

Operating a dilution refrigerator

Last Time: Cooling power

$$\dot{Q} = \dot{n} \left[\overbrace{H_D(T) - H_C(T)}^{\text{enthalpy}} \right]$$

→ molar circulation
rate of ${}^3\text{He}$

Cooling power determined from change in enthalpy when ${}^3\text{He}$ crosses the phase boundary from the concentrated phase to the dilute phase (evaporative cooling)

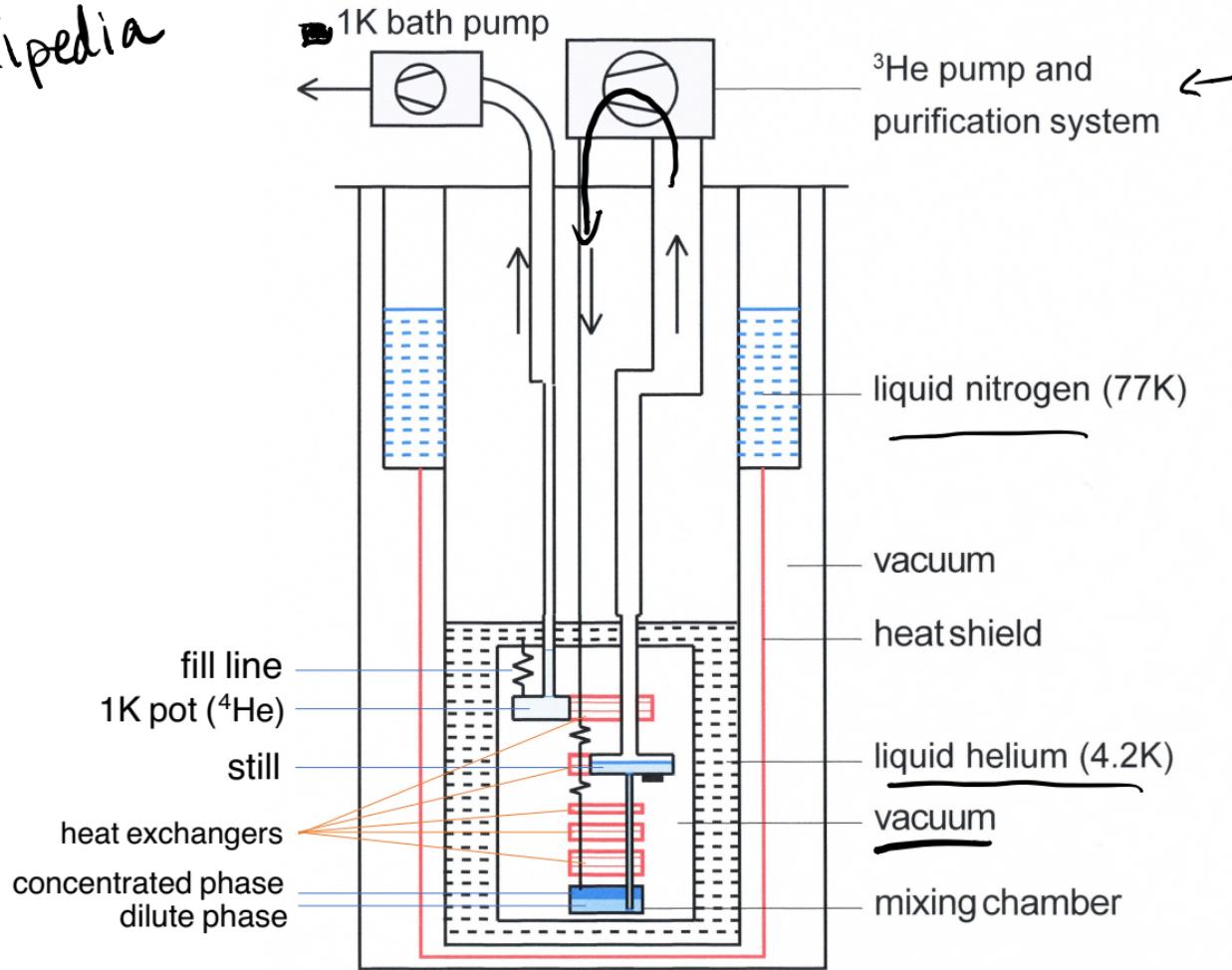
Found $\dot{Q} \approx 83 \dot{n} T^2$ which led to a base temperature of $\approx 14 \text{ mK}$ for

