

eisenberg-noe-2001-debt-model-with-default-costs

November 20, 2021

DEBT MODEL WITH DEFAULT COSTS

ALPHA = 1 and BETA = 1

Running Debt Model to find Greatest Clearing Vector in MANUAL mode with 3 nodes...

Scenario 3 - Firm B defaults in first round, Firm A in second round, MODE == 'MANUAL', NUM_AGENTS = 3, NOMINAL_LIABILITY_MATRIX = np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING_CASHFLOW_BEFORE_SHOCK = [11, 8, 12], and implicitly ALPHA=BETA=1
Shock value is 4

AGENT LABELS

Agent labels ['A', 'B', 'C']

NOMINAL LIABILITY MATRIX Data Frame

i.e. what node i expects (row) to pay node j (column)...

	A	B	C
A	0	2	9
B	7	0	9
C	3	1	0

Nominal liabilities for each node:

Liability of Node A to Node B is 2
Liability of Node A to Node C is 9
Liability of Node B to Node A is 7
Liability of Node B to Node C is 9
Liability of Node C to Node A is 3
Liability of Node C to Node B is 1

NOMINAL LIABILITY MATRIX TRANSPOSED Data Frame
i.e. what node j expects to receive from i...

	A	B	C
A	0	7	3
B	2	0	1
C	9	9	0

Node A expects to receive 7 from Node B
Node A expects to receive 3 from Node C
Node B expects to receive 2 from Node A
Node B expects to receive 1 from Node C
Node C expects to receive 9 from Node A
Node C expects to receive 9 from Node B

OPERATING CASH FLOW VECTOR

Exogenous cash flow for Node A: 7
Exogenous cash flow for Node B: 4
Exogenous cash flow for Node C: 8
[7, 4, 8]

CALCULATING RELATIVE LIABILITIES FOR EACH NODE

Node A

- Liabilities for Node A

Liability of Node A to Node B (i.e. P_{01}) is 2.0

Liability of Node A to Node C (i.e. P_{02}) is 9.0

- Total obligations for Node A

Total Obligation Vector updated in round 1 for Node A with value 11.0

Total nominal liabilities for Node A (i.e. p_{bar}_1) is 11.0

- Relative Liabilities of Node A

Relative Liability of Node A to Node B is 0.181818181818182

Relative Liability of Node A to Node C is 0.81818181818182

Sum of Relative Liabilities for Node A is 1.0

Node B

- Liabilities for Node B

Liability of Node B to Node A (i.e. P_{10}) is 7.0

Liability of Node B to Node C (i.e. P₁₂) is 9.0

- Total obligations for Node B

Total Obligation Vector updated in round 1 for Node B with value 16.0

Total nominal liabilities for Node B (i.e. p_{bar}₂) is 16.0

- Relative Liabilities of Node B

Relative Liability of Node B to Node A is 0.4375

Relative Liability of Node B to Node C is 0.5625

Sum of Relative Liabilities for Node B is 1.0

Node C

- Liabilities for Node C

Liability of Node C to Node A (i.e. P₂₀) is 3.0

Liability of Node C to Node B (i.e. P₂₁) is 1.0

- Total obligations for Node C

Total Obligation Vector updated in round 1 for Node C with value 4.0

Total nominal liabilities for Node C (i.e. p_{bar}₃) is 4.0

- Relative Liabilities of Node C

Relative Liability of Node C to Node A is 0.75

Relative Liability of Node C to Node B is 0.25

Sum of Relative Liabilities for Node C is 1.0

RELATIVE LIABILITY MATRIX Data Frame

	A	B	C
A	0.0000	0.181818	0.818182
B	0.4375	0.000000	0.562500
C	0.7500	0.250000	0.000000

RELATIVE LIABILITY MATRIX Data Frame SANITY CHECK

	A	B	C	Relative Liability Total	CORRECT VALUE?
A	0.0000	0.181818	0.818182	1.0	True
B	0.4375	0.000000	0.562500	1.0	True
C	0.7500	0.250000	0.000000	1.0	True

RELATIVE LIABILITY MATRIX TRANSPOSED Data Frame

i.e. what node i (row) expects to receive from node j (column) in relative terms...

	A	B	C
A	0.000000	0.4375	0.75
B	0.181818	0.0000	0.25
C	0.818182	0.5625	0.00

Expected nominal payments in for Node A - both proportion and total amount

Node A expects to receive proportion 0.4375 from Node B

Node A expects to receive proportion 0.75 from Node C

Total payments in to Node A is 10.0 in round 1.

Expected nominal payments in for Node B - both proportion and total amount

Node B expects to receive proportion 0.18181818181818182 from Node A

Node B expects to receive proportion 0.25 from Node C

Total payments in to Node B is 3.0 in round 1.

Expected nominal payments in for Node C - both proportion and total amount

Node C expects to receive proportion 0.8181818181818182 from Node A

Node C expects to receive proportion 0.5625 from Node B

Total payments in to Node C is 18.0 in round 1.

