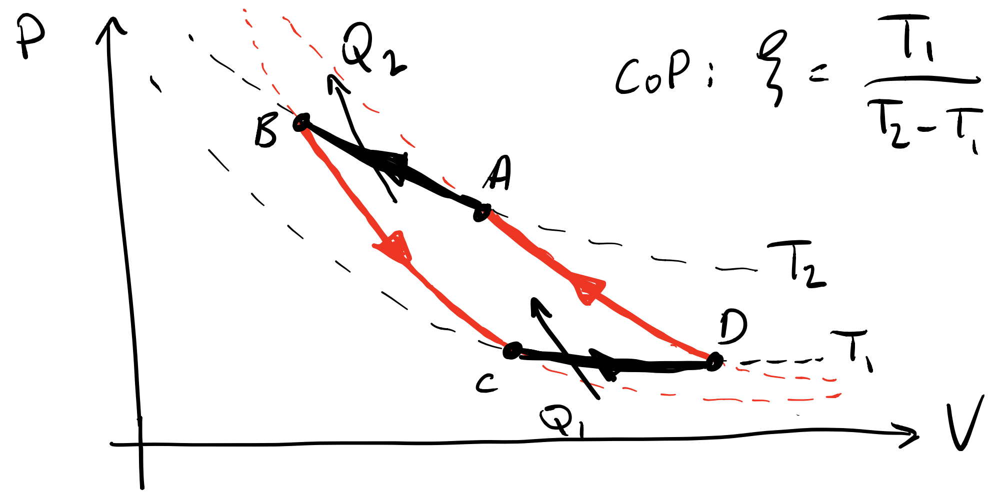
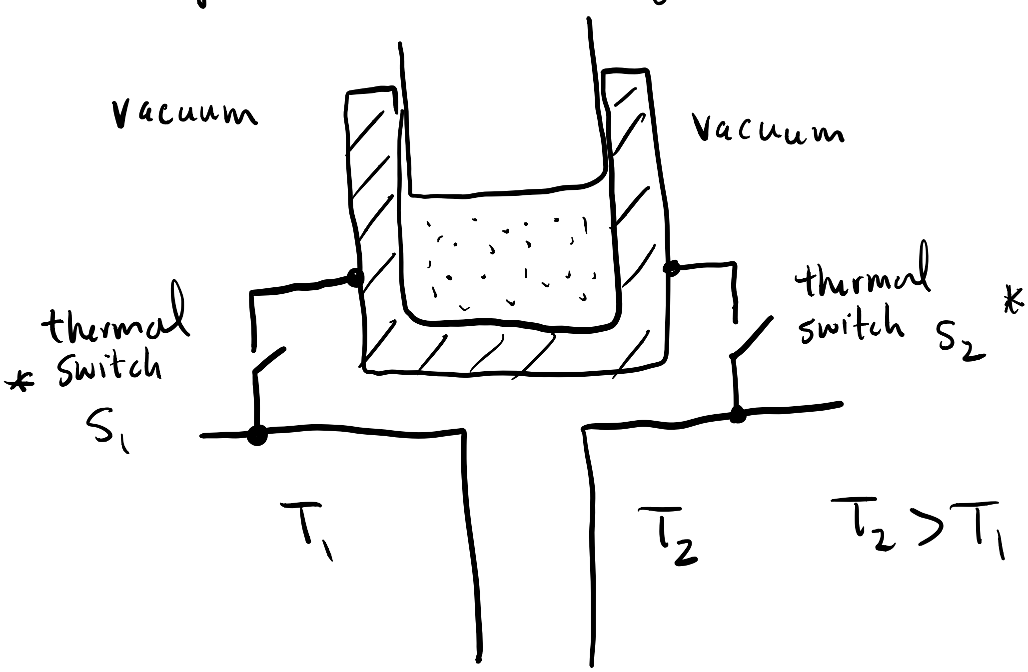
PHYS 425 w11d2

Last time: Refrigeration by the Carnot Cycle



CoP: $\xi = \dfrac{T_1}{T_2 - T_1}$

Design of a Carnot Refrigerator



$T_2 > T_1$

Refrigeration cycle: 4 stages

① $A \to B$ close S2 & isothermally compress gas. Do work on gas, extract heat Q_2 from sys. & dump into thermal reservoir.