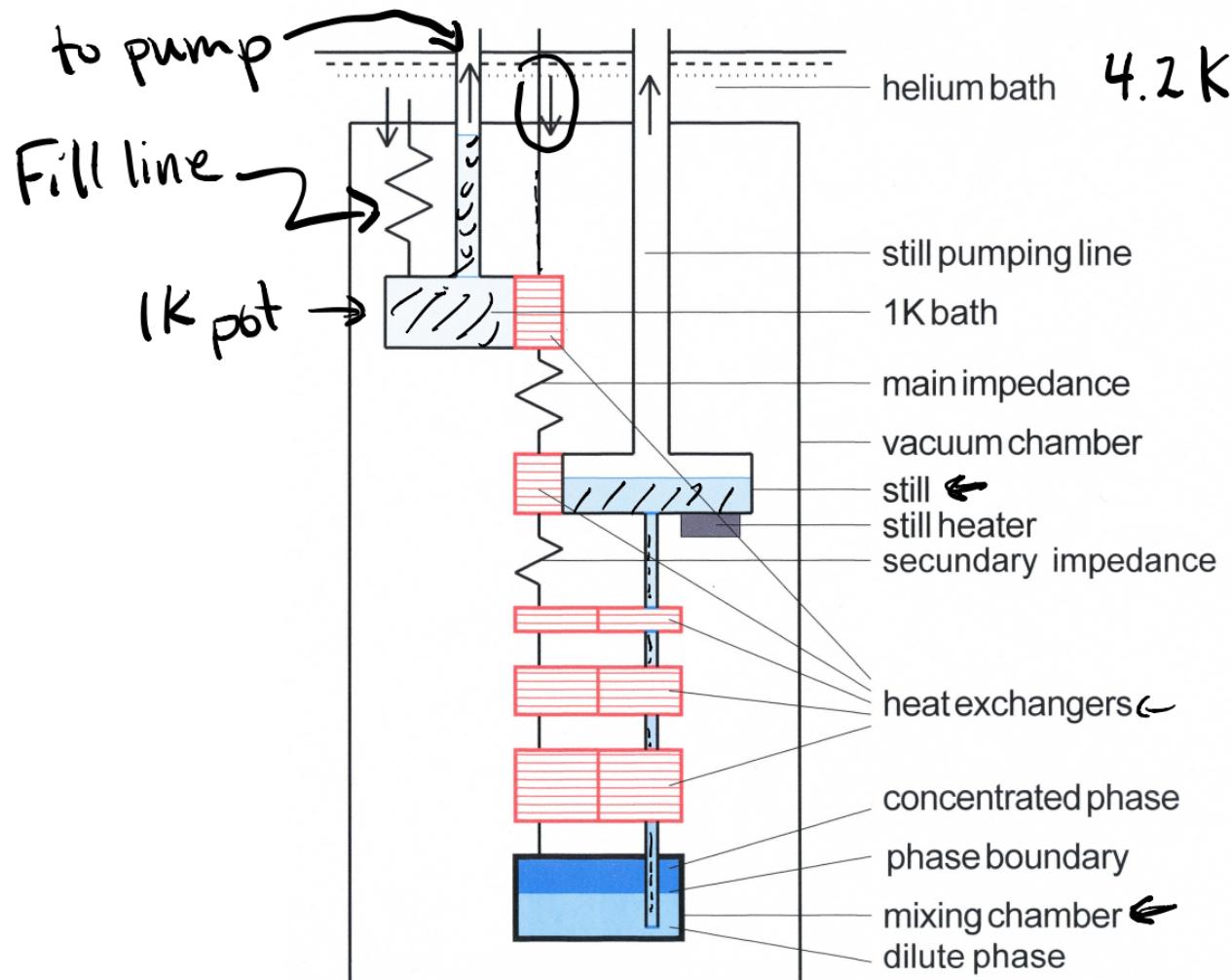
- $\dot{n} = 3 \times 10^{-5} \text{ mol/s}$ and a heat load
- of $0.5 \mu\text{W}$.

Operation of a dilution refrigerator (wet)
→ use cryogenic liquids to pre-cool system

① Fill L. N_2 & L. ${}^4\text{He}$ Dewars.

