Semantic approaches to number

1 Introduction

Noun phrases can denote single objects as well as a multitude of objects. Commonly, languages have formal means to express these differences in the number of objects that the noun phrase denotes. A familiar situation, observed in English and many other languages (at least for most count nouns), is the following: (i) the cardinality of objects that the noun phrase refers to is marked directly on the noun, (ii) the noun is marked for a two-way contrast, singular vs. plural (*the apple* vs. *the apple**s***). Of course, this is by far not the only option. Languages can mark the distinction on other categories than the noun and they can provide a richer contrast than just singular-plural (singular-dual-plural, singular-dual-trial-plural, among others). A language can also express the cardinality of referents in a less direct way. For example, it can contrast underspecified with specified forms, as is the case in a dialect of Fula, in which most nouns have a general number form (a form underspecified with respect to the number), a singular form and a plural form (1) (see Koval (1979)):[1]

(1) nyaari 'cat(s)', nyaarii-ru 'cat' nyaarii-ji 'cats' (a dialect of Fula)

Three questions will be discussed in this chapter:

1. How should we model the capacity of noun phrases to refer to one or more objects?
2. How can the model be used for semantic classification of grammatical categories, in particular, verbs and verbal phrases?
3. How is the semantic model tied to number marking in language?

All of these are empirical questions, but they have a different status. An answer to the first question should provide a general model that includes objects of various cardinality. An answer to the second and the third questions should explain how the model is put to use in individual languages.

[1]See Corbett (2000) for a rich typological study on the realization of number marking.

2 Modeling the domain of entities

2.1 Introduction

As is standard in formal semantics, we assume that natural language expressions denote objects in a model. In this section, we will describe the domain of entities, D, that is part of the model. We will set up the domain of entities in such a way as to reflect the fact that languages allow us to talk about single objects (e.g., *the book*, *John*), as well as multiple objects (e.g., *the books*, *John and Bill*). What differentiates between single and multiple objects is that the former can be part of the latter, but not vice versa. For example, while the individual denoted by the proper name *John* is intuitively part of what is denoted by *John and Bill*, the reverse does not hold. Thus, understanding the properties of the domain that include both single and multiple objects is inseparable from studying parthood and the properties of the part relation. We will start by considering such properties. The properties of the part relation should not be confused with the properties of the actual English word *part*. We are not attempting to represent the meaning of the word, rather, we are trying to represent intuitions about what makes an object part of another object or objects. The part relation is standardly notated as $\leq$ and we will follow that notation here.[2]

The part relation should be transitive and antisymmetric. The transitivity requirement is in accordance with the intuition that, say, if the individual John is part of what is denoted by *John and Bill* and John and Bill is part of what is denoted by *John, Bill and Dave*, then the individual John is also part of John, Bill and Dave. The antisymmetry condition requires that an element is symetrically related by $\leq$ only to itself. Note that any single object as well as a multitude of objects are considered as elements here.

(2) Transitivity of $\leq$:
For any x, y, z, if $x \leq y$ and $y \leq z$, then $x \leq z$

(3) Antisymmetry of $\leq$:
For any x, y, if $x \leq y$ and $y \leq x$, then $x = y$

Finally, the relation is reflexive:

(4) Reflexivity of $\leq$:
For any x, it holds that $x \leq x$

This last requirement might clash with one's intuitions about what constitutes parts since it might be unintuitive to think that, for example, John is part of John. For readers who are hesitant to accept the reflexive relation, we note that we can always recover proper parthood, notated as $<$, from the part relation,

[2] The study of parthood is the domain of mereology. Due to the space limits, the properties of parthood are discussed only briefly here and somewhat informally. For more technical details, readers should consult Partee et al. (1990); Landman (1991); Moltmann (1997); Casati and Varzi (1999); Hovda (2009); Champollion and Krifka (2016).

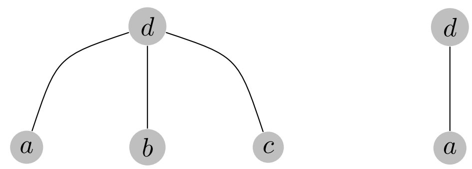

Figure 1: Examples of structures that do not have a unique join

using the definition below. This relation is irreflexive and maybe closer to one's intuitions.

(5) Proper parthood
$x < y = x \leq y$ and not $y \leq x$

Apart from $<$, it is helpful to introduce another concept, that of overlap. According to the definition (and intuitions) x and y overlap if there is an element that is part of both x and y. This captures the intuition that, say John and Bill overlaps with Bill and Sue, and that John overlaps with John and Bill.

(6) Overlap
x overlaps with y if and only if there is z, $z \leq x$ and $z \leq y$

With the help of overlap, let us consider the last concept, that of join of elements that belong to a set, which intuitively could be seen as joining all elements in a set into one (new) element. The exact conditions that joins have to satisfy are specified in (7).

(7) x is the join of (elements in) P if and only if

1.every element in P is part of x, and

2.every part of x overlaps with an element in P

For example, the join of the set consisting of sleeping individuals is an element that has sleepers as its part and whatever is part of this entity must overlap with sleepers (non-sleepers are not part of the join). We will now postulate the last requirement on the domain of entities. This is the uniqueness of join:

(8) Uniqueness of join:
For any non-empty set P of elements in the domain D, it holds that there is a unique element x that is the join of P.

Let us consider the last condition using a graphical representation, see Fig. 1 and 2. As is customary in these representations, every circle represents an element and each line represents the part relation (an element appears below an element that it is part of). Elements that have no proper parts are written at the bottom. We will call such elements atomic. The elements that have proper parts are called pluralities.

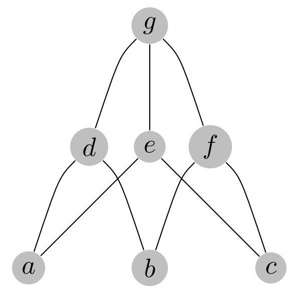

Figure 2: Examples of structures that all have a unique join

Fig. 1 shows examples of structures that lack a unique join. For the left figure, consider the set consisting of a and b. This has no join since d, the only plausible candidate, has c as its part and c does not overlap with a or b. Neither does the set consisting of b and c have any join. In the right representation in Fig. 1, we see a case in which the set consisting of a has two joins: the entity a and the entity d.

Both cases represented in Fig. 1 would be problematic as representations for the domain of entities. More concretely, let us think of a, b and c as three individuals. The structure on the left side would have no element corresponding to objects a together with b, or b together with c. The right figure would have two different objects corresponding to the join of being a, while intuitively, only one object is the join (a itself). In contrast to that, Fig. 2 represents a case in which the uniqueness of joins is satisfied. If we again think of a, b and c as individuals, we see that every join of such individuals is represented and no set has two distinct joins. Such a structure could serve as the domain of entities.

