PHYS 425 - w10l2

Last Time:

In high-T limit

$$\Phi_G \approx -\frac{V}{2\pi^2} k_B T e^{\eta} \int_0^{\infty} k^2 e^{-\varepsilon(k)/k_B T} dk$$

→ For $\varepsilon(k) = \frac{\hbar^2 k^2}{2m}$

use $S = -\left(\frac{\partial \Phi_G}{\partial T}\right)_{V,\mu}$ & $C_V = T \frac{\partial S}{\partial T}$

to find $C_V = \frac{3}{2} N k_B$

In low-T limit

$$\Phi_G \simeq \frac{V}{2\pi^2} k_B T \int_0^{\infty} k^2 \ln\left[1 - e^{-\varepsilon(k)/k_B T}\right] dk$$

For $\varepsilon(k) = \frac{\hbar^2 k^2}{2m}$ find $S \propto T^{3/2}$ & $C_V \propto T^{3/2}$

Meas. & calc. data look very different.

Lev Landau realized that C_v calculated for BEC of dilute gas of particles $\left(\varepsilon(k) = \frac{\hbar^2 k^2}{2m}\right)$ did not match measured C_v of superfluid liquid ^{4}He.

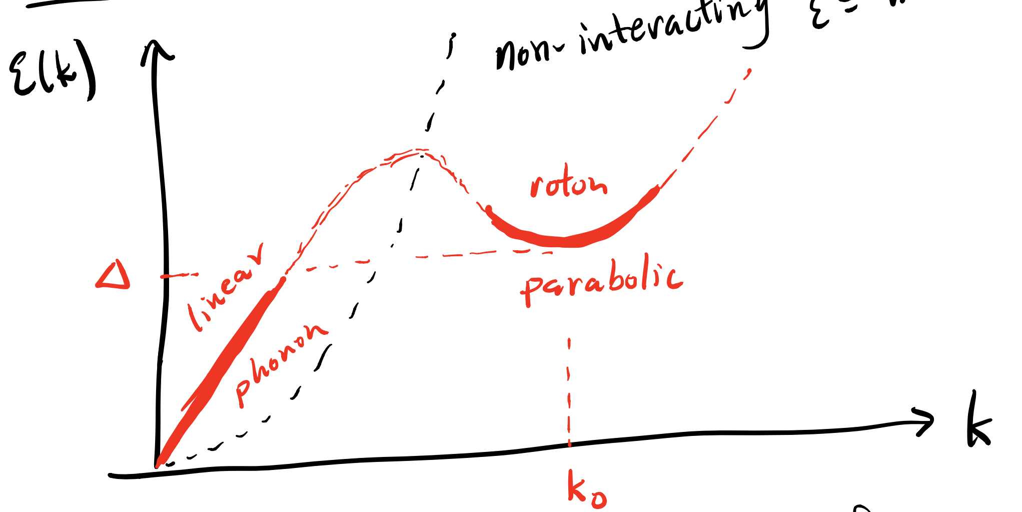
Liquid ^{4}He <u>not</u> collection of free particles

→ they're interacting.

Landau predicted the shape of $\varepsilon(k)$ vs k (energy vs momentum, $p = \hbar k$) dispersion relation. Picked k-dependence of $\varepsilon(k)$ that would give

The correct temp. dependence of thermodynamic properties of superfluid ^4He.

Landau's prediction:

