Deriving Fairness From the Judgement of Others

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Abstract

Humans often deal fairly with one another. Although this has been widely recognized as a puzzle, no clear explanation has yet emerged. I propose a new model for deriving fairness, in which a player may may value the judgement of other players. Applied to the Ultimatum Game, this model can produce fair offers corresponding to those seen in experiments, as well as rejections in the presence of uncertainty. Furthermore, when players are allowed to choose how much value to assign to the other player's judgment, there is a unique equilibrium in which rational players choose a moderate valuation of the other's opinion.

1 Motivation

The existence of fairness is a persistent puzzle in game theory. Although humans often act in a way that maximizes objective payoffs, as predicted by ordinary game theory, there are many circumstances in which they behave "fairly" or "altruistically" in a manner that defies predictions.

A particularly dramatic area of divergence is fairness in bargaining games where little negotiation is possible, such as the Ultimatum Game. In the Ultimatum Game, two players divide a resource. The first player proposes a division, which the second player either accepts or rejects. If the second player accepts, the resource is divided as per the proposal; otherwise both players get nothing.

Conventional analysis predicts that the first player will offer a tiny fraction of the resource and the second player will always accept, because a a tiny fraction is still better than nothing. Experimental studies with the Ultimatum Game and other closely related bargaining games, however, discover that the first player usually offers a significant fraction of the resource (often precisely half!) and that the second player generally rejects small offers (See, for example, [21, 13, 2, 14, 20]). Clearly, utility maximization cannot explain fairness unless utility is modified to include something other than objective payoffs.

I propose a new model which predicts fairness phenomena on the basis of weaker assumptions than pre-existing models. In my model, each player is given the option of assigning value to the judgements of other players. Applying this model to the Ultimatum Game, I show that valuing other players' judgements of fairness leads to fair behavior, and that placing moderate value on the judgement of other players can be a dominant strategy.

2 Valuable Judgement Model

I propose a *Valuable Judgement Model* in which a player's utility function may mix objective payoff with the judgements of other players, consistent with Camerer and Thaler's argument for explaining fairness via learned manners.[5]

To define the model, let us begin with a standard game. There are n players in the game, each of which is assumed to be rational (i.e. acting to maximize utility). The game may be further parameterized by a type $t \in T$ which is observed by a player i as the signal τ_i, which may be distorted.

When the game is played, one action a is selected from the set of all possible actions A, according to the strategies of the players (the action taken by player i is a_i). The payoff function $P(A) \rightarrow \Re^n$ maps each action to a real-valued objective payoff for each player.

In the Valuable Judgement Model, calculation of utility depends on two additional assumptions: a common knowledge standard for judging relative fairness and honest signaling of judgement.

We will take the standard for relative fairness to be a function $F(\theta, p) \rightarrow \Re$ that, given a fair payoff θ and a realized payoff p, outputs a measurement of how "fair" the realized payoff is. Holding θ fixed, $F(\theta, p)$ is a concave function that is equal to 1 at $p = \theta$, denoting maximal fairness.

Each player i, then, has a set of judgement functions $J_{ij}(A_j, \theta_i) \rightarrow \Re$ derived from the relative fairness function which measure the relative fairness of actions taken by each other player j against player i's standard of fairness.

Note that common knowledge of fairness and judgement functions is a relatively weak assumption. In particular, it does not preclude players from holding different beliefs about what payoff is most fair. Rather, it merely guarantees that each player j can make predictions of the form, "If I take action a_j, and player i believes that θ_i is a fair payoff, then i will judge my action to have fairness f."

The assumption of honest signaling of judgement means that judgement is not a strategic behavior. At the end of the game, $J_{ij}(A_j, \theta_i)$ is conveyed faithfully to player j, who may freely use or discard the information. This is also not a strong assumption because $J_{ij}(A_j, \theta_i)$ conveys only an emotional response, which psychological research shows is rather difficult for humans to conceal (e.g. [7, 11]).