As is common, we will notate the element that is the join of a set consisting of a pair of elements using the $\vee$ symbol, for example, $x \vee y$ for the join of $\{x, y\}$.

Since every non-empty set has a a unique join, it follows that any (set consisting of a) pair of elements has a unique join. It is probably worth pointing out that this makes the set of elements with the operation $\vee$ a join semilattice, which is a set of objects that are ordered by the reflexive, antisymmetric and transitive part relation $\leq$ such that for any pair of elements x, y, the join of x and y, $x \vee y$, derived from the part relation, exists.[3]

At this point, we know what properties should be satisfied by the structure that models the domain of entities. The remaining question is how such a structure should be built. There are two standard ways. Either we construct the domain using sets and the part relation corresponds to subset relation or subset and membership relation. This approach is discussed in Sect. 2.2. Alternatively, we think of entities as individuals, such that both atomic and plural entities are

[3]To be more precise, it is a complete join semilattice. For formal properties and technical details, see Landman (1991), Landman (2000), Hovda (2009) and Champollion and Krifka (2016).

individuals. The part relation is then interpreted as individual part relation (see Link 1983). This approach is discussed in Sect. 2.3. The two approaches agree on many aspects of D, the domain of entities, as we will make precise below.[4]

2.2 The domain constructed using sets

A popular approach is to represent a plurality (an entity that has proper parts) as the set of corresponding individuals. Consider the following concrete example. Suppose there are three relevant apples: a, b and c. Using the set-theoretic notions, it seems natural to assume that in this situation *these apples* would denote the set that has a, b and c as its elements, or, in the standard notation: $\{a, b, c\}$. How can we represent that something is part of this set? Set theory includes two part relations, the subset relation $\subseteq$ and the membership relation $\in$.[5] In order to express that the set of two apples $\{a, b\}$ is part of *these apples*, we would have to make use of the first relation: $\{a, b\} \subseteq \{a, b, c\}$. If we want to express that the apple a is part of *these apples*, we have two options. The first option is to say that objects are represented just as objects, e.g., the apple a is represented as a. In that case, the part relation would have to be expressed using the second relation, $\in$, and $a \in \{a, b, c\}$ would represent that the apple a is part of the apples (cf. Bartsch 1973; Bennett 1975).

An alternative option is to let an object and the singleton set with that object count as elements of the same type in the model. This has been discussed in Quine (1969), which shows that one way to reconcile the set-theoretic law of extensionsionality with the existence of various member-less objects (e.g., apple a) is to treat an object and its singleton set (and the singleton set containing that singleton set, and so on), as one and the same thing. In natural language semantics, it has probably been most prominently used in Schwarzschild (1996) (see, in particular, the appendix in that work). In this way, then, the apple a would be the same thing as $\{a\}$, $\{\{a\}\}$ and so on and the part relation can be always represented using the subset relation, $\subseteq$. The intuition that the apple a is part of the apples would be represented as $\{a\} \subseteq \{a, b, c\}$.[6]

[4] Apart from the approaches just mentioned there are other accounts that study the interpretational role of number that will not be covered here. First, some accounts make us of plural logic to analyze the fact that language can express and quantify over plural objects (Boolos, 1984; Schein, 1993; McKay, 2006). For a good introduction into plural logic, see Oliver and Smiley (2013); Linnebo (2017). Yet other approaches employ sets of assignments to model discourse effects of singular and plural noun phrases (van den Berg 1996; Krifka 1996a; Nouwen 2007; Dotlačil 2011; see also Brasoveanu 2007, 2008). Finally, there are approaches that combine the study of parthood with the study of topological relations (see Casati and Varzi 1999; Grimm 2012; Wągiel 2018). These last approaches are related to the account discussed in Sect. 2.3 but not identical to it.

[5] For the membership relation, it holds that $a \in B$ if and only if a is an element of the set B. For the subset relation, it holds that $A \subseteq B$ if and only if all elements in A are also elements of B.

[6] Note that this conclusion is arrived at under one particular way of reconciling the law of extensionsionality with the existence of various member-less objects. Thus, there is nothing contradictory in accepting the law of extensionsionality and not believing that $a = \{a\} = \{\{a\}\} \ldots$, it just requires a different solution to the problem than Quine (1969) discusses.

Is the first option or the second option better? At this point, we will keep both ways of modeling the domain of entities with atoms open, and only note that there is a trade-off here: whereas the second approach stretches our understanding of member-less objects, the first approach complicates our understanding of being part of, which is defined in two ways.

More generally, then, we can describe the domain D of objects in the model as follows:[7]

(9) Suppose there are some objects $X = \{x, y, z \ldots\}$. Then:

a. Option 1 (D with sets & entities)
- For every element in X, D includes that element.
- For any subset of X whose cardinality is greater than 1, D includes that subset.
- Nothing else is in D.

b. Option 2 (D with singleton sets)
- For every element in X, D includes the singleton set of that element.
- For any subset of X whose cardinality is greater than 1, D includes that subset.
- Nothing else is in D.

When we need to distinguish Option 1 and Option 2, we will call the first option 'D with sets & entities' and the second option 'D with singleton sets'. We will come back to distinguishing the two options after we introduce another approach popular in the semantics of pluralities.

2.3 The domain constructed using atomic and plural individuals

A different approach to model D, which is arguably at least as common as the application of sets and has been commonly used since Link (1983), is to think of both atomic and plural entities as individuals. In this view, then, John is an individual, an apple is an individual, and so are any pluralities. For example, the apples together form an individual and so do John, Bill and Dave. Individuals are related by the individual part relation, $\leq_i$, which satisfies the conditions discussed in Sect. 2.1 (it is reflexive, antisymmetric and transitive). We can distinguish between two types of entities, which both are part of D: atomic individuals, which have no proper parts, and plural individuals, which do have proper parts.

The sum of (the individuals in) a set is a (possibly plural) individual that could be thought of as fusion of individuals in the set into one individual. The sum has to satisfy the uniqueness condition, see also discussion in Sect. 2.1.

[7] These are not the only possible ways the set theory can be harnessed in modeling D (see Hoeksema 1983; Landman 1989a,b for another salient alternative; see also Schwarzschild 1996).

