From Building Block Algebras to Adaptive Libraries

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Tutorial at 8th IEEE SASO
Imperial College London
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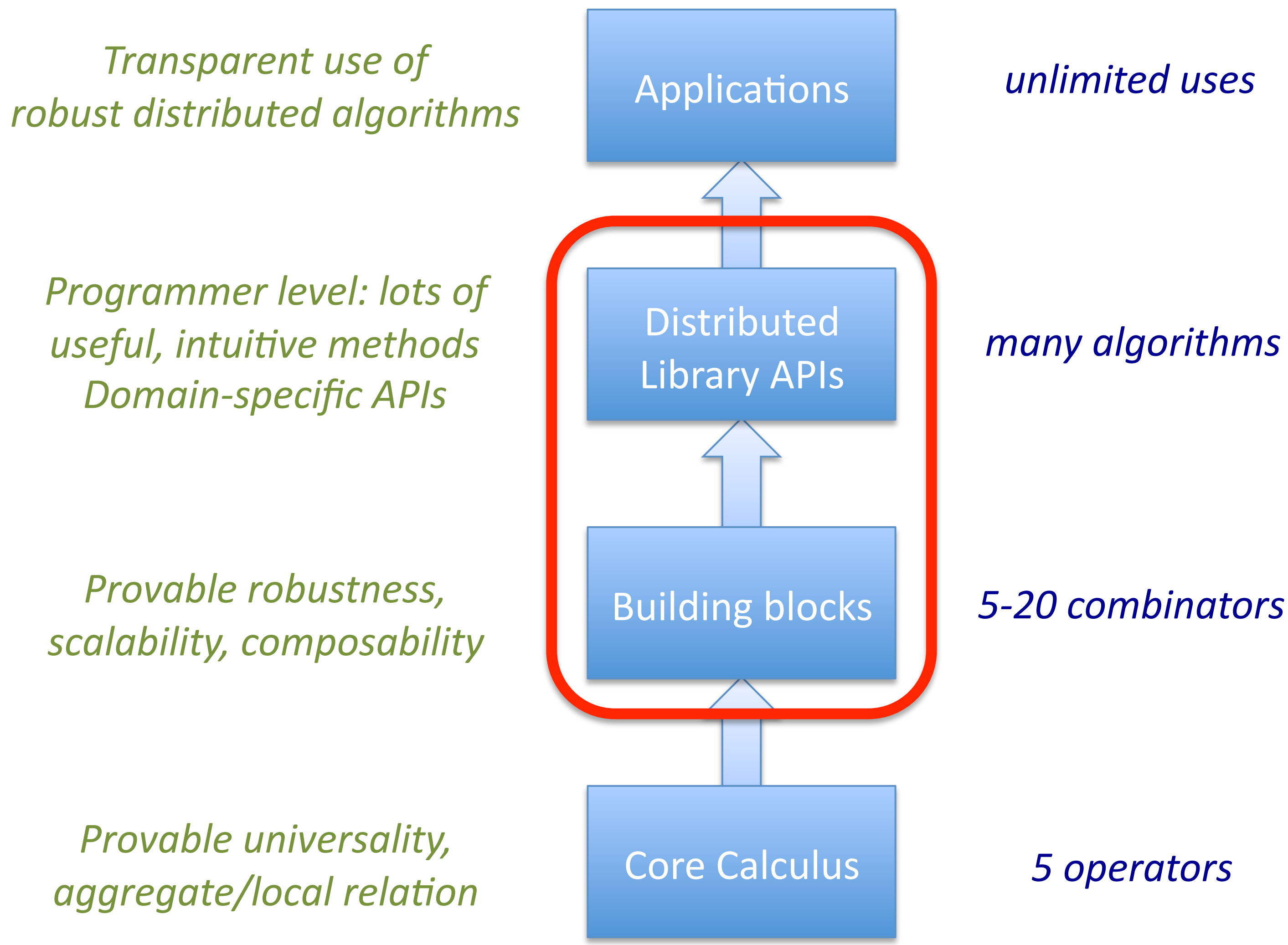
From Principle to Practice:



Outline

- **Building Blocks**
- Space-Time Universality
- Eventual Consistency

Example: Managing Crowd Danger

Foundation: Field Calculus

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Variable　Literal Value　Local Ops　State

Communication

$$
\begin{array}{llr}
\mathtt{e} & ::= \mathtt{x} \mid \mathtt{l} \mid (\mathtt{b}\ \overline{\mathtt{e}}) \mid (\mathtt{f}\ \overline{\mathtt{e}}) & \text{expression} \\
 & \mid (\mathtt{rep}\ \mathtt{x}\ \mathtt{w}\ \mathtt{e}) \mid (\mathtt{nbr}\ \mathtt{e}) \mid (\mathtt{if}\ \mathtt{e}\ \mathtt{e}\ \mathtt{e}) & \text{special constructs} \\
\mathtt{w} & ::= \mathtt{x} \mid \mathtt{l} & \text{variable or value} \\
\mathtt{F} & ::= (\mathtt{def}\ \mathtt{f}(\overline{\mathtt{x}})\ \mathtt{e}) & \text{function} \\
\mathtt{P} & ::= \overline{\mathtt{F}}\ \mathtt{e} & \text{program}
\end{array}
$$

Domain Change

Abstract & Call Functions

- Aggregate/local coherence, space-time universal
- Too low level, no adaptivity guarantees

[Viroli et al., ’13]

Making a library of building blocks:

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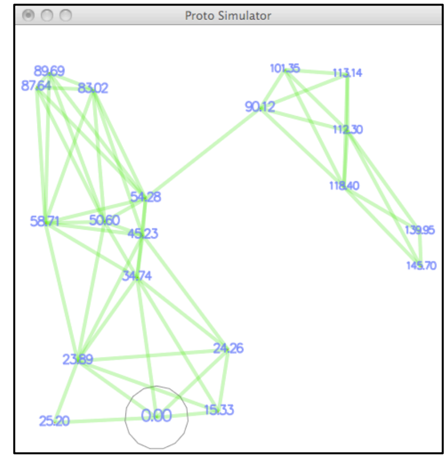
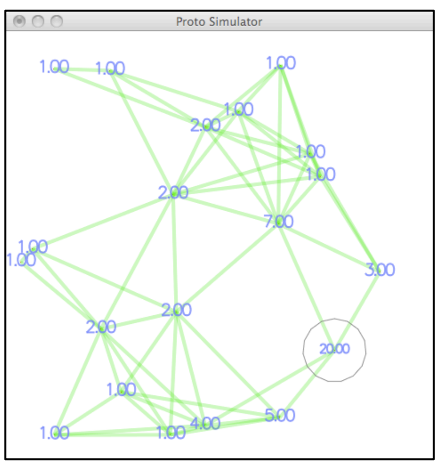
Function	Space	Time
Structure	`nbr-range,...`	`dt,...`
Aggregation	`C`	`T`
Spreading	`G`	
Symmetry breaking	`S`	random
Restriction	`if`	
Compute	computable functions, random	