START OF ROUND 1

TOTAL OBLIGATION VECTOR - round 1

i.e. total nominal obligations for each node i.e. \bar{p}_i ...

Total nominal obligation for Node A (i.e. \bar{p}_1): 11.0

Total nominal obligation for Node B (i.e. \bar{p}_2): 16.0

Total nominal obligation for Node C (i.e. \bar{p}_3): 4.0

TOTAL PAYMENT MADE PER NODE - round 1

i.e. $\min[\text{nominal obligations, cashflow (payments in + exogenous cash flow)}]$ for each node...

Node A

- Total payments in for Node A

Total payments in to Node A is 10.0 in round 1.

- Liabilities for Node A

Liability of Node A to Node B (i.e. P_{01}) is 2.0

Liability of Node A to Node C (i.e. P_{02}) is 9.0

- Total obligations for Node A

Total nominal liabilities for Node A (i.e. p_{bar_1}) is 11.0

Payment out is $\min[\text{payment out, total cash flow}]$ i.e. $\min[11.0, 17.0]$

- Total Dollar Payment Vector for round 1 and Node A

Total Dollar Payment Vector for round 1 and Node A updated with value 11.0

Node B

- Total payments in for Node B

Total payments in to Node B is 3.0 in round 1.

- Liabilities for Node B

Liability of Node B to Node A (i.e. P_{10}) is 7.0

Liability of Node B to Node C (i.e. P_{12}) is 9.0

- Total obligations for Node B

Total nominal liabilities for Node B (i.e. p_{bar_2}) is 16.0

Payment out is $\min[\text{payment out, total cash flow}]$ i.e. $\min[16.0, 7.0]$

Total payments in to Node B is 3.0 in round 1.

Total payments in to Node B is 3.0 in round 1.

Total payments in to Node B is 3.0 in round 1.

Round 1 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 7.0. Default loss for Node B is 0.0

- Total Dollar Payment Vector for round 1 and Node B

Total Dollar Payment Vector for round 1 and Node B updated with value 7.0

Node C

- Total payments in for Node C

Total payments in to Node C is 18.0 in round 1.

- Liabilities for Node C

Liability of Node C to Node A (i.e. P_{20}) is 3.0

Liability of Node C to Node B (i.e. P_{21}) is 1.0

- Total obligations for Node C

Total nominal liabilities for Node C (i.e. p_{bar_3}) is 4.0

Payment out is $\min[\text{payment out}, \text{total cash flow}]$ i.e. $\min[4.0, 26.0]$

- Total Dollar Payment Vector for round 1 and Node C

Total Dollar Payment Vector for round 1 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR - round 1

Total payment by Node A (i.e. p_1): 11.0

Total payment by Node B (i.e. p_2): 7.0

Total payment by Node C (i.e. p_3): 4.0

[11.0, 7.0, 4.0]

UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE - round 1

Node A

Total operating cash flow (exogenous assets) 7.0.

Total payments in to Node A is 10.0 in round 1.

Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0

Equity Vector for round 1 and Node A

Equity Vector for round 1 and Node A updated with value 6.0 i.e. total cash flow 17.0 minus total payments out (liabilities) 11.0.

Node B

Total operating cash flow (exogenous assets) 4.0.

Total payments in to Node B is 3.0 in round 1.

Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0

Equity Vector for round 1 and Node B

Equity Vector for round 1 and Node B updated with value 0.0 i.e. total cash flow 7.0 minus total payments out (liabilities) 7.0.

Node C

Total operating cash flow (exogenous assets) 8.0.

Total payments in to Node C is 18.0 in round 1.

Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0

Equity Vector for round 1 and Node C

Equity Vector for round 1 and Node C updated with value 22.0 i.e. total cash flow 26.0 minus total payments out (liabilities) 4.0.

EQUITY FOR EACH NODE - round 1

Equity for Node A: 6.0
Equity for Node B: 0.0
Equity for Node C: 22.0
[6.0, 0.0, 22.0]

ROUND 1 DEFAULTERS

Node B has defaulted in round 1
{'A': False, 'B': True, 'C': False}
There are defaulters in this round (i.e. round 1), algorithm will proceed for another round.

END OF ROUND 1

START OF ROUND 2

TOTAL OBLIGATION VECTOR - round 2

i.e. total nominal obligations for each node i.e. p_bar_i...
Total nominal obligation for Node A (i.e. p_bar_1): 11.0
Total nominal obligation for Node B (i.e. p_bar_2): 16.0
Total nominal obligation for Node C (i.e. p_bar_3): 4.0

TOTAL PAYMENT MADE PER NODE - round 2

i.e. min[nominal obligations, cashflow (payments in + exogenous cash flow)] for each node...

Node A

- Total payments in for Node A
Relative Payment in to Node A from Node B is 0.4375

Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
 Relative Payment in to Node A from Node C is 0.75
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node A is 6.0625 in round 2.

