PHYS 425 - w8l2

Last Time: Bose-Einstein Condensation
Critical Temperature

$$\boxed{k_B T_c = 3.31 \frac{\hbar^2}{m} n^{2/3}}$$

Δx ~ space occupied by a single particle in the system

→ spatial extent of particle's w.f.

→ de Broglie wavelength $\lambda = \frac{2\pi}{k}$

$$\frac{\hbar^2 k^2}{2m} \sim k_B T \Rightarrow \frac{1}{k} \sim \sqrt{\frac{\hbar^2}{2m k_B T}}$$

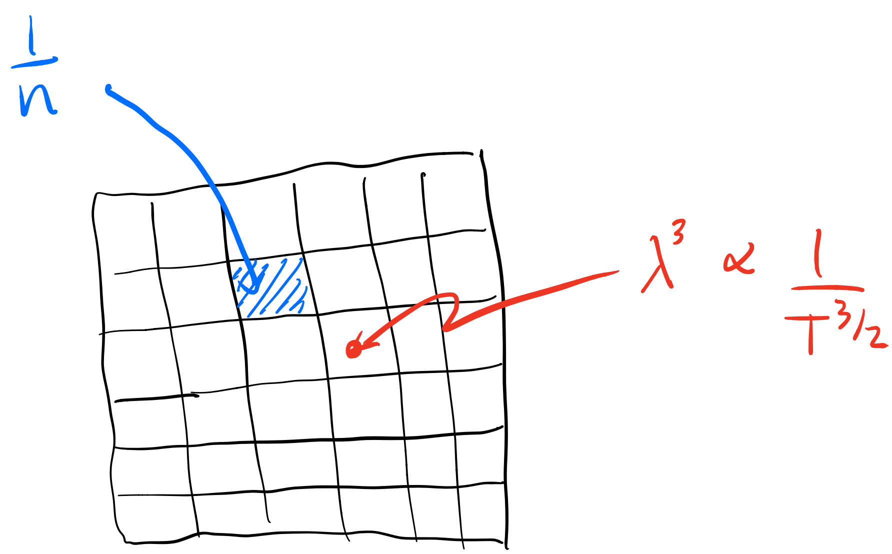
$$\therefore \lambda \sim \sqrt{\frac{(2\pi\hbar)^2}{2m k_B T}}$$

$\therefore$ volume occupied per particle or "Quantum Volume"

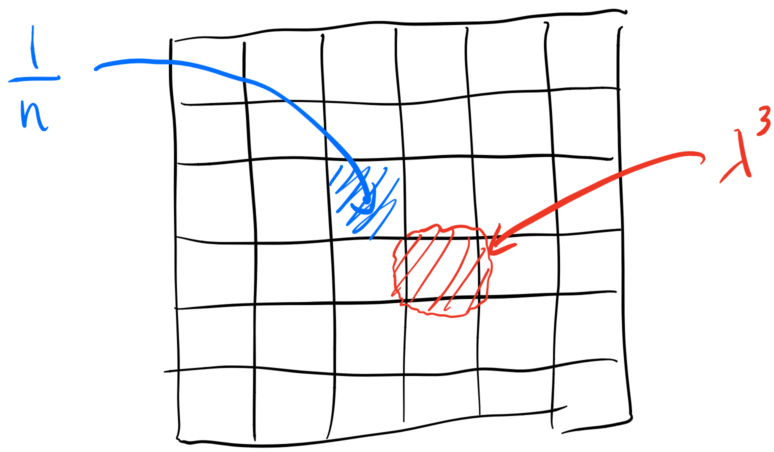
$$(\Delta x)^3 \sim \lambda^3 \sim \left(\frac{\hbar^2}{m k_B T}\right)^{3/2}$$

Consider also the space available per particle in the system of volume V.

Space available per particle is $\dfrac{V}{N} = \dfrac{1}{n}$



high T, classical limit



low T, quantum limit.

When w.f. of adjacent particles overlap, the quantum nature of system becomes apparent on a macroscopic scale

→ new physical phenomena

e.g. superfluidity / supeconductivity.

Particles in BEC behave as if belonging to a single macroscopic wavefun that extends throughout entire volume of system.

check condition

$$\frac{1}{n} = \lambda^3 \implies \frac{1}{n} \approx \left(\frac{\hbar^2}{mk_BT^*}\right)^{3/2}$$

$\left(T^* \text{ is temp at which } \frac{1}{n} = \lambda^3 \right)$

$$\frac{\hbar^2}{mk_BT^*} = \frac{1}{n^{2/3}}$$

$$\therefore k_BT^* \approx \frac{\hbar^2}{m} n^{2/3}$$

same within numerical constants.

c.f. $$\boxed{k_BT_c \simeq 3.31 \frac{\hbar^2}{m} n^{2/3}}$$