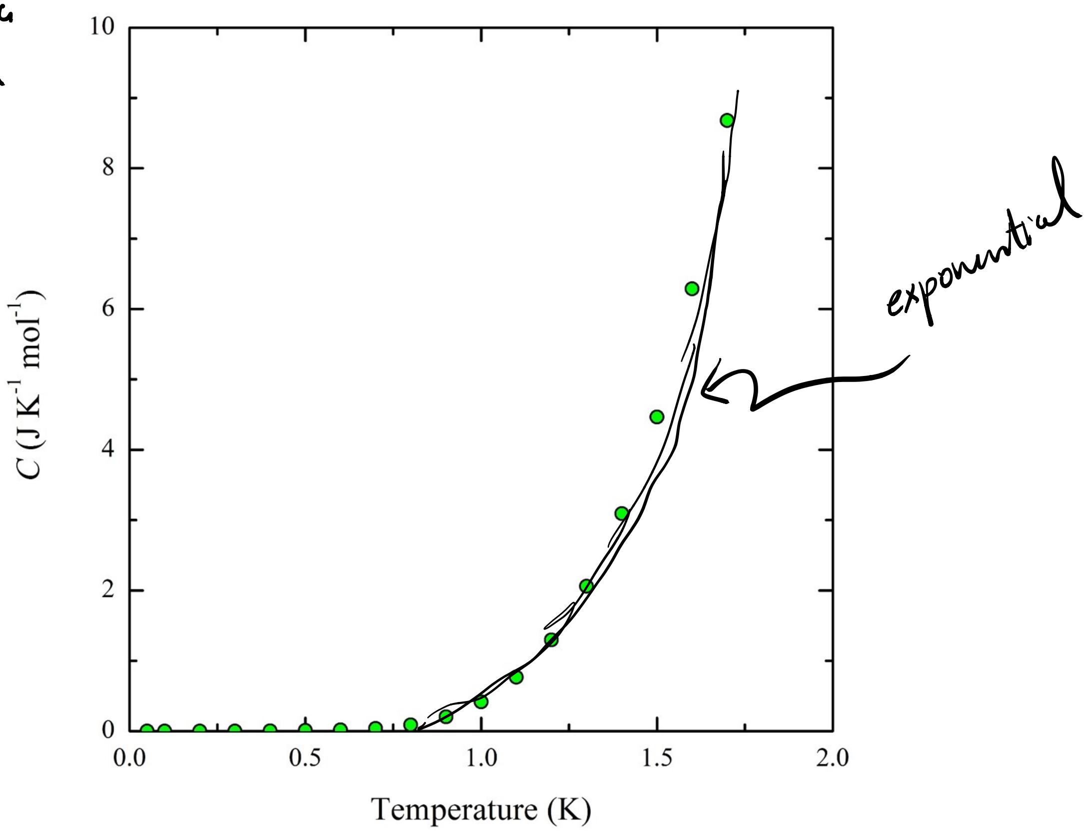


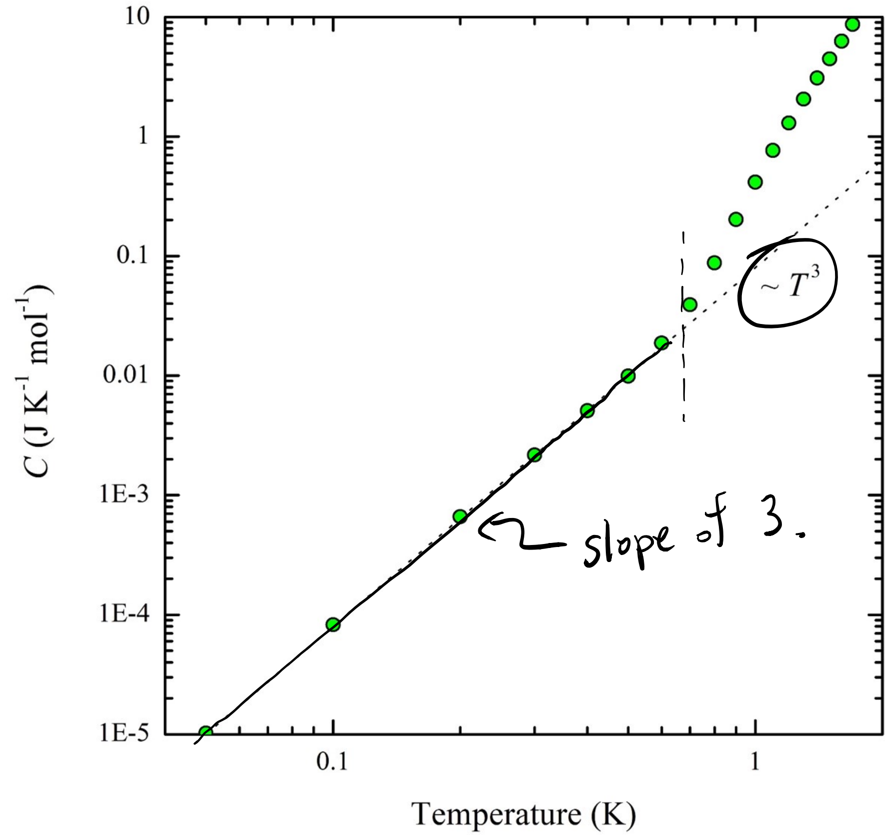
linear part of $\varepsilon(k)$ vs k responsible for the very low-T T^3 dependence of C_V.