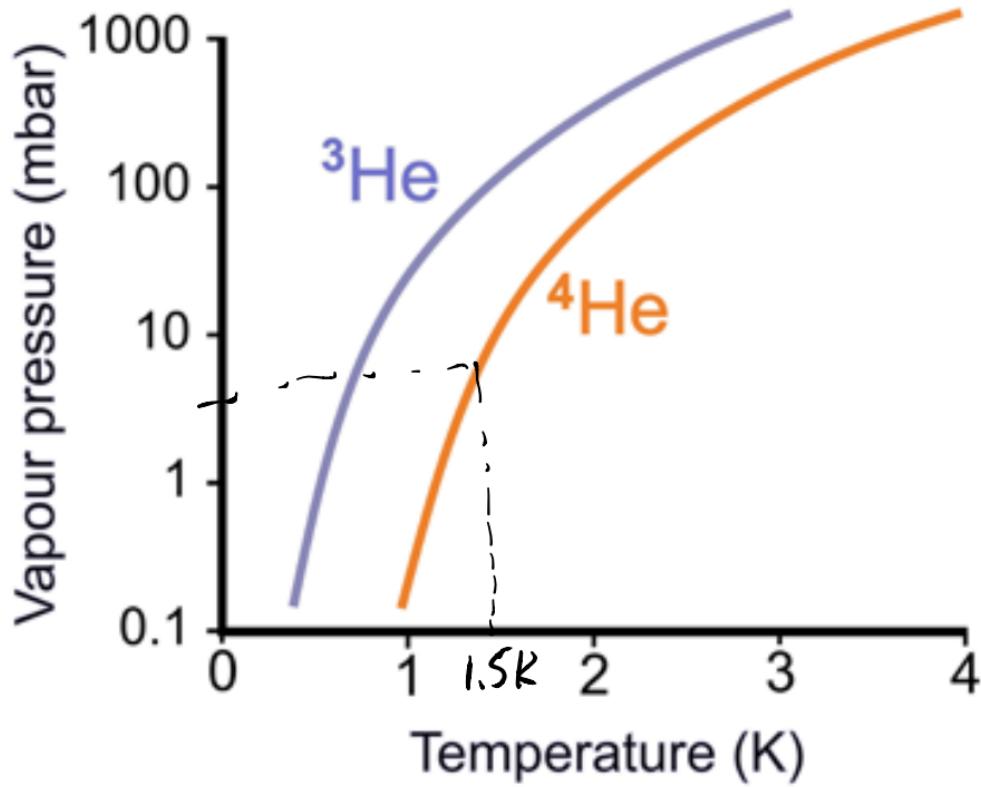
Wikipedia





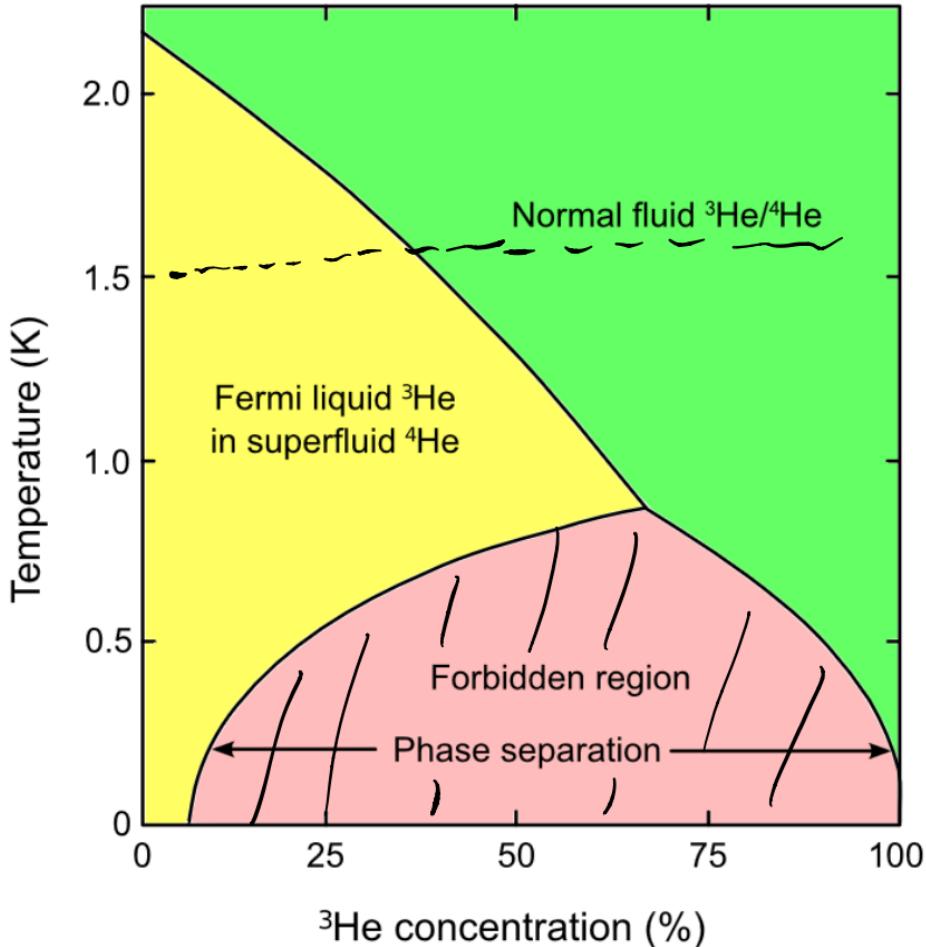
② ${}^4\text{He}$ from outside bath fills a 1K pot via a fill line.

L. ${}^4\text{He}$ in 1K pot is pumped so that it reaches a temp. of 1.2 - 1.5 K.



- ③ Let ^3He - ^4He gas mixture into the dilution refrigerator.

Mixture cools to 1.5 K which is enough to cause gas to condense in liquid. \Rightarrow homogeneous liquid
 \Rightarrow no phase separation yet.

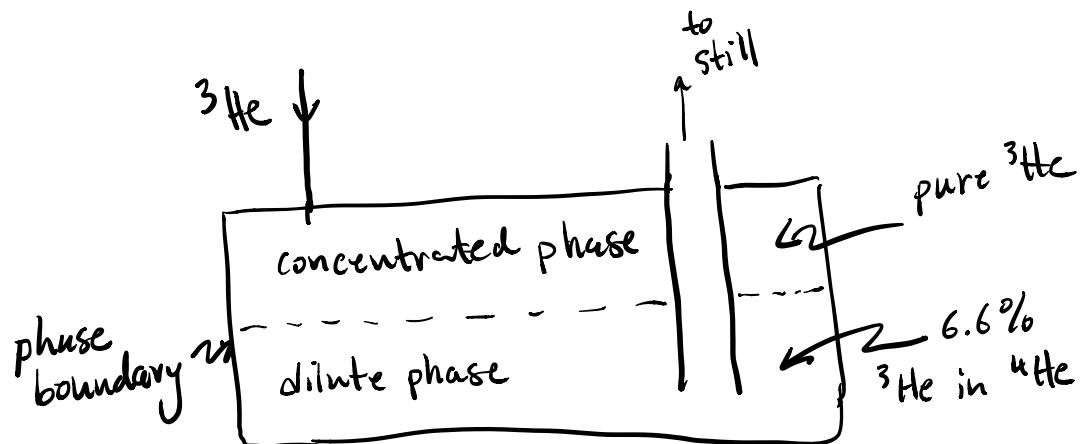


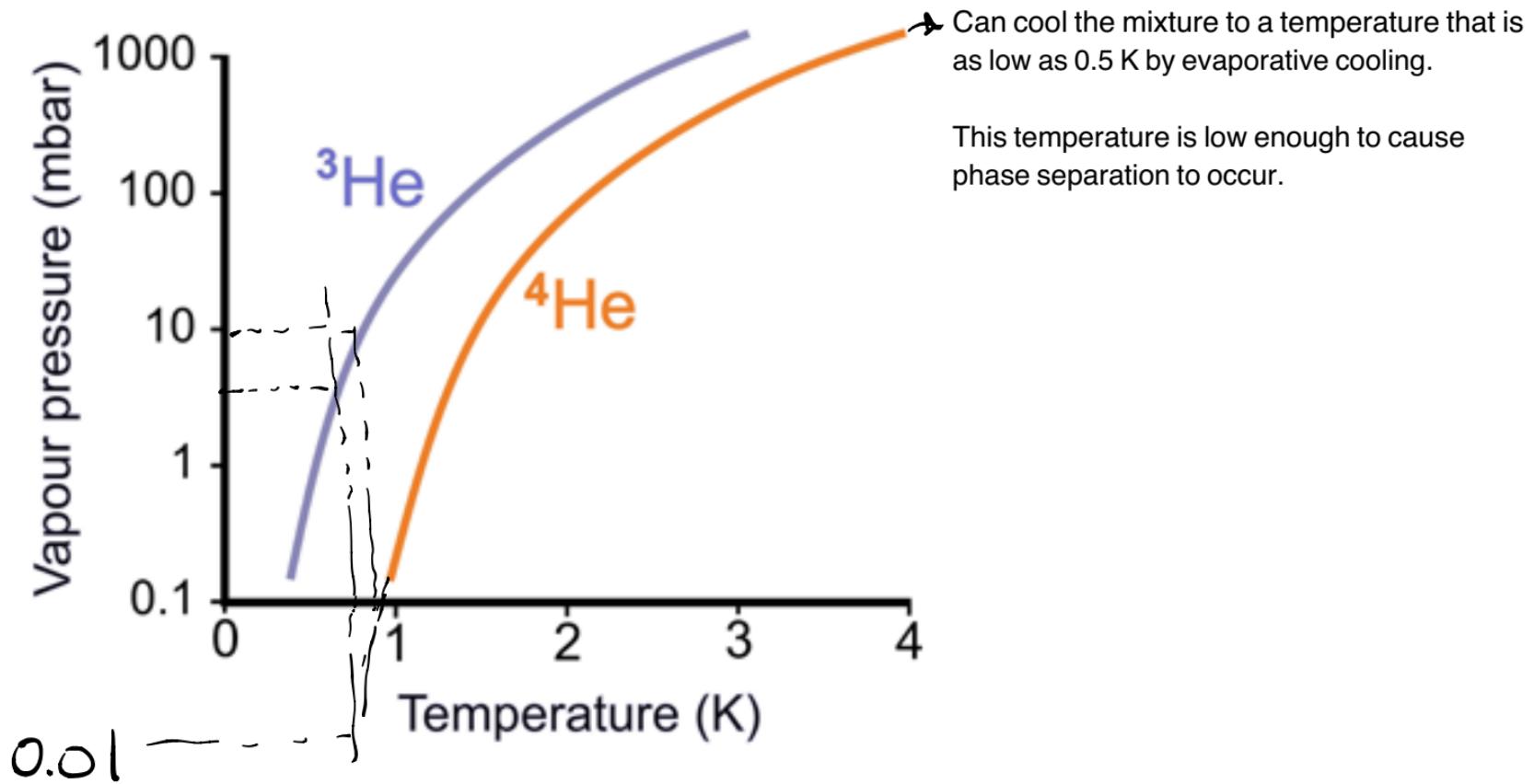
④ Start pumping on still of dilution refrigerator.

