PHYS 425 — w3l2

Heat Transfer by Radiation

First, recall the Stefan-Boltzmann law:

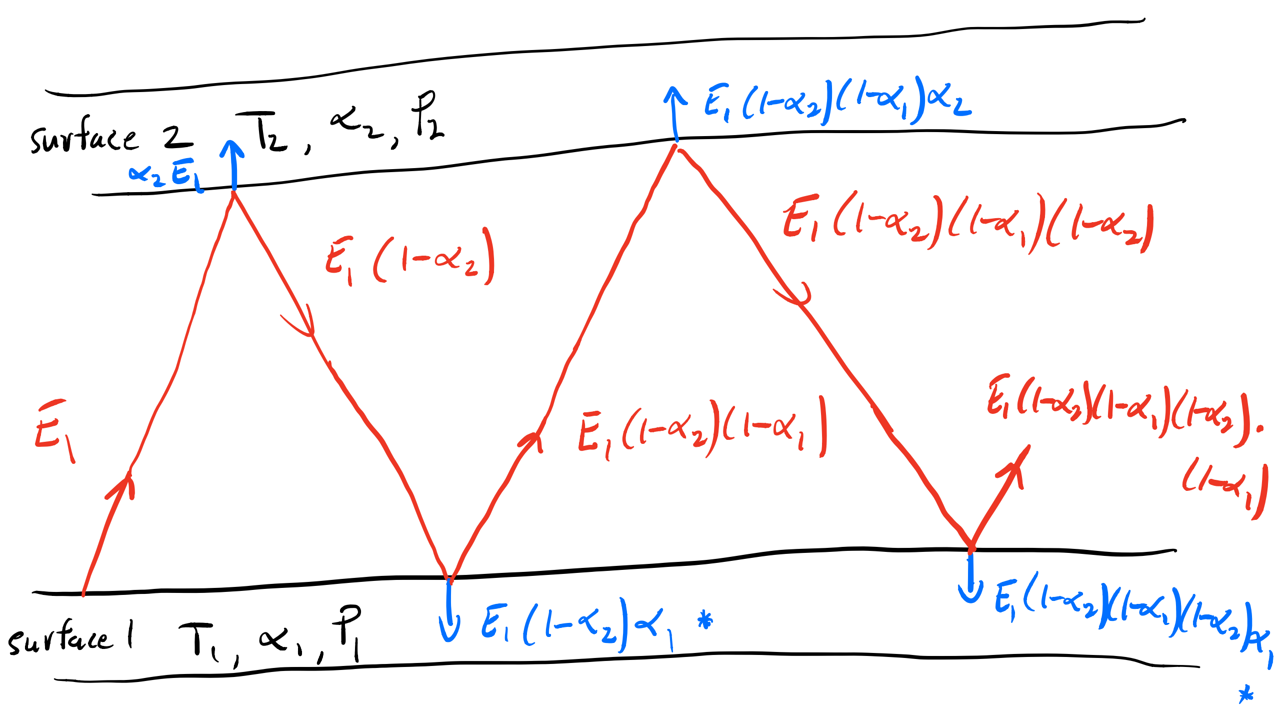
A surface at a temp. T radiates power

$$\rightarrow \quad P = A\varepsilon\sigma T^4$$

A is the area of the surface

ε is the emissivity (for a Blackbody $\varepsilon = 1$)

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2h^2} = 5.67 \times 10^{-8} \frac{W}{m^2K^4}$$ Stefan-Boltzmann const.



Consider two parallel surfaces a diff. temps.

In time Δt surface 1 emits energy

$$E_1 = P_1 \Delta t \quad \text{in the form of photons}$$

Surface 2 absorbs a fraction of E_1 equal to $\alpha_2 E_1$. α is the absorbance of a surface.

