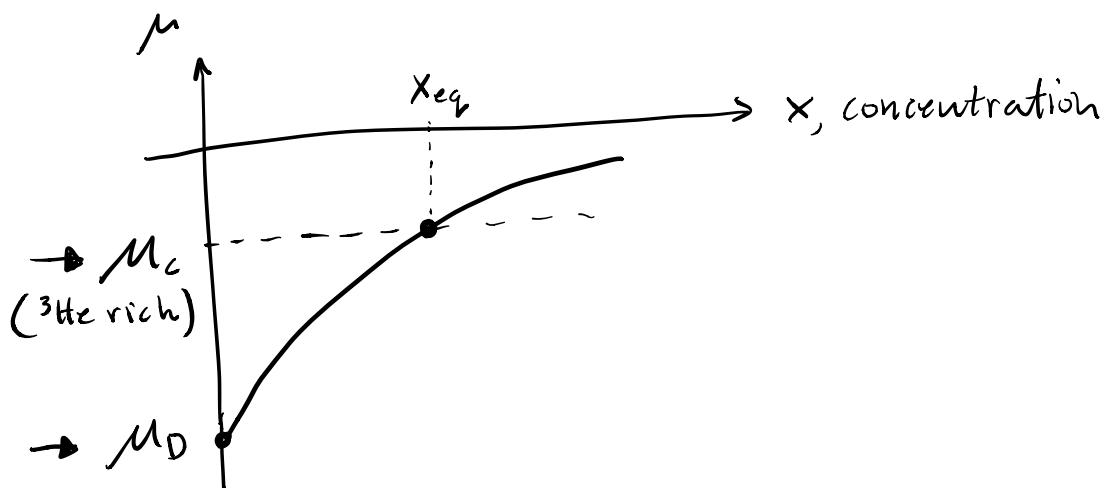


PHYS 425 - w6l3

Heat Capacity & Entropy

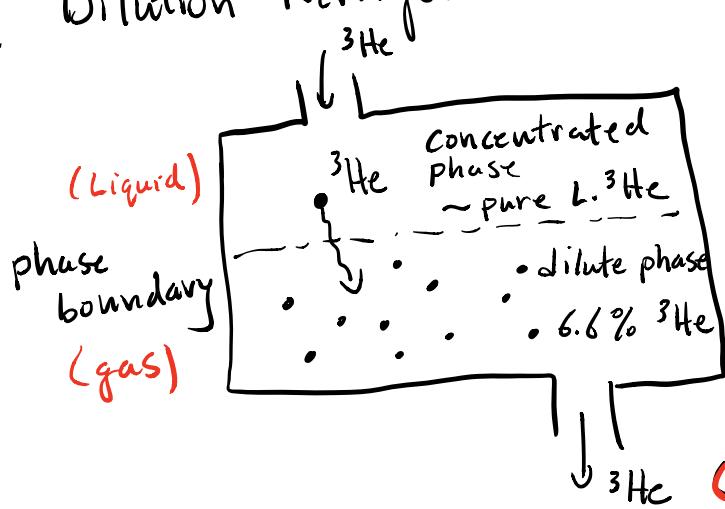
■ $C_V = \frac{\pi^2}{2} N k_B \frac{T}{T_F} = S$

Chemical Potential



Analogy between Evaporative Cooling

{ Dilution Refrigeration

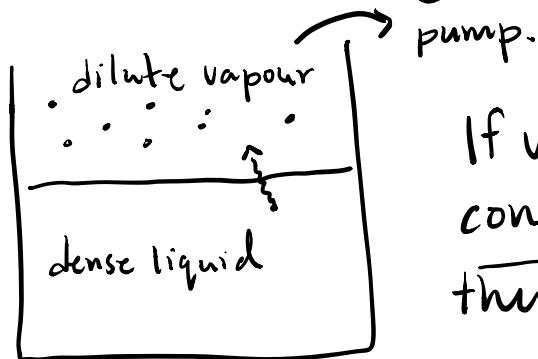


mixing
chamber

"upside-down"
evaporator

In equil. rate of injection = rate of removal.
 To keep concentration in dilute phase fixed,
 require ${}^3\text{He}$ atoms from concentrated
 phase to cross the phase boundary
 at same rate (\dot{N}) as injection/removal.

Recall evaporative cooling



If vapour is at
const. pressure,
 then:

$$H = U + PV$$

$$\begin{aligned} dH &= dU - PdV + PdV + VdP \\ &= dQ + VdP \xrightarrow{\text{O}} \end{aligned}$$

$$\therefore dH = dQ$$

$$\Delta H = \Delta Q$$

latent heat of evaporation
 given by change in
 enthalapy

Need to calc. the enthalpies of the concentrated & dilute phase in the mixing chamber.

