

**PubH 7440: Introduction to Bayesian Analysis**  
**Spring 2020, Homework 3**  
**Due: March 3, 2020**

1. (Carlin and Louis, Chapter 5, Problem 9.) Consider the following hierarchical changepoint model for the number of occurrences  $Y_i$  of some event during time interval  $i$ :

$$Y_i \sim \begin{cases} \text{Poisson}(\theta), & i = 1, \dots, k \\ \text{Poisson}(\lambda), & i = k + 1, \dots, n \end{cases}$$

$$\theta \sim G(a_1, b_1), \quad \lambda \sim G(a_2, b_2), \quad \theta \text{ and } \lambda \text{ independent}$$

$$b_1 \sim IG(c_1, d_1), \quad b_2 \sim IG(c_2, d_2), \quad b_1 \text{ and } b_2 \text{ independent}$$

Where  $G$  denotes the gamma and  $IG$  the inverse gamma distribution.

- (a) Apply this model to the data on coal mining disasters, which gives counts of coal mining disasters in Great Britain by year from 1851 to 1962. (Here, “disaster” is defined as an accident resulting in the death of 10 or more miners.) Set  $a_1 = a_2 = .5$ ,  $c_1 = c_2 = 1$ , and  $d_1 = d_2 = 1$  (a collection of “moderately informative” values). Also assume  $k = 40$  (corresponding to the year 1890). Derive the full conditional of  $\theta$ ,  $\lambda$ ,  $b_1$ , and  $b_2$  and write an R program to obtain marginal posterior density estimates for  $\theta$ ,  $\lambda$ , and  $R = \theta/\lambda$  using outputs from the Gibbs sampler.

- (b) Re-do this in WinBUGS or BUGS. Are your answers comparable?

- (c) Now assume  $k$  is unknown, and adopt the following prior for  $k$ :

$$k \sim \text{Discrete Uniform}(1, \dots, n), \text{ independent of } \theta \text{ and } \lambda$$

Describe an MH-within-Gibbs step to sample  $k$  conditional on other parameters (no coding or real MCMC sampling is needed).