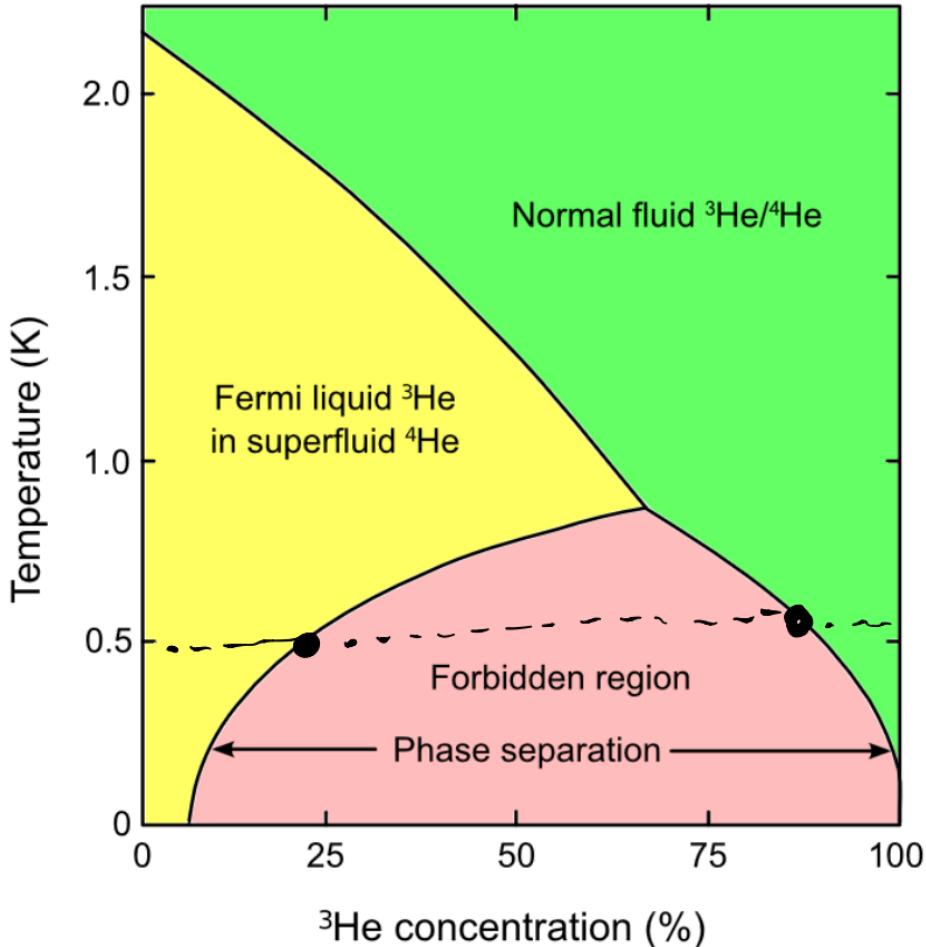
The pumped mixture cools via evaporative cooling.

Can get to temps. low enough to cause phase separation to occur.

Note: Need to select ratio of ^3He to ^4He st. phase separation boundary occurs inside the mixing chamber.







When pumping on the still, removing primarily ${}^3\text{He}$. This is because vapour pressure of ${}^4\text{He}$ component is very low.

Actually need to apply heat to still!

Why? \Rightarrow Raise the vapour of ${}^3\text{He}$ component s.t. ${}^3\text{He}$ circulation rate is relatively large.