- Liabilities for Node A

Liability of Node A to Node B (i.e. P_{01}) is 2.0
 Liability of Node A to Node C (i.e. P_{02}) is 9.0
 Total nominal liabilities for Node A (i.e. p_{bar_1}) is 11.0

Payment out is $\min[\text{payment out}, \text{total cash flow}]$ i.e. $\min[11.0, 13.0625]$

- Total Dollar Payment Vector for round 2 and Node A

Total Dollar Payment Vector for round 2 and Node A updated with value 11.0

Node B

- Total payments in for Node B

Relative Payment in to Node B from Node A is 0.181818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node B from Node C is 0.25
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node B is 3.0 in round 2.

- Liabilities for Node B

Liability of Node B to Node A (i.e. P_{10}) is 7.0
 Liability of Node B to Node C (i.e. P_{12}) is 9.0
 Total nominal liabilities for Node B (i.e. p_{bar_2}) is 16.0

Payment out is $\min[\text{payment out}, \text{total cash flow}]$ i.e. $\min[16.0, 7.0]$

Relative Payment in to Node B from Node A is 0.181818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node B from Node C is 0.25
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node B is 3.0 in round 2.
 Relative Payment in to Node B from Node A is 0.181818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node B from Node C is 0.25
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node B is 3.0 in round 2.
 Relative Payment in to Node B from Node A is 0.181818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node B from Node C is 0.25
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node B is 3.0 in round 2.
 Round 2 and Node B has defaulted due to nominal obligations 16.0 being greater than cash flow 7.0. Default loss for Node B is 0.0

- Total Dollar Payment Vector for round 2 and Node B
Total Dollar Payment Vector for round 2 and Node B updated with value 7.0

Node C

- Total payments in for Node C
Relative Payment in to Node C from Node A is 0.8181818181818182
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Total payments in to Node C is 12.9375 in round 2.

- Liabilities for Node C
Liability of Node C to Node A (i.e. P_20) is 3.0
Liability of Node C to Node B (i.e. P_21) is 1.0
Total nominal liabilities for Node C (i.e. p_bar_3) is 4.0

Payment out is min[payment out, total cash flow] i.e. min[4.0, 20.9375]

- Total Dollar Payment Vector for round 2 and Node C
Total Dollar Payment Vector for round 2 and Node C updated with value 4.0

TOTAL PAYMENT VECTOR - round 2

Total payment by Node A (i.e. p_1): 11.0
Total payment by Node B (i.e. p_2): 7.0
Total payment by Node C (i.e. p_3): 4.0
[11.0, 7.0, 4.0]

UPDATE EQUITY (i.e. BOOK VALUE) FOR EACH NODE - round 2

Node A

Total operating cash flow (exogenous assets) 7.0.
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Relative Payment in to Node A from Node C is 0.75
Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 6.0625 in round 2.
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Equity Vector for round 2 and Node A
Equity Vector for round 2 and Node A updated with value 2.0625 i.e. total cash flow 13.0625 minus total payments out (liabilities) 11.0.

Node B

Total operating cash flow (exogenous assets) 4.0.
 Relative Payment in to Node B from Node A is 0.181818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node B from Node C is 0.25
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Total payments in to Node B is 3.0 in round 2.
 Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
 Equity Vector for round 2 and Node B
 Equity Vector for round 2 and Node B updated with value 0.0 i.e. total cash flow
 7.0 minus total payments out (liabilities) 7.0.

Node C

Total operating cash flow (exogenous assets) 8.0.
 Relative Payment in to Node C from Node A is 0.81818181818182
 Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node C from Node B is 0.5625
 Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
 Total payments in to Node C is 12.9375 in round 2.
 Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
 Equity Vector for round 2 and Node C
 Equity Vector for round 2 and Node C updated with value 16.9375 i.e. total cash
 flow 20.9375 minus total payments out (liabilities) 4.0.

 EQUITY FOR EACH NODE - round 2

Equity for Node A: 2.0625
 Equity for Node B: 0.0
 Equity for Node C: 16.9375
 [2.0625, 0.0, 16.9375]

 ROUND 2 DEFAULTERS

Node B has defaulted in round 2
 {'A': False, 'B': True, 'C': False}
 There are defaulters from earlier rounds but no new defaulters in the current
 round, algorithm will not proceed for another round.

Checking limited liability and absolute priority for Node A

Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
 Relative Payment in to Node A from Node B is 0.4375
 Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
 Relative Payment in to Node A from Node C is 0.75

Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 6.0625 in round 2.

- Limited liability is met. Node A made a payment of 11.0 in round 2 which is less than or equal to the cash flow (payments in + exogenous cash) of 13.0625.
Total nominal obligation for Node A (i.e. p_{bar_1}): 11.0
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node A from Node B is 0.4375
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Relative Payment in to Node A from Node C is 0.75
Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Total payments in to Node A is 6.0625 in round 2.

