

TECHNICAL NOTE

Use of a genetic algorithm to perform reliability analysis of unsaturated soil slopes

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This paper describes the development of a probabilistic design approach to analyse the stability of unsaturated soil slopes. Instability of unsaturated soil slopes is caused by the downwards progression of a wetting front, which reduces the near-surface suctions, increases the weight of soil in the failure zone and reduces the soil strength. Assuming the slip surface forms parallel to the slope surface (at a depth equal to the wetting front depth) the probability of failure is determined in this paper through the reliability index determined using a computationally efficient genetic algorithm method.

KEYWORDS: failure; partial saturation; slopes; suction

La présente communication décrit le développement d'une méthode d'étude probabiliste pour l'analyse de la stabilité de pentes à sol non saturé. L'instabilité des pentes à sol non saturé est causée par la progression vers le bas d'un front mouillant, qui réduit l'aspiration des surfaces proximales, augmente le poids du sol dans la zone de rupture, et réduit la résistance du sol. En supposant que la surface de glissement se forme parallèlement à celle de la pente (à une profondeur égale à celle du front mouillant), la probabilité de la rupture est déterminée, dans la présente communication, à travers l'indice de fiabilité déterminé en appliquant une méthode à algorithme génétique efficace sur le plan computationnel.

INTRODUCTION

Rainfall-induced landslides are a major cause of disturbance to transport networks in many parts of the world. In slopes where the water table is some depth below the ground surface, negative pore water pressure (suctions) develop in the near-surface soils, which contribute significantly to their overall stability. These suctions are, however, transient and reduce as water percolates into the slope (and a wetting front develops) during periods of heavy or prolonged rainfall.

In this note the development of a model for determining the reliability of a slope in which the soil properties are considered as random variables is presented. By transforming the variables into polar coordinates, the complexities associated with defining the limit state function, which have affected many previous attempts at probabilistic analysis of slopes, can be overcome. The minimisation problem is solved in a powerful and efficient genetic algorithm (GA) environment.

ANALYSIS OF THE STABILITY OF UNSATURATED SOIL SLOPES

Planar slip surface method

Fourie *et al.* (1999) note that in most slope failures caused by infiltration, the failure plane forms parallel to the existing slope (see Fig. 1). The failure surface is formed by the downwards migration of a wetting front (caused by infiltration into the partially saturated soil). The current authors

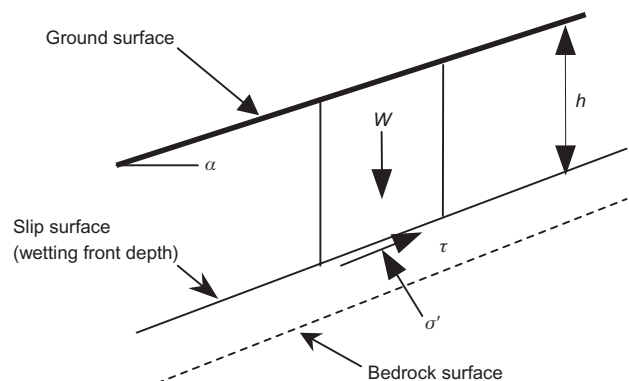


Fig. 1. Slip surface model

suggest using an infinite slope model in which the soil strength is described using an appropriate model. Fredlund *et al.* (1978) expanded the Mohr–Coulomb model to incorporate negative pore-water pressure (matric suction) effects

$$\begin{aligned}\tau &= c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \\ &= C + (\sigma_n - u_a) \tan \phi'\end{aligned}\quad (1)$$

where τ = shear strength of unsaturated soils, c' is the effective cohesion, σ_n is the total normal stress on the failure plane, u_a is the pore-air pressure on the failure plane, ϕ' is the angle of internal friction associated with the net normal stress state variable $\sigma_n - u_a$, u_w is the pore-water pressure on the failure plane, $u_a - u_w$ is the matric suction on the failure plane, and ϕ^b is the angle indicating the rate of increase in shear strength relative to the matric suction. The effects of c' and the contribution of matric suction $(u_a - u_w) \tan \phi^b$ can for convenience be combined into a single parameter (C). The factor of safety (FOS) for the planar slip surface is given by

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$$\text{FOS} = \frac{C + \gamma h \cos^2 \alpha \tan \phi}{\gamma h \cos \alpha \sin \alpha} \quad (2)$$

where γ is the unit weight of soil, h is the wetting front depth and α is the slope angle. C can be measured in the laboratory or using in situ direct shear tests (see Springman *et al.*, 2003).