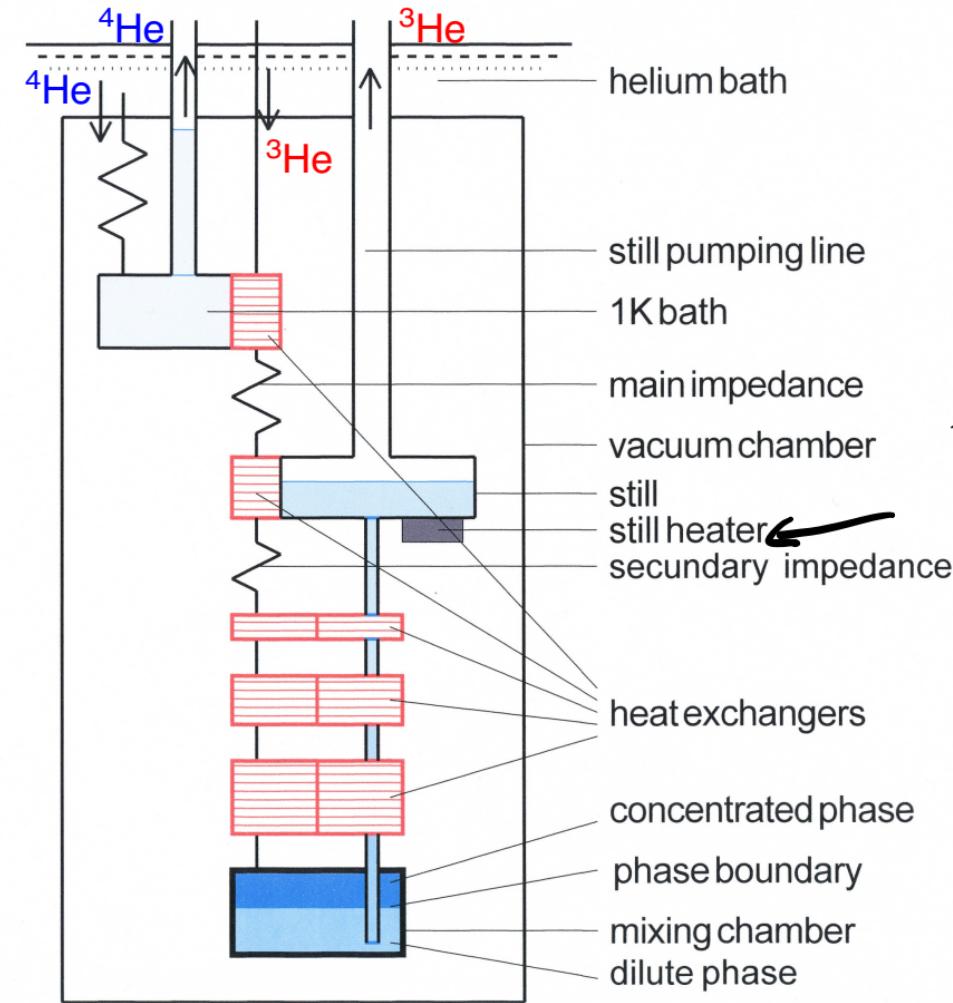
Remember that cooling power

$$\dot{Q} = 83 \dot{n} T_{mc}^2$$

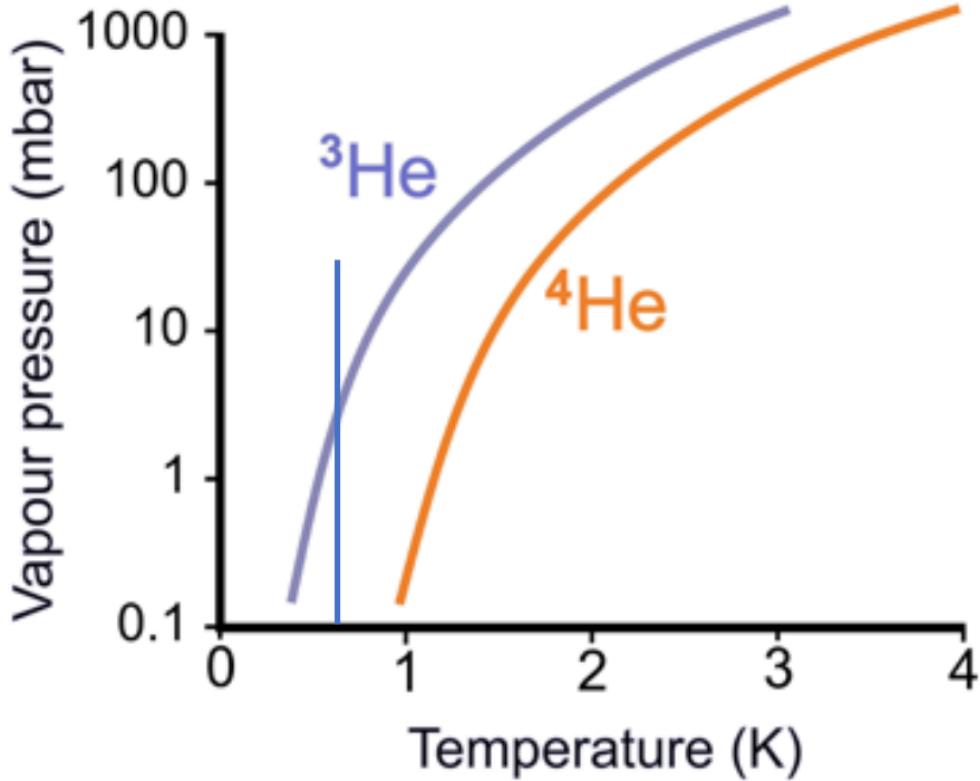
\uparrow mixing chamber temp.
molar circulation rate of ${}^3\text{He}$.

large \dot{n} , low T_{mc}

Can't make still too hot b/c that will raise ${}^4\text{He}$ vapour press. s.t. no longer circulating almost pure ${}^3\text{He}$.



typical still
temp ~0.7 K
(700 mK)



At 0.7 K, the vapour pressure of the ^4He component of the mixture is negligible. Therefore, if we pump on the still when its temperature is ~ 0.7 K, we mainly remove ^3He atoms.

Don't want the still temperature to get much below 0.7 K because then the circulation rate of ^3He is low and the temperature of the mixing chamber will rise.

Don't want the still temperature to get much higher than 0.7 K because then we will have too much ^4He in the circulated gas.