② $B \to C$ open S2 to isolate system. Adiabatically expand gas. $dQ = 0$.
Gas does work to push piston. ∴ Temp $T_2 \to T_1$

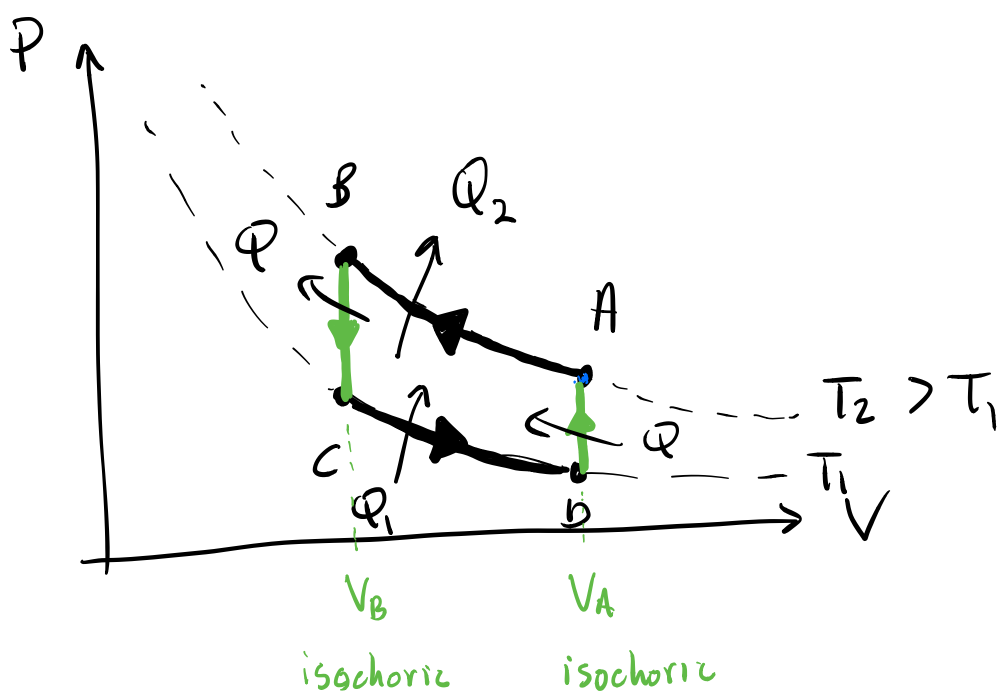
③ $C \to D$ Close S1 & isothermally expand gas. Gas does work & absorbs heat Q_1 from cold reservoir at T_1 ⇒ <u>Refrigeration</u>

④ $D \to A$ open S1 to isolate system. Adiabatically compress gas. Do work on gas & $dQ = 0$,
∴ temp increase from T_1 to T_2.

The Carnot refrigerator is conceptually simple, but hard to implement in practice.

Can we design another type cycle that does work on an ideal gas to extract heat from a cold thermal reservoir?

$\Longrightarrow$ Stirling Cycle (for refrigeration)



Have already analyzed isothermal processes

$A \to B$ & $C \to D$

$A \to B \quad W_{AB} = N k_B T \ln\left(\frac{V_B}{V_A}\right) = Q_2$ neg. work by gas. Q_2 from gas to T_2 res.

$C \to D \quad W_{CD} = Nk_BT \ln\left(\frac{V_A}{V_B}\right) = Q_1$ pos. work by gas Q_1 from T_1 res. to gas.