What is represented?	D with sets & entities	D with singleton sets	D with plural individuals
Atomic entities, e.g., x	entities, e.g., x	singleton sets, e.g., $\{x\}$	atomic individuals, e.g., x
Pluralities, e.g., x and y	sets, e.g., $\{x, y\}$	non-singleton sets, e.g., $\{x, y\}$	plural individuals, e.g., $x \oplus y$
Part-of relation	subset, $\subseteq$, member-of, $\in$	subset, $\subseteq$	individual part, $\leq_i$
Composition of plural entities	set construction + union	union, $\cup$	sum, $\oplus$

Table 1: Table 1: Relations between various ways of modeling D

The sum of pair of entities has a plausible application in natural language semantics. In Link (1983), it is used to represent the meaning of conjoined terms like *John and Bill*. Following Link (1983), we will notate the operation as $\oplus$. The last example would then be represented as John $\oplus$ Bill and would be the plural individual that has John and Bill and no other individual as its parts.[8] Definite descriptions can also be modeled in this framework. Since Sharvy (1980), it is standard to let definite descriptions refer to the supremum of a set P, notated as $\sigma x.Px$ and defined as follows:

(10) $\sigma x.Px$ the unique individual y such that y is in P and $z \leq_i y$ for every individual z in P

For example, the denotation of *the water* can be represented as $\sigma x.\text{water}(x)$, which is the unique entity that is water and has all water chunks as its parts.

2.4 Comparison between the approaches and the role of atoms

Right now, we have several possible ways to model D, discussed in Sect. 2.2 and 2.3. Note that all these models are complete join semilattices, or, in other words, they cannot be differentiated when we just consider the conditions discussed in Sect. 2.1. Correspondences between the models are in Table 1. If we want to distinguish them, we have to consider other arguments.

One such argument comes from the domain of entities that have no atoms, i.e., that lack entities with no proper parts. Such a domain cannot be modeled using sets, which always create atomic structures.[9] This is not so for the approach using sums and plural individuals. While we did specify in Sect. 2.3 that individuals are either pluralities or atomic, nothing forced us to do so. For example, we could consider another domain, the domain of material, which lacks atoms and orders its elements by material part relation. This is in fact what Link (1983) considers for the analysis of mass nouns. In other words, we can assume that the domain of entities consists of two distinct join-semilattices: atomic (which include entities that have no proper parts) and non-atomic (in which every entity has some entity as its proper part). However, while this

[8] In a tradition separate from Link (1983), *and* is analyzed by an operation that is dual of join, meet (cf. Partee and Rooth (1983)). (Meet of two elements returns an element that they share.) See also Winter (2001) and Champollion (2016b).

[9] An option is to change the axioms that define sets to accommodate non-atomic structures, as has been done in the ensemble theory of Bunt (1985).

move is possible, it is not completely clear that using the atom-less structure is the right analysis for mass nouns. For details and more discussion, see REF-CHAPTER ON MASS/COUNT NOUNS.

Let us now turn to the first two columns in Table 1. Can we differentiate between the two approaches that both fall in the category of the set-theoretic accounts of pluralities? That is, how do D with sets & entities and D with singleton sets compare when applied to language? The former approach might seem advantageous when considering number marking. If we maintain that atomic entities are entities, rather than singleton sets, we could maybe postulate a very simple correlation between number and the model: singular number appears with noun phrases that are modeled as entities, plural number appears with noun phrases that are modeled as sets. However, mass nouns would again cause problems. For example, the noun phrase *the furniture in this room* appears as singular, yet, it does not have to be atomic (see REF-CHAPTER ON MASS/COUNT NOUNS for details).

We could weaken the correlation and suggest that while singulars can be represented by both entities and sets of entities, plurals can be only represented by the latter. But this is problematic as well. As we will see later (see Sect. 4), plurals can be used to range over atomic entities, as well.

To sum up, the first approach (D with sets & entities) makes use of two part-of relations and makes a type distinction between atomic and non-atomic entities, but at this point we have no evidence that this distinction would be useful in language. All things being equal, one should prefer an approach with fewer commitments, as is the second and the third approach. With respect to these two, it is largely immaterial (for linguists) which one is chosen when modeling the domain of atomic entities. Indeed, the argument for one or the other approach focuses on philosophical, rather than linguistic issues (see especially Link 1983 and Landman 1989a). However, restricting oneself to sets as models of a complete join semilattice is problematic when modeling the domain that should lack atoms (see also Landman 1989a,b).

3 Collective, distributive and cumulative readings

After introducing semantic models that can represent atomic and plural entities, we now turn to the second question posed in the introduction: How can the model be used for semantic classification of grammatical objects, e.g., verbs and verbal phrases? We will discuss three types of predicates/interpretations of predications that are relevant here: collective, distributive and cumulative ones.

3.1 Collective readings

Collective readings arise from predicates that apply to a semantically plural argument. Examples are *meet* and transitive *gather*, which require a semantically plural subject and object, respectively, see (11) and (12). Notice that the constraints that these predicates impose are interpretational, rather than morphological: both predicates are acceptable with arguments that are morphologically singular, as long as these arguments represent a multitude of entities, (11-c) and (12-c).

(11) a. #John met.
b. John and Mary met.
c. The committee met.

(12) a. #I gathered the apple.[10]
b. I gathered the apples.
c. I gathered the Venetian collection.

How are we to capture the interpretation of these sentences? As a starting point, let's consider first how the interpretation of sentences in which no plural arguments appear could be analyzed.

(13) John is sleeping.

Let's say that John is the entity j and *is sleeping* expresses a set of sleeping entities, $S = \{x \mid x \text{ sleeps}\}$. Then, we say that (13) is true if and only if j is in the set S.

(14) $j \in S$

Since the domain D also includes pluralities, we can follow the same steps when dealing with collective predicates. The sentence *John and Mary met* could have truth conditions described in (15-a) (using atomic and plural individuals) or (15-b) (set approaches).

(15) a. $j \oplus m \in M$
b. $\{j, m\} \in M$

It is one thing to capture the interpretation of (11-b), but we should also be able to explain why (11-a) and (12-a) are degraded. Let's consider the model in which atomic entities are represented as entities and pluralities as sets (D with sets & entities). In that case, we could say that collective predicates are those predicates that can only apply to sets (cf. Bennett 1975; Bartsch 1973). This idea has been formalized in terms of type differences: atomic entities are of type e, non-atomic entities (sets) are of type $\langle e, t\rangle$. Collective predicates are then of type $\langle\langle e, t\rangle, t\rangle$ and the degraded status of (11-a) is straightforwardly explained as a type clash (a function of type $\langle\langle e, t\rangle, t\rangle$ cannot take an object of type e as its argument). The account is simple and straightforward, but it is not without problems. First, it forces one to say that collective noun phrases, like *the committee*, are represented as sets: had they been represented as atomic entities, they should not be compatible with collective predicates. While a set-type analysis of collective noun phrases is assumed in some analyses of pluralities (cf.

[10] This sentence is acceptable if we coerce the basic interpretation of *the apple* and interpret the object noun as mass. This is not the interpreation we are after.