Given these two assumptions, we are ready to define valuable judgement utility as a linear mixture of judgement and objective payoffs,

$$u_i(A_i) = \sum_{j \neq i} \alpha_{ij} J_{ji}(A_i, \theta_j) + (1 - \sum_{j \neq i} \alpha_{ij}) P_i(A_i),$$

where the constants $\alpha_{ij} \in [0, 1]$ sum to a value between zero and one and indicate the relative subjective value player i places on each player j's judgement and the objective payoff.

The key question, then, is what values will α_{ij} assume when set by a rational player interested in maximizing objective payoff? At first blush, it might seem rational to ignore the judgement of others, since it would act as a brake to one's own exploitative behavior. In fact, however, placing some value on the judgement of others can serve as a commitment by which a player unilaterally protects itself from exploitation.

We will not investigate how either a standard for judgement or honest signals of judgement might arise. It is enough for our current purposes that, given the option of using them or ignoring them in a one-shot transaction, rational players will opt to use them.

2.1 Alternate Approaches

There are a number of proposals to modify utility to fit experimental data. One approach is to incorporate beliefs about the other player's intentions into the utility function, using the mechanisms of psychological games.[12] For example, Rabin proposes a "kindness function"[23] that can be incorporated to yield fairness equilibria. Similarly, Falk et al. develop a qualitative model where fairness depends on the set of available alternatives.[9, 10] Beliefs about intentions, however, are properties which cannot be directly or reliable observed. These models also do not explain why an individual player should assign value to beliefs.

Another common approach is direct mixture of behaviors into the utility function. Andreoni and Miller add altruism to utility by adding utility from the objective payoff of other players proportional to an altruism factor.[1] Levine combines Andreoni and Miller with Rabin to produce a model where each player has an altruism/spitefulness factor and values payoffs granted by other players based on estimates of their spitefulness (i.e. money is "blessed" by an altruistic giver and "tainted" by a spiteful giver).[17] Bolton and Ockenfels, on the other hand, add a relative term to the utility function, parameterized by thresholds above and below which "fair" behavior dominates over utility maximizing behavior.[4] These proposals, however, provide little explanatory power since they directly incorporate phenomena in need of explanation, such as altruism and spite.

Evolutionary theories have attempted to fill the explanatory gap. One approach is to find fair evolutionarily stable strategies such as Bolton's FAIRMAN strategy[3] and the Ellingsen's obstinate bargainers.[8] Levine and Pesendorfer provide an ingenious model of how cooperation might evolve through imitation.[18] Although useful in demonstrating that fair behavior can survive, evolutionary approaches have little direct explanatory power: as in the case of biological evolution, a "just-so" story showing of how something might arise does not not mean it arose that way. Recent results bear this out: Dekel, et al. show that any efficient strategies is stable[6] and Heifetz et al. show that almost any distortion of preferences is viable under appropriate circumstances, invalidating evolutionary arguments in all but special cases.[15]

A more subtle approach was recently advanced by Lopomo and Ok,[19] based

Variable	Meaning
n	number of players
$t \in T$	type parameter of a game
τ_i	type parameter as observed by player i
$a \in A$	player actions executing a game
$P(a)$	payoff to players for action a
θ	Payoffs for a fair division
$F(\theta, p)$	fairness of payoff p compared to fairest payoff θ
$J_{ij}(A_j, \theta_i)$	player i's judgement of fairness for player j's action
α_{ij}	Valuation of player j's judgement by player i
$u_i(a_i)$	player i's expected utility for taking action a_i
d	Proposed division in the ultimatum game, in $[0, 1]$
d^*	Ambivalence point of Bob's dominant strategy, in $[0, 1]$
$\hat{d}$	Utility maximizing offer in Alice's dominant strategy, in $[0, 1]$
$Eqv(\alpha_{BA})$	Value of α_{AB} at the boundary of dominant valuation
ϵ	standard deviation of observation noise
γ_{BA}	noisy observation of α_{BA} by Alice
$\hat{\alpha}_{BA}$	Alice's estimate of Bob's valuation

Table 1: Summary of Variables

on negatively interdependent preferences previously investigated by Kockesen, Ok, and Sethi.[22, 16] Unlike previous approaches, Lopomo and Ok require only that it is possible that highly obstinate types exist, from which they derive a "fear of rejection" which causes strictly rational players to propose fair divisions. This approach does not, however, explain altruistic behavior where there is no threat of rejection, nor does it explain why a strictly rational player might reject an offer.