All compositions are self-stabilizing

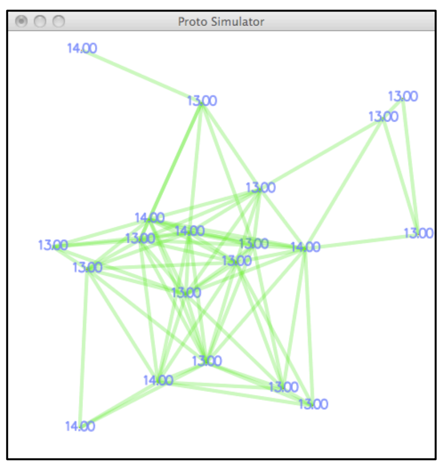


Time-decay

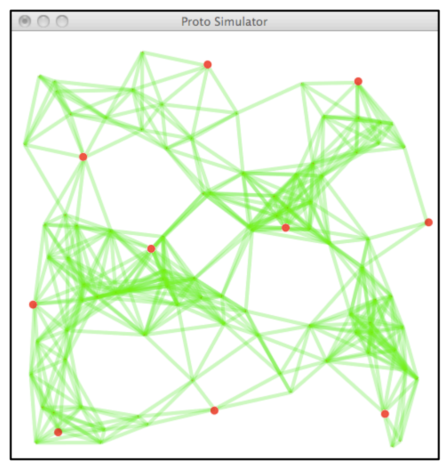
Converge-cast



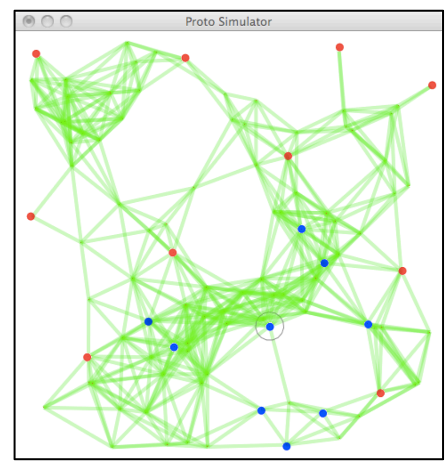
Gradient-path-integral



Sparse-choice



`if`



Building Block: G

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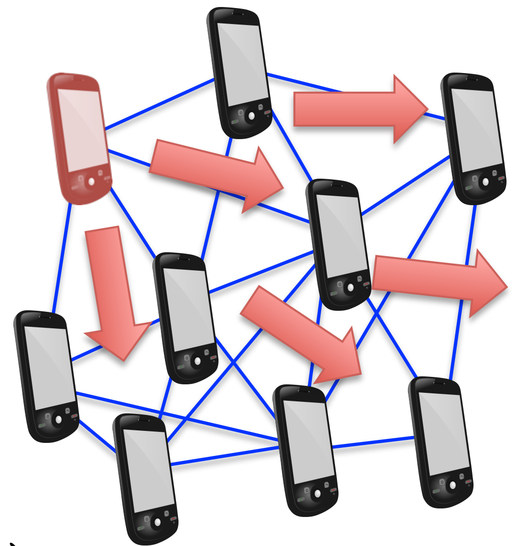
Information spreading



Field Calculus Implementation:

```
(def G (source initial metric accumulate)
 (2nd
  (rep distance-value
   (tuple infinity initial)
   (mux source (tuple 0 initial)
    (min-hood
      (tuple
       (+ (1st (nbr distance-value)) (metric))
       (accumulate (2nd (nbr distance-value)))))))))
```

Library Examples:

```
(def distance-to (source)
 (G source 0 nbr-range (fun (v) (+ v (nbr-range)))))
```

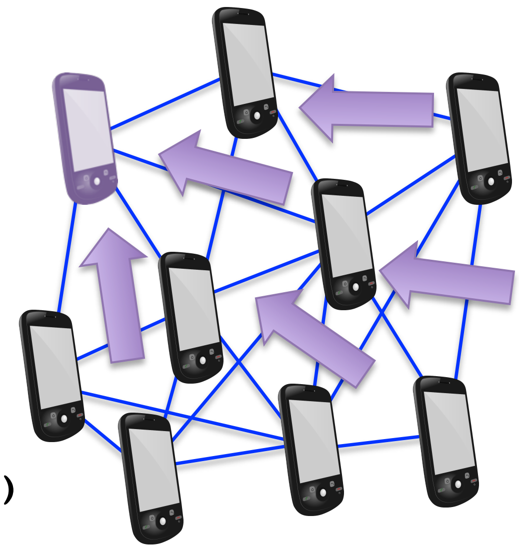
```
(def broadcast (source value)
 (G source value nbr-range identity))
```

Building Block: C

Information collection

Field Calculus Implementation:

```
(def C (potential accumulate local null)
 (rep v local
  (accumulate local
   (accumulate-hood accumulate
    (mux (= (nbr (find-parent potential)) (uid))
     (nbr v) null)))))
(def find-parent (potential)
 (mux (< (1st (min-hood (nbr potential))) potential)
   (2nd (min-hood (nbr (tuple potential (uid)))))
   NaN))
```



Library Examples:

```
(def summarize (sink accumulate local null)
 (broadcast sink
  (C (distance-to sink) accumulate local null)))

(def average (sink value)
  (/ (summarize sink + value 0)
     (summarize sink + 1 0)))
```

Building Block: T

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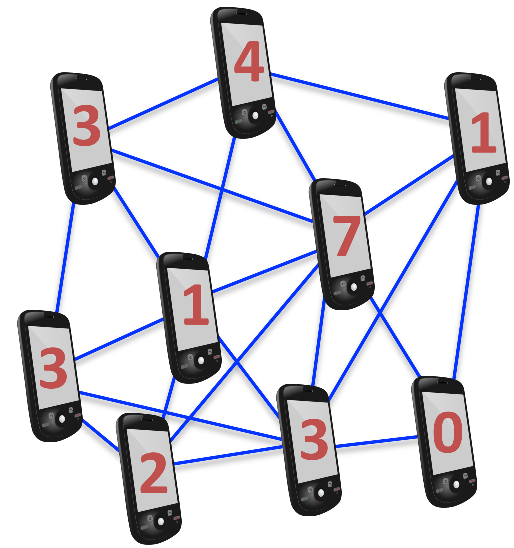
Time-summarization of information

Field Calculus Implementation:

```
(def T (initial decay)
 (rep v initial
  (min initial
   (max 0 (decay v)))))
```



Library Examples:

```
(def timer (length)
 (T length (fun (t) (- t (dt)))))
```

```
(def limited-memory (value timeout)
 (2nd (T (tuple timeout value)
      (fun (t) (tuple (- (1st t) (dt)) (2nd t))))))
```

Building Block: S

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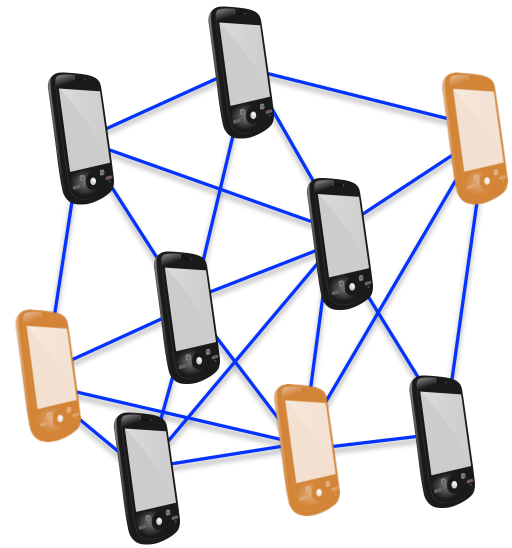
Choice of sparse subset



Field Calculus Implementation:

```
(def S (grain metric)
   (break-using-uids (random-uid) grain metric))

(def random-uid ()
  (rep v (tuple (rnd 0 1) (uid))
   (tuple (1st v) (uid))))

(def break-using-uids (uid grain metric)
 (= uid
   (rep lead uid
    (distance-competition
     (G (= uid lead) 0 metric
      (fun (v) (+ v (metric))))
     lead uid grain metric))))

(def distance-competition (d lead uid grain metric)
  (mux (> d grain) uid
   (mux (>= d (* 0.5 grain)) infinity
    (min-hood
     (mux (>= (+ (nbr d) (metric)) (* 0.5 grain))
      infinity
      (nbr lead))))))
```

Building Block: `if`

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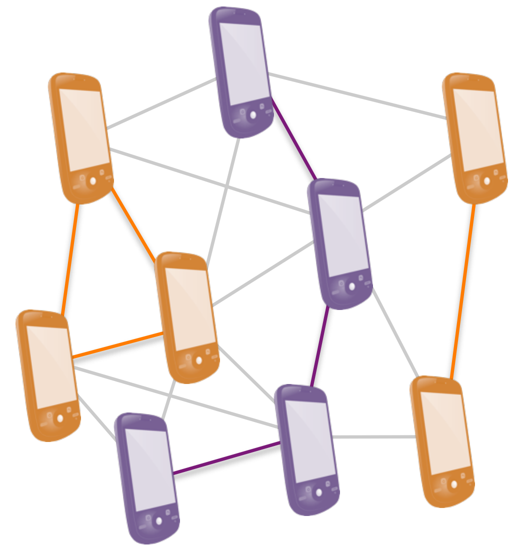
Restrict scope to subspaces

Field Calculus Implementation:

```
(if test
    true-expression
    false-expression)
```



Library Examples:

```
(def distance-avoiding-obstacles (source obstacles)
  (if obstacles
      infinity
      (distance-to source)))


(def recent-event (event timeout)
  (if event true (> (timer timeout) 0)))
```

Applying building blocks:

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Example API algorithms from building blocks:

distance-to (source)	max-likelihood (source p)
broadcast (source value)	path-forecast (source obstacle)
summarize (sink accumulate local null)	average (sink value)
integral (sink value)	region-max (sink value)
timer (length)	limited-memory (value timeout)
random-voronoi (grain metric)	group-size (region)
broadcast-region (region source value)	recent-event (event timeout)
distance-avoiding-obstacles (source obstacles)	

Since based on these 5 building blocks, all programs built this way are self-stabilizing!