Surface reflects a fraction of E_1 equal to $\rho_2 E_1$. ρ is the reflectance of a surface.

$$\rho + \alpha = 1 \Rightarrow \rho = 1 - \alpha$$

Surface 1 absorb $\quad E_1(1-\alpha_2) \cdot \alpha_1$

Surface 1 reflect $\quad E_1(1-\alpha_2) \cdot (1-\alpha_1)$

Surface 2 absorb $\quad E_1(1-\alpha_2)(1-\alpha_1) \cdot \alpha_2$

Surface 2 reflect $\quad E_1(1-\alpha_2)(1-\alpha_1) \cdot (1-\alpha_2)$

Surface 1 absorb $\quad E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2) \cdot \alpha_1$

Surface 1 reflects $\quad E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2) \cdot (1-\alpha_1)$

$\vdots$

Can do the same calc. for surface 2.

Surf. 2 emits E_2

Surf. 1 absorb $E_2\alpha_1$

Surf. 1 reflect $E_2(1-\alpha_1)$

$\vdots$

Amount of E_1 that is absorbed by surface 1 is:

$$E_1(1-\alpha_2)\alpha_1 + E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)\alpha_1$$
$$+ E_1(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)(1-\alpha_1)(1-\alpha_2)\alpha_1 + \ldots$$

$$= E_1(1-\alpha_2)\alpha_1\left[1 + \underbrace{(1-\alpha_1)(1-\alpha_2)}_{\equiv\beta} + (1-\alpha_1)^2(1-\alpha_2)^2 + \ldots\right]$$

$$= E_1(1-\alpha_2)\alpha_1\underbrace{\left[1+\beta+\beta^2+\ldots\right]}_{\sum_{n=0}^{\infty}\beta^n = \frac{1}{1-\beta}}$$

geometric series w/ $0<\beta<1$

$\therefore$ amount of E_1 absorbed by surface 1 is:

$$\frac{E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

$\therefore$ amount of E_1 absorbed by surface 2 is:

$$E_1 - \frac{E_1(1-\alpha_2)\alpha_1}{1-\beta} = \frac{E_1(1-\beta) - E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

$$= \frac{E_1(1-(1-\alpha_1)(1-\alpha_2)) - E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

$$= \frac{E_1 - E_1(1-\alpha_1)(1-\alpha_2) - E_1(1-\alpha_2)\alpha_1}{1-\beta}$$

$$= \frac{\cancel{E_1} - E_1(\cancel{1}-\alpha_2) + \cancel{E_1(1-\alpha_2)}\alpha_1 - \cancel{E_1(1-\alpha_2)}\alpha_1}{1-\beta}$$

$$= \frac{E_1\alpha_2}{1-\beta}$$ Net amount of E_1 absorbed by surf. 2

Likewise, amount of E_2 absorbed by surface 1 is $\dfrac{E_2 \alpha_1}{1-\beta}$

$\therefore$ Net transfer of energy from surface 1 to 2 is:

$$Q_{1\,to\,2} = \underbrace{\frac{E_1 \alpha_2}{1-\beta}}_{\text{amount of } E_1 \text{ absorbed by } 2} - \underbrace{\frac{E_2 \alpha_1}{1-\beta}}_{\text{amount of } E_2 \text{ absorbed by } 1.}$$

Amount of E_2 "lost" to surface 1.

$$\therefore Q_{1\,to\,2} = \frac{E_1 \alpha_2 - E_2 \alpha_1}{1-\beta} \quad \textcircled{*}$$

If have two surfaces at same temp & equal area, then net transfer of heat expected to be zero

$$E_1 \alpha_2 - E_2 \alpha_1 = 0$$

or $$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2}$$

If we make surface 2 a blackbody, then $\alpha_2 = 1$ (blackbody is a surface that absorbs all incident radiation).

then we have

$$\frac{E_1}{\alpha_1} = E_2$$

Know $E_1 = A\varepsilon_1 \sigma T^4 \Delta t$

$E_2 = A\sigma T^4 \Delta t$ ($\varepsilon_2 = 1$ for blackbody)

$$\therefore \frac{\cancel{A}\varepsilon_1 \cancel{\sigma} \cancel{T^4} \cancel{\Delta t}}{\alpha_1} = \cancel{A}\cancel{\sigma}\cancel{T^4}\cancel{\Delta t}$$

$$\left.\frac{\varepsilon_1}{\alpha_1} = 1 \right\} \quad \varepsilon_1 = \alpha_1$$

Kirchhoff's Law

absorbance = emissivity

Return to (A) & assume parallel surfaces of equal area.

