

PHYS 425 w6l1

Last Time: Low Temp. heat capacity
of System of Identical Fermions

$$C_V = \frac{\pi^2}{2} N k_B \frac{T}{T_F}$$

$$dQ = T dS$$

divide by dT

$$\therefore \frac{dQ}{dT} = C_V = T \left(\frac{\partial S}{\partial T} \right)_V$$

Same.

alternatively

$$\therefore S = \int_0^T \frac{C_V}{T} dT = \int_0^T \frac{\pi^2}{2} N k_B \frac{1}{T_F} dT$$

$$\therefore S = \frac{\pi^2}{2} N k_B \frac{T}{T_F}$$

${}^3\text{He}$ vs ${}^4\text{He}$ Recap:

- ${}^3\text{He}$ & ${}^4\text{He}$ electrically identical
- ${}^3\text{He}$ has larger zero-pt energy

$$E_0 \propto \frac{1}{m}$$

\therefore at $T=0$, ${}^3\text{He}$ atoms have more K.E. \nmid take up more space per particle

	${}^3\text{He}$	${}^4\text{He}$
Density	$0.082 \frac{\text{g}}{\text{cm}^3}$	$0.1451 \frac{\text{g}}{\text{cm}^3}$

Latent Heat	$30 \frac{\text{J}}{\text{mol}}$	$85 \frac{\text{J}}{\text{mol}}$
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- ${}^3\text{He}$ is a Fermion. Fermi energy is

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{L^3} \right)^{2/3}$$

Chemical Potential μ

First law of thermodynamics

$$dU = TdS - PdV$$

assumes that no. of particles in system is fixed.

What if have a system w/ variable no. of particles?

Eg. One electron in H atom

Have one particle distributed among many atomic energy levels of H atom. $\rightarrow N$ fixed.

However, if take a single energy level of H atom as system, then $N=0$ or 1 \rightarrow particle no. in system variable.

If add (or remove) particles from system,
then clearly expect internal energy to
change.

$$U = U(S, V, N)$$

$$\textcircled{1} \quad dU = \underbrace{\left(\frac{\partial U}{\partial S}\right)_{V,N}}_T dS + \underbrace{\left(\frac{\partial U}{\partial V}\right)_{S,N}}_{-P} dV + \underbrace{\left(\frac{\partial U}{\partial N}\right)_{S,V}}_{\mu} dN$$

chemical potential

$$\rightarrow \therefore dU = TdS - PdV + \mu dN = dQ - dW$$

heat mechanical work ↑ chemical work

$$[\mu] = \text{energy}$$

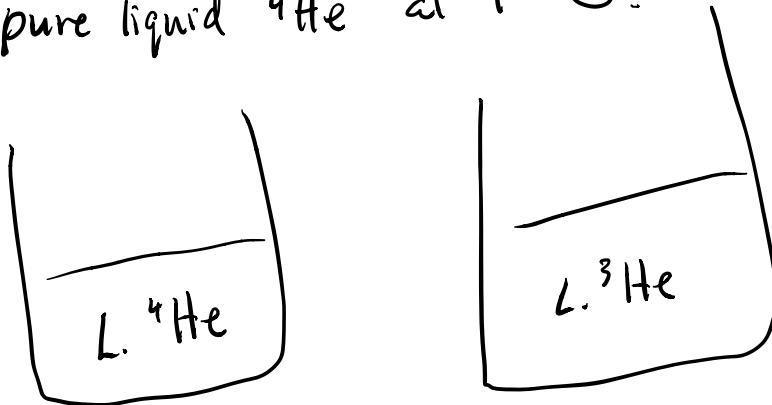
μdN = energy increase of system
when dN particles added

