Antenna Theory

Fundamentals

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A.1 Introduction

What is an antenna?

the system component that is designed to radiate or receive EM waves

$\Rightarrow$ Transducer (Tx): guided wave to radiated wave

$\Rightarrow$ Receiver (Rx): radiated wave to guided wave

A.1 Introduction

A brief history…

1873: Maxwell, unified electricity and magnetism
1886 : Hertz, first wireless radiation system (dipole)
1901 : Marconi, transmitted signals over long distance
WW2 : waveguide apertures, horns, reflectors, etc. were developed
1960’s: numerical methods for solving Maxwell’s Equations
1970’s: patch antennas were developed

A.2 Electromagnetic Radiation

How do antennas radiate?

Free charge…

$\Rightarrow$ no motion: electrostatics

$\Rightarrow$ constant velocity: magnetostatics

$\Rightarrow$ with acceleration: electrodynamics

acceleration includes:

1) change in velocity

2) change in direction

A.2 Electromagnetic Radiation

Maxwell's Equations in frequency domain ($e^{i\omega t}$ assumed):

$$\nabla \times \mathbf{E} = -\mathbf{M}_i - i\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = \mathbf{J}_i + i\omega\varepsilon\mathbf{E}$$
$$\nabla \cdot (\varepsilon\mathbf{E}) = q_{ve}$$
$$\nabla \cdot (\mu\mathbf{H}) = q_{vm}$$

the continuity equation:

$$\nabla \cdot \mathbf{J}_i = -i\omega q_{ve}$$
$$\nabla \cdot \mathbf{M}_i = -i\omega q_{me}$$

A.2 Electromagnetic Radiation

Boundary conditions to be satisfied

$$-\hat{n} \times \mathbf{E}_{\boldsymbol{d}} = \mathbf{M}_{\boldsymbol{s}}$$
$$\hat{n} \times \mathbf{H}_{\boldsymbol{d}} = \mathbf{J}_{\boldsymbol{s}}$$
$$\hat{n} \cdot (\varepsilon \mathbf{E}_{\boldsymbol{d}}) = q_{se}$$
$$\hat{n} \cdot (\mu \mathbf{H}_{\boldsymbol{d}}) = q_{sm}$$

radiation condition must also be satisfied: in an infinite homogeneous medium that waves travel outwardly from the source should vanish at infinity.

A.2 Electromagnetic Radiation

The first-order coupled equations can be de-coupled

$$\nabla^2\mathbf{E} + \beta^2\mathbf{E} = \frac{1}{\varepsilon}\boldsymbol{\nabla} q_{ve} + \boldsymbol{\nabla} \times \mathbf{M}_{\boldsymbol{i}} + i\omega\mu\mathbf{J}_{\boldsymbol{i}}$$

$$\nabla^2\mathbf{H} + \beta^2\mathbf{H} = \frac{1}{\mu}\boldsymbol{\nabla} q_{vm} - \boldsymbol{\nabla} \times \mathbf{J}_{\boldsymbol{i}} + i\omega\varepsilon\mathbf{M}_{\boldsymbol{i}}$$

where $\beta = \omega^2\mu\varepsilon.$

Task: solve the unknown field values from the source terms and boundary conditions

A.2 Electromagnetic Radiation

Direct integration may be complex, so the process can be broken into two steps by introducing the magnetic and electric vector potentials (**A** and **F**)

$$\nabla^2\mathbf{A} + \beta^2\mathbf{A} = -\mu\mathbf{J}_i$$
$$\nabla^2\mathbf{F} + \beta^2\mathbf{F} = -\varepsilon\mathbf{M}_i$$

where

$$\mathbf{A} = \frac{4\pi}{\mu}\int_v \mathbf{J}_i \frac{e^{-i\beta R}}{R}dv'$$
$$\mathbf{F} = \frac{4\pi}{\varepsilon}\int_v \mathbf{M}_i \frac{e^{-i\beta R}}{R}dv'$$