What about the isochoric processes (const. vol.)?

$$dU = dQ - \cancel{dW}^{\,0} \qquad dW = PdV, \text{ but } dV = 0$$

$$\therefore dW = 0$$

$$dU = dQ$$

$$B \to C \quad \Delta U = \frac{f}{2}Nk_B(T_1 - T_2) = Q < 0$$

$\therefore$ gas losses heat $\underline{\underline{Q}}$.

$$D \to A \quad \Delta U = \frac{f}{2}Nk_B(T_2 - T_1) = Q' > 0$$

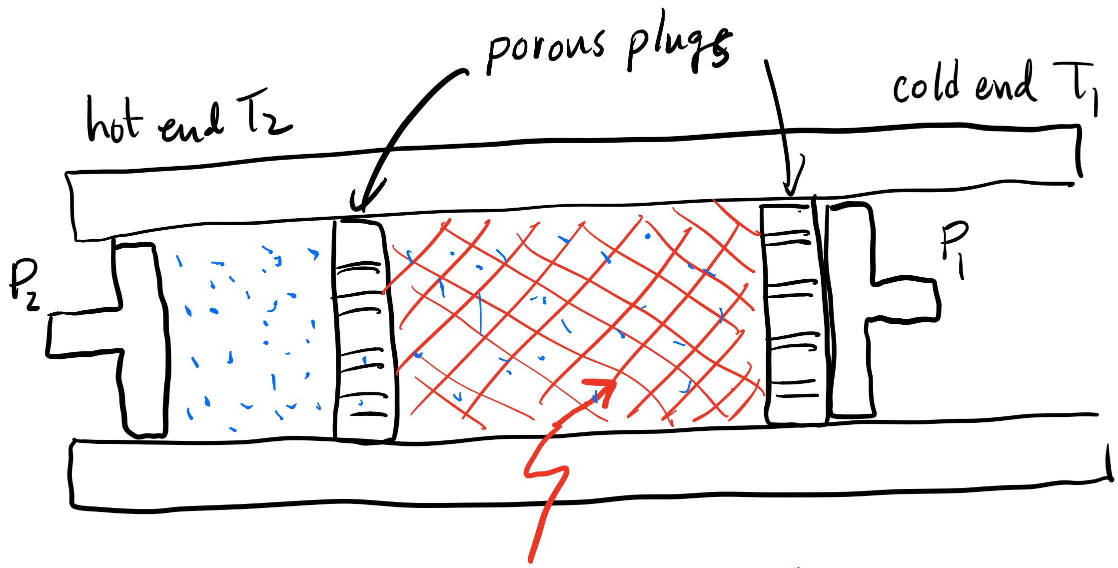
$\therefore$ gas absorbs heat $\underline{\underline{Q'}}$.

Note: $Q' = -Q$

Where does the heat go to (come from) during the isochoric processes $B \to C$ ($D \to A$)?

The Stirling refrigerator (cryocooler) requires a "regenerator" which is used to temporarily store heat absorbed from the gas & then to release it back to the gas.

One simple version of a Stirling Cryocooler
⇒ two pistons.



corresponds to pt. A on P-V diagram.

porous regenerator

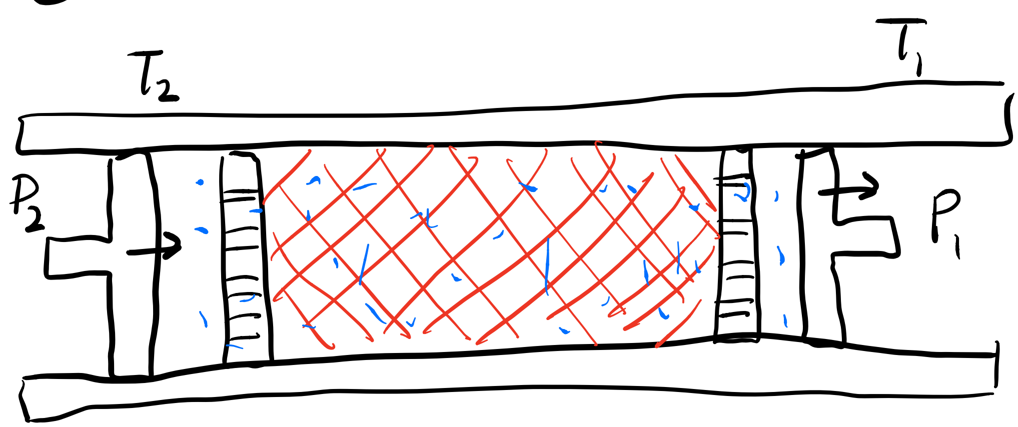
→ gas passes through
→ high heat capacity
• can absorb/store/release heat w/o signif. change to its temp.