Basic variables in the problem

The variables in equation (2) include parameters, C , h , ϕ' and γ , which continue to vary throughout a rainfall event. Although Hassan & Wolff (1999) note these parameters are most likely log normally distributed, Whitman (1984) notes that the reliability index (β) is not very sensitive to the distribution of the parameters provided that $\beta < 2.5$ and the standard deviation of the parameters is not very large. In this paper the variables are therefore assumed to be normally distributed.

RELIABILITY MODEL FOR UNSATURATED SOIL SLOPES

At the limit state ($\text{FOS} = 1$), using equation (2), the limit state function can be written as

$$g(\mathbf{X}) = C + \gamma h \cos^2 \alpha \tan \phi - \gamma h \cos \alpha \sin \alpha \quad (3)$$

and the probability of failure (P_f) is given as

$$P_f = P(g(\mathbf{X}) \leq 0) \quad (4)$$

$$= \int_{C + \gamma h \cos^2 \alpha \tan \phi - \gamma h \cos \alpha \sin \alpha \leq 0} f(\mathbf{X}) d\mathbf{X}$$

in which $f(\mathbf{X})$ is the probability density function (range of likely values) of the random variables, (C , ϕ , γ , h). Integrating equation (4) is rarely practicable, partly owing to its non-linear form in addition to difficulties accessing the probability density distribution functions of the random variables. The reliability index, which allows comparative reliability to be assessed for a system, where the probability distributions are unknown, is often employed to solve such problems.

Reliability index

Hasofer & Lind (1974) propose an invariant method to calculate the reliability index in which the random variables are transformed into non-dimensional form

$$\bar{X}_i = \frac{X_i - E[X_i]}{\sigma[X_i]} \quad (i = 1, 2, \dots, n) \quad (5)$$

where $E[X_i]$ and $\sigma[X_i]$ are the mean value and standard deviation of the variable X_i . The limit state function can be written in terms of the reduced variables

$$g(\bar{\mathbf{X}}) = g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_n) \quad (6)$$

The reliability index (β_{HL}) is the minimum distance from the origin of the reduced variable space ($\bar{\mathbf{X}}$) to the limit state surface ($g(\bar{\mathbf{X}}) = 0$) (Fig. 2). The distance is normally calculated using a cosine directional search; however, Val *et al.* (1996) note a tendency for the identification of local minima when using this method on highly non-linear functions. They suggest transforming the rectangular coordinates to polar coordinates to avoid this problem. Given the limit state function as defined with equation (3) and taking (C , ϕ , γ , h) as random variables, the polar coordinates of the design point for a number of variables can be described using the

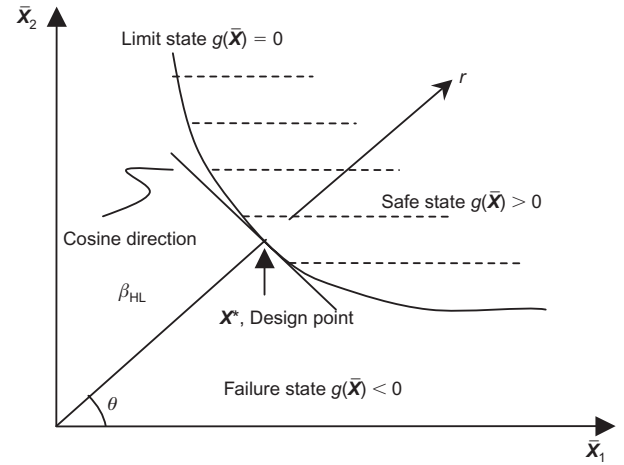


Fig. 2. Definition of reliability index in polar coordinates

radial distance (r) and the polar angle (θ), (Val *et al.*, 1996), as

$$\begin{aligned} \bar{C} &= \frac{C - E(C)}{\sigma(C)} = r \cos \theta_1 \cos \theta_2 \cos \theta_3 \\ \bar{\phi} &= \frac{\phi - E(\phi)}{\sigma(\phi)} = r \cos \theta_1 \cos \theta_2 \sin \theta_3 \\ \bar{\gamma} &= \frac{\gamma - E(\gamma)}{\sigma(\gamma)} = r \cos \theta_1 \sin \theta_2 \\ \bar{h} &= \frac{h - E(h)}{\sigma(h)} = r \sin \theta_1 \end{aligned} \quad (7a)$$