Bennett 1975; Kratzer 2008; Magri 2012), several influential works argued for treating collective noun phrases as atoms (Barker 1992; Schwarzschild 1996). Other approaches assumed that collective noun phrases are ambiguous between being atoms and sets (or non-atomic joins in lattice-theoretic terms) (cf. Landman 1989a,b, 2000; see also REF-CHAPTER ON COLLECTIVE NOUNS for more discussion). This last position would be compatible with the approach discussed here. However, if collective nouns could be shifted between e-type and $\langle e, t\rangle$-type, it is not clear what would block the same type ambiguity for atomic entities like j.

Second, the account introduces massive ambiguity into language. Most predicates do not impose constraints on the cardinality of their arguments. Consider a predicate like *sleep*, which can apply to atoms like j as well as non-atomic entities like the entity denoted by *John and Mary*. We would have to say that this predicate, as well as any other non-collective predicate, is ambiguous (between $\langle e, t\rangle$ and $\langle\langle e, t\rangle, t\rangle$ types).

An alternative is to consider the model that postulates no type difference between atoms and non-atomic entities. Collective predicates like *meet* and *gather* would then be restricted to arguments that consist of easily identifiable proper parts. What would count as easily identifiable proper parts? Parts that can be found using the part-of relation ($<_i$ / $\subset$) would belong here, but not only those, as we will see below. Non-collective predicates (e.g., *sleep*) would have no such restriction. Since no type difference is assumed between collective and non-collective predicates, the second problem does not arise. Regarding the first problem, the crucial question is whether a collective noun phrase like *the committee* consists of easily identifiable parts. Intuitively, this is so: the (atomic) parts are individuals that form the committee. But how to model that? If we treat collective noun phrases as plural entities, then finding their parts is no different than finding parts of coordinations: in both cases it boils down to the $<_i$ relation ($\subset$ if we work with sets). If we treat collective noun phrases as atoms, something more has to be said. One prominent analysis of collections as atoms is presented in detail in Landman (1989a,b); Link (1991); Landman (2000). These works argue that collections are impure atoms. Impure atoms, unlike pure atoms like John, can be mapped to an entity that is the sum of all entities that constitute it. If such an entity itself has proper parts then the constraint on collective predicates is satisfied. Let's notate the mapping from impure atoms to plural entities as $\downarrow$. (It is also common to assume the inverse of the function, $\uparrow$.) One way to understand this analysis is to notice that it introduces another part relation, see (16), using the lattice-theoretic notation. Apart from the part relation introduced in the last section, (16-a), we also have the part relation shown in (16-b), which tells us what easily identifiable parts a collective noun phrase has. Notice that we end up with two relations, but this definitely does not mean that we sneak in the distinction that was present in D with entities & sets from the start. D with entities & sets distinguish between membership relation and subset relation, and these relations are different from (16-a) and (16-b).

(16) Part-of relations for pure and impure atoms: x is an easily identifiable part of y iff

a. $x \leq_i y$, or
b. y is an impure atom and $x \leq_i \downarrow y$

Even the analysis postulating (16) is not without problems. One issue is that sentences like (11-b) become ambiguous. They could be interpreted as (17-a) or as (17-b).

(17) a. $j \oplus m \in M$
b. $\uparrow (j \oplus m) \in M$ (i.e., the group consisting of j and m)

This leads Landman to a modification of the presented account (see Landman 1989a). The modification is to restrict all non-pluralized predicates (on pluralization, see Sect. 3.2) only to atoms. This is true of non-collective predicates as well as collective predicates. Because of this condition, there is only one way that (11-b) can be represented, namely as (17-b), where the singular predicate denoting *to meet* takes the group made up of John and Mary as an argument. Furthermore, collective predicates are still required to apply only to arguments with easily identifiable parts, and because of that, sentences like (11-a) are excluded.

The difference between impure atoms and pluralities that can be mapped to impure atoms is freed to explain a different fact about collective predicates: that they come in at least two types. We will discuss this empirical fact in the rest of the section.

First, notice that quantifiers that are syntactically singular like *each boy* or *each apple* cannot be arguments of collective predicates. This can be explained by pointing to the fact that *each boy*/*each apple* range over pure atoms. In other words, the oddness of these examples has the same source as #*The boy met*/#*I gathered the apple*. Interestingly, other quantificational noun phrases, like *most/all the boys/apples*, are possible here, see (18-c) and (18-d).

(18) a. #Each boy met in the hall.
b. #I gathered each apple.
c. All the boys met in the hall.
d. I gathered most/all the apples.

As has been known since Dowty (1987), there is a subset of collective verbs that cannot combine even with such quantificational noun phrases. They are restricted to plural referential noun phrases (e.g., plural definites, coordinations of proper names). An example is the relation *outnumber*.

(19) a. The boys outnumber the girls.
b. #All the boys/most boys outnumber all the girls/most girls.

There are several approaches to explain the contrast between the two types of collective predicates (see Dowty 1987; Brisson 1998; Winter 2001; Brisson 2003; Champollion 2010; Dotlačil 2013; Minor 2017). We will discuss one such approach, presented in Winter (2001, 2002). Using the terminology of that work, let us call predicates like *outnumber* atom predicates. Such predicates in their non-pluralized versions range over (pure or impure) atoms. This is in contrast to the remaining predicates, so-called set predicates, which are argued to range over non-atoms. An example of a set predicate is *meet*.[11] The contrast in (18) and (19) is then explained by assuming that the plurality *the boys* can shift to an impure atom (through $\uparrow$) but quantificational phrases as *all the N/each N/most N* cannot do so, and, furthermore, the determiner *each*, unlike *all* and *most*, ranges only over atoms. For more details, see the cited work.

3.2 Distributive predicates

In this section, we discuss another class of predicates, which are compatible with pure atoms, in contrast to collective predicates, and when these predicates take a plural argument, they often give rise to inferences that atoms or sub-groups carried out the action described by the predicate. Due to these inferences, that is, due to the fact that the meaning of the predicate is distributed down to parts of the plural argument, this class of predicates is known as distributive predicates.

We are mainly interested in understanding how our model of pluralities can be used to capture the distributive property. For reasons to be discussed, we will need different tools to capture distributivity of purely lexical predicates and distributivity of complex predicates. An example of the first type is verb-only predicates, as is *sleep* in the sentence *John and Mary sleep*. An example of the second type is a syntactically composed predicate, as is *drank a coffee* in *John and Mary drank a coffee*. We discuss the first type, lexical predicates, in the next section.

3.2.1 Lexical predicates

Consider the following example:

(20) John and Mary sleep.

From what we have said so far, we should conclude that (20) is true if the plural entity of John and Mary is in the set of S of sleepers, $j \oplus m \in S$ (or, using the set-theoretic domain, $\{j, m\} \in S$).

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[11] Note that this position differs from that of Landman discussed above, in which non-pluralized predicates range over atoms. That is, translating this position to lattice-theoretic terms, we would conclude that *John and Mary met* is captured by (i-a), and not by (i-b), and collective noun phrases can shift to pluralities.

