Variables:

$green = initial, determined\ constant\ values$
$blue = initial\ values\ pulled\ from\ Hummel\ Thesis$

$D = magnitude\ of\ drag\ force$
$\rho = density\ of\ air$
$v = magnitude\ of\ velocity$
$A = area\ of\ frisbee\ top$
$C_D = 'coefficient'\ of\ drag\ (not\ really\ a\ coefficient)$
$C_{D0} = form\ drag$
$C_{D\alpha} = induced\ drag$
$\alpha_0 = ideal\ angle\ of\ attack$
$\alpha = angle\ of\ attack$
$C_L = 'coefficient'\ of\ lift\ (not\ really\ a\ coefficient)$
$C_{L0} = form\ lift$
$C_{L\alpha} = induced\ lift$
$\overline{F_L} = Lift\ Force\ Vector$
$\overline{F_D} = Drag\ Force\ Vector$
$\overline{F_g} = Gravitational\ Force\ Vector$
$r = radius$
$m = mass$
$\bar{n} = normal\ vector\ of\ frisbee\ plane$

Drag:

[1] Drag Force Magnitude:

$$D = \frac{\rho v^2 A C_D}{2}$$

[2] Coefficient of Drag:

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2$$

[3] Drag Direction:

$$\overline{F_D} = D * -\hat{v}$$

Lift:

[4] Lift Force Magnitude:

$$L = \frac{\rho v^2 A C_L}{2}$$

[5] Coefficient of Lift:

$$C_L = C_{L0} + C_{L\alpha} * \alpha$$

[6] Lift Direction:

$$\overline{F_L} = L * \hat{n}$$

Gravity:

$$\overline{F_g} = m * -9.81 * \hat{k}$$

General Equations Used in the code:

[7] Area:

$$A = \pi r^2$$

[8] N2L:

$$\sum \bar{F} = m\bar{a} \qquad \bar{a} = \frac{\sum \bar{F}}{m}$$

[9] Dot & Cross Product:

$$\bar{u} \cdot \bar{v} = u_1v_1 + u_2v_2 + u_3v_3 \qquad \bar{u} \times \bar{v} = \begin{bmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{bmatrix}$$

[10] Projection (a onto v):

$$proj_{\bar{v}}\bar{a} = \frac{\bar{a} \cdot \bar{v}}{\|\bar{v}\|^2}\bar{v}$$

[11] Projection onto a plane:

$$proj_{Plane}\bar{a} = \bar{a} - \frac{\bar{a} \cdot \bar{n}}{\|\bar{n}\|^2}\bar{n}$$

[12] Angle between two vectors:

$$\theta = arccos\left(\frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| * \|\bar{v}\|}\right)$$

[13] Position and its derivatives:

$$\overline{p(t)} = \int \overline{v(t)}dt \overset{[b]}{=} \iint \overline{a(t)}dt = position$$

$$\overline{p'^{(t)}} = \overline{v(t)} = \int \overline{a(t)}dt \overset{[a]}{=} velocity$$

$$\overline{p''(t)} = \overline{v'(t)} = \overline{a(t)}dt = acceleration$$

What the code does:

1. Prompts user for input on Initial Velocity (vector), height thrown (float), mass(float), tilt angle above Y-axis (float, degrees), tilt angle above X-axis (float, degrees). (Note that angle on Z doesn't matter because that would be the spin of the frisbee)

2. Cycles the loop until 3 minutes is up or the frisbee hits the floor. Does this by making a for loop on 0 up to 180000 milliseconds (1 cycle = 1 ms), and breaks the loop if the height of the frisbee goes below 0 m. **Here's the loop:**

    a. Computes angle of attack. Does this by finding the angle between the frisbee plane and frisbee velocity (initial velocity or velocity from last cycle) **[11] -> [12]**

    b. Computes drag coefficient (as angle of attack changed) **[2]**

    c. Computes drag force magnitude (as coefficient changed) **[1]**

    d. Multiplies this magnitude by negative normal vector of velocity (initial velocity or from last cycle) **[3]**

