1 and Node A updated with value 3.0

***Node B***

- Total payments in for Node B
Total payments in to Node B is 7.0

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 3.0
Liability of Node B to Node C (i.e. P_12) is 1.0

- Total obligations for Node B
Total nominal liabilities for Node B (i.e. p_bar_2) is 4.0
Payment out is min[payment out, total cash flow] i.e. min[4.0, 11.0]

 - Total Dollar Payment Vector for round 1 and Node B
Total Dollar Payment Vector for round 1 and Node B updated with value 4.0

***Node C***

- Total payments in for Node C
Total payments in to Node C is 2.0

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 2.0
Liability of Node C to Node B (i.e. P_21) is 5.0

- Total obligations for Node C
Total nominal liabilities for Node C (i.e. p_bar_3) is 7.0
Payment out is min[payment out, total cash flow] i.e. min[7.0, 4.0]
Round 1 and Node C has defaulted due to nominal obligations 7.0 being greater
than cash flow 4.0

 - Total Dollar Payment Vector for round 1 and Node C
```

```
Total Dollar Payment Vector for round 1 and Node C updated with value 4.0

-----------------------------------------------------

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 3.0
Total payment by Node B (i.e. p_2): 4.0
Total payment by Node C (i.e. p_3): 4.0
[3.0, 4.0, 4.0]

-----------------------------------------------------

UPDATE EQUITY FOR EACH NODE

Total payments in to Node A is 5.0
Total dollar payment by Node A (i.e. p_1) is 3.0

 - Equity Vector for round 1 and Node A
Equity Vector for round 1 and Node A updated with value 10.0
Total payments in to Node B is 7.0
Total dollar payment by Node B (i.e. p_2) is 4.0

 - Equity Vector for round 1 and Node B
Equity Vector for round 1 and Node B updated with value 11.0
Total payments in to Node C is 2.0
Total dollar payment by Node C (i.e. p_3) is 4.0

 - Equity Vector for round 1 and Node C
Equity Vector for round 1 and Node C updated with value 4.0

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EQUITY FOR EACH NODE

Equity for Node A: 10.0
Equity for Node B: 11.0
Equity for Node C: 4.0
[10.0, 11.0, 4.0]

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ROUND 1 DEFAULTERS

Node C has defaulted in round 1
{'A': False, 'B': False, 'C': True}
There are defaulters in this round (i.e. round 1), algorithm will proceed for
another round.
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END OF ROUND 1

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START OF ROUND 2

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TOTAL OBLIGATION VECTOR

i.e. total nominal obligations for each node i.e. p_bar_i…
Total nominal obligation for Node A (i.e. p_bar_1): 3.0
Total nominal obligation for Node B (i.e. p_bar_2): 4.0
Total nominal obligation for Node C (i.e. p_bar_3): 7.0

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TOTAL PAYMENT MADE PER NODE

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for
each node…

***Node A***

- Total payments in for Node A
Relative Payment in to Node A from Node B is 0.75
Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node A from Node C is 0.2857142857142857
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 4.142857142857142

- Liabilities for Node A
Liability of Node A to Node B (i.e. P_01) is 2.0
Liability of Node A to Node C (i.e. P_02) is 1.0
Total nominal liabilities for Node A (i.e. p_bar_1) is 3.0
Payment out is min[payment out, total cash flow] i.e. min[3.0,
9.142857142857142]

 - Total Dollar Payment Vector for round 2 and Node A
Total Dollar Payment Vector for round 2 and Node A updated with value 3.0

***Node B***

- Total payments in for Node B
Relative Payment in to Node B from Node A is 0.6666666666666666
```

```
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.7142857142857143
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 4.857142857142858

- Liabilities for Node B
Liability of Node B to Node A (i.e. P_10) is 3.0
Liability of Node B to Node C (i.e. P_12) is 1.0
Total nominal liabilities for Node B (i.e. p_bar_2) is 4.0
Payment out is min[payment out, total cash flow] i.e. min[4.0,
8.857142857142858]