Therefore

$$\begin{aligned} C &= E(C) + r \cos \theta_1 \cos \theta_2 \cos \theta_3 \sigma(C) \\ \phi &= E(\phi) + r \cos \theta_1 \cos \theta_2 \sin \theta_3 \sigma(\phi) \\ \gamma &= E(\gamma) + r \cos \theta_1 \sin \theta_2 \sigma(\gamma) \\ h &= E(h) + r \sin \theta_1 \sigma(h) \end{aligned} \quad (7b)$$

where

$$\begin{aligned} (-1/2)\pi &\leq \theta_i \leq (1/2)\pi \quad (i = 1, 2, \dots, n-2), \\ 0 &\leq \theta_{n-1} \leq 2\pi \end{aligned}$$

The reliability index can be expressed as the minimum radius (r) of the design point (see Fig. 2)

$$\beta_{\text{HL}} = \min r = \min(\bar{\mathbf{X}}^T \bar{\mathbf{X}})^{1/2} \quad (8)$$

This constrained optimisation problem with the limit state as the performance function can be stated as

Minimise r , Subject to

$$g(r, \theta) = 0 \quad (9)$$

If the expressions in equation (7) are substituted into equation (3), then a cubic expression in terms of r is produced during the expansion, and solution of the problem will be difficult. To avoid this set

$$Y_1 = \gamma h \quad (10)$$

$$Y_2 = Y_1 \cos^2 \alpha \tan \phi$$

Equation (3) can then be rewritten as

$$g(\mathbf{X}) = C + Y_2 - Y_1 \cos \alpha \sin \alpha \quad (11)$$

According to the Taylor series of multi-variables (Harr, 1987)

$$E[F(x_1, x_2, \dots, x_n)] = F[E(x_1), E(x_2), \dots, E(x_n)]$$

$$\sigma[F(x_1, x_2, \dots, x_n)] = \left[\sum \left(\frac{\partial F}{\partial x_i} \sigma(x_i) \right)^2 \right]^{1/2} \quad (12)$$

Assuming C , ϕ' , γ and h are normally distributed and uncorrelated

$$E(Y_1) = E(\gamma)E(h)$$

$$\sigma(Y_1) = \left[(E(h)\sigma(\gamma))^2 + (E(\gamma)\sigma(h))^2 \right]^{1/2}$$

$$E(Y_2) = E(Y_1) \cos^2 \alpha \tan(E(\phi)) \quad (13)$$

$$\sigma(Y_2) = \left[[\cos^2 \alpha \tan(E(\phi))\sigma(Y_1)]^2 + [E(Y_1)\sigma(\tan \phi) \cos^2 \alpha]^2 \right]^{1/2}$$

Thus the reduced variables of C , Y_1 and Y_2 can be expressed in polar coordinates as

$$C = E(C) - r\omega_1\sigma(C)$$

$$Y_1 = E(Y_1) - r\omega_2\sigma(Y_1) \quad (14)$$

$$Y_2 = E(Y_2) - r\omega_3\sigma(Y_2)$$

In which

$$\omega_1 = \cos \theta_1 \cos \theta_2$$

$$\omega_2 = \cos \theta_1 \sin \theta_2 \quad (15)$$

$$\omega_3 = \sin \theta_1$$

Substituting equation (14) into equation (3) and taking $g(X) = 0$ gives

$$E(C) - r\omega_1\sigma(C) + E(Y_2) - r\omega_3\sigma(Y_2) - [E(Y_1) - r\omega_2\sigma(Y_1)] \cos \alpha \sin \alpha = 0 \quad (16)$$

So r can be obtained by rewriting equation (16)

$$r = \frac{E(C) + E(Y_2) - E(Y_1) \cos \alpha \sin \alpha}{\omega_1\sigma(C) + \omega_3\sigma(Y_2) - \omega_2\sigma(Y_1) \cos \alpha \sin \alpha} \quad (17)$$

In which the mean values and standard deviations of Y_1 and Y_2 are described using equation (13).

Constraints of the non-linear programming problem

To prevent negative value being assigned for the variables, lower bound (non-negative) values must be set for C , Y_1 and Y_2

$$C = E(C) - r\omega_1\sigma(C) \geq 0$$

$$Y_1 = E(Y_1) - r\omega_2\sigma(Y_1) \geq 0 \quad (18)$$

$$Y_2 = E(Y_2) - r\omega_3\sigma(Y_2) \geq 0$$

The reliability of an unsaturated soil slope can be obtained by minimising equation (17) subject to the constraints in equation (18). This constrained non-linear programming problem is solved in this paper using the GA method (Xue & Gavin, 2007). Although at this stage the model considers

only variables that affect the slope resistance. The development of the model to include correlated variables will be particularly important if the model is to consider both the load and resistance components, as variables such as the precipitation rate will affect both the load and resistance (i.e. the wetting front depth). At this stage it is recommended that a likely range of wetting front depths be calculated using simple empirical models (Gavin & Xue, 2008) or finite element analyses (Ng & Shi, 1998), and the likely values of wetting front depth and the uncertainty associated with the assessment of these can be included in the reliability analysis.