Complex Example: Crowd Management

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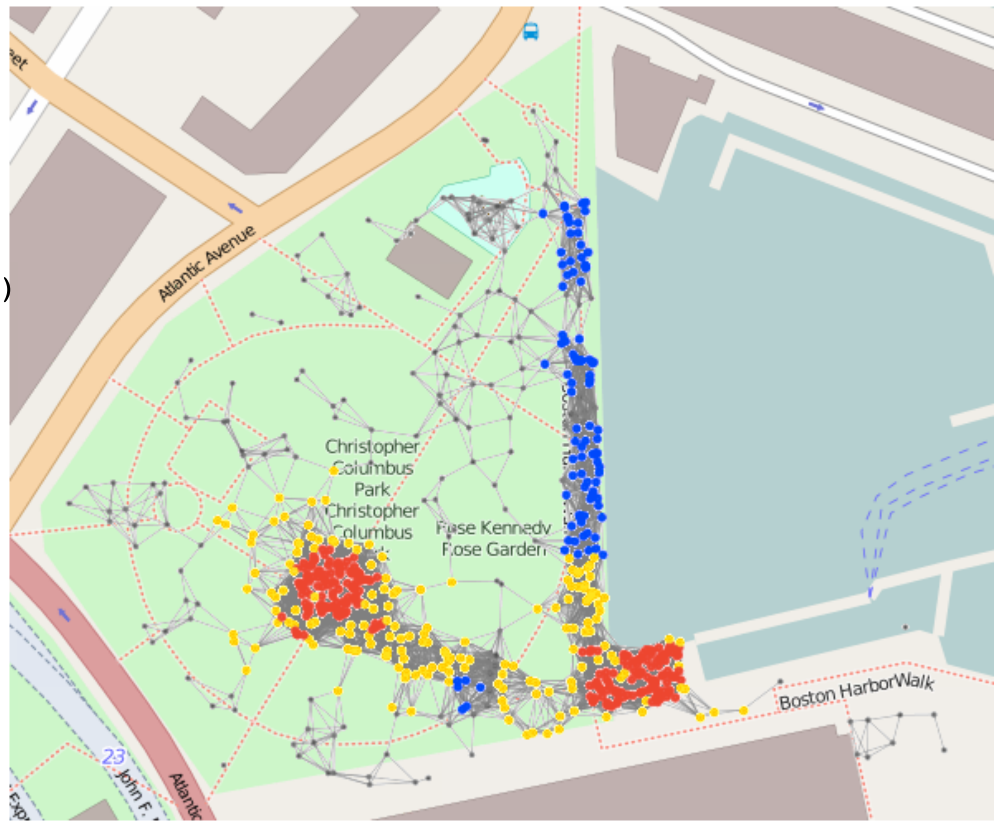
```
(def crowd-tracking (p)
 ;; Consider only Fruin LoS E or F within last minute
 (if (recently-true (> (density-est p) 1.08) 60)
    ;; Use S to break into "cells" and estimate danger of each
    (+ 1 (dangerous-density (S 30) p))
    0))

(def recently-true (state memory-time)
 ;; Make sure first state is false, not true...
 (rt-sub (not (T 1 1)) state memory-time))
(def rt-sub (started s m)
 (if state 1 (limited-memory s m)))

(def dangerous-density (partition p)
 ;; Only dangerous if above critical density threshold...
 (and
  (> (average partition (density-est p)) 2.17)
  ;; ... and also involving many people.
  (> (summarize partition + (/ 1 p) 0) 300)))
```

```
(def crowd-warning (p range)
  (> (distance-to (= (crowd-tracking p) 2))
   range)

(def safe-navigation (destination p)
  (distance-avoiding-obstacles
   destination (crowd-warning p)))
```



18 lines non-whitespace code

10 library calls (18 ops)