Aside: Osmotic Pressure

$$\pi = i M_{\text{solute}} RT$$

↑
osmotic pressure

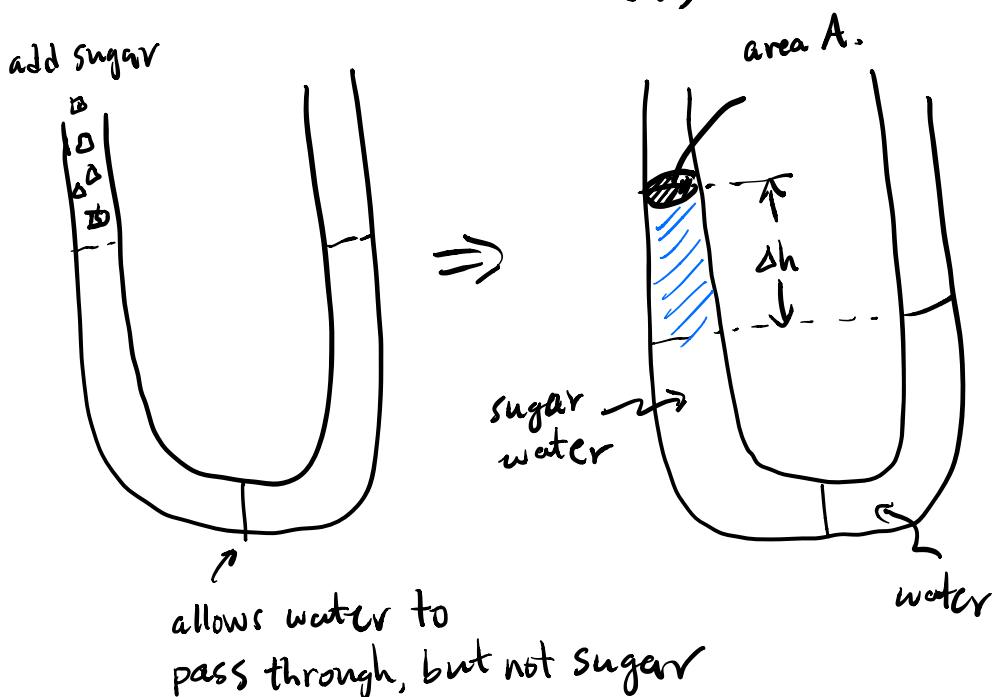
↑
molar concentration

$$\frac{N_{\text{solute}}}{V} \quad \begin{matrix} \curvearrowleft \\ \# \text{ moles of} \\ \text{solute} \end{matrix}$$

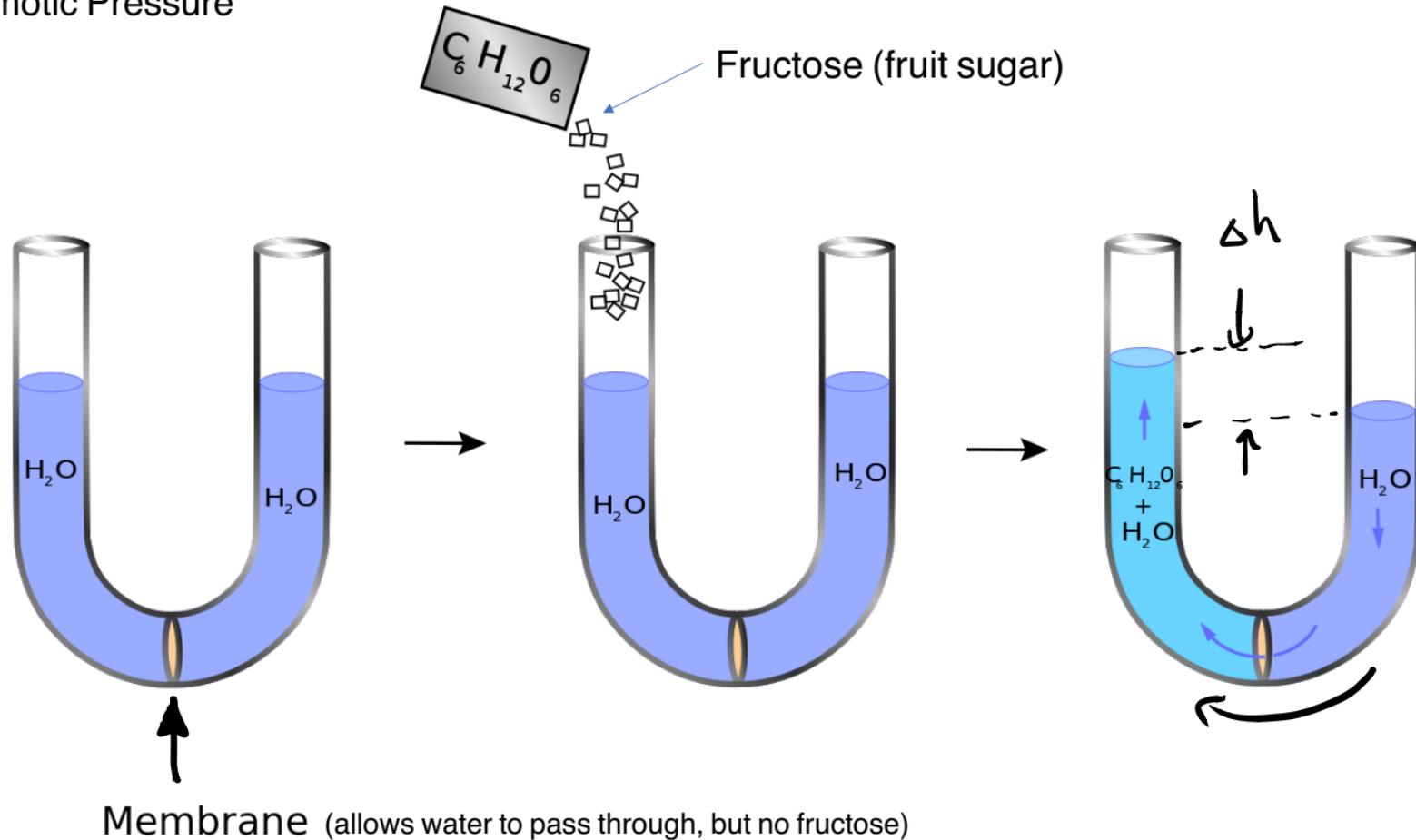
i : dimensionless constant called the Van't Hoff index ($i \approx 1$)