 - Total Dollar Payment Vector for round 2 and Node B
Total Dollar Payment Vector for round 2 and Node B updated with value 4.0

***Node C***

- Total payments in for Node C
Relative Payment in to Node C from Node A is 0.3333333333333333
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.25
Total dollar payment by Node B (i.e. p_2) is 4.0
Total payments in to Node C is 2.0

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 2.0
Liability of Node C to Node B (i.e. P_21) is 5.0
Total nominal liabilities for Node C (i.e. p_bar_3) is 7.0
Payment out is min[payment out, total cash flow] i.e. min[7.0, 4.0]
Round 2 and Node C has defaulted due to nominal obligations 7.0 being greater
than cash flow 4.0

 - Total Dollar Payment Vector for round 2 and Node C
Total Dollar Payment Vector for round 2 and Node C updated with value 4.0

------------------------------------------------------

TOTAL PAYMENT VECTOR

Total payment by Node A (i.e. p_1): 3.0
Total payment by Node B (i.e. p_2): 4.0
Total payment by Node C (i.e. p_3): 4.0
[3.0, 4.0, 4.0]

------------------------------------------------------

UPDATE EQUITY FOR EACH NODE
```

```
Relative Payment in to Node A from Node B is 0.75
Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node A from Node C is 0.2857142857142857
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 4.142857142857142
Total dollar payment by Node A (i.e. p_1) is 3.0

 - Equity Vector for round 2 and Node A
Equity Vector for round 2 and Node A updated with value 9.142857142857142
Relative Payment in to Node B from Node A is 0.6666666666666666
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.7142857142857143
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 4.857142857142858
Total dollar payment by Node B (i.e. p_2) is 4.0

 - Equity Vector for round 2 and Node B
Equity Vector for round 2 and Node B updated with value 8.857142857142858
Relative Payment in to Node C from Node A is 0.3333333333333333
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.25
Total dollar payment by Node B (i.e. p_2) is 4.0
Total payments in to Node C is 2.0
Total dollar payment by Node C (i.e. p_3) is 4.0

 - Equity Vector for round 2 and Node C
Equity Vector for round 2 and Node C updated with value 4.0

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EQUITY FOR EACH NODE

Equity for Node A: 9.142857142857142
Equity for Node B: 8.857142857142858
Equity for Node C: 4.0
[9.142857142857142, 8.857142857142858, 4.0]

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ROUND 2 DEFAULTERS

Node C has defaulted in round 2
{'A': False, 'B': False, 'C': True}
There are defaulters from earlier rounds but no new defaulters in the current
round, algorithm will not proceed for another round.

***Checking limited liability and absolute priority for Node A***
```

```
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node A from Node B is 0.75
Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node A from Node C is 0.2857142857142857
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 4.142857142857142

- Limited liability is met. Node A made a payment of 3.0 in round 2 which is
less than or equal to the cash flow (payments in + exogenous cash) of
9.142857142857142.
Total nominal obligation for Node A (i.e. p_bar_1): 3.0
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node A from Node B is 0.75
Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node A from Node C is 0.2857142857142857
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 4.142857142857142

 -Checking absolute priority for Node A in round 2. Nominal obligations is 3.0
and Dollar payments is 3.0
-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 2 i.e. either obligations are paid
in full or all available cash flow (i.e. sum of the payments received by the
node plus the exogenous operating cash flow) is paid to creditors. Nominal
obligations were 3.0,  Dollar payment was 3.0 and Total cash flow was
9.142857142857142

Node A in round 2 passes candidate clearing vector payment entry checks.