A.2 Electromagnetic Radiation

Now the electric and magnetic field can be represented as

$$\mathbf{E} = \mathbf{E_A} + \mathbf{E_F} = \left[-i\omega\mathbf{A} - i\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A})\right] + \left[-\frac{1}{\varepsilon}\nabla\times\mathbf{F}\right]$$

$$\mathbf{H} = \mathbf{H_A} + \mathbf{H_F} = \left[\frac{1}{\mu}\nabla\times\mathbf{A}\right] + \left[-i\omega\mathbf{F} - i\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{F})\right]$$

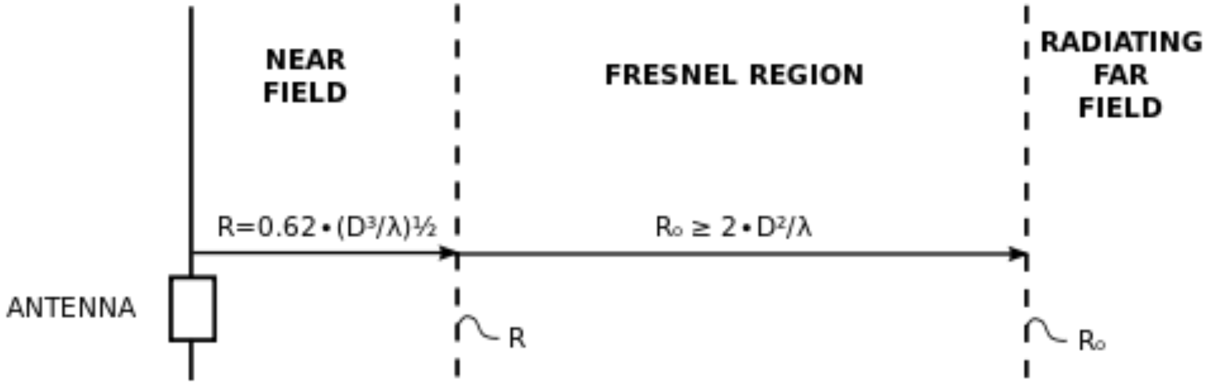
where Lorenz gauge can be applied

$$\nabla\cdot\mathbf{A} = -i\omega\mu\varepsilon\phi_m$$
$$\nabla\cdot\mathbf{F} = -i\omega\mu\varepsilon\phi_e$$

A.2 Electromagnetic Radiation

Observing the radiation solution around an antenna, the field can be divided into three regions

1) Reactive near-field region: reactive near-field predominates
2) Radiating near-field (Fresnel) region: radiated field predominates, and the angular field distribution is dependent upon the distance from the antenna
3) Far-field (Fraunhofer) region: the angular field distribution is independent upon the distance from the antenna



Ref: https://en.wikipedia.org/wiki/Near_and_far_field

A.2 Electromagnetic Radiation

In the far-field region, the radial distance can approximate by

$R = \begin{cases} r - r' \cos\psi \\ r \end{cases}$ for phase and amplitude terms

Now the vector potentials can be simplified as

$$\mathbf{A} \cong \frac{\mu e^{-i\beta R}}{4\pi r}\mathbf{N}$$

$$\mathbf{F} \cong \frac{\varepsilon e^{-i\beta R}}{4\pi r}\mathbf{L}$$

with

$$\mathbf{N} = \int_v \mathbf{J}_i e^{i\beta r' \cos\psi}\, dv'$$

$$\mathbf{L} = \int_v \mathbf{M}_i e^{i\beta r' \cos\psi}\, dv'$$

A.2 Electromagnetic Radiation

The electric and magnetic field can thus be represented in the far-field region as

$$E_r \cong 0, H_r \cong 0$$

$$E_\theta \cong -i\frac{\beta e^{-i\beta R}}{4\pi r}(L_\phi + \eta N_\theta)$$

$$E_\phi \cong i\frac{\beta e^{-i\beta R}}{4\pi r}(L_\theta - \eta N_\phi)$$

$$H_\theta \cong i\frac{\beta e^{-i\beta R}}{4\pi r}(N_\phi - L_\theta/\eta)$$

$$H_\phi \cong -i\frac{\beta e^{-i\beta R}}{4\pi r}(N_\theta + L_\phi/\eta)$$

where the intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$

A.3 Antenna Theorems

Duality theorem:

From the Maxwell's Equations, one can observe that the equations may be grouped into two categories - the electric and the magnetic part, and a systematic interchange of symbols changes the first equation into the second and vice-versa.