(i) a. $j \oplus m \in M$
    b. $\uparrow (j \oplus m) \in M$

What is crucial for us is that this approach also gives up the ambiguity of (17), albeit going in the opposite direction than Landman's account. It should also be noted that this position is not far off from later works of Landman, see (Landman, 2000, Chapter 5).

But what does it mean for a plural entity to sleep? One reasonable thing to say is that a plural entity sleeps if and only if each of its atoms sleeps. In this respect, a predicate like *sleep* is clearly different from collective predicates. For example, *John and Mary met* does not imply that John met and that Mary met (the last two sentences are not even interpretable).

In taxonomic terms, *sleep* in (20) belongs to the class of predicates for which it holds that whenever they apply to a plural entity, they apply to every relevant part of the plural entity. Such predicates are called distributive. We will not discuss the question of what counts as a relevant part (see Gillon 1987; Schwarzschild 1996 on that). We only note that when the plural entity is built from pure atoms, the atoms count as such relevant parts, and we will restrict our attention to such cases in the following discussion.

Note that the terminology is slightly confusing here, since collective predicates can also distribute. For example, given our world knowledge (at the time of writing), (21) does not describe one joint meeting. Rather, *meet* distributes down to impure atoms that make up the subject, and the sentence expresses that two separate meetings took place. But *meet* still qualifies as a collective predicate, since it does not and cannot distribute down to pure atoms.

(21) The North Korean committee and the South Korean committee met.

How are we to analyze distributive predicates? One option is to say that the inference is simply a lexical property of the predicate *sleep*. When learning the meaning of *sleep*, we also learn that for that word, the propositions in (22) express roughly the same. (Such pairs are sometimes called pseudo-equivalent, see Scha and Winter 2015; de Vries 2015.)

(22) x sleeps $\leftrightarrow$ every atomic entity part of x sleeps

To put it differently, this line of reasoning would mean that (20) does not need to be represented differently from a case of collective predication. The only difference is that *sleep*, unlike collective predicates, does not come with restrictions on what its argument can be, and it is endowed with a pseudo-equivalence that implies a distributive reading.

This line of analysis is strengthened by the following contrast (from Yoon 1996):

(23) a. Are the toys clean?
b. Are the toys dirty?

Suppose that someone asks (23) in a nursery for small children. Answering 'Yes' to (23-a) implies that every toy is clean. But in order to answer 'Yes' to (23-b), it suffices that some of the toys are dirty. Given that, as was just claimed, the inferences to sub-parts are lexical, it is fully expected that different lexical predicates would yield different inferences (see Yoon 1996 for details). It is then natural to let these inferences be part of lexical semantics (Scha, 1981; Dowty, 1987; Casati and Varzi, 1999). The following pair of pseudo-equivalences can be considered here:

(24) a. x is dirty $\leftrightarrow$ some part of x is dirty
b. x is clean $\leftrightarrow$ every part of x is clean

Besides lexical information, pragmatics is also known to influence the choice between the 'every' and the 'some' interpretation (see Krifka 1996b; Malamud 2012).

To sum up, up to now we can maintain that the relation between the predicate and its argument is the same irrespective of whether the predicate applies to atomic entities (e.g., the entity denoted by *John*) or plural entities (e.g., $\sigma x.\text{toys}(x)$ or $j \oplus m$). Once the application is licit, it is also irrelevant whether we are dealing with collective or distributive predicates: in both cases the predicate directly applies to an entity, and it is only lexical specification that will tell us how that application is to be understood. We will now turn to cases which complicate the picture.

3.2.2 Complex predicates

Consider the following case, from Winter (2001):

(25) The girls are wearing a dress.

The pragmatically most plausible interpretation could be paraphrased as 'every girl is wearing a dress'. This is a distributive reading of (25). The reading might be slightly marginal but it is possible and plausible. The predicate *wearing a dress* applies to each atom in the plural entity denoted by *the girls*. Intuitively, it is parallel to the readings discussed above (e.g., *the toys are clean*). Could we account for them in the same way? This is doubtful. For example, we could try to specify the following lexical conditions:

(26) x wears y $\leftrightarrow$ every atom in x wears y

But this is not going to work. If we plug in the plural entity corresponding to *the girls* as x, and we let y be existentially bound, all we would get is that every girl wears the same dress.

(27) $\exists y$(every atom in $\sigma x.\text{girls}(x)$ wears y)

The problem is that dresses covary with the girls, and (27) does not deliver this covariation.[12]

There are two closely related approaches to account for (26) and both of them postulate a special constant, an operator, that enriches the domain D to account for distributive interpretations.

One line of research relates distributive readings to pluralization. Constructively speaking, the pluralization of a predicate P (a set of entities) collects all the sums created from subsets of P. The definition is below and one illustrative

[12] But see de Vries (2015) for a possible approach in which (26) is analyzed through inferences present in lexical semantics.

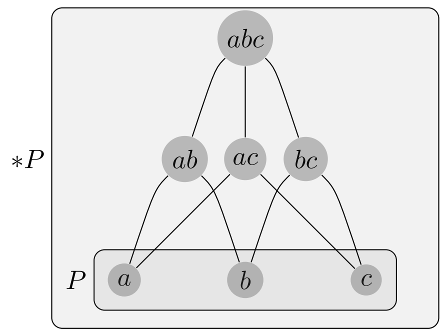

Figure 3: Pluralization of P

example in Fig. 3. One way to think about this is to notice that $^*P$ is the smallest set that is a superset of P and $\forall x, y \in {}^*P[x \oplus y \in {}^*P]$.

(28) $\quad {}^*P = \{\sigma x.Qx \mid Q \subseteq P$ and Q is not empty$\}$

The example shows why we talk about pluralization. For instance, while *girl* can be understood as the set consisting of a, b and c, *girl would be the set consisting of a, b and c and all sums (plural entities) formed out of these three entities. Intuitively, this is what we would like the meaning of the plural expression *girls* range over (see also Sect. 4).

Pluralization is commonly used to model morphological pluralization. However, it can also be used for distributive readings. Distributive interpretations are analyzed as cases of predicate pluralization (see, in particular, Kratzer 2003, 2008; see also Landman (2000)). The distributive reading of (25) is represented by specifying that the plurality *the girls* is in the pluralized predicate *wearing a dress*. If wearing a dress is true only for individual girls, say a, b and c, then *the girls* will correspond to $a \oplus b \oplus c$ and this plural entity will be part of the pluralized predicate, Fig. 3. Notice that while pluralizing a predicate allows a distributive reading it does not <u>force</u> a distributive reading of the predicate. For example, if the girls only wore one joint, ballooning, big dress, then *(*wearing a dress*) would also hold of the entity the girls. This might be a problem if we assume that distributive readings are separate interpretations, distinct from other ones (in particular, collective readings). Whether this is so is an open question, see Schwarzschild (1996); Lasersohn (1995); Frazier et al. (1999) for differing positions. See also Champollion (2014) for discussion.