***Checking limited liability and absolute priority for Node B***

Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node B from Node A is 0.6666666666666666
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.7142857142857143
Total dollar payment by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 4.857142857142858

- Limited liability is met. Node B made a payment of 4.0 in round 2 which is
less than or equal to the cash flow (payments in + exogenous cash) of
8.857142857142858.
Total nominal obligation for Node B (i.e. p_bar_2): 4.0
Total dollar payment by Node B (i.e. p_2) is 4.0
Relative Payment in to Node B from Node A is 0.6666666666666666
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node B from Node C is 0.7142857142857143
Total dollar payment by Node C (i.e. p_3) is 4.0
```

```
Total payments in to Node B is 4.857142857142858

 -Checking absolute priority for Node B in round 2. Nominal obligations is 4.0
and Dollar payments is 4.0
-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 2 i.e. either obligations are paid
in full or all available cash flow (i.e. sum of the payments received by the
node plus the exogenous operating cash flow) is paid to creditors. Nominal
obligations were 4.0,  Dollar payment was 4.0 and Total cash flow was
8.857142857142858

Node B in round 2 passes candidate clearing vector payment entry checks.

***Checking limited liability and absolute priority for Node C***

Total dollar payment by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.3333333333333333
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.25
Total dollar payment by Node B (i.e. p_2) is 4.0
Total payments in to Node C is 2.0

- Limited liability is met. Node C made a payment of 4.0 in round 2 which is
less than or equal to the cash flow (payments in + exogenous cash) of 4.0.
Total nominal obligation for Node C (i.e. p_bar_3): 7.0
Total dollar payment by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.3333333333333333
Total dollar payment by Node A (i.e. p_1) is 3.0
Relative Payment in to Node C from Node B is 0.25
Total dollar payment by Node B (i.e. p_2) is 4.0
Total payments in to Node C is 2.0

 -Checking absolute priority for Node C in round 2. Nominal obligations is 7.0
and Dollar payments is 4.0
Dollar payments less than nominal obligations. Now checking if all value is paid
to creditors, i.e. Total cash flow for Node C
All value i.e. cash flow available to Node C is 4.0
-Absolute priority is satisfied for Node C

- Absolute priority is met by Node C in round 2 i.e. either obligations are paid
in full or all available cash flow (i.e. sum of the payments received by the
node plus the exogenous operating cash flow) is paid to creditors. Nominal
obligations were 7.0,  Dollar payment was 4.0 and Total cash flow was 4.0

Node C in round 2 passes candidate clearing vector payment entry checks.