Similar to ideal gas law $PV = nRT$

$$P = \left(\frac{N}{V}\right)RT$$



Osmotic Pressure



Weight of shaded column is

$$mg = \rho A \Delta h g$$

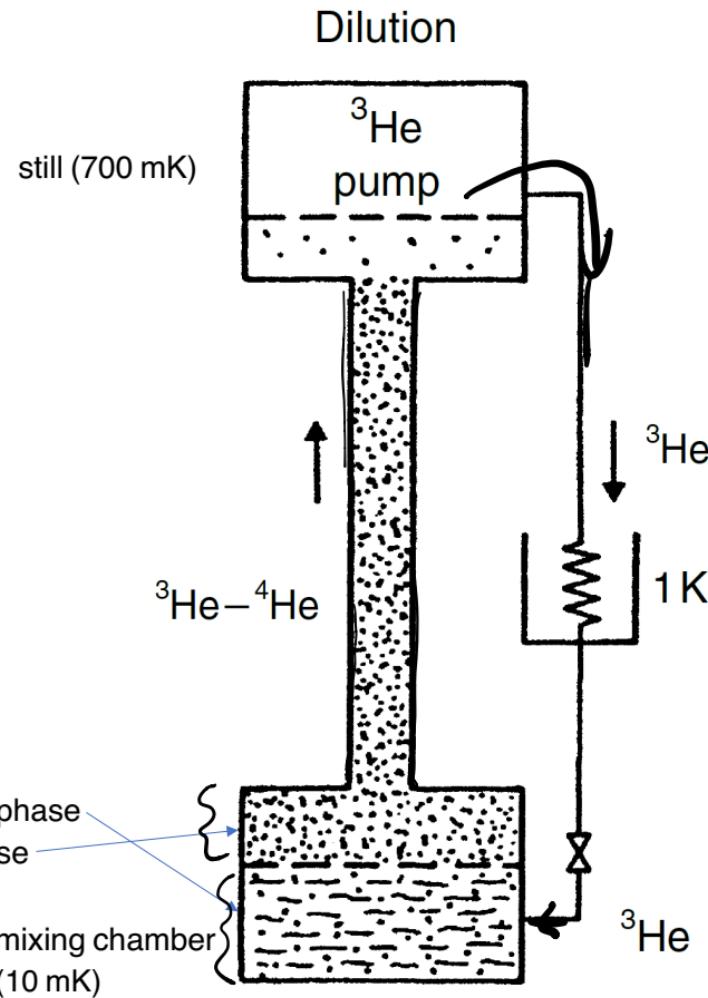
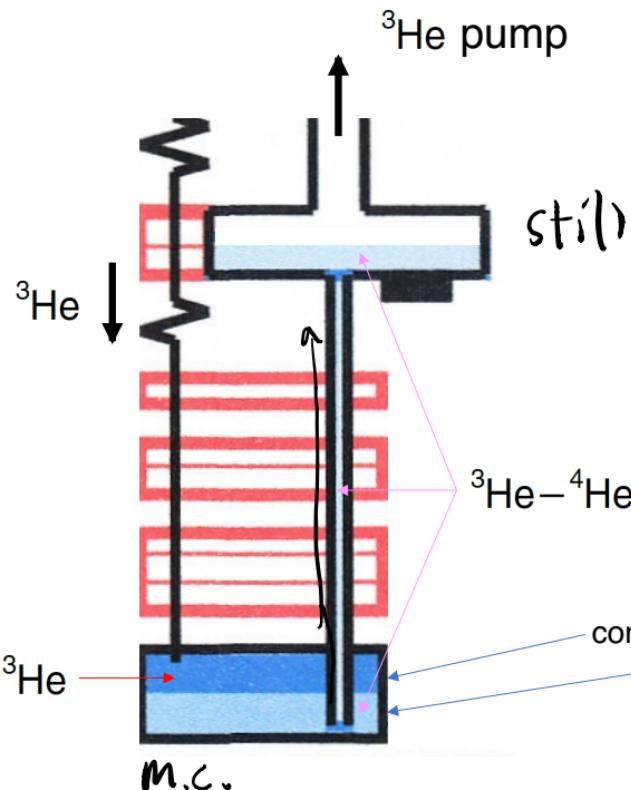
\therefore pressure diff.

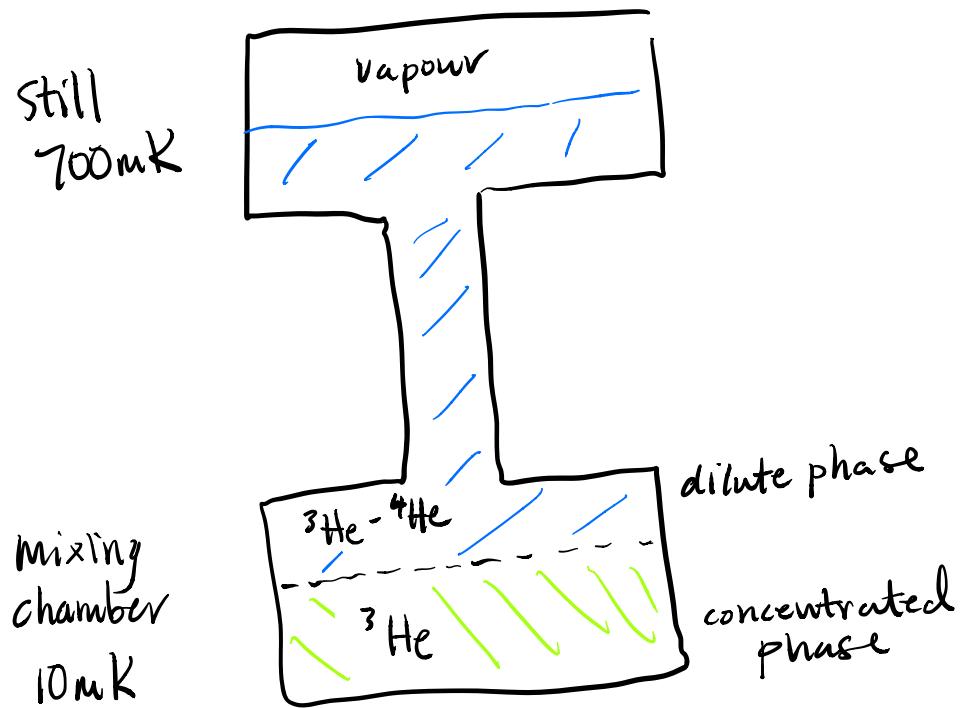
$$\Delta P = \frac{mg}{A} = \rho g \Delta h$$

At equil. $\rho g \Delta h = M_{\text{solute}} RT$ (assuming $i=1$)

$$\therefore \Delta h = \frac{M_{\text{solute}} RT}{\rho g}$$

Osmotic pressure difference between the mixing chamber and still drives ${}^3\text{He}$ up the column of mixture





If treat ${}^3\text{He}$ in dilute phase as the solute
in ${}^4\text{He}$:

$$\text{mixing chamber} \quad \overline{T}T_{mc} = M_{mc} R T_{mc}$$

$$\text{Still} \quad \overline{T}T_{st} = M_{st} R T_{st}$$

$$\text{In equil.} \quad \overline{T}T_{mc} = \overline{T}T_{st} \Rightarrow M_{mc} T_{mc} = M_{st} T_{st}$$

$$\text{or} \quad M_{st} = M_{mc} \frac{T_{mc}}{T_{st}}$$

If $\frac{T_{mc}}{T_{st}} = \frac{10}{700} \approx \frac{1}{70}$

$$x_{mc} = 6.6\%$$

$$x_{st} \approx 0.1\%$$

If we remove ${}^3\text{He}$ atoms from the still using a pump, then we lower x_{st} & create an osmotic pressure diff.