The second approach postulates a distributivity operator (see Link 1983; Roberts 1990; Schwarzschild 1996; Lasersohn 1995, 1998; Champollion 2016a), often notated as Dist or D. We will use the former notation to avoid a clash with the label we use for the domain of entities. It has been noted (see especially Landman 1989a) that the Dist operator can be defined in terms of pluralization, (29).

(29) $\mathrm{Dist}(P) = {}^*\{x \in Atom : Px\}$

That is, the Dist operator restricts pluralization to cases in which the original, non-pluralized predicate is true of atoms. This would correctly work for the example above. Furthermore, unlike the * operator, the distributivity operator Dist forces the distributive reading. For instance, Dist(*wearing a dress*) would not hold of the entity the girls if the girls only wore one joint big dress.

Connecting pluralization and distributivity is theoretically appealing as we only have to work with one concept underlyingly – distributivity is just pluralization, or restricted pluralization in the sense of (29). The connection helps to explain some equivalences present in the interpretation of English (and other languages). One such case is shown below (from Landman 1989a). Assuming that the plural form of *boy*, *boys*, is modeled as **boy*, we can derive the implication. The case in (30-b) follows in the same way, the only difference is that * (or Dist) applies to the predicate *carried the piano upstairs*.

(30) a. If it is true that John is a boy and that Bill is a boy then it holds that John and Bill are boys.
b. If it is true that John carried the piano upstairs and that Bill carried the piano upstairs then it holds that John and Bill carried the piano upstairs. (due to the distributive reading of *carried the piano upstairs*)

Before moving to the last type of reading, one last issue should be mentioned. It is possible that some readers might struggle with accepting sentences such as (25) in its distributive interpretation. Indeed, it has been shown repeatedly that the distributive reading with complex predicates is marginal (see Dotlačil 2010 and references therein for evidence). If the distributive reading is the only pragmatically plausible reading, as is the case in (25), readers might find the sentence itself hard to accept. This can be explained in several ways. First, it is possible that inserting the extra operator, * (or Dist), yields lower acceptability, which would match with the findings of Frazier et al. (1999); Boylan et al. (2011) that collective interpretations are preferred for definite plurals and conjunctions of proper names. Alternatively, it is possible that readers prefer the distributive reading to be marked overtly (e.g., by the (floating) quantifier *each*). If the marker is absent, it is assumed that the sentence was used to convey the collective reading. For details, see Dotlačil (2010) and Pagliarini et al. (2012).

3.3 Cumulative readings

The last reading we will consider is a cumulative reading (the reading has also been called 'serially distributive' in Kroch 1974 and 'codistributive' in Sauerland 1998 and Winter 2000). The interpretation can be shown on the example (31).

(31) Two chickens laid three eggs.

The most prominent reading is that two chickens were involved in laying eggs and three eggs were laid in total. We could analyze this reading as follows: there are two pluralities (*two chickens*, *three eggs*) and these are related by *lay*; it is then lexical meaning of the verb that specifies how chickens and eggs are related. Alternatively, we could generalize pluralization (or distributivity operator) to apply to relations (see also Scha 1981; Sternefeld 1998):

(32) $^*R :=$ the smallest set such that it is a superset of R and $\forall x, x', y, y'[\langle x, x'\rangle, \langle y, y'\rangle \in {}^*R \rightarrow \langle x \oplus y, x' \oplus y'\rangle \in {}^*R]$.

Perhaps, pluralizing relations might seem excessive for simple examples like (31). But things change once we consider more complex sentences, like (33) (from Beck and Sauerland 2000). The most natural reading is that every politician received a bribe from one of the five companies and each of the five companies gave a bribe to one of the politicians. Crucially, bribes covary with polician-company pairs. The covariation is hard to account for if we simply assumed that the cumulative reading is a lexical property of the verb (for the same reason that covariation was hard to account for in examples in Sect. 3.2.2).

(33) The politicians have taken a bribe from the five companies.

On the other hand, the interpretation follows if we assume that (33) involves a pluralized relation:

(34) a pair of pluralities *the politicians*, *five companies* is in the relation: $^*\{\langle x, y\rangle : x$ have taken a bribe from $y\}$.

While we can derive the right reading for (33) using (34), this analysis is not without problems. First, we have to assume that somehow the relation *have taken a bribe from* is an available object that can be pluralized. Following Beck and Sauerland (2000), it would mean that the relation is a constituent (since the pluralization operator in that work applies only to syntactic constituents). In Beck and Sauerland (2000), this is achieved by quantifier raising (QR). However, the QR used is an unorthodox one. Normally, an argument after QR has, as its scope, a set of entities. Let's show this on one example. The sentence (35-a) is most naturally interpreted in inverse scope, expressing that for every corner, there was a policeman who stood on that corner. This interpretation can be delivered if we assume that the argument *every corner* undergoes QR. In that case, the argument has, as its scope, a set of objects that one or other policeman stood on, as shown in (35-b).

(35) a. A policeman stood on every corner.
b. every corner ($\{x \mid$ a policeman stood on $x\}$)

In contrast to that, arguments in (33) do not have a set of entities as their scope. Rather, a more complex object, a set of pairs, has to be created, which the two arguments combine with at the same time. Appealing to QR does not explain how this more complex process is allowed in grammar.

Second, the analysis along the lines of (34) would make us expect that if A and B are independently known to be able to QR, then a pair A, B could be among the pairs of the pluralized relation, and so could a pair B, A. But this is not so.[13] We will show this on two examples, which involve the quantifier *every N* and numerals, both of which should be compatible with QR in the sentences that we will investigate. The examples come from Kratzer (2003) and are based on Schein (1993).

First, consider the baseline case, in (36). This allows for a cumulative reading: between them, three copy editors caught all the mistakes in the manuscript.

(36) Three copy editors caught every mistake in the manuscript.

However, a variant of (36) in which the quantifiers in the subject and the object are reversed, lacks a cumulative interpretation, see (37). That interpretation should express that between them all the copy editors caught 500 mistakes. For instance, this would be true if there were five copy editors and each of them caught 100 mistakes. This is not a possible interpretation.

(37) Every copy editor caught 500 mistakes in the manuscript.

But why not? After all, in both cases, we just have to pluralize the relation *caught in the manuscript* and let the subject-object pair combine with that.

The final and related problem is that (34) requires that the pair of arguments combining with the pluralized relation co-vary with the indefinite as a pair. This works fine for that example, but there is evidence that only one of the arguments might co-vary with an indefinite in cumulative readings. For details, see Chapter 10 of Landman (2000). All the issues discussed have lead semanticists to several alternatives to Beck and Sauerland (2000), see Landman (2000), Kratzer (2003), Champollion (2010), Dotlačil (2010), Brasoveanu (2013).

