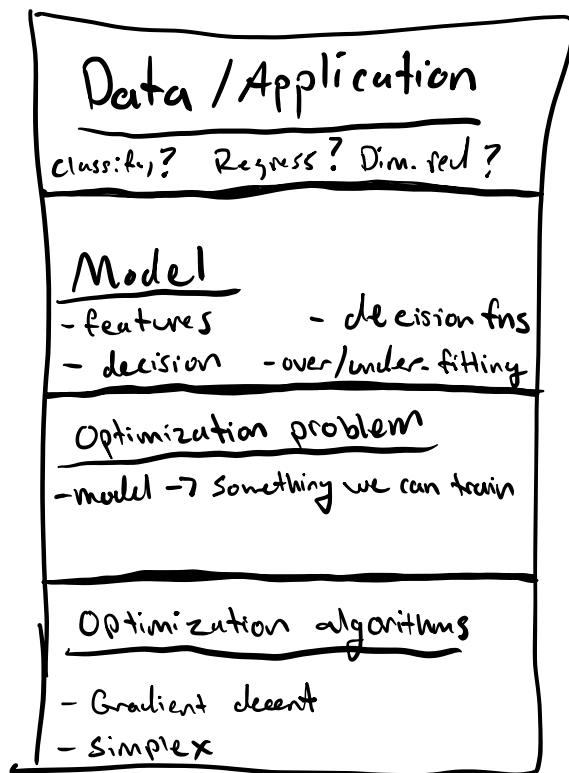


Abstraction + optimization algorithms



Optimization problems

Unconstrained optimization:

- objective function $f(x)$, smooth
 - find w^* that minimizes f
 - f has global + local minima
- $f(x)$ is convex if $\forall x, y \in \mathbb{R}^d$ a line segment $(x, f(x))$ to $(y, f(y))$ never goes below $f(\cdot)$
- Continuous conv. fn. has one of:
- no minimum
 - one unique global minimum (strong convexity)
 - connected set of local minima w/ equal $f(\cdot)$

Sum of f_i , f_i convex $\forall i$, still convex

Algos for general smooth f :

- Gradient decent
 - ↳ blind (w/ learning rate)
 - ↳ stochastic blind
 - ↳ w/ line search
- Newton's Method (requires Hessian)
- Non linear Conjugate gradient

Algos for non-smooth f :

- gradient decent
- BFGS (Broyden-Fletcher-Goldfarb-Shanno)

line search

- project down to 1D and use that result to inform your step size
 - ↳ in 1D its easy to use secant method / Newton's method
 - ↳ direct search

Constrained optimization

- given $f(x)$ and constraints $g(x)=0$
- ↳ use Lagrangian to make it unconstrained

LP's

$$\min_x c^T x \quad : \quad Ax \leq b$$

$$x \in \mathbb{R}^d \quad \left| \quad A \in \mathbb{R}^{m \times d} \quad \left| \quad b \in \mathbb{R}^m \quad \left| \quad c \in \mathbb{R}^d \right. \right.$$

- optima lie on vertices (when some constraints hold w/ equality)
↑ are "active"

Hard-margin SVM: support vectors are active constraints

Algos:

- Simplex: walk vertices to find optimum
- interior point methods

QPs

$$\min_x x^T Q x + c^T x \quad : \quad Ax \leq b$$

$$Q \succeq 0 \quad \text{ex. SVM} \quad \min_w \|w\|^2 \leq \gamma I w \quad I \succeq 0$$

Algos:

- simplex (good in general)
- Sequential minimal optimization (SMO)
- coordinate descent