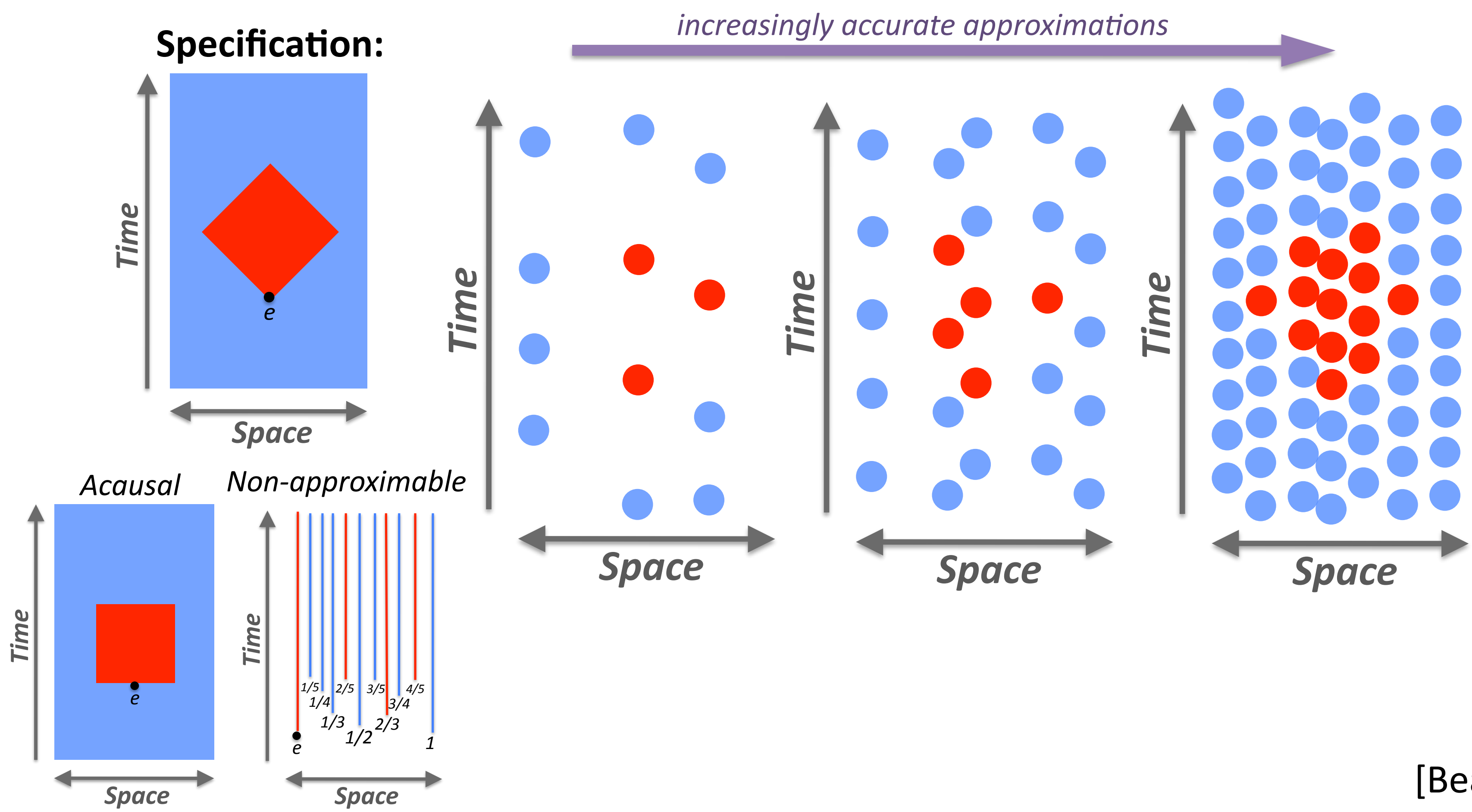
if: 3 G: 9 C: 3 T: 2 S: 1

Outline

- Building Blocks
- **Space-Time Universality**
- Eventual Consistency

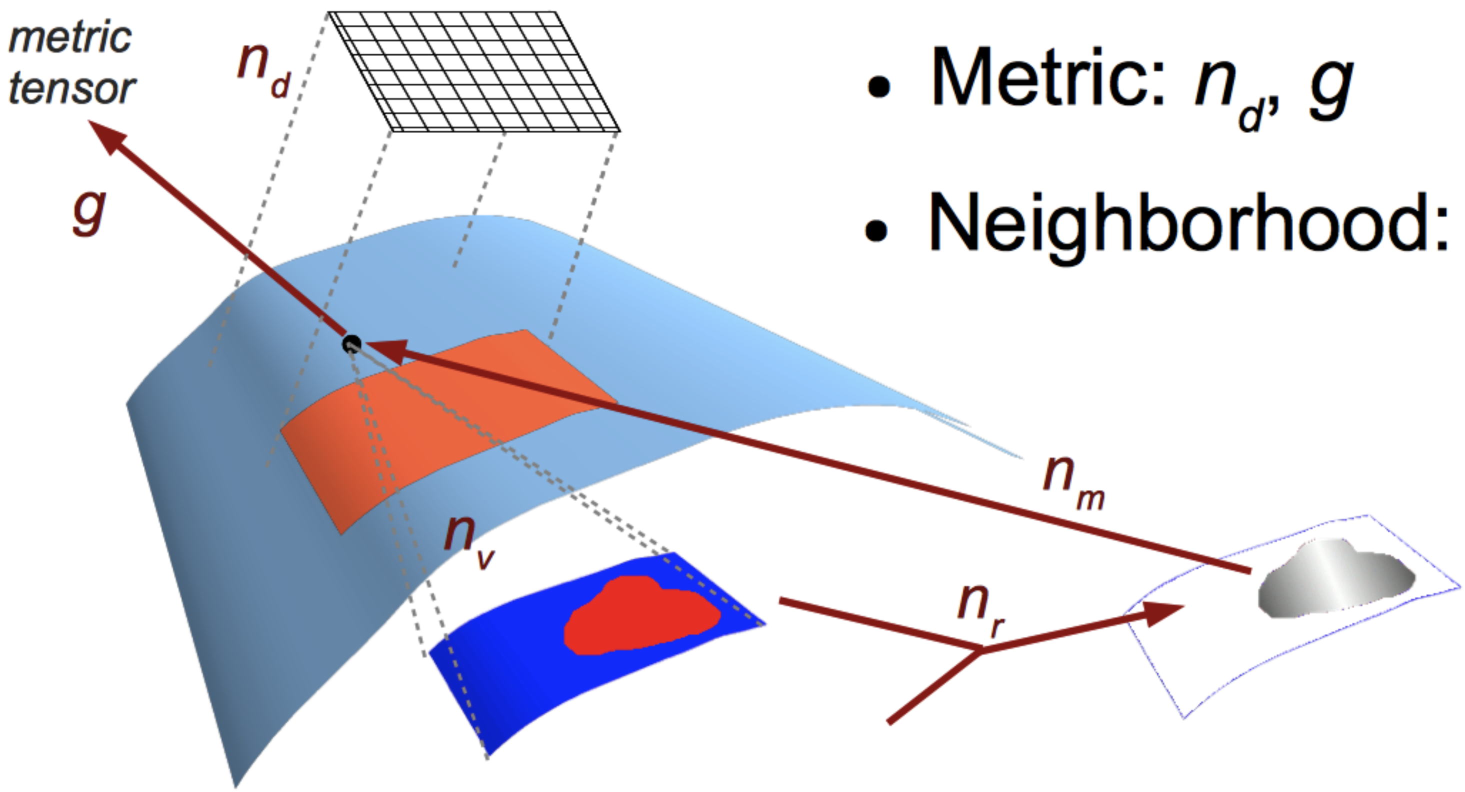
Space-Time Universality

Space-time Universal = arbitrarily good approximation of any causal, finitely-approximable computation



[Beal, '10]

Proposed Universal Operators



- Pointwise Turing-universal: P
  - Metric: n_d, g
  - Neighborhood: n_v, n_r, n_m

[Beal, '10]

Proof sketch

Field Calculus

2. Field calculus can approximate those Proto programs

1. Some Proto programs are finitely approximable

Proto

3. This subset is enough to get universality

Space-Time Universality

Field Calculus

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Variable   Literal Value   Local Ops

State

Communication

e	::= x | l | (b $\overline{\text{e}}$) | (f $\overline{\text{e}}$)	expression
	| (rep x w e) | (nbr e) | (if e e e)	special constructs
w	::= x | l	variable or value
F	::= (def f($\overline{\text{x}}$) e)	function
P	::= $\overline{\text{F}}$ e	program

Domain Change

Abstract & Call Functions

[Viroli et al., ’13]

Proto

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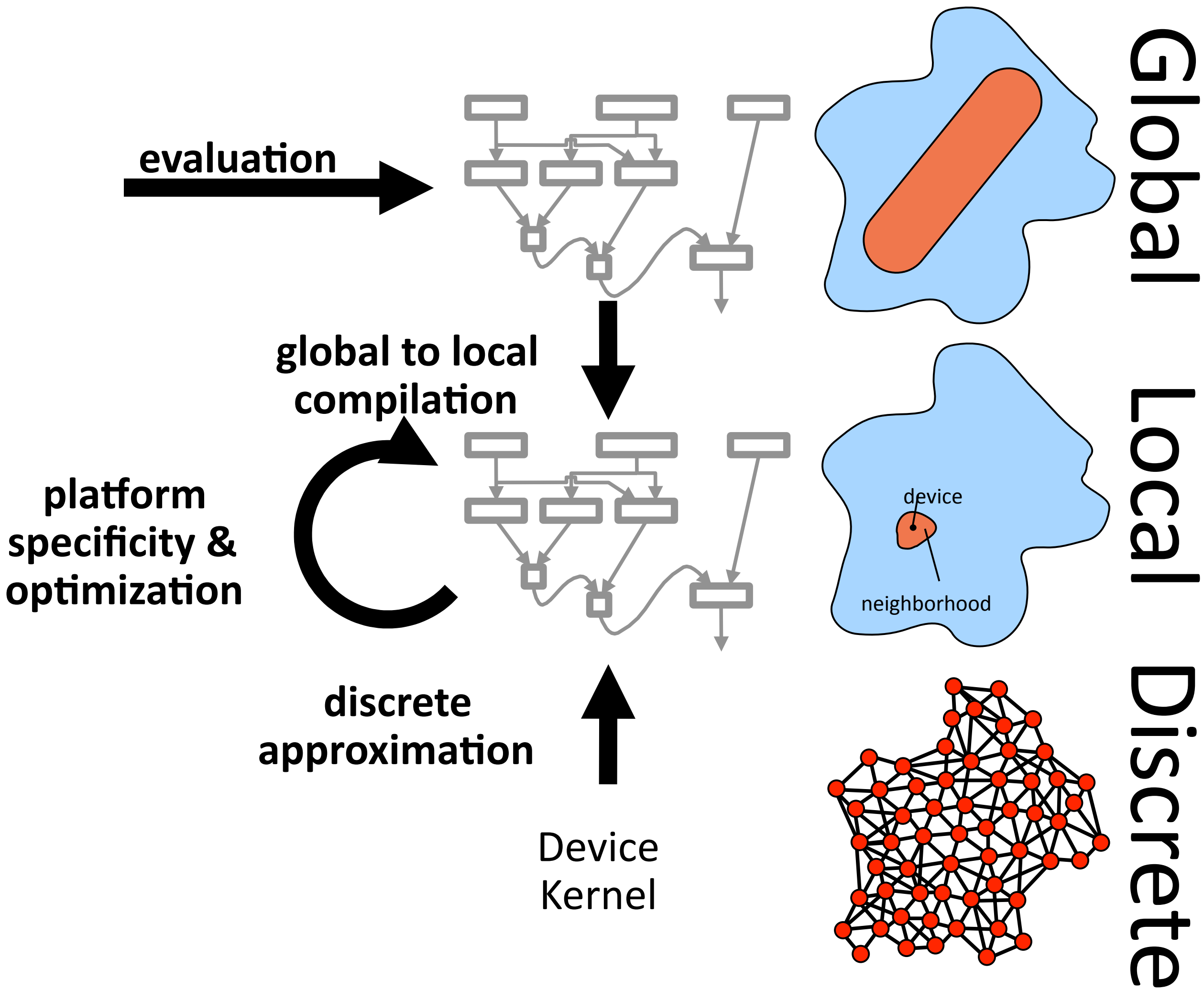
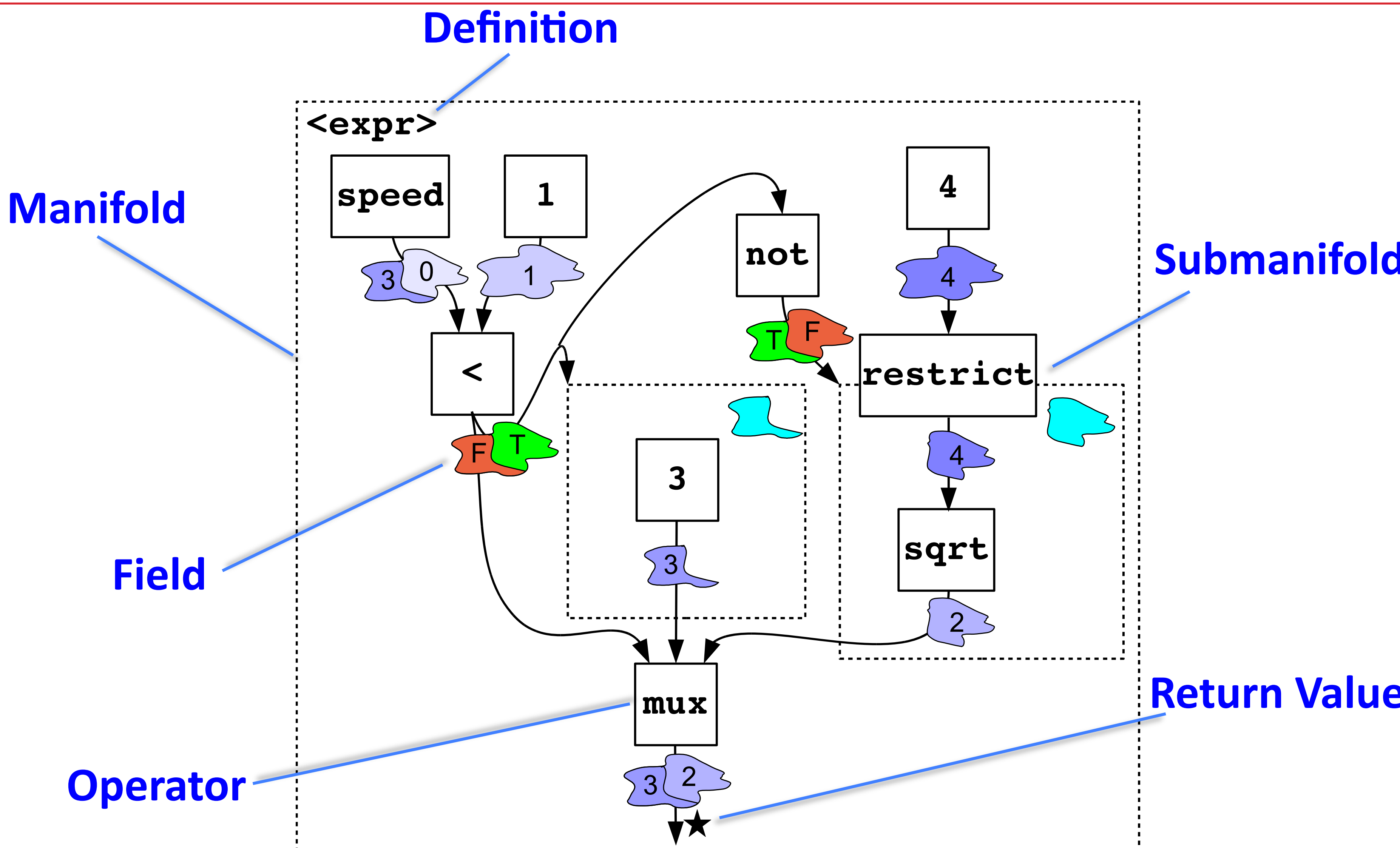
```
(def gradient (src) ...)
(def distance (src dst) ...)
(def dilate (src n)
  (<= (gradient src) n))
(def channel (src dst width)
  (let* ((d (distance src dst))
         (trail (<= (+ (gradient src)
                       (gradient dst))
                    d)))
    (dilate trail width)))
```