3.4 Summary

We have discussed three types of readings arising with predicates, collective, distributive and cumulative ones, and we have seen evidence to complicate the domain of entities. In particular, arguments were given to introduce impure atoms into the system, as well as several operators (* or Dist, ↑ and ↓). We have also seen that contrary to first impressions, it is not beneficial to distinguish atoms and non-atoms in the model as objects of different types, as D with sets & entities would have it. In the next section we consider another domain in which the atomic/non-atomic distinction plays a role: that of grammatical number.

[13] However, Beck and Sauerland (2000) also discuss arguments from QR, not covered here, that do support their account.

4 Mapping the domain of entities to grammatical number

After introducing semantic models that can be used to represent the domain of entities, and after discussing how the properties of the domain help us classify various readings/predicates, we will turn to a separate issue: how is the semantic model tied to number marking in natural languages?

Our main focus will be on English, even though we will make a few observations about other languages.

Probably, the most natural starting point is to claim that singular and plural distinction corresponds to the atomic/non-atomic difference. We will label this the partition hypothesis (since it partitions D into two parts, the atomic part for singular nouns and the non-atomic part for plural nouns) and spell it out in (38).

(38) Hypothesis 1: Partition on D by number marking
  a. Singular noun phrase – entities that have no proper part in D (entities are atomic)
  b. Plural noun phrase – entities that have proper part in D (entities are non-atomic)

There are two challenges for this hypothesis, and they both come from plural nouns.

First, it has been observed that when bare plurals appear in a clause with other plural arguments, they can have a reading that one would expect with singular indefinites. The example (39-a), from Chomsky (1975) is a case at hand, as it seems to express what (39-b) does. Notice that when the condition on another plural argument is not satisfied, as in (39-c), the reading parallel to (39-b) does not arise.

(39) a. Unicycles have wheels.
  b. Unicycles have a wheel.
  c. #Every unicycle has wheels.

This interpretation, also known as a dependent plural reading (cf. de Mey 1981), seems to fly in face of the partition hypothesis: here, we seem to have a case in which both singular and plural forms are used to denote the same objects.

However, the fact that singulars and dependent plurals collapse is more of a lucky coincidence in (39), as already observed in Kamp and Reyle (1993). In (40), the argument *ingredients* can have a dependent plural reading, yet it is distinct from (40-b) even in that reading.

(40) a. Most boys bought ingredients that were sufficient to bake the cake.
  b. Most boys bought an ingredient that was sufficient to bake the cake.

To see this, consider the following scenario (based on Kamp and Reyle 1993).

There are five boys and four of them go shopping. The first two boys each buy just one ingredient that is enough to bake the cake (a pre-packaged baking set). The other two boys each buy several ingredients that are needed to bake the cake. In that case, (40-b) is false since there are only two boys out of five that bought one ingredient sufficient to bake the cake. But (40-a) is true. This indicates that dependent plurals are number-neutral, rather than atomic.

The second problem is related. In some contexts, plurals are sometimes used as underspecified with respect to number information. For example, a parent can answer (41-a) in a positive way even though they have only one child. But if the plural form was used to mark non-atomic entities, we would expect that people should respond with 'No' when they have only one child. Following Farkas and de Swart (2010), we will call this use an inclusive reading of plurals. That plurals in inclusive readings are underspecified, rather than identical to singulars, can be seen from the fact that (41-b) differs in meaning from (41-a) (from Krifka 1989). The contrast is even sharper in (41-c) (from Farkas 2006).

(41) a. Do you have children?
b. Do you have a child?
c. Does Sam have a Roman nose/#Roman noses?

To sum up, we see two cases which seem to show that plurals do not express any number information, unlike singulars. We will now discuss these cases in turn.

4.1 Dependent plurals

We have seen above some evidence that dependent plurals are number-neutral. It turns out that dependent plurals do impose cardinality restrictions, just not in the way one might have expected. This can be seen in (42), from Zweig (2008).

(42) a. Ten students live in New York boroughs.
b. Ten students live in a New York borough.

While both (42-a) and (42-b) can be interpreted as stating that each of ten students lives in just one New York borough, (42-a) expresses something more: it requires that not all students live in the same borough, that is, at least two boroughs have to be considered in (42-a). This shows that the requirement that the plural morphology denotes non-atomic entities is true for dependent plurals, as well. However, this condition does not have to be satisfied in the scope of the licensor of the dependent reading (per student in (42)). It has to be satisfied outside of it (for ten students as a whole).

This suggests that dependent plurals do not need to be a problem for the partition hypothesis. Of course, it remains to be seen how they are to be accounted for.

One prominent line of approach is to relate dependent plurals to cumulative readings (cf. de Mey 1981; Roberts 1990; Beck 2000). Indeed, there is a clear parallelism between cumulative and dependent readings. For example, the cumulative reading of (43), repeated from above, states that every chicken laid some eggs and every egg was laid by some chicken and there were three eggs laid overall and two chickens were involved in this. We can see that in this case, the cardinality requirement of the object is not satisfied in the scope of the subject but outside of it, as is the non-atomic requirement of dependent plurals.

(43) Two chickens laid three eggs.

Unfortunately, analyzing dependent plurals as cumulative readings undergenerates. A problem is that cumulative readings are not licensed by quantificational elements: noun phrases headed by *all*, *most*, quantificational adverbs like *always* etc. Yet, these elements do license dependent plurals. The contrast can be seen quite clearly in (44), based on (Zweig, 2008, ex. 46). (44-a) can have the dependent plural reading, expressing that John ordered one or more pizzas per Friday dinner and in total, more than one pizza was ordered last month; but (44-b) clearly lacks a cumulative interpretation, which would express that John ordered one or more pizzas and in total he ordered fourteen pizzas last month.

(44) a. Last month, John always ordered pizzas for Friday dinner.
b. Last month, John always ordered fourteen pizzas for Friday dinner.

One generalization appearing in literature (Champollion 2010; Minor 2017) is that dependent plurals are licensed by the same noun phrases that are compatible with *meet*-type collective predicates (see Sect. 3.1). The fact that dependent plurals are related but not identical to cumulative readings significantly complicates their analysis. Currently, the most promising analyses that can capture contrasts such as (44) are Champollion (2010), Ivlieva (2013) and Minor (2017), which, however, significantly go beyond the concepts introduced in this handbook article. The last work also observes a second, related generalization: only noun phrases that allow inclusive readings (see Sect. 4.2) can show dependent plural readings in the scope of quantificational noun phrases like *all NP*. The evidence is unfortunately scarce, it is only based on English and European Portuguese, on one hand, and Brazilian Portuguese, on the other hand (cf. Martí 2008, Minor 2017). In Minor (2017), inclusive readings are analyzed as number neutral, and the same interpretation is required for dependent plural readings in the scope of quantifiers (e.g., *all*). If this analysis is on the right track, it provides an interesting twist to dependent plurals: dependent plurals might, after all, provide evidence against the partition hypothesis and for number neutrality of plurals; that is, plurals would be number neutral when they give rise to dependent plural readings in the scope of *all*. However, this analysis is only tenable if one can show that inclusive readings are due to number neutrality of plurals. We turn to that question next.