3 Ultimatum Game

We will now apply an instantiation of the Valuable Judgment Model to the Ultimatum Game to show how valuing the judgements of others can result in fair behavior.

In the Ultimatum Game, there are two players, who we will designate Alice and Bob, who are dividing 1 unit of objective payoff. First Alice proposes a division of the payoff $d \in [0, 1]$ in which Alice gets d units of payoff and Bob gets $(1 - d)$ units of payoff. Bob their either accepts or rejects the proposed division. If Bob accepts the proposal, then Alice gets d units and Bob gets $(1-d)$ units of payoff. If Bob rejects the proposal, then both Alice and Bob get zero units of payoff.

For this analysis, the type t of a game will be common knowledge[1] of the

[1] This implies that $\tau_A = \tau_B = t$.

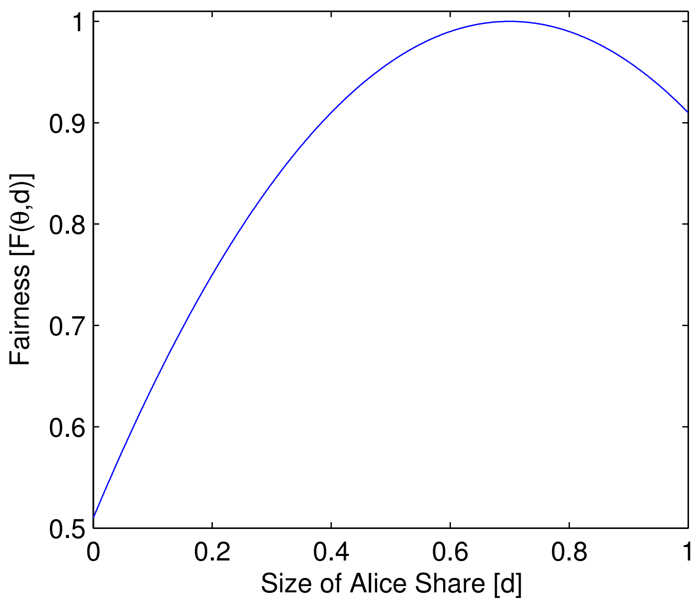

Figure 1: Example fairness function, evaluated for a fair distribution $\theta = 0.7$.

fair division $\theta \in [0, 1]$,[2] the value α_{AB} that Alice assigns Bob's judgement and the value α_{BA} that Bob assigns Alice's judgement.

Let us explore the behavior of the game using simple definitions for the fairness and judgement functions. First, let us take fairness to be a simple quadratic, satisfying the concavity requirement,

$$F(\theta, p) = 1 - (\theta - p)^2.$$

Bob judges Alice on the fairness of her offer,

$$J_{BA}(d, \theta) = F(\theta, d).$$

Alice, on the other hand, judges Bob based on the idea that rejection is always fair when offered nothing and acceptance is most fair when offered a fair split,

$$J_{AB}(reject, \theta) = F(1, d),$$

$$J_{AB}(accept, \theta) = F(\theta, d)$$

Note that this particular model also judges Bob poorly when he accepts a split too unbalanced in his favor, but this will not affect our analysis.

