CSCI 4408/5408, Professor E. Gethner
Assignment 3
12 February 2018
Quiz 3 will be on Thursday, 22 February 2018.

Please feel free to collaborate with one another on this assignment. As practice for writing the quiz, write up the solutions on your own. **IMPORTANT: Be neat, write complete sentences, and SHOW ALL OF YOUR WORK. The way you communicate the solution to your answer is as important as the answer itself.** Good luck! [Unless explicitly stated otherwise, you may assume that the graphs in each problem are finite and undirected.]

1. **(Non-Isomorphic Trees)**

   Before resorting to *Mathematica* or the web for the answer, think this problem through with pen and paper.

   (a) Think of a by-hand method to give a list of all non-isomorphic trees on exactly five vertices. Display your results using *Mathematica*.

   (b) Use your results from (a) to give a list of all non-isomorphic trees on exactly six vertices. Display your results using *Mathematica*.

   Be sure to explain in detail the method you came up with to acquire your lists in (a) and (b).

2. **(More About Degree Sequences)**

   (a) Find two non-isomorphic trees with the same degree sequence.

   (b) A mysterious tree T has degree sequence $\mathbf{d}$= (5,4,3,2,1,...,1). Determine the number of 1's in the sequence. Think about this one before resorting to the use of *Mathematica*. Then display a tree with degree sequence $\mathbf{d}$ using *Mathematica*.

3. **(How Many Vertices?)** I have a tree T whose average vertex degree is exactly 1.99. How many vertices does T have?

4. **(About Bipartite Graphs)** You may use the Theorem below to help in your investigations of this problem.

   Theorem 1 *An undirected finite graph G is bipartite if and only if it contains no cycles of odd length.*

   Do the following problems with your brain, pen, and paper.

   (a) Prove that if T is a tree, then T is bipartite. Is the converse true? Why or why not?

   (b) Which simple paths P_n are bipartite? Explain.

(c) Which complete graphs K_n are bipartite? Explain.

(d) Is the Peterson Graph bipartite? Explain.

(e) Let G be an arbitrary bipartite graph. What is the smallest possible girth of G? Explain.

5. **Extra Credit: 30 quiz points added to your quiz total.** Present a proof (thorough with all details) of Theorem 1 (problem four in this homework assignment) in class.