[Beal & Bachrach, '06]

Proto as Dataflow Graph



Well-defined program = no inconsistencies in domains, graph structure

Proof sketch

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Field Calculus

2. Field calculus can approximate those Proto programs

1. Some Proto programs are finitely approximable

Proto

3. This subset is enough to get universality

Space-Time Universality

Theorem 1: Proto approximability

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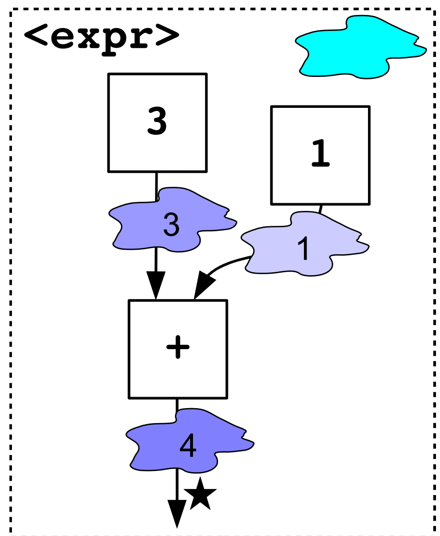
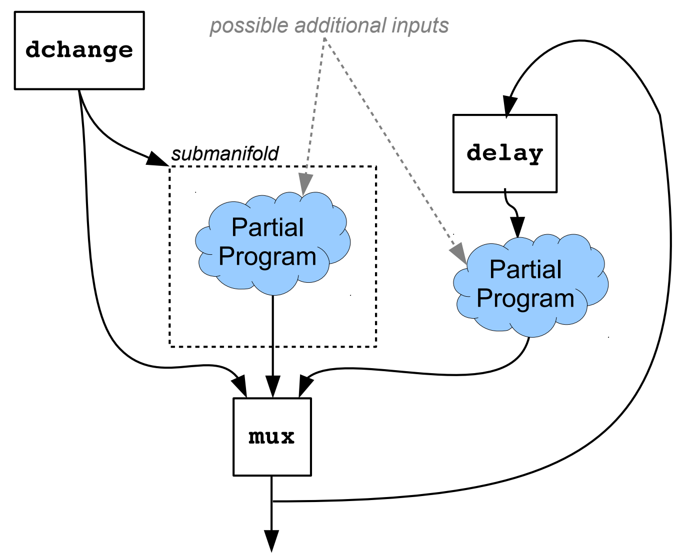
Any well-defined Proto program P = (M, F, O, R, D) composed only of finitely approximable operator instances is finitely approximable.

If inputs converge, then output converges





Intuition: feed-forward composition + special forms

Note: some surprising things (e.g., '=') aren't approximable!

Neighborhood Independence

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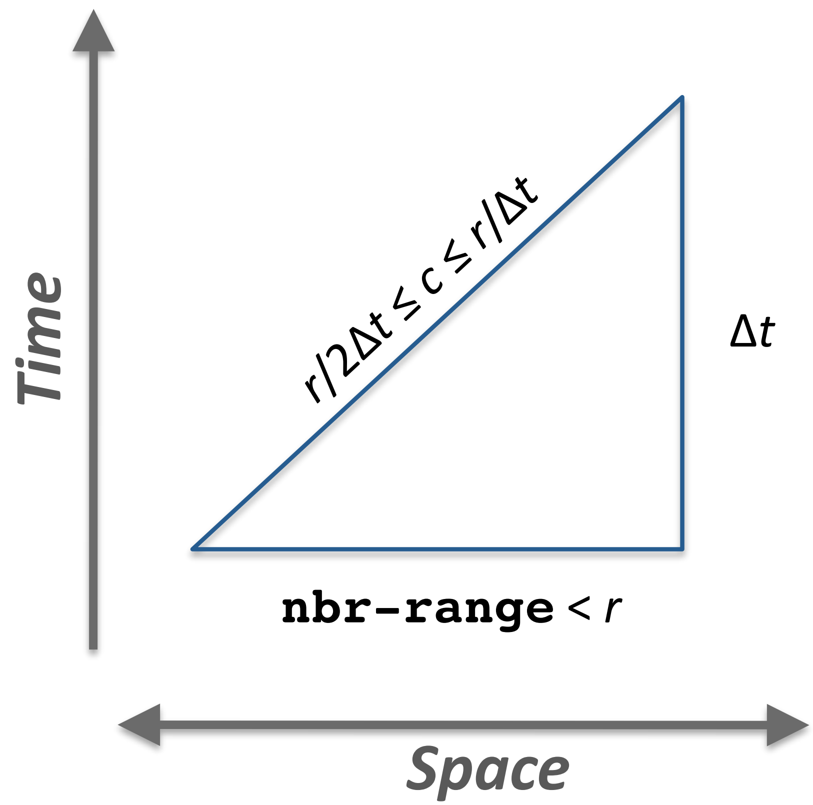
- For consistent communication speed, neighborhood must shrink with time-step.