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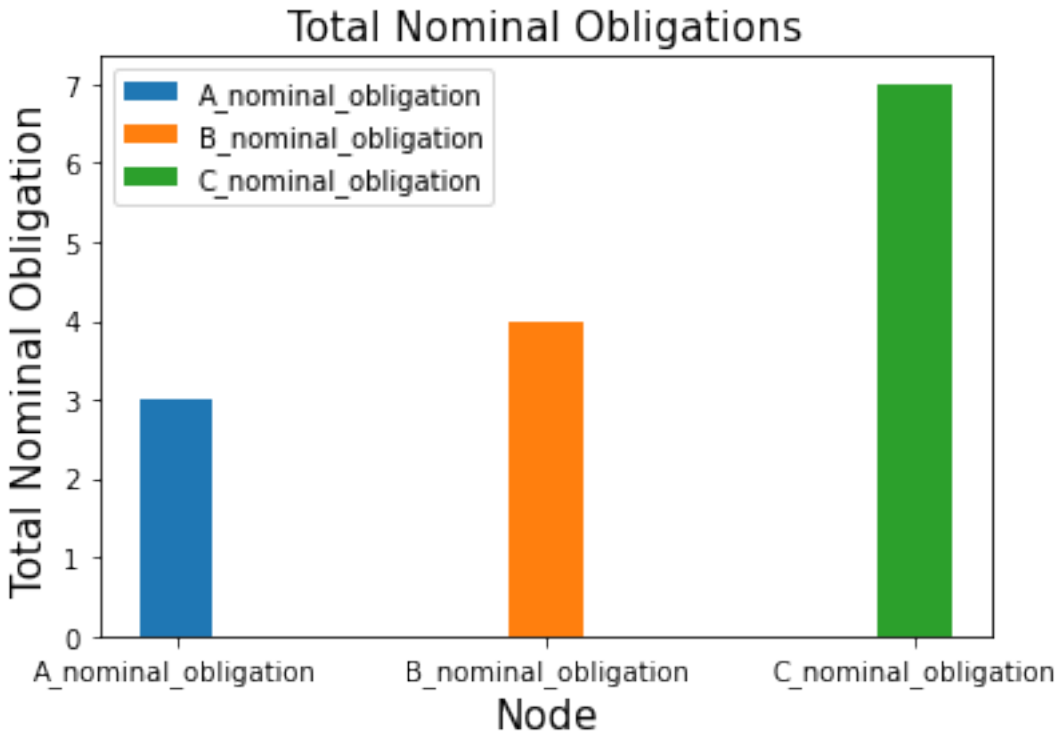
```

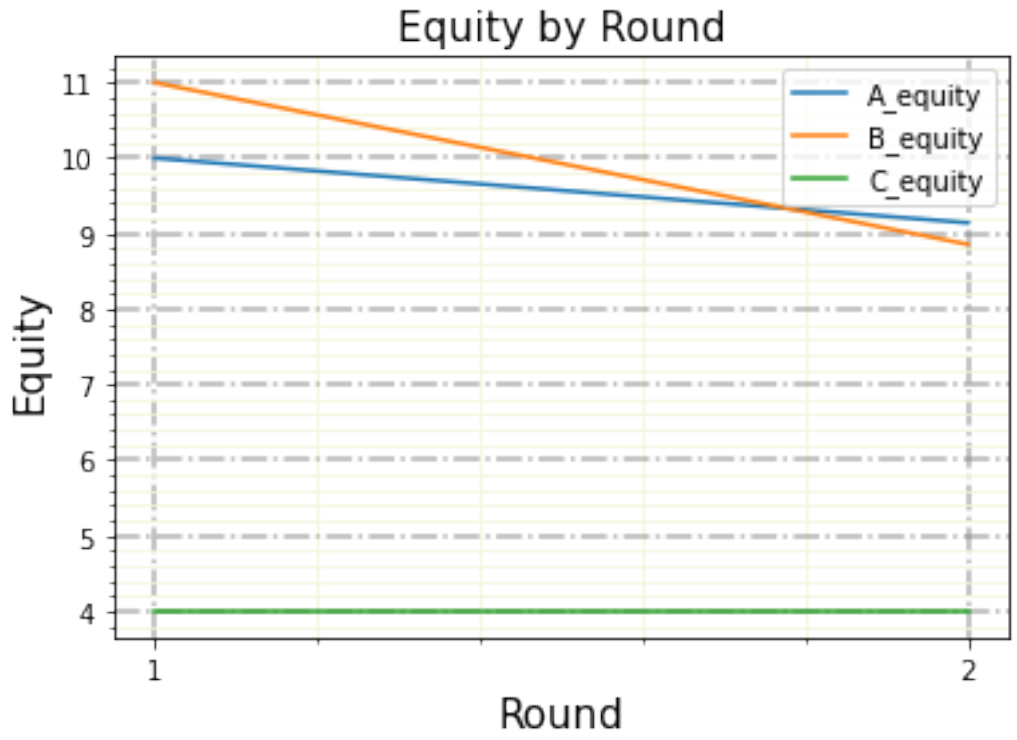
```
CLEARING_PAYMENT_VECTOR