$$O = TdS + \mu dN \quad (\text{const } U \{ V)$$

$$\therefore \mu = -T \left(\frac{\partial S}{\partial N} \right)_{U,V}$$

Mixtures of ${}^3\text{He}$ & ${}^4\text{He}$

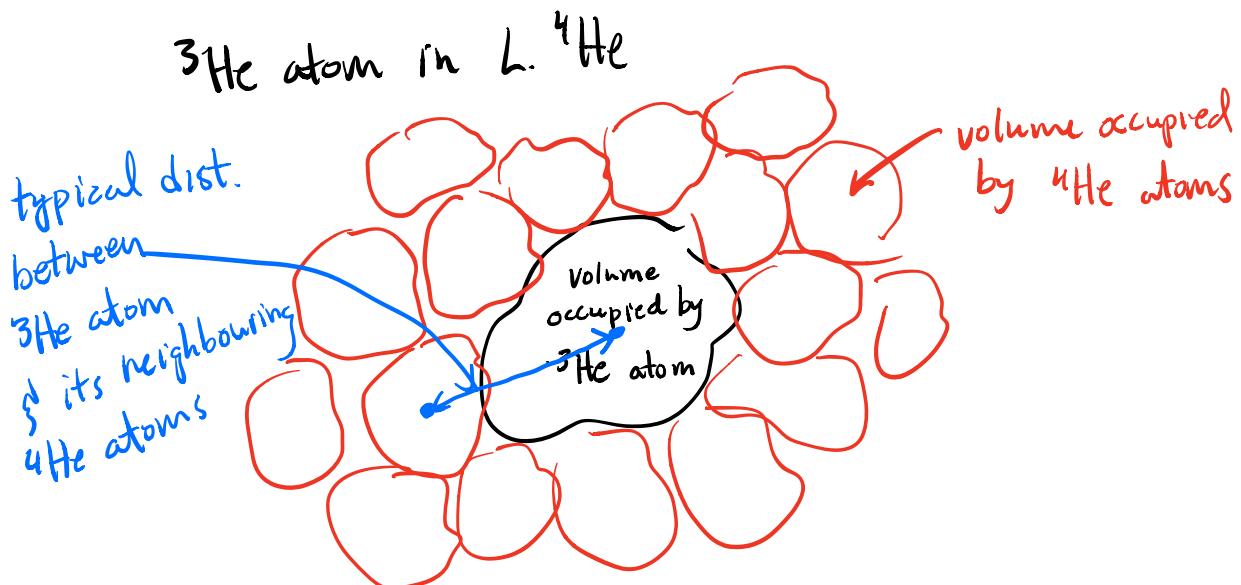
Imagine that we have pure liquid ${}^3\text{He}$ & pure liquid ${}^4\text{He}$ at $T=0$.



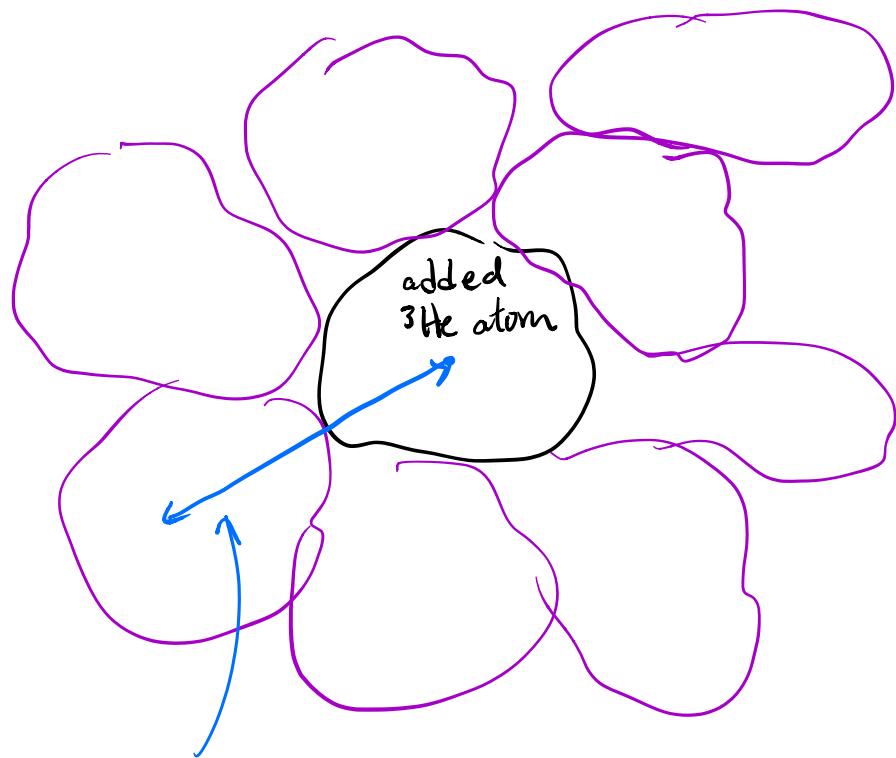
We want to add one ${}^3\text{He}$ particle.
Is it energetically favourable to add it
to L. ${}^3\text{He}$ or L. ${}^4\text{He}$?

Because ${}^4\text{He}$ has lower zero-pt energy, it is more dense. \therefore If add ${}^3\text{He}$ atom to L. ${}^4\text{He}$, it will be closer to neighbouring ${}^4\text{He}$ atoms. \therefore Attraction due to van der Waals force between ${}^3\text{He}$ & surrounding ${}^4\text{He}$ is relatively strong.