-Checking absolute priority for Node A in round 2. Nominal obligations is 11.0 and Dollar payments is 11.0

-Absolute priority is satisfied for Node A

- Absolute priority is met by Node A in round 2 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 11.0, Dollar payment was 11.0 and Total cash flow was 13.0625

Node A in round 2 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node B

Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Relative Payment in to Node B from Node A is 0.181818181818182
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 3.0 in round 2.

- Limited liability is met. Node B made a payment of 7.0 in round 2 which is less than or equal to the cash flow (payments in + exogenous cash) of 7.0.
Total nominal obligation for Node B (i.e. p_{bar_2}): 16.0
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Relative Payment in to Node B from Node A is 0.181818181818182
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node B from Node C is 0.25
Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Total payments in to Node B is 3.0 in round 2.

-Checking absolute priority for Node B in round 2. Nominal obligations is 16.0 and Dollar payments is 7.0

Dollar payments less than nominal obligations. Now checking if all value is paid to creditors, i.e. Total cash flow for Node B

All value i.e. cash flow available to Node B is 7.0

-Absolute priority is satisfied for Node B

- Absolute priority is met by Node B in round 2 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 16.0, Dollar payment was 7.0 and Total cash flow was 7.0

Node B in round 2 passes candidate clearing vector payment entry checks.

Checking limited liability and absolute priority for Node C

Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.81818181818182
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Total payments in to Node C is 12.9375 in round 2.

- Limited liability is met. Node C made a payment of 4.0 in round 2 which is less than or equal to the cash flow (payments in + exogenous cash) of 20.9375.
Total nominal obligation for Node C (i.e. p_{bar}_3): 4.0
Total dollar payment (i.e. liabilities) by Node C (i.e. p_3) is 4.0
Relative Payment in to Node C from Node A is 0.81818181818182
Total dollar payment (i.e. liabilities) by Node A (i.e. p_1) is 11.0
Relative Payment in to Node C from Node B is 0.5625
Total dollar payment (i.e. liabilities) by Node B (i.e. p_2) is 7.0
Total payments in to Node C is 12.9375 in round 2.

-Checking absolute priority for Node C in round 2. Nominal obligations is 4.0 and Dollar payments is 4.0

-Absolute priority is satisfied for Node C

- Absolute priority is met by Node C in round 2 i.e. either obligations are paid in full or all available cash flow (i.e. sum of the payments received by the node plus the exogenous operating cash flow) is paid to creditors. Nominal obligations were 4.0, Dollar payment was 4.0 and Total cash flow was 20.9375

Node C in round 2 passes candidate clearing vector payment entry checks.

CLEARING_PAYMENT_VECTOR

Clearing payment vector found in round 2.

[11.0, 7.0, 4.0]

