CSE-217: Theory of Computation

NON-DETERMINISM

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FORMAL DEFINITION

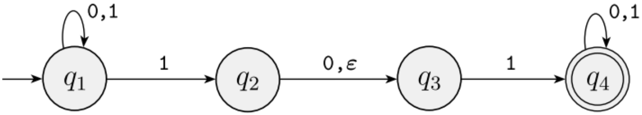
NON-DETERMINISM

DEFINITION 1.37

A ***nondeterministic finite automaton*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set of states,
2. Σ is a finite alphabet,
3. $\delta \colon Q \times \Sigma_\varepsilon \longrightarrow \mathcal{P}(Q)$ is the transition function,
4. $q_0 \in Q$ is the start state, and
5. $F \subseteq Q$ is the set of accept states.

NON-DETERMINISM



NON-DETERMINISM

The formal description of N_1 is $(Q, \Sigma, \delta, q_1, F)$, where

1. $Q = \{q_1, q_2, q_3, q_4\}$,
2. $\Sigma = \{0,1\}$,
3. δ is given as

	0	1	ε
q_1	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\{q_4\}$	$\emptyset$
q_4	$\{q_4\}$	$\{q_4\}$	$\emptyset$,

4. q_1 is the start state, and
5. $F = \{q_4\}$.

Equivalence of NFA and DFA

1. Deterministic and nondeterministic finite automata recognize the same class of languages.

2. Such equivalence is both surprising and useful.

Equivalence of NFA and DFA

3 It is surprising because NFAs appear to have more power than DFAs, so we might expect that NFAs recognize more languages.

4 It is useful because describing an NFA for a given language sometimes is much easier than describing a DFA for that language.

5 Say that two machines are equivalent if they recognize the same language.

Equivalence of NFA and DFA

Theorem

Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

Equivalence of NFA and DFA

PROOF IDEA

1. If a language is recognized by an NFA, then we must show the existence of a DFA that also recognizes it.
2. The idea is to convert the NFA into an equivalent DFA that simulates the NFA.
3. Recall the “reader as automaton” strategy for designing finite automata.

Equivalence of NFA and DFA

4. How would you simulate the NFA if you were pretending to be a DFA?
5. What do you need to keep track of as the input string is processed?
6. In the examples of NFA's, you kept track of the various branches of the computation by placing a finger on each state that could be active at given points in the input.

Equivalence of NFA and DFA

7 You updated the simulation by moving, adding, and removing fingers according to the way the NFA operates.

8 All you needed to keep track of was the set of states having fingers on them.

9 If k is the number of states of the NFA, it has 2^k subsets of states.

Equivalence of NFA and DFA

10 Now we need to figure out which will be the start state and accept states of the DFA.

11 What will be its transition function.

12 We can discuss this more easily after setting up some formal notation.

Equivalence of NFA and DFA

PROOF

- Let N = (Q, Σ, δ, q_0, F) be the NFA recognizing some language A
- We construct a DFA M = (Q', Σ', δ', q_0', F') recognizing A

Equivalence of NFA and DFA

- Before doing the full construction, lets first consider the easier case wherein N has no ϵ arrows.
- Later we take the ϵ arrows into account.

Equivalence of NFA and DFA

1. $Q' = P(Q)$.
   - Every state of M is a set of states of N.
   - Recall that P(Q) is the set of subsets of Q.

Equivalence of NFA and DFA

2 For $R \in Q'$ and $a \in \Sigma$, let

$$\delta'(R, a) = \{q \in Q | q \in \delta(r, a) \text{ for some } r \in R\}$$

- If R is a state of M, it is also a set of states of N.
- When M reads a symbol a in state R, it shows where a takes each state in R.
- Because each state may go to a set of states, we take the union of all these sets

Equivalence of NFA and DFA

3 $q_0' = \{q_0\}$

- M starts in the state corresponding to the collection containing just the start state of N.

Equivalence of NFA and DFA

4 $F' = \{R \in Q' | R$ contains an accept state of N$\}$

- The machine M accepts if one of the possible states that N could be in at this point is an accept state.