    e. Computes lift coefficient **[5]**

    f. Computes lift force magnitude **[4]**

    g. Multiplies this magnitude by normal vector of frisbee plane **[6]**

    h. Sums up force vectors (Lift, Drag, Gravity) **[8]**

    i. Divides sum of vectors by mass of frisbee to get acceleration vector **[8]**

    j. Takes current velocity and adds the product of the acceleration and time change (v = v + a * 0.001s = v + a*dt*) **[13a]**

k. Takes the current position and adds the product of the velocity and time change (p = p + v * 0.001s = p + v*dt*) **[13b]**

l. Appends the current time (***i*** in the loop) to a time array, appends current position to position a position tracking loop (x, y and z) and appends current velocity magnitude to a velocity array. These are used for the plotting.

m. Checks if the frisbee height is less than or equal to 0. If it is, breaks the loop

n. If not, restarts from a)

3. Plots the data

The ODE:

Calculating Drag:

$$\overline{F_D} = -D * \hat{v}$$

$$\overline{F_D} = -D * \frac{\bar{v}}{\|\bar{v}\|}$$

$$\overline{F_D} = -\frac{1}{2}\rho\|\bar{v}\|^2 A C_D * \frac{\bar{v}}{\|\bar{v}\|} = -\frac{1}{2}\rho\|v\|^{\cancel{2}} A C_D * \frac{\bar{v}}{\cancel{\|\bar{v}\|}}$$

$$\overline{F_D} = -\frac{\rho\|v\| A C_D}{2} * \bar{v}$$

$$\overline{F_D} = \underbrace{-\frac{\rho A}{2}}_{constant} * \underbrace{C_D \overline{v(t)}\left\|\overline{v(t)}\right\|}_{variable}$$

Calculating Lift:

$$\overline{F_L} = L * \hat{n}$$

$$\overline{F_L} = \frac{\rho\|\bar{v}\|^2 A C_L}{2} * \hat{n}$$

$$\overline{F_L} = \underbrace{\frac{\rho A}{2}\hat{n}}_{constant} * \underbrace{C_L\|\bar{v}\|^2}_{variable}$$

Gravity:

$$\overline{F_g} = \underbrace{m * -9.81 * \hat{k}}_{constant}$$

Grouping together and coming up with the ODE:

$$\overline{p(t)} = \int \overline{v(t)}dt = \iint \overline{a(t)}dt = position$$

$$\overline{p'^{(t)}} = \overline{v(t)} = \int \overline{a(t)}dt = velocity$$

$$\overline{p''(t)} = \overline{v'(t)} = \overline{a(t)}dt = acceleration$$

$$\overline{a(t)} = \frac{\sum \bar{F}}{m} = \left(\frac{1}{m}\right)\sum \bar{F}$$

$$\overline{a(t)} = \left(\frac{1}{m}\right)\left(\overline{F_D} + \overline{F_L} + \overline{F_g}\right)$$

$$\overline{a(t)} = \left(\frac{1}{m}\right)\left(-\frac{\rho A}{2} * C_D\overline{v(t)}\left\|\overline{v(t)}\right\| + \frac{\rho A}{2}\hat{n} * C_L\|\bar{v}\|^2 + m * -9.81 * \hat{k}\right)$$

$$\overline{a(t)} = -\frac{\rho A}{2m} * C_D\overline{v(t)}\left\|\overline{v(t)}\right\| + \frac{\rho A}{2m}\hat{n} * C_L\|\bar{v}\|^2 - 9.81 * \hat{k}$$

$$\overline{p''(t)} = -\frac{\rho A}{2m} * C_D\overline{p'^{(t)}}\left\|\overline{p'^{(t)}}\right\| + \frac{\rho A}{2m}\hat{n} * C_L\left\|\overline{p'^{(t)}}\right\|^2 - 9.81 * \hat{k}$$

Assumptions:

- Constant air distribution
- Center of mass and center of pressure are the same:
  - No moments will occur
  - Angle of attack remains the same due to the lack of moments
- Gyroscopic effects are strong enough to keep the frisbee in the same orientation
- The frisbee stops as soon as it hits the floor