Node A pays: 11.0

Node B pays: 7.0

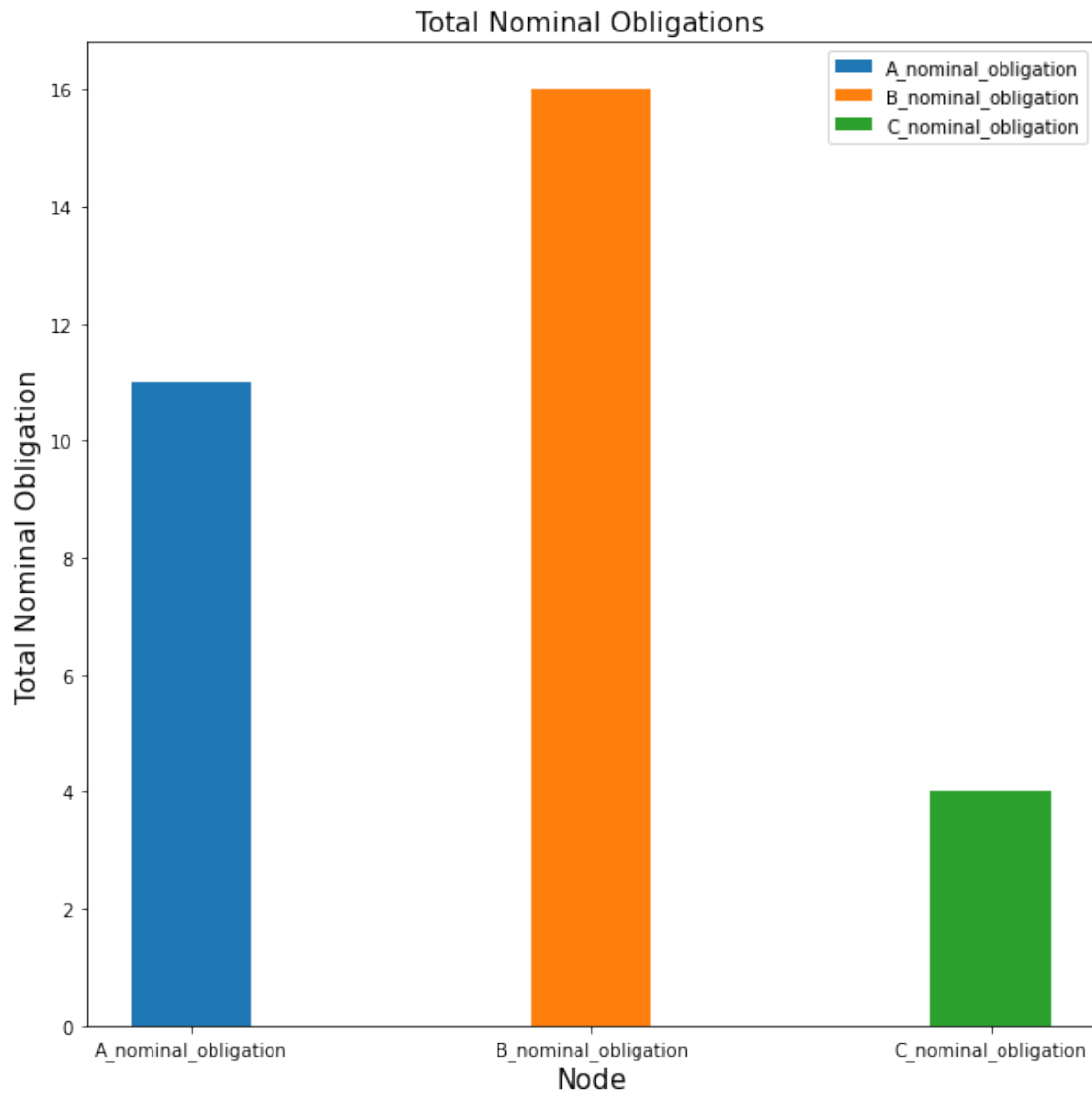
Node C pays: 4.0

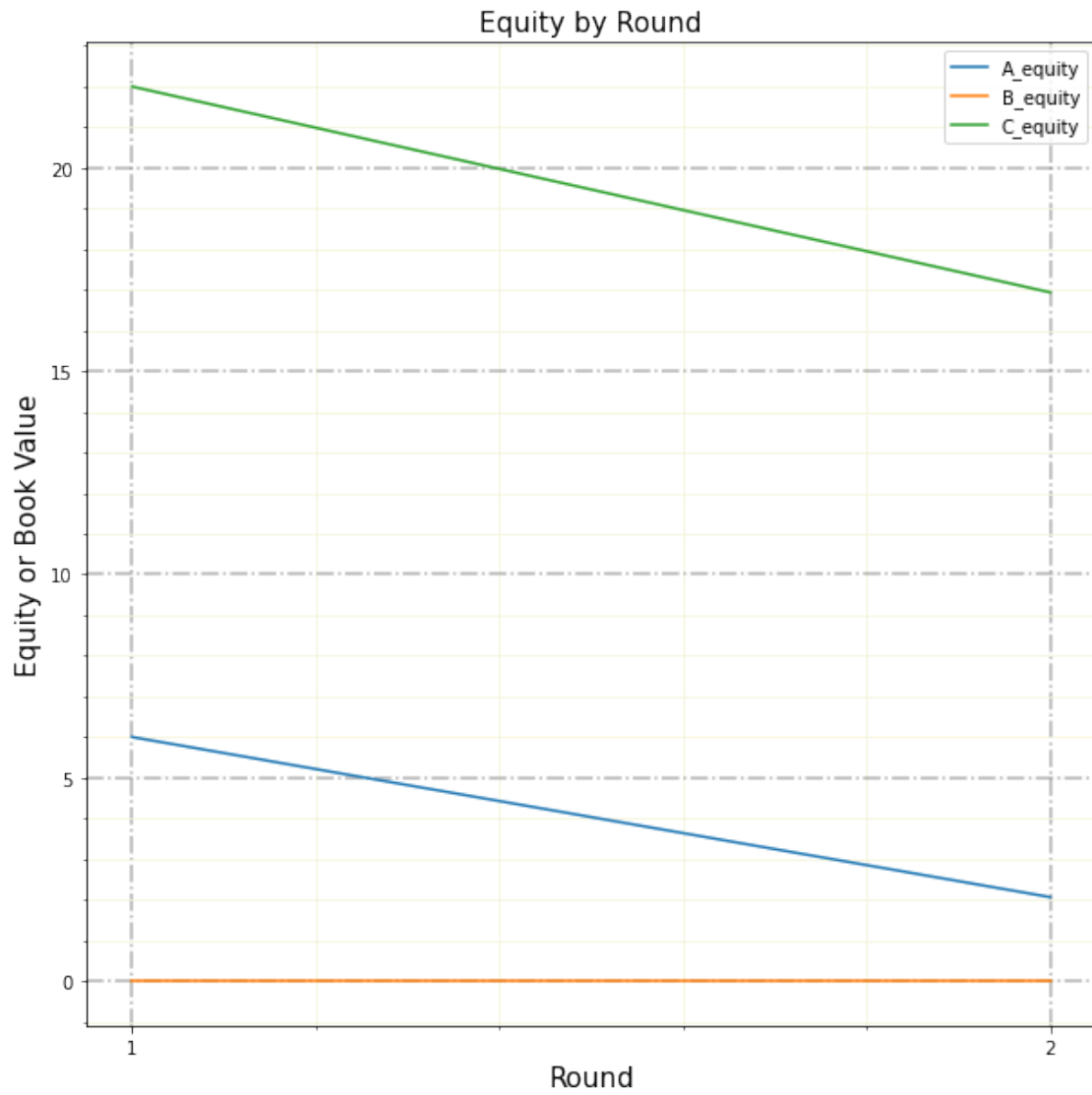
Default loss incurred by Node A is: 0
Default loss incurred by Node B is: 0.0
Default loss incurred by Node C is: 0