Equivalence of NFA and DFA

- Now we need to consider the ϵ arrows.
- To do so, we set up an extra bit of notation.
- For any state R of M, we define E(R) to be the collection of states that can be reached from members of R by going only along ϵ arrows, including the members of R themselves

Equivalence of NFA and DFA

- Formally, for $R \subseteq Q$ let $E(R) = \{q | q$ can be reached from R by traveling along 0 or more ϵ arrows$\}$
- Then we modify the transition function of M to place additional fingers on all states that can be reached by going along ϵ arrows after every step.
- Replacing $\delta(r, a)$ by $E(\delta(r, a))$ achieves this effect

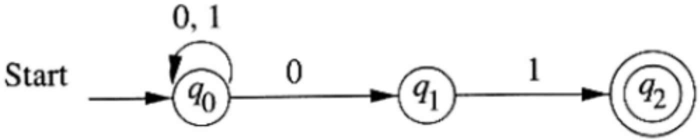
Equivalence of NFA and DFA

- Thus $\delta'(R, a) = \{q \in Q | q \in E(\delta(r, a))$ for some $r \in R\}$
- Additionally, we need to modify the start state of M to move the fingers initially to all possible states that can be reached from the start state of N along the ϵ arrows.
- Changing q_0' to be $E(\{q_0\}))$ achieves this effect

Equivalence of NFA and DFA

- We have now completed the construction of the DFA M that simulates the NFA N.
- The construction of M obviously works correctly.
- At every step in the computation of M on an input, it clearly enters a state that corresponds to the subset of states that N could be in at that point.
- Thus our proof is complete.

Example



An NFA accepting all strings that end in 01

Example-continued

	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_1\}$	$\emptyset$	$\{q_2\}$
$*\{q_2\}$	$\emptyset$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$*\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$
$*\{q_1, q_2\}$	$\emptyset$	$\{q_2\}$
$*\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$

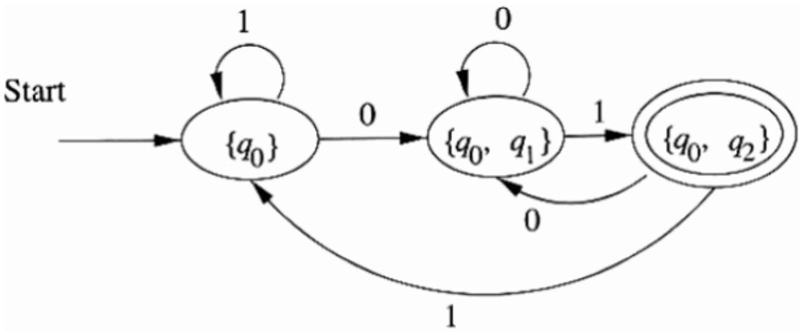
Figure 2.12: The complete subset construction from Fig. 2.9

Example-continued

	0	1
A	A	A
$\rightarrow B$	E	B
C	A	D
$*D$	A	A
E	E	F
$*F$	E	B
$*G$	A	D
$*H$	E	F

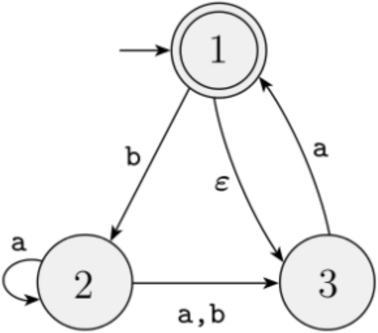
Renaming the states

Example-continued

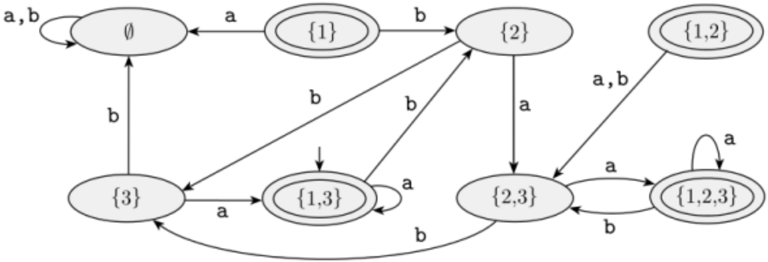


The DFA constructed from the NFA

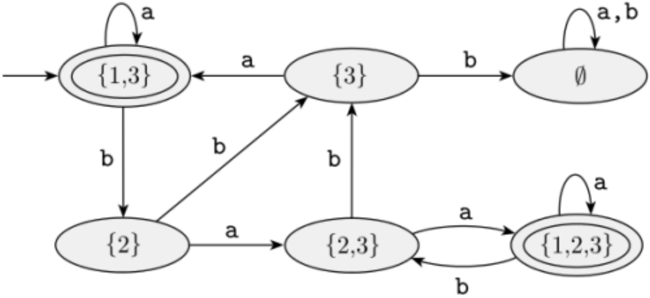
Example-2



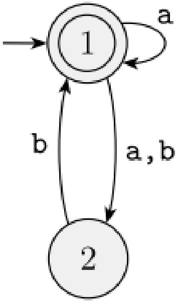
Example-2 continued



Example-2 continued



Exercise-1 Convert the following NFA to equivalent DFA



Exercise-2 Convert the following NFA to equivalent DFA