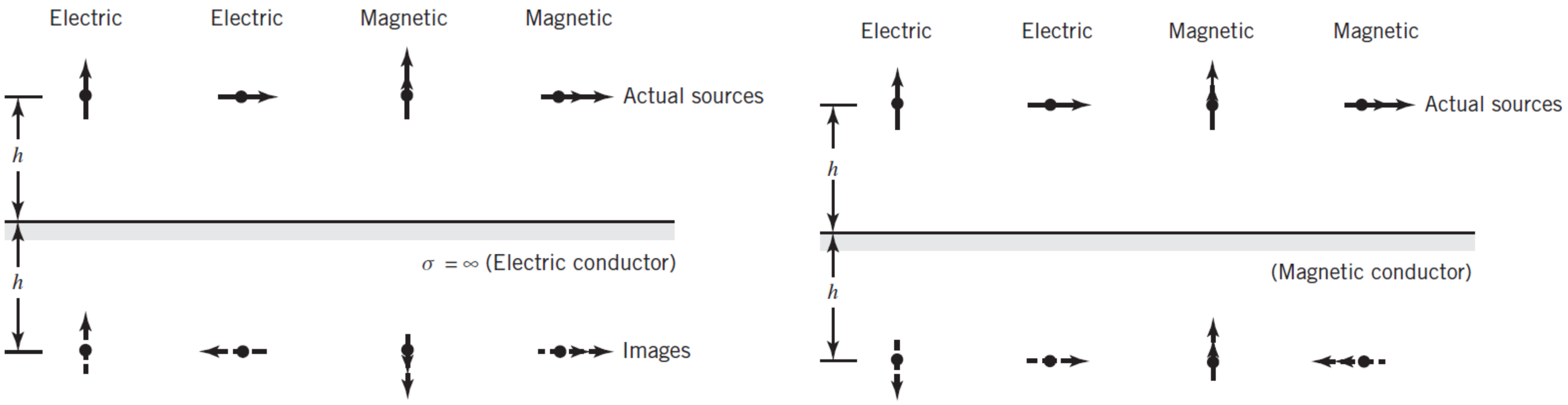
Electric sources ($\mathbf{J} \neq 0$, $\mathbf{M} = 0$)	Magnetic sources ($\mathbf{J} = 0$, $\mathbf{M} \neq 0$)
$\mathbf{E}_A$	$\mathbf{H}_F$
$\mathbf{H}_A$	$-\mathbf{E}_F$
$\mathbf{J}$	$\mathbf{M}$
$\mathbf{A}$	$\mathbf{F}$
ε	μ
μ	ε
β	β
η	$1/\eta$
$1/\eta$	η

Ref: Balanis, Advanced Engineering Electromagnetics, 2nd, p.312.

A.3 Antenna Theorems

Image theory :

A special kind of boundary value problem (BVP) when the boundary is PEC or PMC. The full-space calculation can be simplified into a half-space problem.



Ref: Balanis, Advanced Engineering Electromagnetics, 2nd, p.318.

A.3 Antenna Theorems

Reciprocity theorem :

Within a linear and isotropic medium, the existed two set of sources $\mathbf{J}_1$, $\mathbf{M}_1$ and $\mathbf{J}_2$, $\mathbf{M}_2$ which produce the radiated field $\mathbf{E}_1$, $\mathbf{H}_1$ and $\mathbf{E}_2$, $\mathbf{H}_2$, the following should be satisfied:

$$-\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2$$

When integration is applied on a sphere with infinite radius, one obtains

$$\int_v (\mathbf{E}_1 \cdot \mathbf{J}_2 - \mathbf{H}_1 \cdot \mathbf{M}_2)\, dv = \int_v (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_2 \cdot \mathbf{M}_1)\, dv$$

A.3 Antenna Theorems

Uniqueness theorem:

The fields **E**, **H** created by some sources **J**, **M** in a lossy volume are unique if any one of these is true:

1) Tangential **E** over the enclosed surface is known
2) Tangential **H** over the enclosed surface is known
3) Tangential **E** over some part of the enclosed surface and tangential ***H*** over the remain part are known

A.3 Antenna Theorems

Equivalence principle:

From the uniqueness theorem, when the surface currents on a selected imaginary surface are known, the fields inside the surface or outside the surface can be deduced from the imaginary currents.