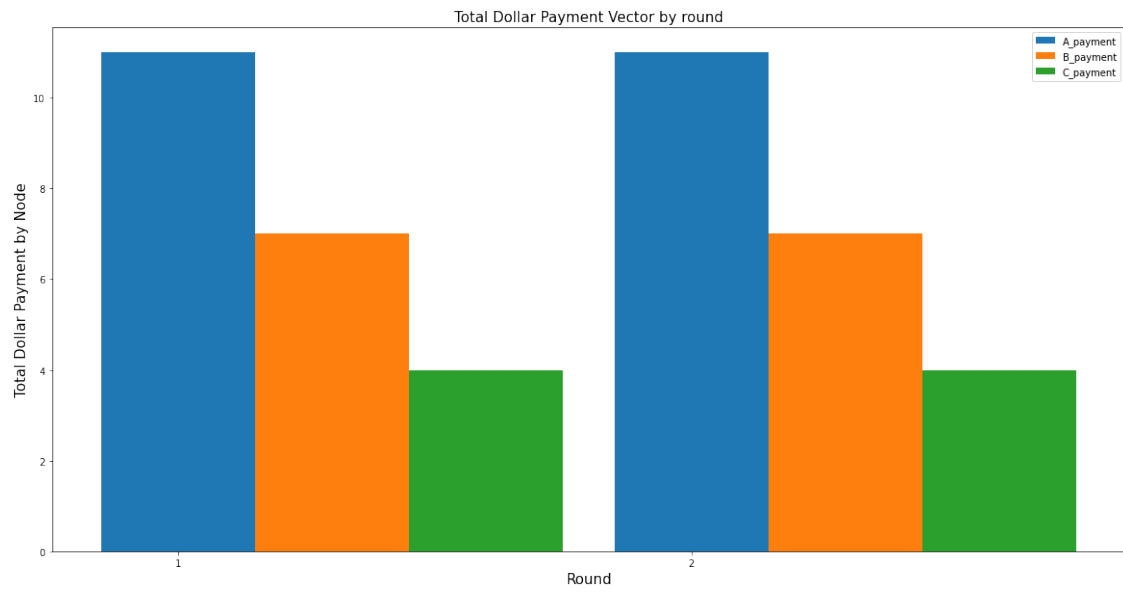
END OF ROUND 2

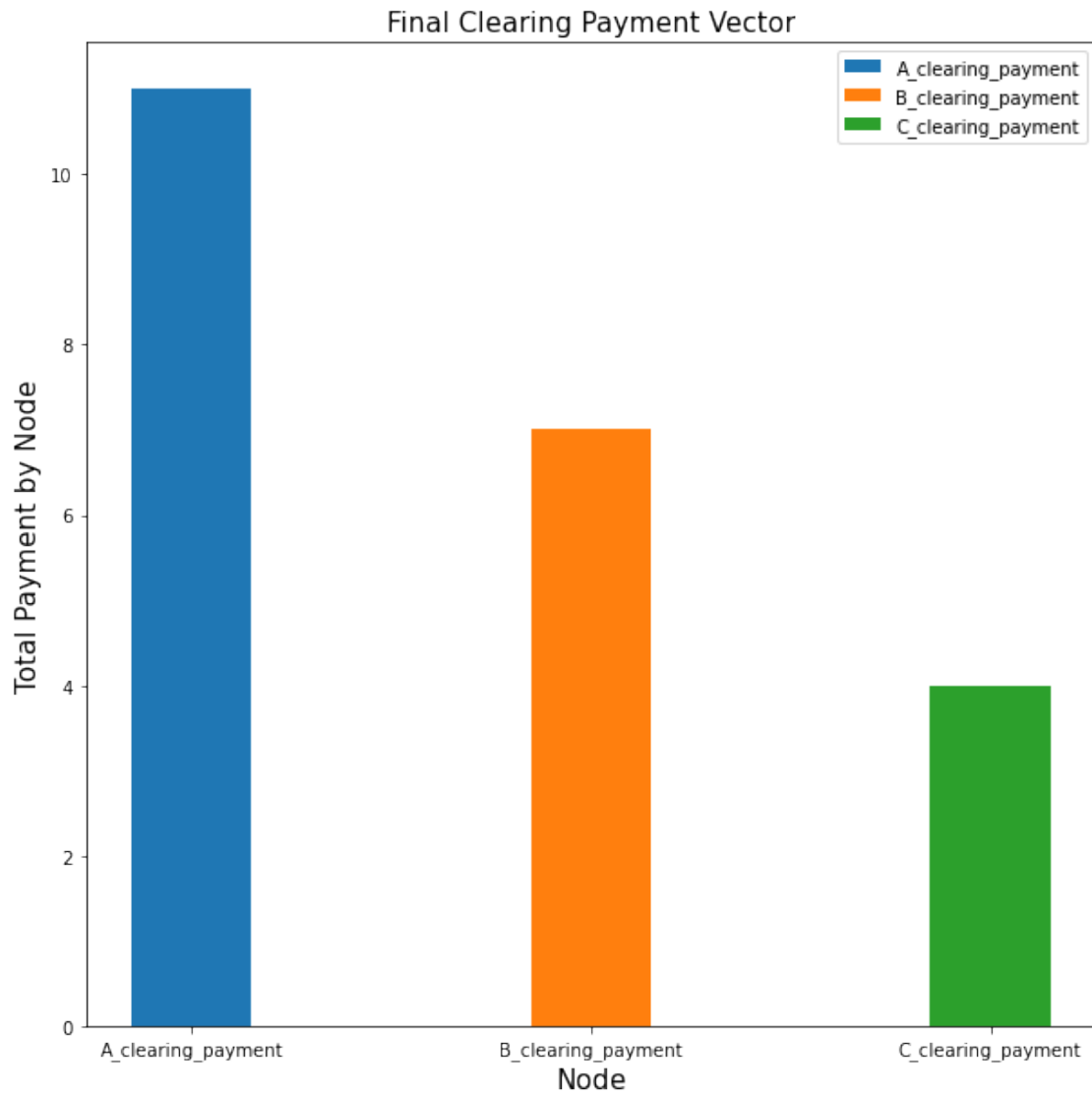
- Systemic Risk: Node B has defaulted in round 1. The number of prior default waves is 0. There are 3 nodes in the system (0 of which have defaulted i.e. []).
- Systemic Risk: Node A has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).