"Cooling Power" of mixing chamber

$$\dot{Q} = \dot{n} [H_D(T) - H_C(T)]$$

molar
circulation
rate of ${}^3\text{He}$

Calculate Enthalpy of Concentrated Phase.

If liquid ${}^3\text{He}$ is considered to incompressible, the a process at const. pressure will also be at constant volume.

$$\Rightarrow C_p \approx C_v$$

$$dH = dQ + VdP, \text{ for const pressure}$$

$$dH = dQ.$$

$$\therefore C_p = \left(\frac{\partial Q}{\partial T} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p$$

$$\text{or } \Delta H = \int_0^T C_p dT$$

$$\therefore H(T) = H(0) + \int_0^T C_p dT$$

Know $C_v = C_p = \frac{\pi^2}{2} N k_B \frac{T}{T_F} = \frac{\pi^2}{2} n R \frac{T}{T_F}$

no. of moles

$$(R = N_A k_B)$$

\therefore molar specific heat of the concentrated phase is

$$C_c = \frac{\pi^2}{2} R \frac{T}{T_F} = S_c \quad \left. \begin{array}{l} \text{molar specific heat} \\ \text{molar entropy} \end{array} \right\}$$

Below 40mK, specific heat of pure L.³He experimentally found to be $(24T) \frac{J}{\text{mol} \cdot K}$

where T is in Kelvin

$$\therefore H_c = H_c(0) + \int_0^T 24 T dT$$

$$H_c(0) + (12 T^2) \frac{J}{mol} \left. \begin{array}{l} \text{molar} \\ \text{enthalpy} \\ \text{of pure } {}^3\text{He} \end{array} \right\}$$

(concentrated
phase)

Enthalpy of the dilute phase.

molar specific heat is

$$C_D = \frac{\pi^2}{2} R \frac{T}{T_F}$$

At a concentration of 0.066, $T_F = 0.38 \text{ K}$

s.t.

$$C_D = S_D = (107 T) \frac{J}{mol \cdot {}^3\text{He} \cdot K}$$

Due to binding energy between ${}^3\text{He}$ atoms
and surrounding ${}^4\text{He}$ in dilute phase,
enthalpy of this phase measured to

different than that of non-interacting Fermi system. \therefore cannot calc. H_D from $\int C_D dT$.

Need another method.

Consider the Gibb's Free energy

$$G \equiv H - TS = U + PV - TS$$

$$\begin{aligned} \therefore dG &= dU + PdV + VdP - TdS - SdT \\ &= \cancel{TdS} - \cancel{PdV} + \mu dN + \cancel{PdV} + \cancel{VdP} \\ &\quad - \cancel{TdS} - \cancel{SdT} \end{aligned}$$

$$dG = \mu dN + VdP - SdT$$

$$\therefore \mu = \left(\frac{\partial G}{\partial N} \right)_{P,T} \quad \#$$

Recall that for a Fermi system at low Temp.

$$\mu = E_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{2/3}$$

$$\mu \propto \left(\frac{N}{V} \right)^{2/3}$$

If halve N , then also halve V

$\therefore \frac{N}{V}$ remains const. $\Rightarrow \mu$ unchanged.

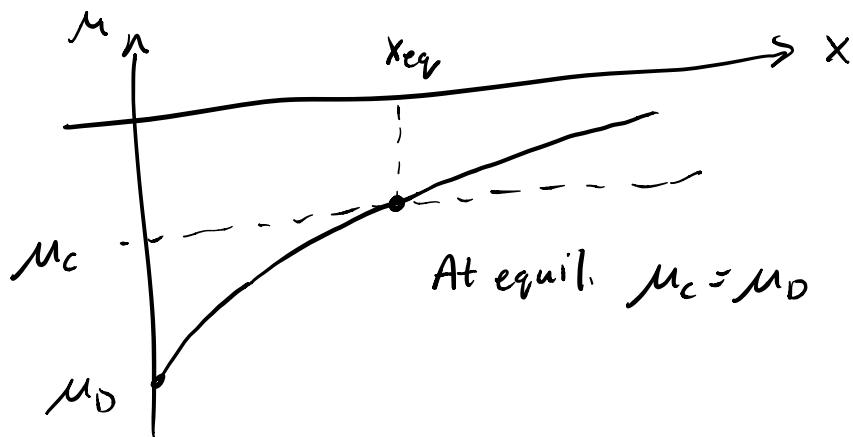
$\Delta G = \int \mu dN$, since μ is indep. of N

$$\Delta G = \mu \Delta N \Rightarrow \underline{G_f} - \underline{G_i} = \underline{\mu N_f} - \underline{\mu N_i}$$

$$\therefore G = \mu N \quad \text{or} \quad \mu = \frac{G}{N}$$

the chemical potential
is equal to the Gibb's
free energy per particle.