A problem program:

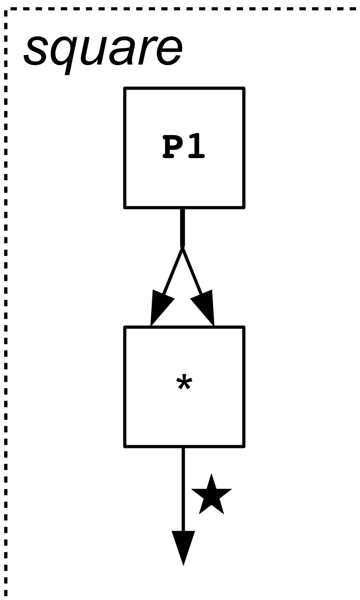
```
(def use-speed-and-radius (bool)
  (* (any-hood (nbr bool))
     (distance-to bool)))
```

- Neighborhood-independent programs aren't affected by this problem



Theorem 2: Field Calculus → Proto

Any well-defined neighborhood-independent Proto program P = (M, F, O, R, D) composed only of finitely approximable operator instances can be approximated using field calculus.




```
(def function_1 (p_1)
  (function_1_1 p_1))

(def function_1_1 (p_1)
  (function_1_2 p_1 (* p_1 p_1)))

(def function_1_2 (p_1 v_1) v_1)
```

Intuition: construct equivalent graph in field calculus

Theorem 3: Field Calculus Universality

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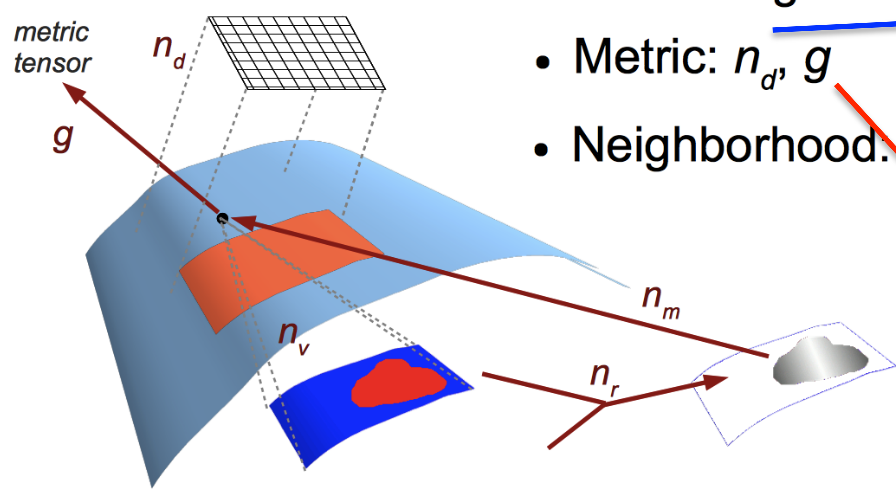
Field Calculus is Space-Time Universal

Corollary: Any well-defined finitely approximable Proto program can be approximated by field calculus.



- Pointwise Turing-universal: P
  - Metric: n_d, g
  - Neighborhood: n_v, n_r, n_m

Approximable subset of Proto covers these

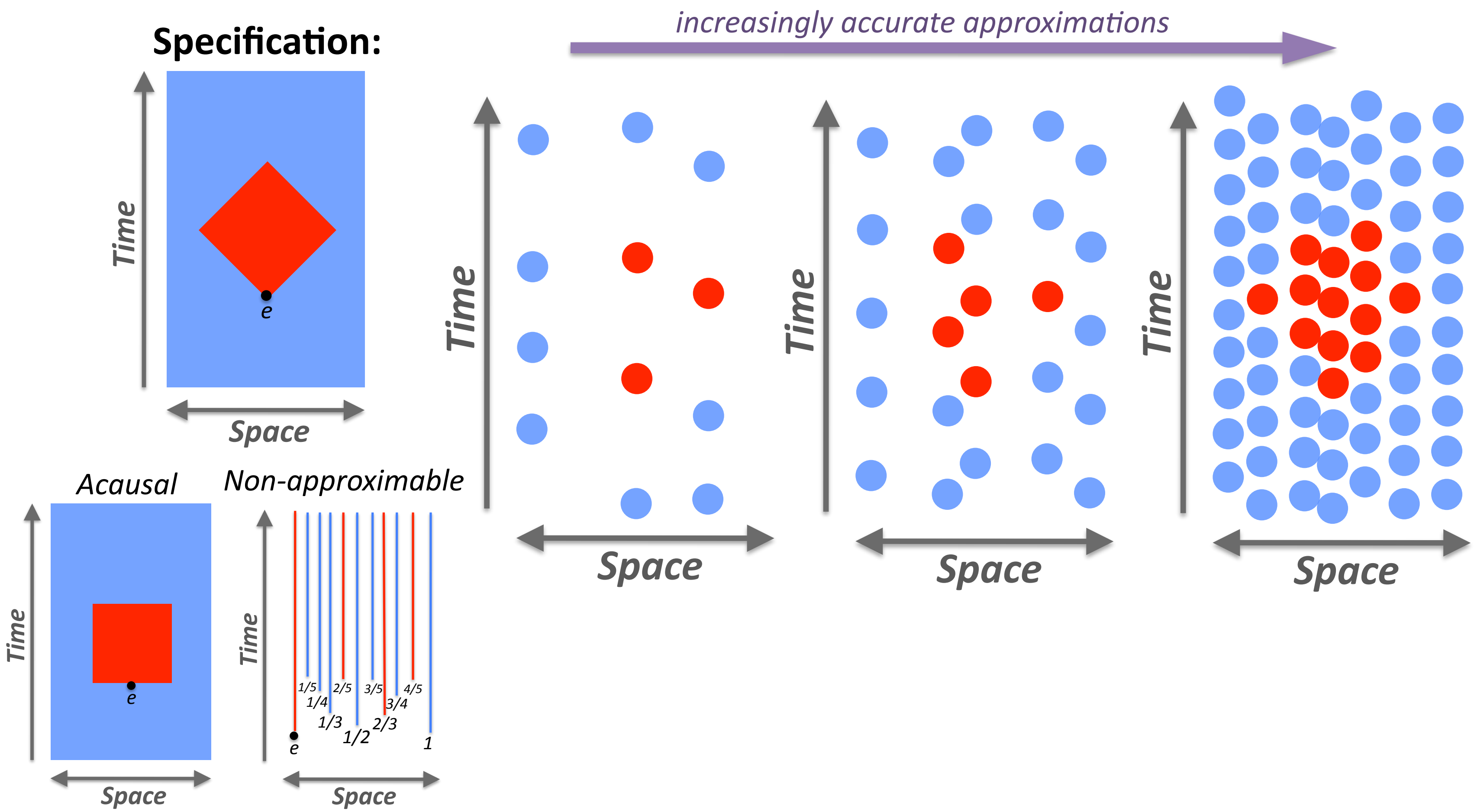
Metric tensor can be a built-in

Intuition: Proto is nearly space-time universal.

Outline

- Building Blocks
- Space-Time Universality
- **Eventual Consistency**

Causality & Finite Approximability



[Beal, ’10]

Eventual Consistency

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- **Consistent Program:** Let P be a space-time program, e be an evaluation environment, and e_i a countable sequence of ε-approximations that approximate field e. Program P is consistent if $P(e_i)$ approximates $P(e_i)$ for every e and e_i.

- **Eventually Consistent Program:** Consider a causal program P evaluated on environment e with domain M. Program P is eventually consistent if, for any environment e in which there is a spatial section S_M such that the values of e do not change at any device in the time-like future $T^+(S_M)$, there is always some spatial section S_M' such that P is consistent on the time-like future $T^+(S_M')$

Intuition: resilience against scale and discretization

[Beal et al, submitted]

What are the threats to consistency?