GENETIC ALGORITHM

In the GA, the parameters in the optimisation problem (variables such as the soil properties and wetting front depth) are translated into chromosomes with a data string (binary or real). A range of possible solutions is obtained from the variable space and the fitness of these solutions is compared with some predetermined criteria. If a solution is not obtained, a new population is created from the original (parent) chromosomes. This is achieved using 'crossover' and 'mutation' operations. Crossover involves gene exchange from two random (parent) solutions to form a child (new solution). Mutation involves the random switching of a single variable in a chromosome, and is used to maintain population diversity, as the process converges towards a solution. The key advantages of GA are

- it is a population-based approach and thus considers a wide range of possible solutions
- the mutation process restricts the solution falling into local minima that can occur in alternative solution techniques.

The analysis uses real-coded methods to encode the chromosomes with the variables. The fitness of each chromosome is determined using the objective function, and the fitness of all solutions is compared. The analysis is implemented using a program, Genetic Algorithm for Slope Stability Analysis (GASSA), written in Visual C++ (Xue, 2007). A flow chart detailing the operation of the program is shown in Fig. 3.

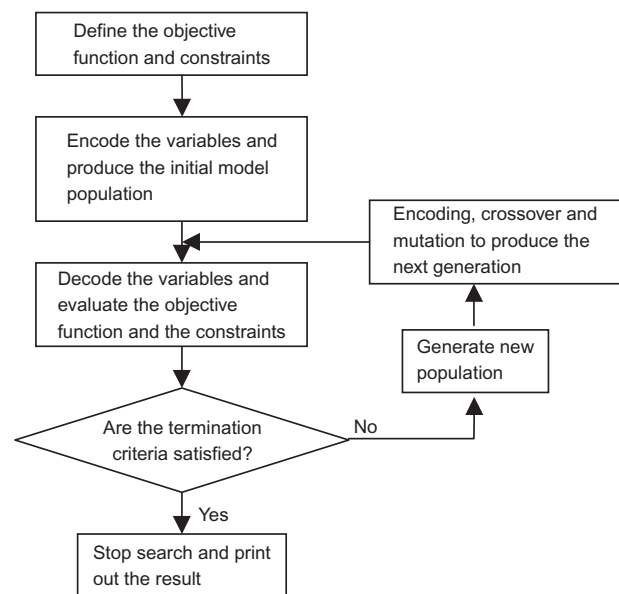


Fig. 3. Flow chart used in genetic algorithm (after Xue & Gavin 2007)

APPLICATION TO A CASE HISTORY

Springman *et al.* (2003) report data from field tests performed to investigate the effect of infiltration on the stability of two alpine slopes. Both slopes were subjected to a two-day artificial rainfall event. The rainfall intensity was 16 mm/h for the first 24 h, reducing to 12 mm/h thereafter. While one slope, which had an average slope angle of 31°, remained stable throughout the test, the second 42° slope developed a 0.5 m deep, translational slip after 45 h of rainfall. Instrumentation suggested that the degree of saturation (S_r) was 85–95% at failure. A back analysis of the failure using a planar slip surface model with ϕ' varying from 39 to 41°, and two wetting front depths of 0.2 m and 0.5 m was performed by Springman *et al.* (2003). They calculated values of total cohesion at limiting equilibrium ($FOS = 1$), which ranged from 0.1 to 0.5 kPa. The reliability index of the slope was determined using the new method implemented in GASSA, the FOSM approach and a Monte Carlo simulation, the latter being an alternative full-population search technique to the GA approach. The following parameters were used in the analyses: mean ϕ' value of 40°, with a coefficient of variation (COV) of 0.035, mean unit weight = 20.2 kN/m³ and COV = 0.05. Using values of in-situ suction and soil water content measured on the 31° slope, Springman *et al.* (2003) report values of available total cohesion at depths of 0.16 to 0.4 m of between 1.8 to 4.1 kPa. A mean C value of 2.93 kPa and COV of 0.39 were adopted in this paper to carry out the reliability analysis. The wetting front depth was also treated as a variable in these analyses with COV = 0.05, and the mean value was varied incrementally from 0.2 m to 1.0 m to assess the effect of wetting front depth on the reliability index.