parabolic part of $\varepsilon(k)$ vs k gives the "activated" exponential part of C_V vs T at higher temps. (still $T < T_\lambda \sim 2.17K$)

Expt data for liquid 4He.

$T_\lambda = 2.2\,K$





V. Low temp. $C_v \propto T^3$

about ~ 0.8 - 1K, C_v increase rapidly

Lev Landau (1908–1968)

Scientific American article
"The Top-Secret Life of Lev Landau"

Nobel Prize 1962 – for his pioneering theories in condensed matter, esp. liquid helium.

Landau:
- qualified for University at 13 (parents held back)
- published first research paper at 18
- PhD at 21.
- Made series of textbooks w/ former student Lifshitz. "Course of Theoretical Physics" 10 volumes.
- Created a series of exams called "Theoretical Minimum".
  From 1934–1961, 43 candidates passed.

- Believed in Socialism but was arrested in 1938 for comparing Stalinish rule w/ Hilter & Moussolini.
- Forced to stand for 7 hrs/day
- Colleague Kapitsa wrote to Stalin urging for Landau's release.
- After release from prison, Landau developed theory to explain Kapitza's experimental results on superfluid ^4He
  ↳ discovery of rotons.

- Later Landau worked on Soviet Nuclear bomb & would win "Stalin Prize" & get "Hero of Socialist Labour" honour.

- Landau admitted to psychiatric hospital 6 times (involuntarily)
- 1962 car accident
  - coma 2 months
  - declared dead several times.

- died 1968 due to complications associated w/ accident.

Landau's Productivity Scale (log scale)

Newton 0 ← best

Einstein 0.5

$\left.\begin{array}{l}\text{Bohr}\\ \text{Heisenberg}\\ \text{Dirac}\\ \text{Schrodinger}\end{array}\right\}$ 1

Landau 2-2.5

Investigate what happens if try $\varepsilon(k) \propto k$

$\varepsilon(k) = u_1 \hbar k$ ... phonon contribution

$$\Phi_{G,ph} = \frac{V}{2\pi^2} k_B T \int_0^\infty k^2 \ln\left[1 - e^{-u_1 \hbar k / k_B T}\right] dk$$

$$x = \frac{u_1 \hbar k}{k_B T} \qquad dk = \frac{k_B T}{u_1 \hbar} dx$$

$$\Phi_{G,ph} = \frac{V}{2\pi^2} k_B T \left(\frac{k_B T}{u_1 \hbar}\right)^3 \underbrace{\int_0^\infty x^2 \ln\left[1 - e^{-x}\right] dx}_{\text{Maple } -\frac{\pi^4}{45}}$$

$$\Phi_{G,ph} = -\frac{V\pi^2}{90} \frac{(k_B T)^4}{(u_1 \hbar)^3}$$

$$S_{ph} = -\left(\frac{\partial \Phi_G}{\partial T}\right)_{V,\mu} = \frac{2\pi^2 V}{45} \frac{k_B}{(u_1 \hbar)^3} (k_B T)^3$$

$$\boxed{C_{V,ph} = T\left(\frac{\partial S}{\partial T}\right) = \frac{2\pi^2 V}{15}\frac{k_B}{(u_1\hbar)^3}(k_B T)^3}$$

linear portion of Landau's $\varepsilon(k)$ vs k gives $C_V \sim T^3$ which agrees w/ experimental data at very low Temps.

→ still need to produce the "activated" or enhance C_V at temps near but below the superfluid transition temp. T_λ.