- Systemic Risk: Node C has not defaulted after 2 rounds. There are 3 nodes in the system (1 of which have defaulted i.e. ['B']).

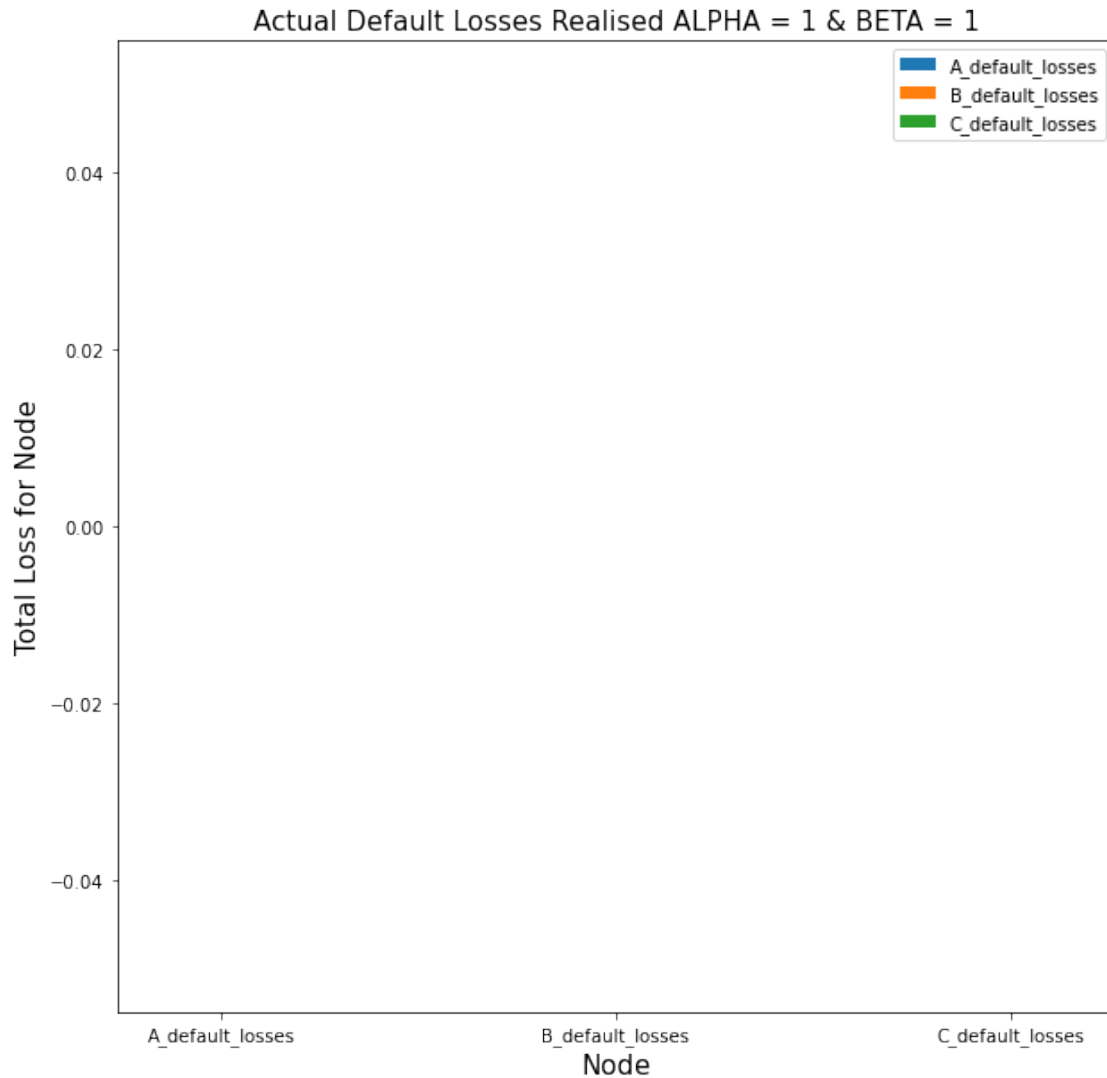
Scenario 3 - Firm B defaults in first round, Firm A in second round, MODE == 'MANUAL', NUM_AGENTS = 3, NOMINAL_LIABILITY_MATRIX = np.array([[0,2,9],[7,0,9],[3,1,0]]), OPERATING_CASHFLOW_BEFORE_SHOCK = [11, 8, 12], and implicitly ALPHA=BETA=1





