1. calculate ||𝑓 − 𝑔||1 and ||𝑓 − 𝑔||∞ for the functions 𝑓 and 𝑔 that are de ned as

• 𝑓(0) = 2,𝑓(1) = −4,𝑓(2) = 8,𝑓(3) = −4 and

• 𝑔(0) = 5,𝑓(1) = 1,𝑔(2) = 7,𝑔(3) = −3

**Answer:**

We know that,

||𝑓||1 ∶= ∑ |𝑓(𝑥)|

𝑥∈𝑀

So for,

||𝑓 − 𝑔||1 = ||𝑓− 𝑔||1 ∶= ∑ |𝑓(𝑥) - 𝑔(𝑥)|

𝑥∈𝑀

= |2-5| + |-4-1| + |8-7| + | -4+3|

= 3+5+1+1

= 10

And,

||𝑓 − 𝑔||∞ = ∑ max { |𝑓 − 𝑔|: i….n}

= max { |2-5| , |-4-1| , |8-7| , | -4+3|}

= max { 3, 5, 1, 1 }

= 5

2. proof that all three axioms for norms hold for the 𝐿1-norm.

**Positive Definite:**

To prove the axioms positive definite, it has to be proven that,

If, ||𝑓 ||∞ = 0 then 𝑓= 0

For L-1 norm it will be if ||𝑓||1 = 0 then 𝑓= 0

Here we have to prove that,

if, ||𝑓− 𝑔 ||1 = 0 then 𝑓− 𝑔 = 0

Normally we take the summation of the function values.

Here, if ||𝑓− 𝑔 ||1 = 0 , it we can assume that if the summation of all the values which are non - negative is 0 then the values for the function are also 0.

So,

𝑓 =0

(Proved)

**Homogeneous :**

||α(𝑓 − 𝑔)||1 = α ||𝑓− 𝑔||1

Let the value be α=5, then

L.H.S (Left hand side)

||5𝑓− 5𝑔||1 = |2\*5 - 5\*5| + |-4\*5 - 1\*5| + |8\*5 - 7\*5| + | -4\*5 + 3\*5|

= |10 - 25| + | -20 - 5| +|40 - 35|+| -20 + 15|

= 15 + 25 + 5 + 5 = 50

R.H.S (Right hand side)

α ||𝑓− 𝑔||1 = 5 ||𝑓− 𝑔||1

= 5\*10 = 50

(Proved)

**Triangular Inequality:**

||𝑓 + 𝑔||1 ≤ ||𝑓 ||1 + || 𝑔||1

L.H.S ||𝑓 + 𝑔||1

= ∑ |𝑓(x) + 𝑔(x) |

= |2+5| + |-4+1| + |8+7| + | -4-3|

= 7+ 3+ 15 + 7

= 32

R.H.S = ||𝑓 ||1 + || 𝑔||1

= ∑ |𝑓(x) | + ∑ |g(x) |

= ( 2+ 4+ 8+ 4 ) + (5+1+ 7+ 3 )

= 18 + 16

= 34

Here, ||𝑓 + 𝑔||1 < ||𝑓 ||1 + || 𝑔||1

(Proved)