This leads to two kinds of source modeling specifically used in antenna analysis:

$\Rightarrow$ Thin wire (ex. dipole): applies $\mathbf{J}$ only

$\Rightarrow$ Aperture (ex. waveguide): applies $\mathbf{M}$ only

A.4 Antenna Parameters

- Radiation pattern: far-field angular distribution of the radiated fields.
- Main lobe: the radiation lobe containing the direction of maximum radiation.
- Side lobe: a radiation lobe in any direction other than that of the major lobe.
- Side lobe level (SLL): amplitude level of the maximum side lobe relative to the main lobe.
- Input impedance: the impedance presented by an antenna at its terminals.
- Radiation efficiency: the ratio of the total power radiated by an antenna to the net power accepted by antenna.

A.4 Antenna Parameters

- Power density: the power density of the fields radiated by the antenna.

$$\mathbf{S} = (\mathbf{E} \times \mathbf{H}^*)/2$$

- Radiation intensity: the power radiated from an antenna per unit solid angle.

$$\mathbf{U} = r^2\mathbf{S}$$

- Directivity: the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D = 4\pi U/P_r$$

A.4 Antenna Parameters

- Gain: the ratio of the radiation intensity in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$G = \frac{4\pi U}{P_a} = e_r D$$

- Beamwidth: the angular separation between two directions in which the radiation intensity is identical.
- Antenna polarization: the polarization of the fields radiated by the antenna. In general, the polarization of an antenna is classified as linear, circular or elliptical.

A.5 Numerical Methods

Two approaches to tackle the Maxwell's Equations:

$\Rightarrow$ Time-domain approach
$\Rightarrow$ Frequency-domain approach

And also, Maxwell's Equations in two forms:

$\Rightarrow$ Integral form
$\Rightarrow$ Differential form

A.5 Numerical Methods

Well known methods are listed:

	Integral form	Differential form
Frequency-domain	MOM	FEM
Time-domain	TDIE	FDTD

A.5 Numerical Methods

Asymptotic methods can be applied for the electrically-large cases :

$\Rightarrow$ Geometrical optics (GO)
$\Rightarrow$ Physical Optics (PO)

And their advanced version:

$\Rightarrow$ Geometrical Theory of Diffraction (GTD)
$\Rightarrow$ Physical Theory of Diffraction (PTD)

A.6 Basic Array Theory

Consider a 1-D phased array, the total field can be expressed as

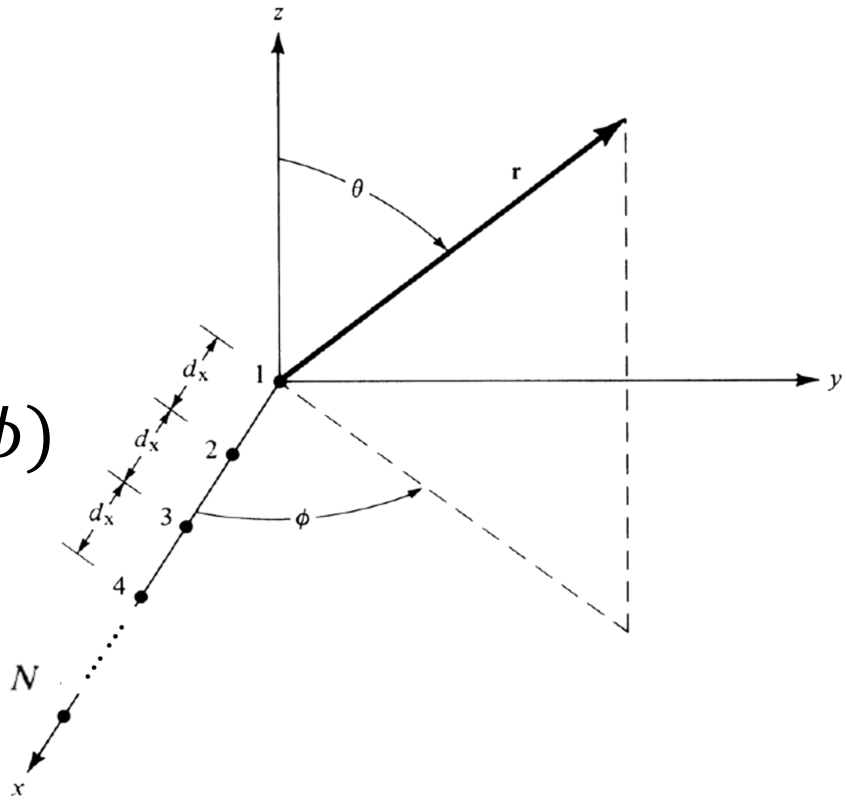
$$\mathbf{E}_{tot}(\mathbf{r}) = \sum_{n=0}^{N-1} A_n e^{i\varphi_n} \mathbf{E}_n(\mathbf{r})$$