Recall chemical pot. of dilute phase vs.
concentration x of ${}^3\text{He}$.



\therefore require molar Gibb's free energies of concentrated & dilute phases to be equal

$$G_c = G_D$$

$$H_c - TS_c = H_D - TS_D$$

know $H_c = H_c(0) + 12T^2$

$$S_c = 24 T$$

$$S_D = 107 T$$

$$H_c(0) + 12T^2 - \underbrace{24T^2}_{TS_c} = H_D - \underbrace{107T^2}_{TS_D}$$

$$\therefore H_D(T) = H_c(0) + (95T^2) \frac{J}{\text{mol. of } {}^3\text{He}}$$

dilute phase

$$H_c(T) = H_c(0) + (12T^2) \frac{J}{\text{mol}}$$

concentrated phase.

Cooling power of mixing chamber
(upside down evaporator)

$$\dot{Q} = \dot{n} [H_D(T) - H_c(T)]$$

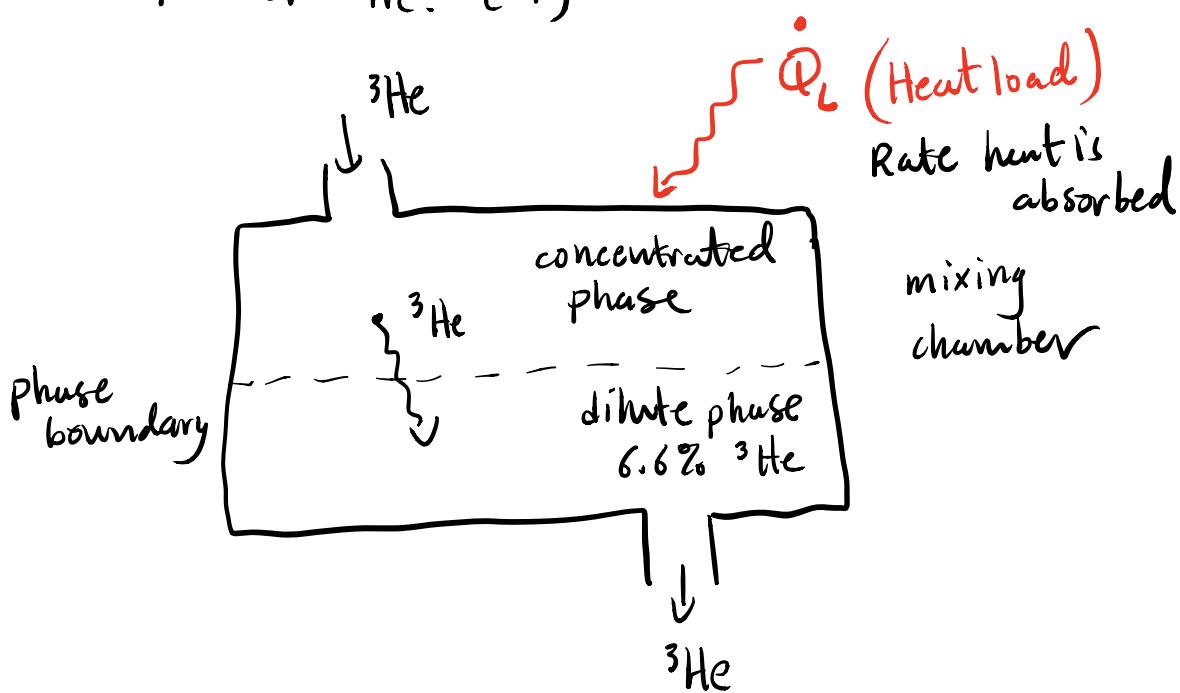
$$= \dot{n} \left[H_c(0) + 95T^2 - \cancel{H_c(0)} - 12T^2 \right]$$

Rate
heat is
removed

$$\dot{Q} = \dot{n} 83T^2 = 85\dot{n}T^2$$

cooling
power of
mixing chamber
of dilution
refrigerator

depends on circulation
rate of ${}^3\text{He}$. (\dot{n})



$$\text{In equil. } \dot{Q}_L = \dot{Q}$$

$$\dot{Q}_L = 83 \dot{n} T^2$$

For a practical dilution refrigerator

$$\dot{n} = 3 \times 10^{-5} \text{ mol/s}$$

$$\dot{Q}_L = 0.5 \mu\text{W}$$

$$\therefore T = \sqrt{\frac{\dot{Q}_L}{83 \dot{n}}} \approx 14 \text{ mK}$$

Base Temp!