Razširjanje zaupanja za lokalizacijo senzorskih omrežji

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Cilj

Lokalizacija senzorskega omrežja

Omrežje si predstavljamo kot utežen graf, kjer so naprave vozlišča. Dve vozlišči sta povezani, če sta pripadajoči napravi dovolj blizu, da lahko izmerita razdaljo. Utež povezav je izmerjena razdalja, kjer pri merjenju pride do napak.
Vemo lokacije od nekaterih naprav, tem rečemo sidra. Izračunati želimo približke za lokacije ostalih, tem rečemo agenti.

Predpostavimo, da je omrežje naprav povezano.

Oznake

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- Vozlišča označujemo z črkami $u, v \in V$.
- $x_u \in \mathbb{R}^n$ lokacija naprave u.
- Za $A = \{v_1, v_2, \ldots, v_k\} \subseteq V$ je $\underline{x}_A = (x_{v_1}, x_{v_2}, \ldots, x_{v_k})$
- Za $uv \in E$ je $d_{u,v}$ izmerjena razdalja.

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Omejimo na $S \subseteq \mathbb{R}^{n \cdot |V|}$ s končnim volumnom.

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$\underline{X}_V$ zvezno porazdeljen slučajni vektor z gosto $\tilde{P}$.

Markovska polja

Pogojna neodvisnost

Nakjučna vektorja $\underline{X}, \underline{Y}$ sta pogojno neodvisna ob $\underline{Z}$, če velja:

$$f_{\underline{X},\underline{Y}|\underline{Z}}\left(\underline{x}, \underline{y} \mid \underline{Z} = \underline{z}\right) = f_{\underline{X}|\underline{Z}}\left(\underline{x} \mid \underline{Z} = \underline{z}\right) \; f_{\underline{Y}|\underline{Z}}\left(\underline{y} \mid \underline{Z} = \underline{z}\right)$$

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Definicija

Graf $G = (V, E)$, in slučajne spremenljivke $\underline{X}_V$ tvorijo Markovsko polje, če velja Markovska lastnost po parih:
Za vsaka nesosedna $u, v \in V$ sta X_u, X_v pogojno neodvisna ob vseh ostalih $\underline{X}_{V \setminus \{u,v\}}$

$\underline{X}_V$ je Markovsko polje

Za nesosednja $u, v \in V$ je

$$f_{X_u, X_v | \underline{X}_{V\setminus\{u,v\}}}\left(x_u, x_v | \underline{X}_{V\setminus\{u,v\}} = \underline{y}_{V\setminus\{u,v\}}\right) \propto$$

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$$\propto \prod_{su \in E} \psi_{s,u}\left(y_s, x_u\right) \prod_{sv \in E} \psi_{s,v}\left(y_s, x_v\right) \prod_{\substack{\{s,t\} \in E \\ \{s,t\} \cap \{u,v\} = \emptyset}} \psi_{s,v}\left(y_s, y_t\right) \propto$$

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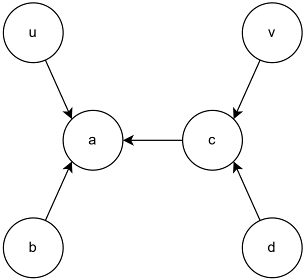
$$\propto g(x_u)\, h(x_v) \Rightarrow X_u, X_v \text{ pogojno neodvisna ob } \underline{X}_{V \setminus \{u,v\}}$$

Razširjanje zaupanja na drevesih

Poglejmo si primer, ko je graf drevo.



$$\int_{\underline{X}_{V_a\setminus\{a\}}} f_{\underline{X}_{V_a}|\underline{X}_{V_s}}\left(\underline{x}_{V_a}|\underline{x}_{V_s}=\underline{y}_{V_s}\right) d\underline{x}_{V_a\setminus\{A\}} \propto$$

$$\int_{X_b,X_d,X_c} \psi_{u,a}\psi_{b,a}\psi_{c,a}\psi_{v,c}\psi_{d,c}\, dx_b dx_d dx_c =$$