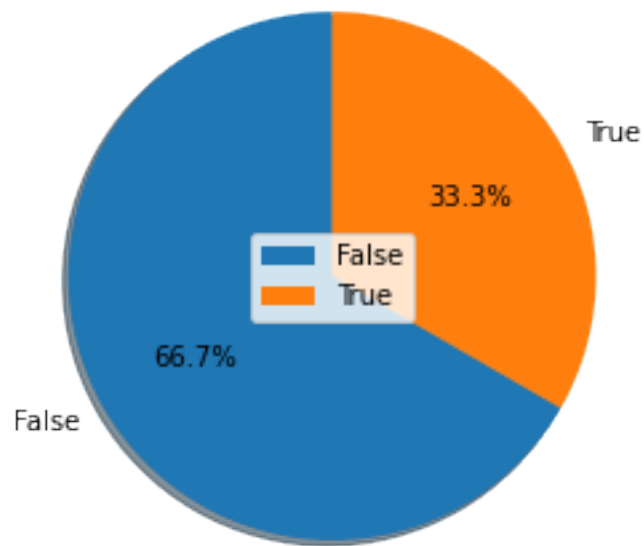






ALPHA and BETA are the fraction of exogenous assets (outside financial network) and endogenous assets (inside financial network i.e. interbank obligations) that are realised on liquidation in the event of default. The two fractions may conceivably be different; we would typically expect that ALPHA would be low, because the bank would be having to sell off its loan portfolio, probably at a knock-down price or fire sale. In contrast, BETA might be much closer to 1, because an obligation from a solvent bank would probably be paid in full (though perhaps with some negotiated discount to compensate for the inconvenience of early repayment).

Percentage of Defaulters After Round 1

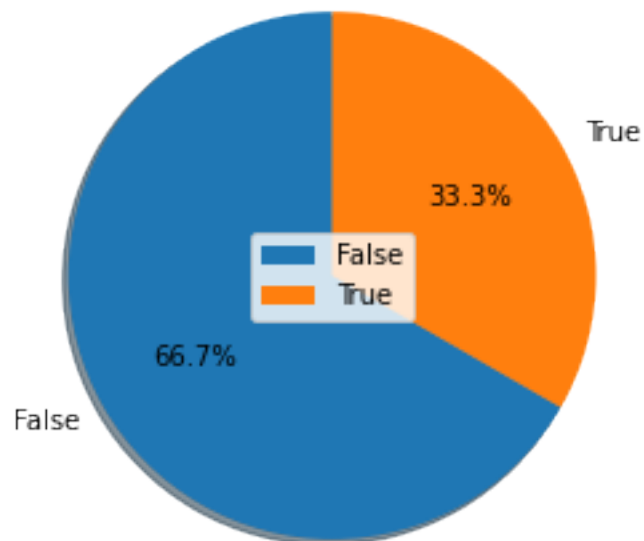


Node A has NOT defaulted in round 1

Node B has defaulted in round 1

Node C has NOT defaulted in round 1

Percentage of Defaulters After Round 2



Node A has NOT defaulted in round 2

Node B has defaulted in round 2
Node C has NOT defaulted in round 2