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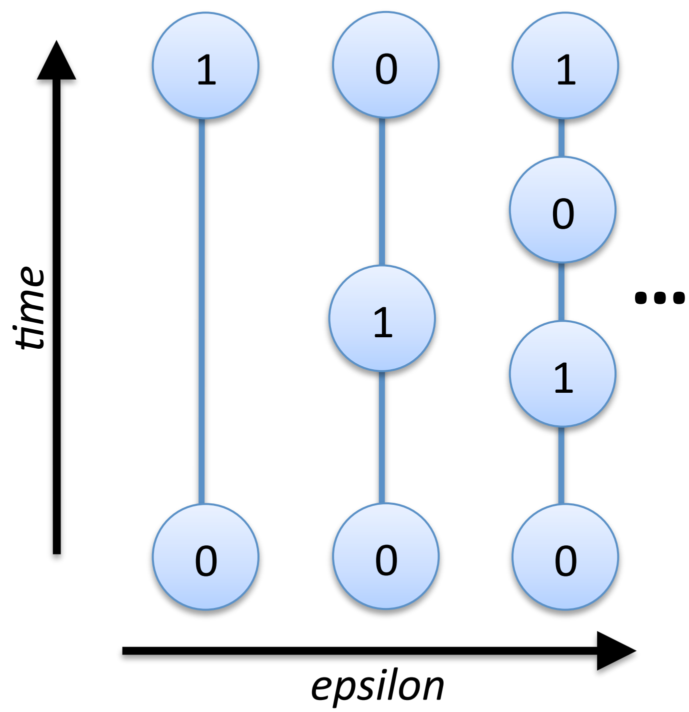
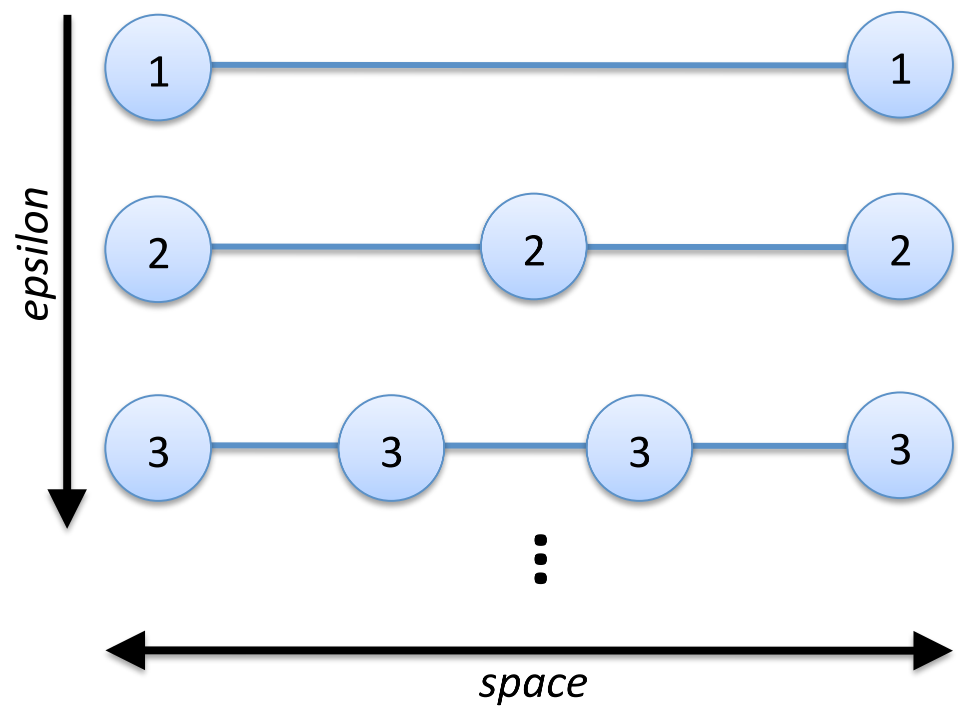
- Unbounded recursion
- Direct use of **rep**, **nbr** constructs

`(rep x 0 (- 1 x))`



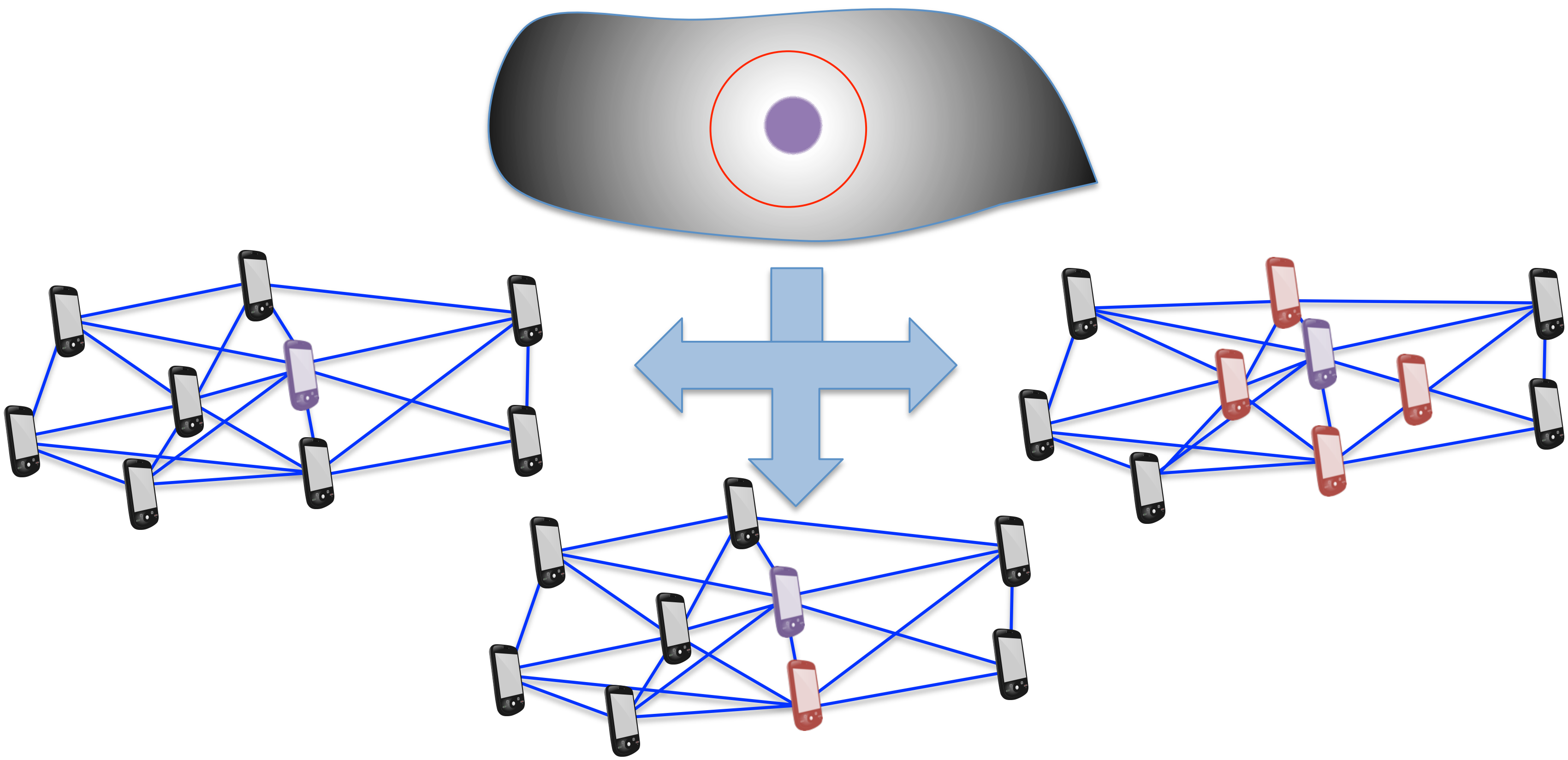
`(/ 1 (min-hood (nbr-range)))`



What are the threats to consistency?

