CSCI 4408/5408, Professor E. Gethner
Assignment 2
30 January 2018
Quiz 2 is on Tuesday 13 February 2018

Please feel free to collaborate with one another on this assignment. As practice for writing the quiz, write up the solutions on your own. **IMPORTANT: Be neat, write complete sentences, and SHOW ALL OF YOUR WORK. The way you communicate the solution to your answer is as important as the answer itself.** Good luck! [Unless explicitly stated otherwise, you may assume that the graphs in each problem are finite and undirected.]

1. **(Degree Sequences)**

   (a) In class we showed that the degree sequence $\mathbf{d}$=$(6, 5, 5, 4, 3, 3, 2, 2, 2)$ was graphic by using the Havel-Hakimi algorithm to reduce $\mathbf{d}$ to $(1, 1, 1, 1, 1, 1)$. The latter sequence is graphic by way of the graph G consisting of three vertex-disjoint edges (see class notes). Use the method given in the proof of correctness of the Havel-Hakimi algorithm to modify G to attain a graph G' with degree sequence $\mathbf{d}$. Although homework is not handed in, for practice, you may want to display G, all of the intermediate graphs, and the final graph G' using *Mathematica*. Be sure to explain your solution in detail.

   (b) Prove or disprove: the following sequence is graphic. $\mathbf{d}$=$(4, 3, 2, 2, 2, 1, 1, 1, 1, 0)$. If the sequence *is* graphic, then follow the procedure in part (a) and, for practice, show all steps of your construction with *Mathematica* output. Be sure to explain your solution in detail.

2. **(More About Degree Sequences)**

   (a) Prove that for every integer x with $0 \leq x \leq 5$, the sequence consisting of $\{1, 2, 3, 5, 5, x\}$ is not graphic. It is your job to put each particular value of x in the correct position in the sequence.

   (b) If the sequence consisting of $\{x, 7, 7, 5, 5, 4, 3, 2\}$ is graphic, then what are all possible values for x? You may assume that $0 \leq x \leq 7$. It is your job to put each particular value of x in the correct position in the sequence.

3. **(Self-Complementary Graphs)**

   Definition 1 *Let G be a finite simple graph with $|V(G)| = n$. Then the* complement of G, *written $\overline{G}$, is given by the graph with the same vertex set as G, but $uv \in E(G)$ if and only if $uv \notin E(\overline{G})$.*

   Hint: think of removing the edges of G from K_n to get $\overline{G}$.

Definition 2 *A graph G is said to be* **self-complementary** *if $G \approx \overline{G}$. (That is, to be self-complementary, G must be isomorphic to its complement).*

(a) Find a graph on four vertices that is self-complementary.

(b) Find a graph on four vertices that is not self-complementary.

(c) Find a graph on five vertices that is self-complementary.

(d) Find a graph on five vertices that is not self-complementary.

Explain your work in detail, and use *Mathematica* to display your output.

4. **(Using the Adjacency Matrix to Determine Connectivity)**

In class we will prove the following theorem.

Theorem 1 *Let G be a finite simple graph with n vertices labeled by $v_1, v_2, \ldots, v_n$. Let A denote the adjacency matrix of G with respect to the given labeling of vertices. Let B=(b_{ij}) be given by the matrix*

$$B = A + A^2 + \cdots + A^{n-1}. \tag{1}$$

Then G is connected if and only if for every $i \neq j$, we have $b_{ij} \neq 0$. That is, the matrix B must have non-zero entries off of the main diagonal.

Your mission, should you choose to accept it, is to take the adjacency matrix A of a mysterious graph G

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and use *Mathematica* to find the smallest positive integer k for which $A + A^2 + \cdots + A^k$ shows that G is connected by way of Theorem **??**, or else prove that no such k exists.

[The following is an example of how to multiply two 3×3 matrices in *Mathematica*:

- $A = \{\{1,2,3\},\{4,5,6\},\{7,8,9\}\}$ and $B = \{\{1,0,0\},\{0,1,0\},\{0,0,1\}\}$
- If you want to see A and/or B listed in rows, then after defining them as above, then type A//TableForm and/or B//TableForm in a new cell and evaluate the cell.
- To multiply A by B (left-to-right) in a new cell type $A.B$ or $A.B$//TableForm and evaluate.

Notice that you should **not** use the * to do matrix multiplication—it won't work!!!]

For more documentation on how to do matrix multiplication using *Mathematica* go to

http://reference.wolfram.com/mathematica/tutorial/MultiplyingVectorsAndMatrices.html