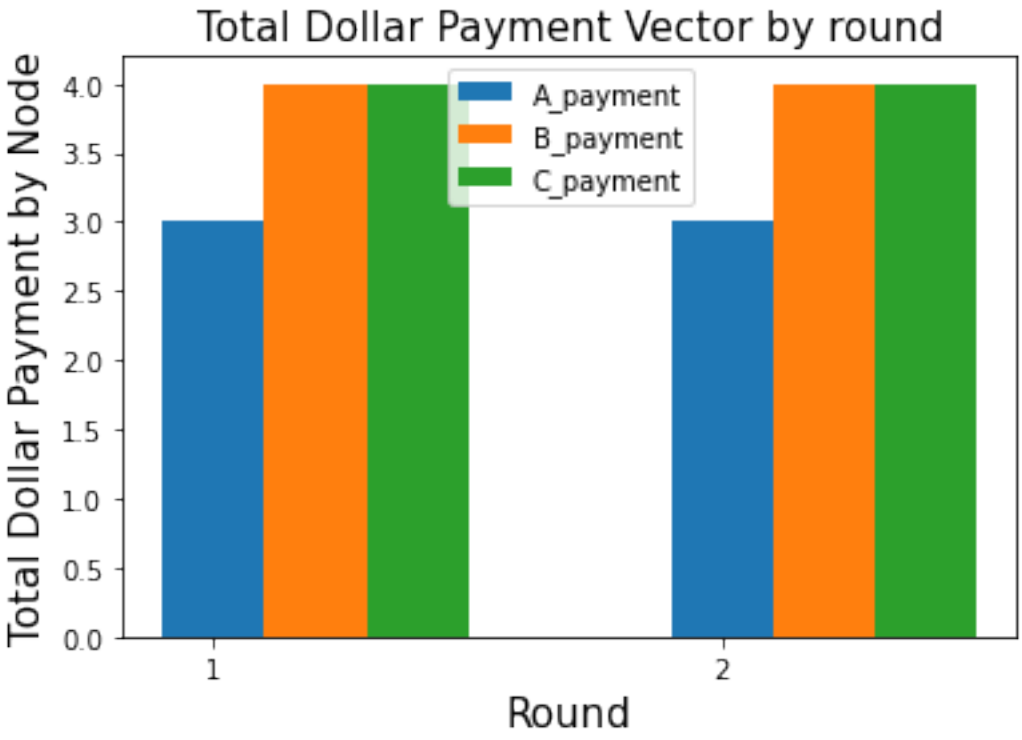
Clearing payment vector found in round 2.
[3.0, 4.0, 4.0]
Node A pays: 3.0
Node B pays: 4.0
Node C pays: 4.0

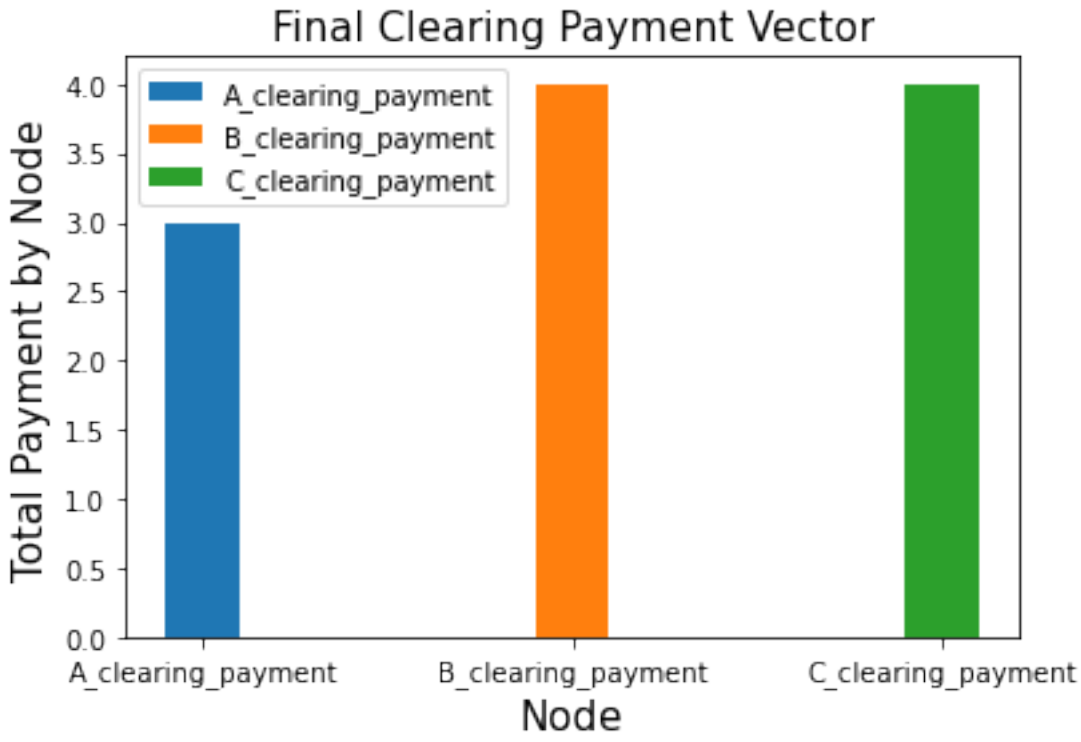
------------------------------------------------------------