- Fragile values (measure zero sets)

```
(def ring (src)
  (= (distance-to src) 15.0)))
```



GPI-calculus

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Special "Boundary" value

Only integers and reals

Restricted built-in ops

$$
\begin{array}{lll}
\mathtt{l} ::= \mathbb{B} \mid \mathbb{Z} \mid \mathbb{R} & & \text{;; Literals} \\
\mathtt{b} ::= \mathtt{m} \mid \mathtt{mux} \mid \mathtt{<} & & \text{;; local operators} \\
\mathtt{e} ::= \mathtt{x} \mid \mathtt{l} \mid (\mathtt{b}\ \overline{\mathtt{e}}) \mid (\mathtt{f}\ \overline{\mathtt{e}}) \mid (\mathtt{sense}\ \mathbb{Z}^{+}) & & \text{;; expression} \\
\quad \mid (\mathtt{if\ e\ e\ e}) \mid (\mathtt{GPI\ e\ e\ e\ e}) & & \text{;; special constructs} \\
\mathtt{F} ::= (\mathtt{def}\ \mathtt{f}(\overline{\mathtt{x}})\ \mathtt{e}) & & \text{;; function} \\
\mathtt{P} ::= \overline{\mathtt{F}}\ \mathtt{e} & & \text{;; program}
\end{array}
$$

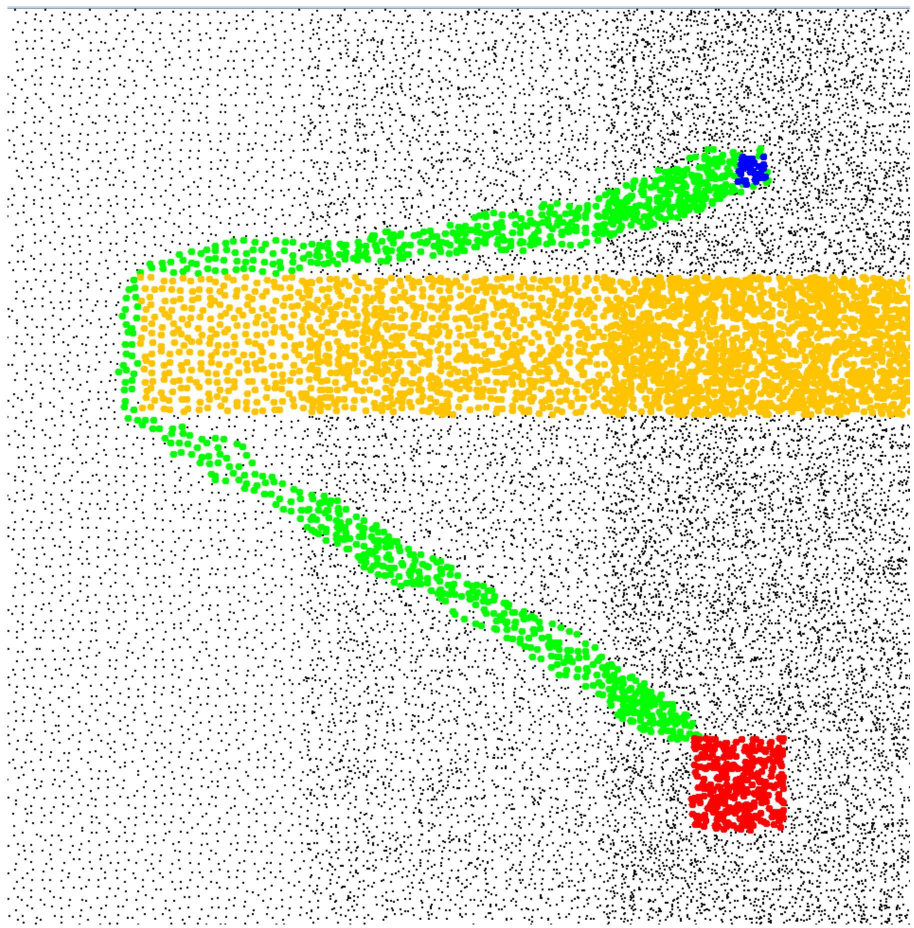
GPI replaces nbr, rep

Semantics prohibits recursion

- Restriction of field calculus to consistent subset
  - Real # comparison produces "Boundary" for equality
  - GPI = Gradient-Path-Integral

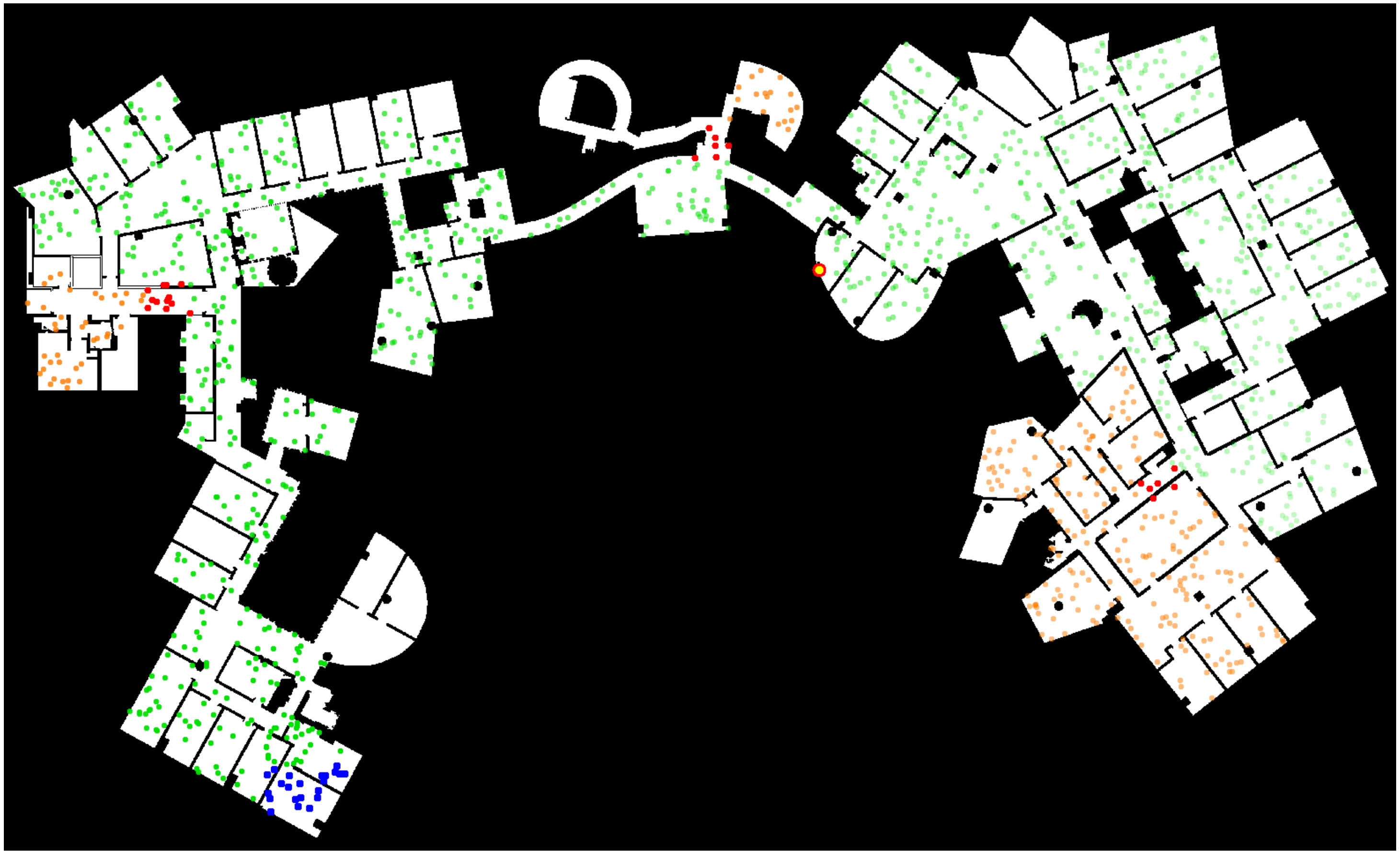
    G, except accumulation always integral, Boundary discarded

Example: Heterogeneous Density



GPI → path avoiding obstacles

Example: Building Evacuation



GPI → obstacle forecast: hazards (red) force orange areas to use alternate routes to blue safe zone

Summary

- Building-block algebras can be used to create aggregate APIs for implicitly adaptive programs
- 5-operator algebra gives implicit self-stabilization
- Field calculus is space-time universal
- GPI-calculus also resilient to device distribution

Summary and Open Problems

Jacob Beal, Mirko Viroli

Tutorial at 8th IEEE SASO
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Summary

- Major technological trends are all driving towards a world filled with distributed systems
- Aggregate programming aims at rapid and reliable engineering of complex distributed systems
- Field calculus provides a universal theoretical foundation for aggregate programming
- Resilient systems design can be simplified by an emerging self-organization toolbox
- Functional composition allows modulation, predictable convergence

Lots of open problems…

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- Many field calculus results may be more generally applicable to all distributed systems
- Extent of basis set needed for various domains
- What implicit propeties are needed/possible for building block algebras and libraries?
- Extension to mobile devices, ongoing changes
- Prediction of approximation quality – “Nyquist”
- Higher order functions, first class functions
- Open environments and security

But we’ve already enough for many applications…

Bibliography

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