PHYS 425 - w8l3

Statistical Mechanics

Intrepretation of Entropy

ideas: (a) Specify state of a system as completely as possible (position, momentum, energy, spin, ... of each particle in system.)

Call each possible state of sys. a quantum state or microstate.

(b) Image an ensemble of systems that are identically prepared (same no. of particles, energy, $\vec{E}$, $\vec{B}$, ...)

Large no. of quantum states consistent w/ these conditions. ($\rightarrow$ called the microcannonical ensble)

(c) If there are W possible quantum states that satisfy constraints on energy, volume, N, ... then <u>assume</u> that all quantum states equally likely.
Prob. of being in any particular quantum state is $\frac{1}{W}$

Eg. $N=4$ spins w/ 2 spin up $E_\uparrow$
2 spin down $E_\downarrow$

$$\frac{N!}{n_\uparrow!\,(N-n_\uparrow)!} = \frac{4!}{2!\,2!} = \frac{4\cdot 3\cdot \cancel{2}\cdot\cancel{1}}{2\cdot 1\cdot \cancel{2}\cdot\cancel{1}} = 6$$

$\uparrow\ \uparrow\ \downarrow\ \downarrow \qquad \downarrow\ \uparrow\ \uparrow\ \downarrow$

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$W = 6$ different microstates w/ $N = 4$

and $E = 2E_\uparrow + 2E_\downarrow$.

Assume each state equally likely

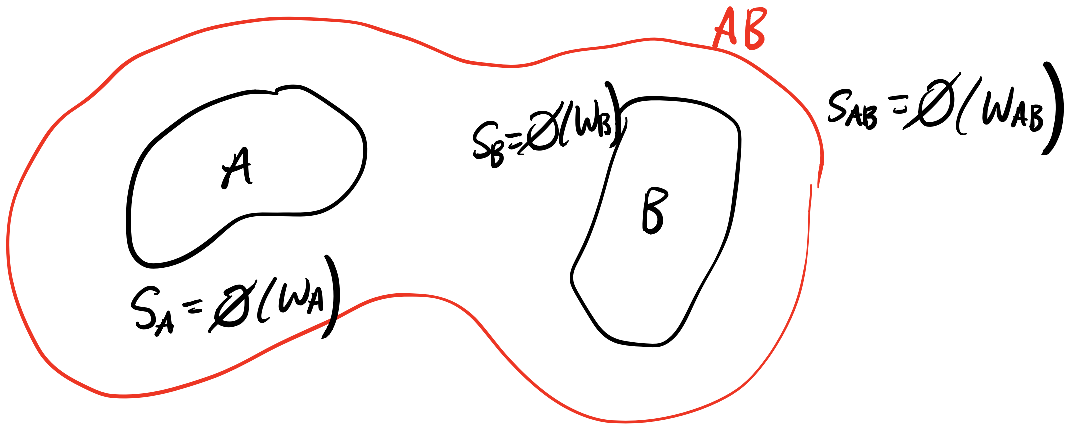
$$p = \frac{1}{W} = \frac{1}{6}$$

Boltzmann's Hypothesis: Entropy of a system is related to the prob. of being in a quantum state $\quad S = \phi(W)$

Next argument follows that of Einstein (1905)

Miracle Year: Photoelectric effect
Brownian Motion
Special Relativity
$E = mc^2$

Consider two indep. systems A & B as well as the composite system AB



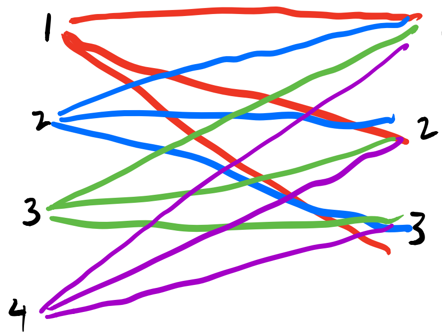
For two indep. systems the total combined entropy is $S_{AB} = \underbrace{\emptyset(W_A)}_{S_A} + \underbrace{\emptyset(W_B)}_{S_B}$

For A & B indep., the total no. of quantum states of sys. AB is:

$$W_{AB} = W_A W_B$$

$$S_{AB} = \emptyset(W_{AB}) = \emptyset(W_A W_B)$$

States of A   states of B



$W_A = 4$

$W_B = 3$

$W_{AB} = 12$

$$\emptyset(W_A W_B) = \emptyset(W_A) + \emptyset(W_B)$$

only sol'n is $\emptyset(W) = k_B \ln(W)$

check $\emptyset(W_A W_B) = k_B \ln(W_A W_B)$

$\emptyset(W_A) = k_B \ln W_A$

$\emptyset(W_B) = k_B \ln W_B$

$$\therefore \emptyset(W_A) + \emptyset(W_B) = k_B \ln(W_A) + k_B \ln(W_B)$$

$$= k_B \ln(W_A W_B) = \emptyset(W_A W_B)$$

✓

Entropy defined to be

$$\boxed{S = \varnothing(W) = k_B \ln W}$$

k_B is Boltzmann's const.

Does this result make sense? Try a simple example.

$N = 4$

$E = 4E_\uparrow$

$\uparrow \ \uparrow \ \uparrow \ \uparrow$

only one possible quantum state consistent w/ $E = 4E_\uparrow$ & $N = 4$

$\therefore W = 1$

$S = k_B \ln 1 = 0$

For a perfectly ordered state $S = 0$ ✓

$N = 4$

$E = 2E_\uparrow + 2E_\downarrow$

already saw that $W = 6$

$\therefore S = k_B \ln 6$

comparitively large entropy for "disordered" state.

Lots of available quantum states

$\Rightarrow$ high entropy