4.2 Inclusive readings

Inclusive readings were probably first observed in formal linguistics in connection with *no*, as in (45), which would be judged as false if one chair was available (cf. Hoeksema 1983; van Eijck 1983). These papers, however, were mainly concerned with the question of what such readings imply for the analysis of the negative determiner.

(45) No chairs are available.

Sauerland (2003) is the first account to fully focus on the analysis of plurals based on (45) and similar examples. The account is modified and discussed in more detail in Sauerland et al. (2005). Both articles argue that there is an asymmetry between singular and plural, proposing a different hypothesis than we have encountered, see (46). Let's call it a subset-superset hypothesis (since plurals would form supersets of singulars).

(46) Hypothesis 2: Subset-superset on D by number marking

a. Singular noun phrase – entities that have no proper part in D (entities are atomic)

b. Plural noun phrase – all entities (atomic and non-atomic)

It should be clear how this hypothesis explains the contrast between the following pair of questions, repeated from above, and in particular, why the first question can be answered positively even if you have only one child.

(47) a. Do you have children?

b. Do you have a child?

But in most cases, there clearly is some interpretational effect attributable to plurals. For example, if I say (48), then I do not want to express that Mary has one or more children. Suppose I do not know precisely how many children she has and later on I learn that she has only one kid. Then I should correct my statement in (48) with something like 'Actually, she only has one child'.

(48) Mary has children.

How could that be explained?

In Krifka (1989) it has been argued that the-more-than-one interpretation, present in (48), arises through a competition between singulars and plurals, and this idea has been worked out in details since (cf. Sauerland et al. (2005); Spector (2007); Zweig (2008); Ivlieva (2013)). The crucial observation is that plurals are weaker in their meaning than singulars are. Hence, plurals can be strengthened to exclusive, more-than-one reading in actual communication. This explanation parallels the standard analysis of weak quantifiers like *some*, which are analyzed as meaning some, possibly all. In normal statements, this weak interpretation is often strengthened to the some-but-not-all interpretation. Notice that the parallelism between the interpretation of *some* and the strengthening of plurals, carries over to questions. As we can see in (49), a question similar to (47) does not show strengthening: (49) would most likely be answered positively even if the addressee read all the assigned books.

(49) Did you read some books on the list?
  a. Answer: Yes, in fact I read them all.
  b. Answer: #No, I read them all.

More generally, it has been observed that plurals are standardly not strengthened to the more-than-one reading in any downward entailing context (cf. Zweig 2008 for discussion). Probably the most well-known examples of downward entailing contexts are negation and the antecedent of conditionals, which, indeed, favor inclusive interpretations of plurals, see (50). Both examples are taken from Sauerland et al. (2005).

(50) a. Kai hasn't found any eggs.
  b. If John had eaten any apples from the basket, there would be at least one/#two less in the basket.

While the subset-superset hypothesis has a promise to explain inclusive readings as well as why plurals do often carry the more-than-one interpretation, it is not without problems. An alternative approach, in which singulars, rather than plurals, are the less specific category, is provided in Farkas and de Swart (2010). One argument in favor of this alternative is that it keeps semantic markedness and morphological markedness aligned (see also Sauerland 2008 and Bale et al. 2011 for discussion). Let us elaborate on the last point. Plural is morphologically marked in English and many other languages, as we can see from the fact that it is associated with an extra sound (e.g., a plural suffix in English), unlike singular (cf. Bale et al. 2011). And suppose, for the sake of argument, that semantic markedness can be measured as extension, so that a phrase with a wider extension (a superset) is less marked than a phrase with a narrower extension (a subset) (for other ways of defining semantic markedness, see Farkas and de Swart 2010). Then, indeed, the subset-superset hypothesis would be a case of mis-aligned markedness: plural forms are morphologically marked, but they are interpreted as supersets of singulars and hence, they are semantically unmarked.

Another issue that the subset-superset hypothesis faces is accounting for number markings in languages different from English. First of all, there are languages with the singular-plural dichotomy in which inclusive plurals are absent. That is, plurals in sentences similar to (49) and (50) would be understood as ranging only over plural entities. Examples of such languages are Brazilian Portuguese (Müller, 2002) and Western Armenian (Bale et al., 2011). For example, the following sentence, which includes a plural, is interpreted as asking about two or more children (from Bale et al. 2011).

(51) bəzdig-ner unis?
  child-indef,pl have$_{2sg}$

'Do you have children? (two or more children)' (Western Armenian)

This state of affairs could be accommodated if we assumed that Western Armenian is a case of language in which the partition hypothesis holds (see (38)). But that opens up a question why languages would differ with respect to the way they carve up the domain of entities.

Another question is how either hypothesis should be applied to languages with other than the singular-plural opposition. The subset-superset hypothesis has been used to analyze languages with dual (Sauerland, 2008). However, the work has been criticized on empirical grounds (see also REF-CHAPTER ON DUAL IN SLOVENE, THIS VOLUME). It would be especially interesting to see how the subset-superset theories can be matched to languages that have a specifically designed form for number underspecification, see (1), see also discussion in Dalrymple and Mofu (2012), and how such languages compare to English.

5 Conclusion

We discussed semantic frameworks that model the general capability of language to refer to atomic, as well as non-atomic entities. Two mathematical concepts were used in the formalization: a set-theoretical approach and an approach in which entities were modeled as atomic and plural individuals. The two approaches are common in semantics, therefore, it is good to be aware of them both.

We saw that there was little difference between what was called D with singleton sets and D with atomic/plural individuals, at least as far as the count domain is concerned. On the other hand, an approach in which atomic entities are treated as objects and pluralities as sets (D with sets & entities) complicates the model by operating with two part relations and two types of objects in D and the complication does not seem to bring advantages when applied to natural language.

The developed models were used to explicitly address the questions as to how three types of predicates, collective, distributive and cumulative ones, are interpreted, as well as what number marking expresses. As we saw, answering the first question led semanticists to extend the framework with new operators and a new distinction (between pure and impure atoms). The second question opened up an issue whether the space of entities is partitioned or exists in the subset-superset relation. Even though the first possibility might seem more intuitive, it is the second possibility that is becoming the standard analysis, at least for some languages.

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