$\alpha_1 = \varepsilon_1 \qquad E_1 = A\varepsilon_1 \sigma T_1^4 \Delta t$

$\alpha_2 = \varepsilon_2 \qquad E_2 = A\varepsilon_2 \sigma T_2^4 \Delta t$

$$Q_{1\,to\,2} = \frac{A\varepsilon_1\sigma T_1^4\Delta t\,\varepsilon_2 - A\varepsilon_2\sigma T_2^4\Delta t\,\varepsilon_1}{1-(1-\varepsilon_1)(1-\varepsilon_2)}$$

$$= \frac{A\Delta t\,\sigma\varepsilon_1\varepsilon_2(T_1^4 - T_2^4)}{\cancel{1}-\cancel{1}+\varepsilon_2+\varepsilon_1-\varepsilon_1\varepsilon_2}$$

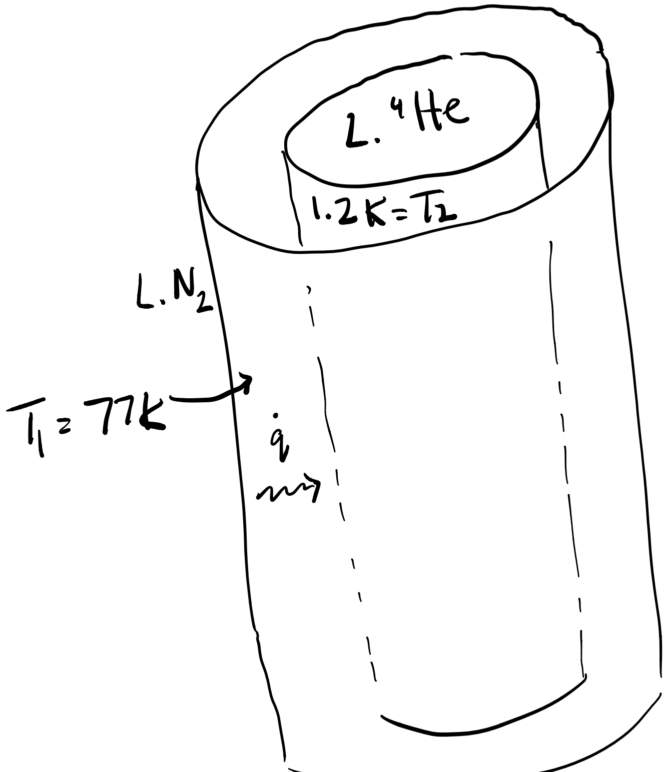
divide by $\varepsilon_1\varepsilon_2$

$$Q_{1\,to\,2} = \frac{A\Delta t\,\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

heat flux from 1 to 2 is

$$\boxed{\dot{q}_{1\,to\,2} = \frac{Q_{1\,to\,2}}{A\Delta t} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}}$$

Eg



$$\dot{q}_{1 \text{ to } 2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

First, note $(77K)^4 >> (1.2K)^4$

$$\therefore \; T_1^4 - T_2^4 \simeq T_1^4$$

To make $\dot{q}_{1 \text{to} 2}$ small, want ε_1 & ε_2 to be small.

to make ε small, Dewar walls are "silvered"

$\varepsilon \approx 0.03$ If $\varepsilon_1 \approx \varepsilon_2 = 0.03$

then $$\dot{q}_{1to2} = \frac{\sigma T_1^4}{\frac{2}{\varepsilon} - 1} = \frac{\sigma T_1^4}{\frac{2-\varepsilon}{\varepsilon}}$$

$$\approx \frac{\varepsilon \sigma T_1^4}{2} = \frac{30\,mJ}{m^2 s}$$

$$= \frac{30\,mW}{m^2}$$

If Dewar is 1.5 m tall & 12 cm in diamter

$A \approx 5.4 \times 10^{-3}\, m^2$

$P_{1to2} = \dot{q}_{1to2} A = 0.16\,mW = 160\,\mu W.$