

# Homework 5

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## Question One

- a) The Fisher linear discriminant method projects a set of sample points of  $d$  dimensions, onto a projected line that has the most separation in  $d-1$  dimensions.

This is done through finding some  $w$ , which will project our points and satisfy the criterion function for its maximum. Using the projected means and within class scatter, as well as the between class scatter, we see the ratio between these projected values gives us

$$J(w) = \frac{w^T S_B w}{w^T S_w w}, \text{ where } w_{B,} \text{ the solution from our } w = S_w^{-1}(m_1 - m_2) \text{ to maximize } J(\cdot).$$

This  $w$  is used for our projected means, points, and is only relevant for its direction it tells us, not magnitude.

b) First we find our means,  $w_1 = \begin{pmatrix} 1.833 \\ 2.00 \end{pmatrix}$

$$w_2 = \begin{pmatrix} 3.833 \\ 3.00 \end{pmatrix}$$

Next we find our scatters, scatter is the sum of

$$S_i = \sum_{x \in D_i} (x - m_i)(x - m_i)^T, \text{ Here } S_1 = \begin{bmatrix} 4.833 & 1 \\ -1.00 & 8 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 8.833 & 3 \\ 3.00 & 12 \end{bmatrix}$$

$$S_w = S_1 + S_2 = \begin{bmatrix} 13.66 & 2 \\ 2.00 & 20 \end{bmatrix}$$

Then we solve for  $w$

$$w = S_w^{-1} (m_1 - m_2)$$

$$w = \begin{pmatrix} -.141089 \\ -.03589 \end{pmatrix}$$

(Finally for our  $S_B$ , between class scatter

$$S_B = (m_1 - m_2)(m_1 - m_2)^T = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$$

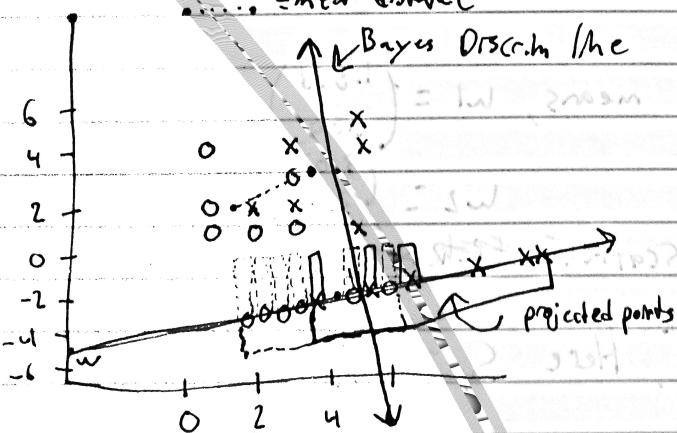
$$\text{Our } J(w) = .03589 \cdot \begin{pmatrix} -.141089 \\ -.03589 \end{pmatrix}^T \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -.141089 \\ -.03589 \end{pmatrix} = 3180.693$$

$$\begin{pmatrix} -.141089 \\ -.03589 \end{pmatrix}^T \begin{pmatrix} 13.66 & 2.00 \\ 2.00 & 20.00 \end{pmatrix} \begin{pmatrix} -.141089 \\ -.03589 \end{pmatrix} = (w)$$

i) Our projection line has been set to  $w = 0$

Simply uses .3180693 as a coef for each  $x$   
 ..... = mean distance added an intercept value for clarity.

Bayes Discr. line  $-4$  for each  $y$  value



If we increase our  $w$  magnitude by 10 when we project our points, as well as move them down -4 to match  $y = -4$  (our  $w$  change).

We assume uniform distribution within our max/min for all points

Pr(x) = P(x)