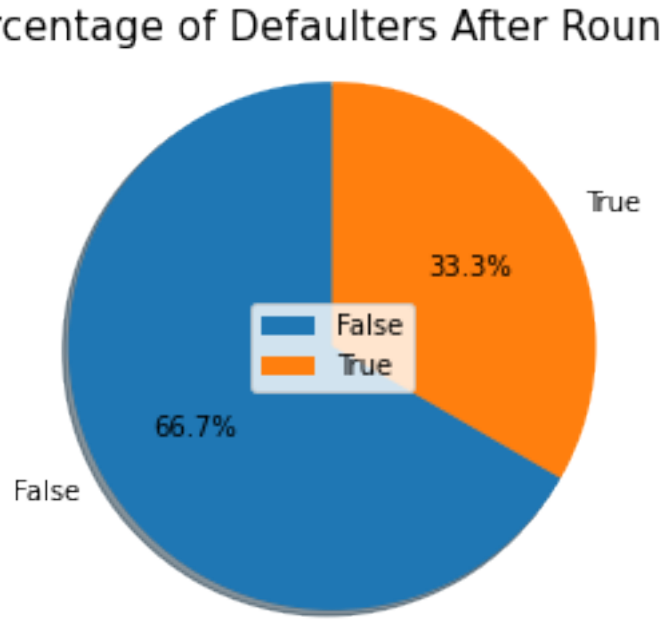
END OF ROUND 2
- Systemic Risk: Node C has defaulted in round 1. The number of prior default
waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
- Systemic Risk: Node A has not defaulted after 2 rounds. There are 3 nodes in
the  system (1 of which have defaulted i.e. ['C']).
- Systemic Risk: Node B has not defaulted after 2 rounds. There are 3 nodes in
the  system (1 of which have defaulted i.e. ['C']).
```




Equity by Round
A_equity
B_equity
C_equity
Equity
Round
11
10
9
8
7
6
5
4
1
2

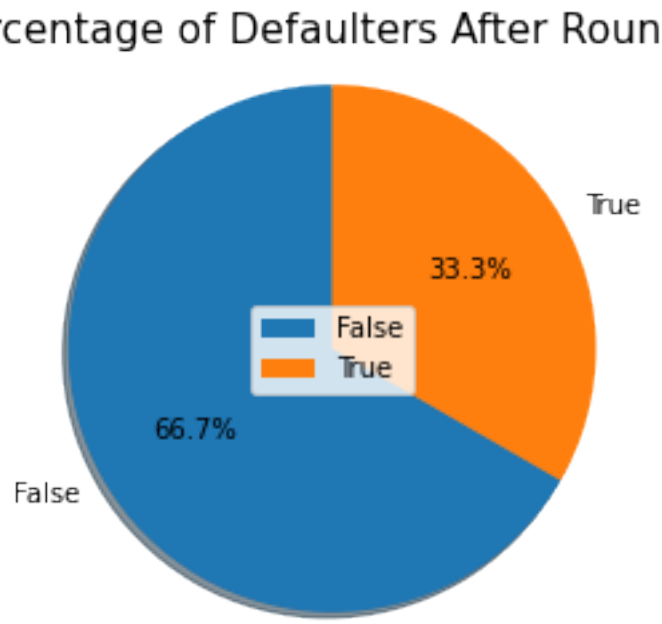

Total Dollar Payment Vector by round
A_payment
B_payment
C_payment
Total Dollar Payment by Node
Round
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
2






```
Node A has NOT defaulted in round 1
Node B has NOT defaulted in round 1
```

```
Node C has defaulted in round 1
```




```
Node A has NOT defaulted in round 2
Node B has NOT defaulted in round 2
Node C has defaulted in round 2
```