Stirling Refrigeration Cycle

① $A \to B$ Use P_2 to compess gas in the hot end. Const. Temp @ T_2.
Keep P_1 stationary.

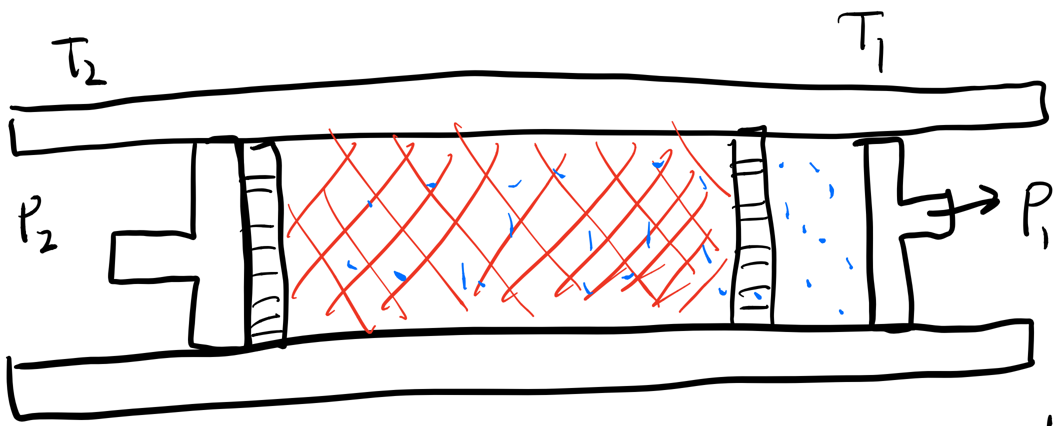
Neg. work by gas. Q_2 from gas to res. at T_2.

② $B \to C$ Constant Volume.



Move P_2 & P_1 simultaneously to right. Push warm gas through regenerator to cold end. Gas cools from T_2 to T_1. Heat released to regenerator.

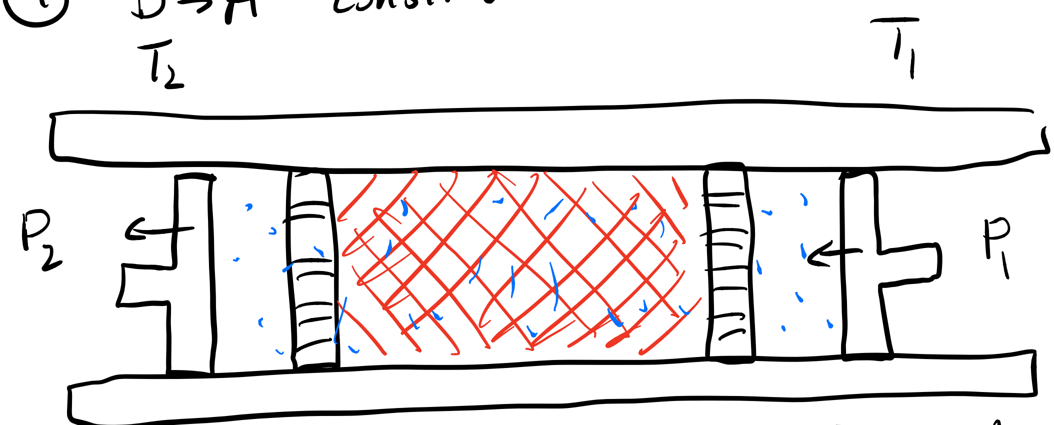
③ $C \to D$



Keep P_2 stationary. Expand gas isothermally at T_1 by moving P_1 to right.

Gas does work. Q_1 from res. T_1 to gas.

④ $D \to A$ const. volume



Move P_2 & P_1 in tandem to left. Push cold gas through regenerator to hot end. Gas warms from T_1 to T_2 by absorbing the heat Q that was previously stored in reg. during the $B \to C$ process.