In contrast, if add ${}^3\text{He}$ atom to L. ${}^3\text{He}$, its neighbours are farther away & van der Waals attraction is weaker.



${}^3\text{He}$ atom in L. ${}^3\text{He}$



large dist. between
neighbouring ${}^3\text{He}$ atoms

In L. ${}^4\text{He}$ avg. dist. between ${}^3\text{He}$ atom and
neighbouring ${}^4\text{He}$ atoms is small { there are
neighbours.

Since ${}^3\text{He}$ { ${}^4\text{He}$ electrically identical,
attraction between ${}^3\text{He}$ - ${}^4\text{He}$ { ${}^3\text{He}$ - ${}^3\text{He}$
determined by same van der Waals force.

${}^3\text{He}$ more strongly bond in ${}^4\text{He}$ b/c
more neighbours w/ shorter distances.

Chemical Potential

1 ${}^3\text{He}$ atom in $L. {}^3\text{He}$

The energy to remove one ${}^3\text{He}$ atom
from $L. {}^3\text{He}$ is, by definition, determined
by the latent heat of evaporation

$$\mu_{{}^3\text{He}} = - \frac{L_3(0)}{N_0}$$

zero temp.
limit.

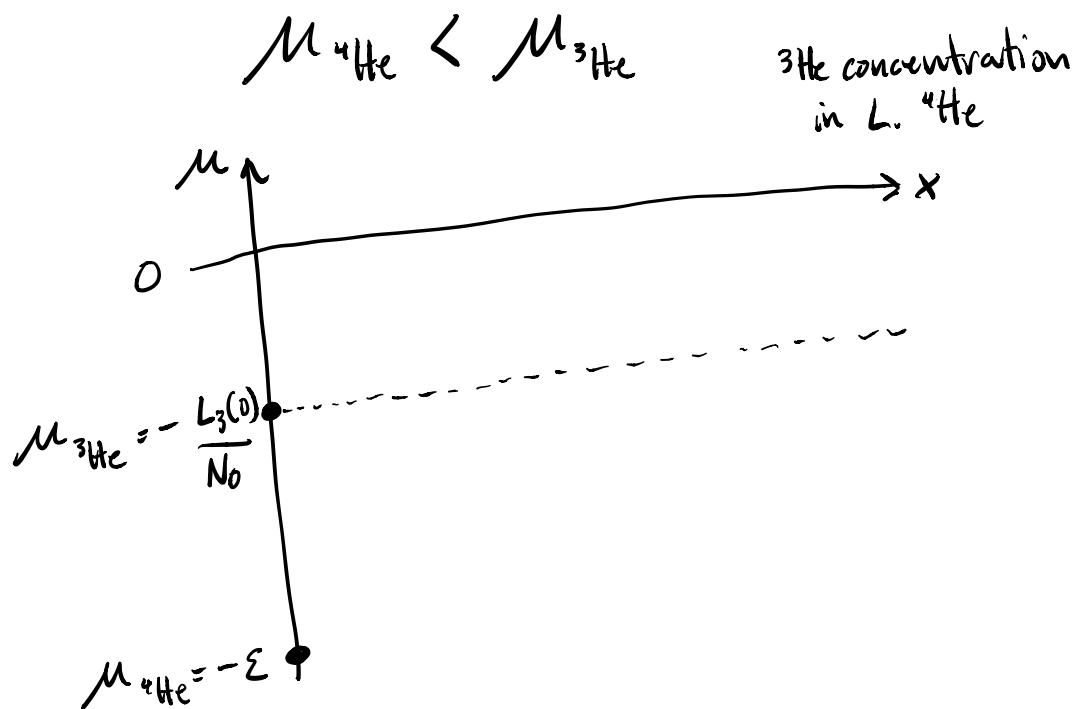
\uparrow
 ${}^3\text{He}$ in ${}^3\text{He}$

energy of $N_0 + 1$ ${}^3\text{He}$ atoms
lower than energy of N_0 ${}^3\text{He}$
atoms and one free ${}^3\text{He}$ atom

1 ^3He atom in L. ^4He

The binding energy of a single ^3He atom
in L. ^4He is ϵ s.t. $\mu_{^4\text{He}} = -\epsilon$
 \uparrow
 ^3He in ^4He

b/c of lower zero pt. energy of ^3He



How do the energetic of ^3He in L. ^4He change
as the concentration x increases.