In far field, the radiation pattern has the form

$$\mathbf{F}_{tot}(\theta, \phi) = \sum_{n=0}^{N-1} A_n e^{i\varphi_n} \mathbf{F}_n(\theta, \phi)$$

where

$$\mathbf{F}_n(\theta, \phi) = e^{i\mathbf{k}\cdot\mathbf{r}_n} \mathbf{P}_n(\theta, \phi)$$



A.6 Basic Array Theory

A further simplification can be made when the element patterns are identical

$$\mathbf{F}_{tot}(\theta, \phi) = \mathbf{P}(\theta, \phi) \sum_{n=0}^{N-1} A_n e^{i\varphi_n} e^{i\mathbf{k}\cdot\mathbf{r}_n}$$

We define the array factor as the summation of the excitation

$$\mathrm{AF} = \sum_{n=0}^{N-1} A_n e^{i\varphi_n} e^{i\mathbf{k}\cdot\mathbf{r}_n}$$

A.6 Basic Array Theory

For the configuration considered in this example (uniformly spaced along the *x*-axis)

$$\mathbf{k} \cdot \mathbf{r}_n = knd_x \sin\theta \cos\phi$$

The array factor can then be represented as

$$\mathrm{AF} = \sum_{n=0}^{N-1} A_n e^{i\varphi_n} e^{iknd_x \sin\theta \cos\phi}$$

A.6 Basic Array Theory

Considered a uniformly excited array with $A_n = 1$, $\varphi_n = 0$, and express the spacing distance in terms of the wavelength for the $\phi = 0$ cut,

$$\mathrm{AF} = \sum_{n=0}^{N-1} e^{in2\pi d_{\lambda x}\sin\theta} = \frac{1 - e^{iN2\pi d_{\lambda x}\sin\theta}}{1 - e^{i2\pi d_{\lambda x}\sin\theta}}$$

where $d_{\lambda x} = d_x/\lambda$. The maximum of the array factor occurs when

$$2\pi d_{\lambda x}\sin\theta = \pm 2m\pi,$$
$$m = 0, 1, 2 \ldots$$

A.6 Basic Array Theory

A trivial case is $m = 0,\ \theta = 0$ for arbitrary $d_{\lambda x}$.
For the general case

$$d_{\lambda x} \sin\theta = \pm m,$$
$$m = 0, 1, 2 \ldots$$

One can show that in order to avoid grating lobes entering the visible space, the selection of the spacing should satisfy the criteria

$$d_{\lambda x} < 1$$

A.6 Basic Array Theory

If one wants to scan the array pointing at the direction (θ_0, ϕ_0), the phasing of each element is given by

$$\varphi_n = -2\pi n d_{\lambda x} \sin\theta_0 \cos\phi_0$$

Following similar derivations, to avoid grating lobes, the selection of the element spacing should satisfy the following

$$d_{\lambda x} < 0.5$$

A.6 Basic Array Theory

Extension to 2-D case is trivial. In order to scan the beam at the direction (θ_0, ϕ_0), the phase progression given to the elements can be separated in *x*- and *y*-directions independently:

$$\beta_x = -kd_x \sin\theta_0 \cos\phi_0$$
$$\beta_y = -kd_y \sin\theta_0 \cos\phi_0$$

The phasing of each element is given by

$$\varphi_{n,m} = n\beta_x + m\beta_y$$