The results of the analyses are shown in Fig. 4, which reveals that all methods predict similar β values for shallow

(less than 0.5 m deep) wetting front depths. As the wetting front depth increased, the FOSM method gave the highest β values, and the predictions from GASSA and Monte Carlo simulation were comparable. Although this is not a rigorous evaluation of the method, the determination of lower β using the new model indicates improved performance than FOSM. When the wetting front reached 0.5 m, which is the depth at which the failure occurred, the reliability index predicted with both GASSA and the Monte Carlo simulation decreases to 2, and the performance of the slope can be described as poor according to United States Army Corps Engineers (USACE, 1999). In contrast the factor of safety calculated with the estimated in-situ mean total cohesion is 1.51, which would be considered quite safe.

CONCLUSION

The stability of unsaturated soil slopes is critically dependent on the response of the slope to rainfall infiltration. The development of a failure surface depends on the slope angle, wetting front depth, unit weight and strength of the soil and the rainfall characteristics. With the exception of the slope angle, all of these parameters are highly variable, and some are interdependent (i.e. the wetting front depth and the rainfall characteristics).

An invariant probabilistic analysis approach for analysing the stability of unsaturated soil slopes was presented. The method, implemented using a computationally highly efficient GA was seen to predict comparable reliability indices to FOSM and Monte Carlo methods when applied to a case history slope failure. While the wetting front depth was not considered to be a variable in the analyses presented, the formulation of the method includes the possibility of including this in the analysis. To model the infiltration response properly, a range of likely wetting front depths should be considered. While the method will be improved by further development, for example to include for the specification of correlated variables, the framework offers a more rational approach to the assessment of unsaturated slope stability than traditional deterministic methods.

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NOTATION

C	total cohesion
COV	coefficient of variation
c'	effective cohesion of soil
$E(X)$	mean value of vector X
FOS	factor of safety
$g(X)$	limit state function defined with vector $[X]$
$g(\bar{X})$	limit state function defined with reduced variables
h	wetting front depth
P_f	probability of failure
r	radius in polar coordinate
u_a	pore air pressure
u_w	pore water pressure
W	the weight of a slice of the slope
X	vector of variables
\bar{X}	vector of reduced variables
α	slope angle
β	reliability index
β_{HL}	reliability index defined with Hasofer–Lind method;

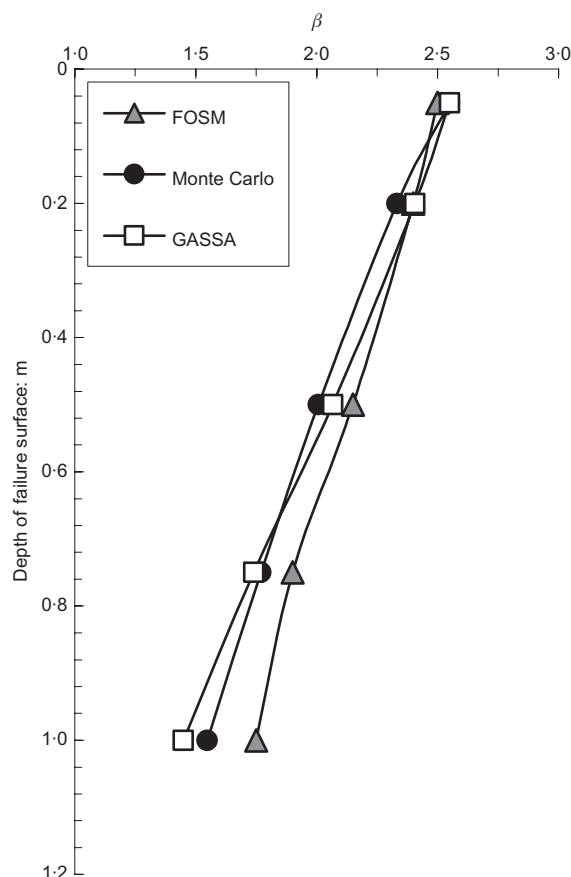


Fig. 4. Variation of reliability index with wetting front depth

γ unit weight of soil
 ∂ partial differential
 ϕ' friction angle of soil
 ϕ^b angle indicating the rate of increase in shear strength relative to the matric suction
 θ angle in polar coordinates
 $\sigma(X)$ standard deviation of vector X
 σ_n total normal strength on the failure plane
 τ shear strength of unsaturated soils

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