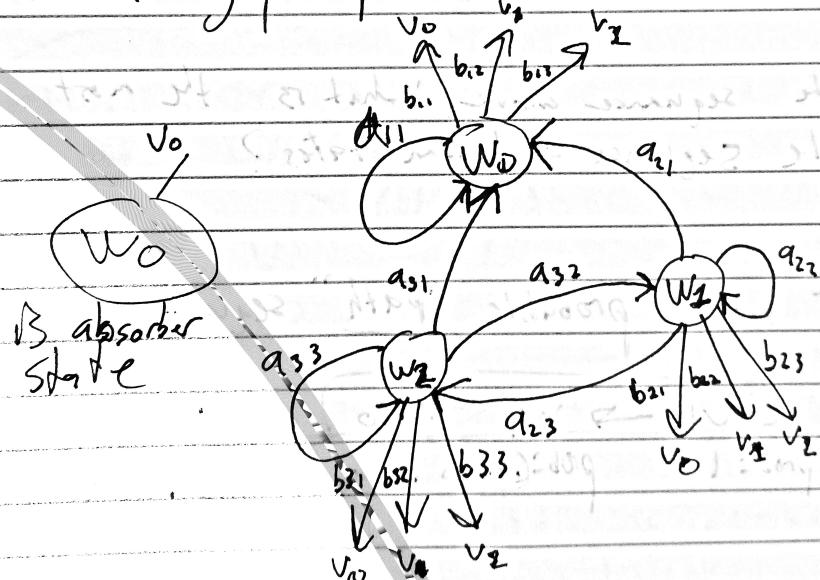
Our  $y = w^T x$  projected points are passed decision criterion  $J(w)$  to give us our one dimensional mapping on the projection line.

## Question 2

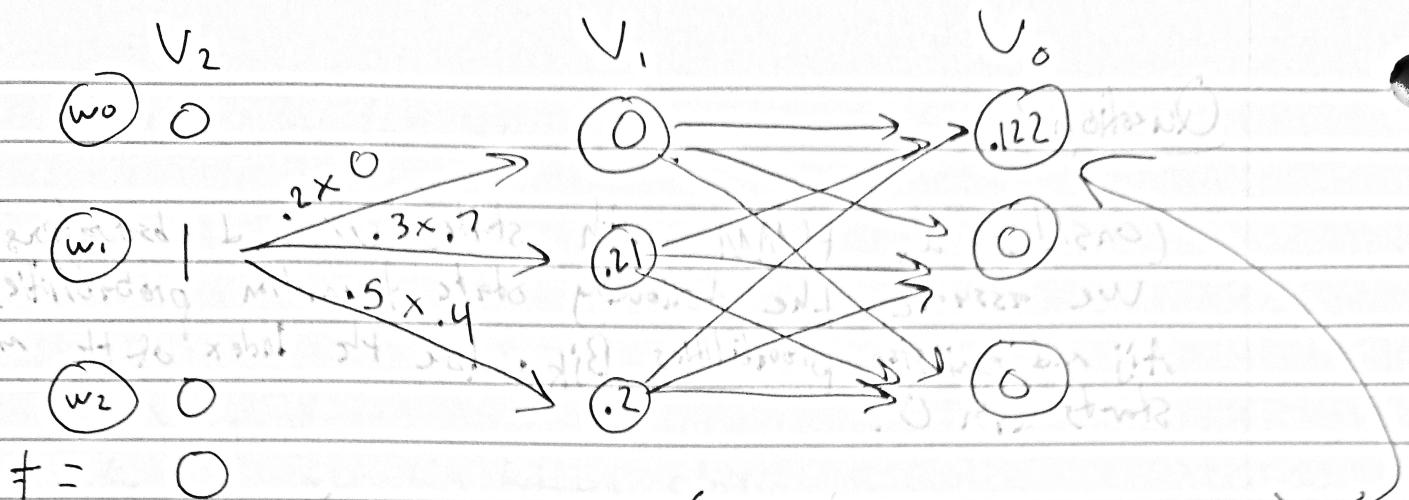
Consider an HMM with states  $w$  and observations  $v$ .  
 We assume the following state transition probabilities  
 $A_{ij}$  and emission probabilities  $B_{jk}$  where the index of the matrices  
 starts with 0.

$$A_{ij} = \begin{pmatrix} 1.0 & 0 & 0 \\ 0.2 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.1 \end{pmatrix} \quad B_{jk} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

a) Show a graph representation of this hidden Markov Model!



b) suppose  $t=0, j = w_1$ , what is the prob it generates the particular sequence  $V = \{v_2, v_1, v_0\}$



the prob of this sequence  $V = \{V_2, V_1, V_0\} = .122$

- (c) Given the sequence above, what is the most probable sequence of hidden states?

Our most "probable" "path" sequence of hidden states

B  $w_1 \rightarrow w_1 \rightarrow w_0$  as  
 $\text{prob} = 1$      $\text{prob} = (.21) \text{ vs } .2 w_2$  and  $.122 \text{ vs } 0$

We used forward algorithm.