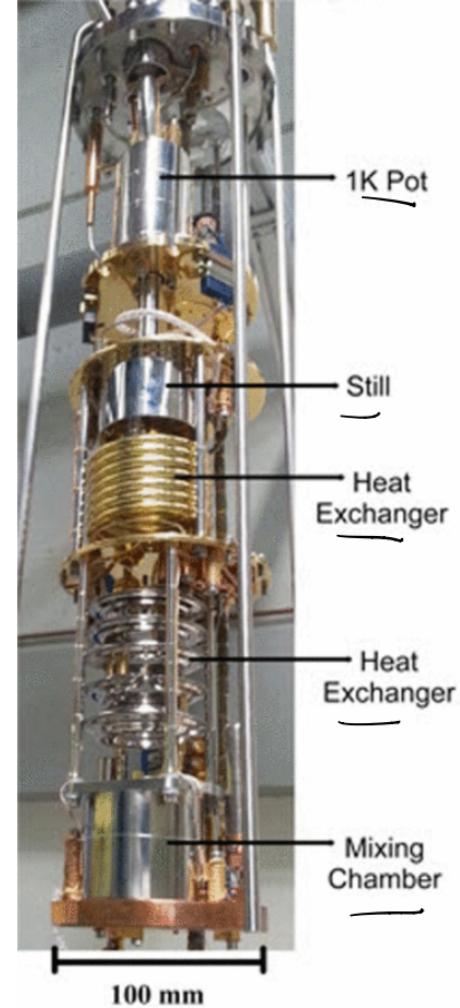
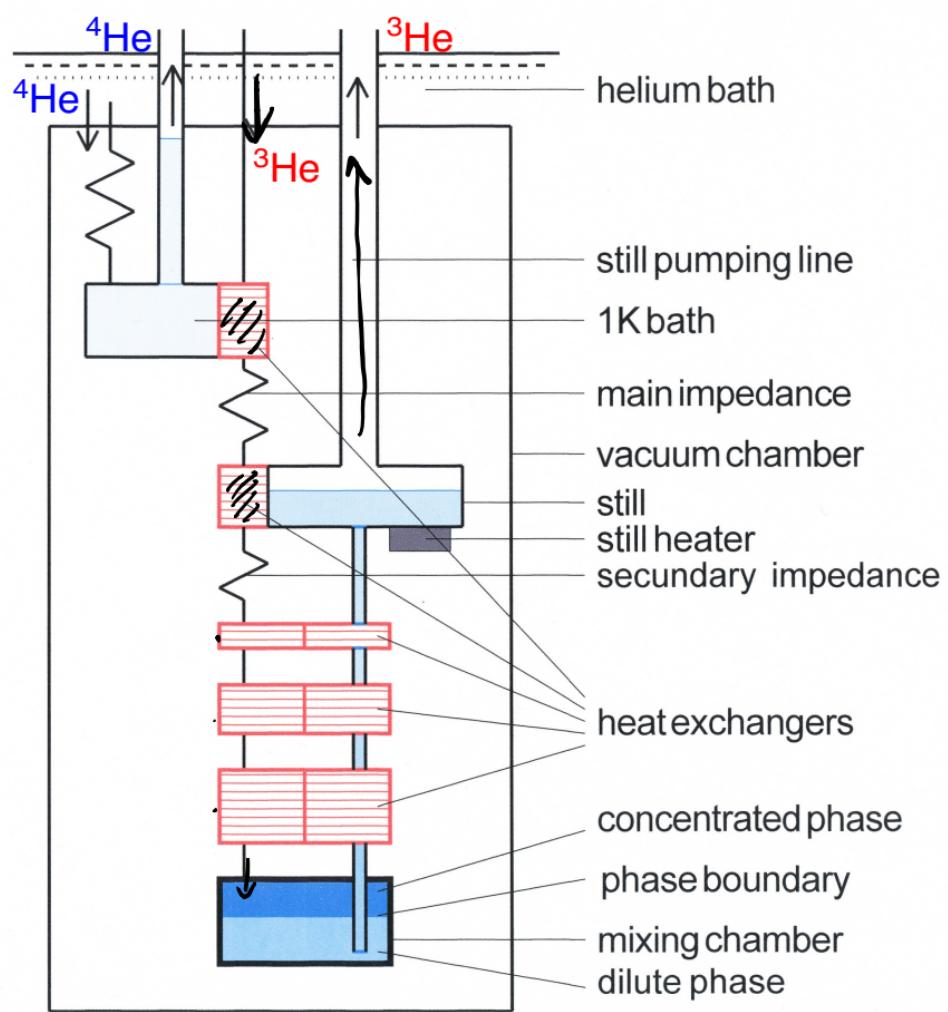
$$\Pi_{\text{mc}} - \Pi_{\text{st}} = \left(M_{\text{mc}} T_{\text{mc}} - M_{\text{st}} T_{\text{st}} \right) R$$

which drives ${}^3\text{He}$ from mixing chamber to still.

But now M_{mc} is too low & ${}^3\text{He}$ from concentrated phase crosses the phase boundary & enters the dilute phase \Rightarrow we get the desired cooling

$$\dot{Q} = \dot{n}_3 (H_D - H_C)$$

- ⑥ ${}^3\text{He}$ removed from still is cooled via heat exchangers & re-introduced to concentrated phase (pure l. ${}^3\text{He}$). Can run dilution refrigerator continuously for long periods of time.



A dry dilution refrigerator uses a two-stage pulse tube cryocooler to pre-cool the dilution refrigerator. This system does not require any external cryogenic liquids (only the mixture inside the refrigerator).

- One drawback is that the pulse tube cryocooler introduces vibrations into the system which can be undesirable for some experiments.