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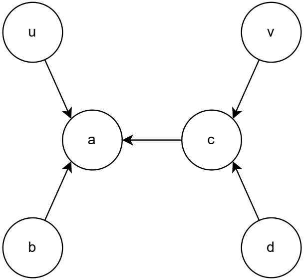

$$\int_{\underline{X}_{V_a \setminus \{a\}}} f_{\underline{X}_{V_a}|\underline{X}_{V_s}} \left(\underline{x}_{V_a} | \underline{x}_{V_s} = \underline{y}_{V_s} \right) d\underline{x}_{V_a \setminus \{A\}} \propto$$

$$\int_{X_b, X_d, X_c} \psi_{u,a} \psi_{b,a} \psi_{c,a} \psi_{v,c} \psi_{d,c} dx_b dx_d dx_c =$$

$$\psi_{u,a} \int_{X_b} \psi_{b,a} dx_b \int_{X_c} \psi_{c,a} \psi_{v,c} \int_{X_d} \psi_{d,c} dx_d dx_c$$

$$\text{Za } u, v \in V_a\text{: } \mu_{u \to v}(x_v) := \int_{X_u} \psi_{u,v}(x_u, x_v) \prod_{t \in N(u) \setminus v} \mu_{t \to u}(x_u)\, dx_u$$

Za $u \in V_s$ in $v \in V_a$: $\mu_{u \to v}(x_v) := \psi_{u,v}(y_u, x_v)$

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$$\mu_{u\to a}(x_a)\, \mu_{b\to a}(x_a) \int_{X_c} \psi_{c,a}\mu_{v\to c}(x_c)\, \mu_{d\to c}(x_c)\, dx_c =$$

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$$\mu_{u \to a}(x_a)\, \mu_{b \to a}(x_a)\, \mu_{c \to a}(x_a) =$$

$$\prod_{t \in N(a)} \mu_{t \to a}(x_a) =: M_a(x_a)$$

Pravilnost razširjanja zaupanja na drevesih

Izrek

Naj bo $G = (V, E)$ drevo in V_a, V_s, $\underline{X}_V$, $\underline{y}_{V_s}$, $M_u(x_u)$ kot prej. Zaupanje je proporcionalno robni gostoti

$$M_u(x_u) \propto f_{X_u|\underline{X}_{V_s}}\left(x_u|\underline{X}_{V_s} = \underline{y}_{V_s}\right).$$

Ideja dokaza so poddrevesa osnovnega drevesa s korenom v u.

Za grafe s cikli

Naj bosta $u, v \in V_a$, $i \in \mathbb{N}$ pa število iteracije.

$$M_u^{(i)}(x_u) \coloneqq \prod_{t \in N(u)} \mu_{t \to u}^{(i)}(x_u)$$

$$\mu_{v \to u}^{(i)}(x_u) \coloneqq \int_{X_v} \psi_{v,u}(x_v, x_u) \frac{M_v^{(i-1)}(x_v)}{\mu_{u \to v}^{(i-1)}(x_v)} dx_v$$

Za $v \in V_s$, $u \in V_a$ je pa $\mu_{v \to u}^{(i)}(x_u) \coloneqq \psi_{v,u}(y_v, x_u)$
Za drevesa to konvergira k pravim vrednostim.

Izboljšave

- Za u, v, ki $d(u, v) \geq 2$, dodamo potenciale.
- Potenciale popravimo zaradi velikih odstopanj.
- Sporočilo $\mu_{u \to v}$ aproksimiramo z $\sum_{i=1}^{n} \frac{w_i}{\sqrt{2\pi}} exp\left[-\frac{|x - x_i|^2}{2\sigma_i^2}\right]$.
- Algoritem izvajamo večkrat na vpetih drevesih.