

Bayesian Repulsive Mixtures for Probabilistic Topic Modelling

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Presentation Outline

1 Latent Dirichlet Allocation

2 Repulsive Mixture Models

3 A Novel Topic Model

4 Conclusion

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Problem Overview

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- ▶ Naive approaches analyse words/phrases individually.
- ▶ Can we instead analyse the **latent themes** in the dataset?
- ▶ Then we can search for topics rather than words/phrases!

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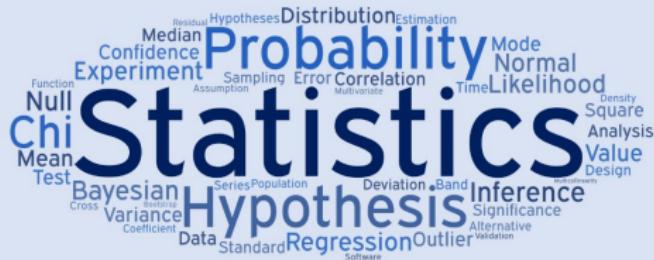


Figure: Potential word distribution of *statistics* topic.

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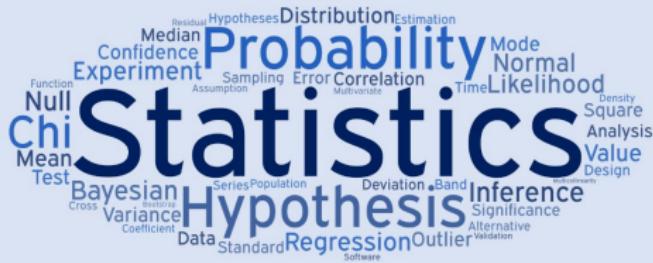


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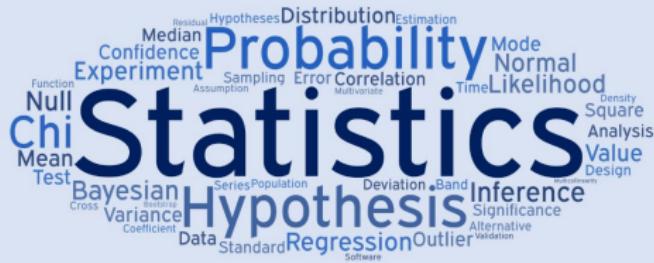


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 - Example: combination of *statistics* and *biology* for abstract on biostatistics.

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- ▶ *How else could we generate the latent variables?*

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where:

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- $\pi = (\pi_1, \dots, \pi_J)$ is a random probability vector.
- ▶ Typically $\pi \sim \text{Dirichlet}_J(\gamma)$, some $\gamma > 0$, and $\mu_j \sim \text{i.i.d. } p(\cdot)$.
- ▶ Well-understood & predictable - any more interesting choices?

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Normalised Random Measures²

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- ▶ Strength of repulsion is controlled by kernel hyper-parameter K .

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- ▶ Might not be appropriate in some situations, leading to misspecification.
- ▶ Repulsive mixture model uses very modern framework - never applied to topic modelling.
- ▶ *How does the behaviour of LDA change if we introduce a repulsive mixture component?*

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- Density is proportional to $y \mapsto K(y, y)$, where K is DPP kernel.

3. Informative predictive distribution:

- Predictive distribution of θ_{N+1} given observed $\theta_1, \dots, \theta_N$ has two interesting behaviours.
- θ_{N+1} can coincide with one of the θ_n .
- θ_{N+1} can be distinct from the θ_n - tends to be very different.

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- ▶ Studied how theoretical properties compared to LDA.
- ▶ **Main advantage:** can achieve well-spaced clusters with customisable cluster locations.
- ▶ **Main disadvantage:** MCMC for posterior inference. Sampler proposed only for posterior of topics and topic mixtures.
- ▶ Future work:
 - Constructing kernel K on simplex.
 - Test efficacy of marginal sampler on real-life dataset.

References

-  Blei, D. M., Ng, A. Y. & Jordan, M. I. (2003)
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