[2] Alice gets θ, Bob gets $1 - \theta$

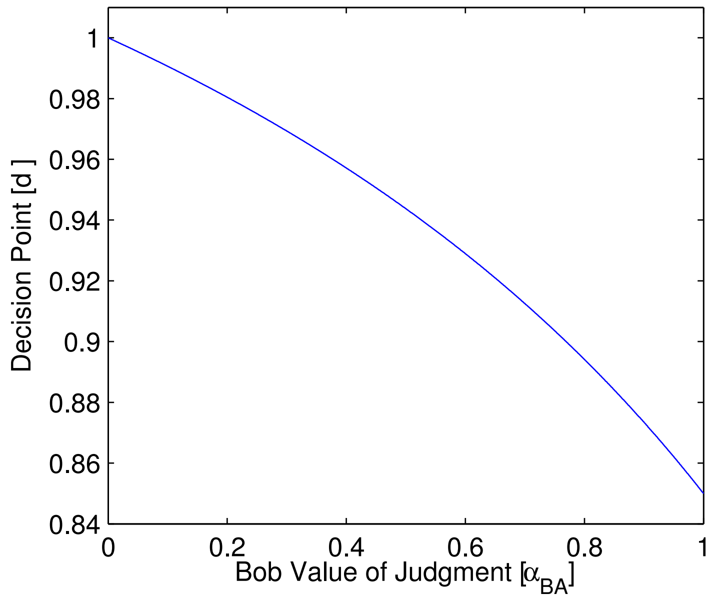

Figure 2: The more that Bob values Alice's judgement, the more proposals he is willing to reject (Figure calculated for $\theta = 0.7$).

3.1 Intuition

Before proceeding with the full analysis, let us work out some of the intuitions of how Alice and Bob behave in this model. If both players do not care about judgement at all ($\alpha_{AB} = \alpha_{BA} = 0$) then it reduces to the standard ultimatum game, where Alice keeps everything and Bob accepts. If, on the other hand, both players care solely about judgement ($\alpha_{AB} = \alpha_{BA} = 1$) then Alice will always offer the fair division θ and Bob will always accept. Since we are mixing the two utility functions linearly, we should expect the equilibrium for intermediate values of α to transition smoothly from fair division to Alice keeping everything, while Bob always accepts.

3.2 Analysis

We will solve the game using backward induction. Given an offer of d, there is a decision point d^* such that Bob's dominant strategy is to reject offers where $d > d^*$ and accept offers where $d < d^*$. We can find d^* by solving for the point when accept and reject produce equal utility:

$$J_{AB}(reject, \theta)\alpha_{BA} = J_{AB}(accept, \theta)\alpha_{BA} + (1 - d^*)(1 - \alpha_{BA}).$$

We can reduce this by plugging in function definitions,

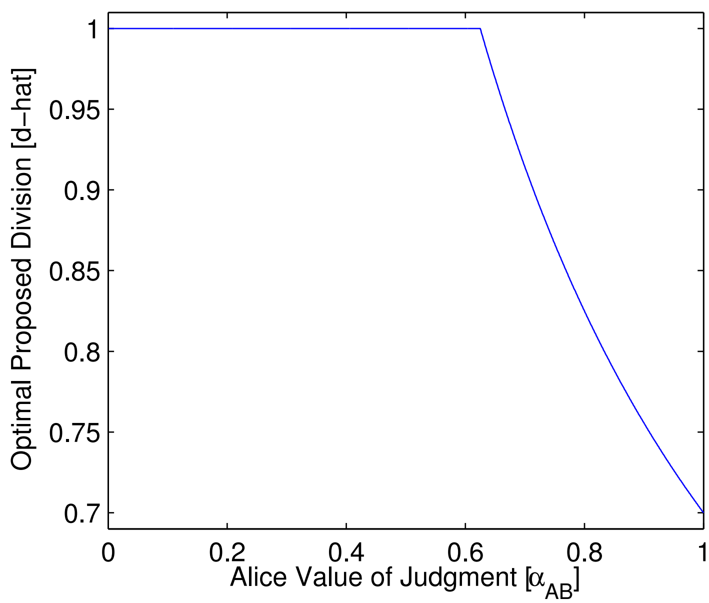

Figure 3: When Alice values Bob's judgement highly, she will propose more fair divisions (Figure calculated for $\theta = 0.7$). Alice's proposals are also constrained by Bob's valuation of her judgement, which limits what he is willing to accept.

$$F(1, d^*)\alpha_{BA} = F(\theta, d^*)\alpha_{BA} + (1 - d^*)(1 - \alpha_{BA})$$

$$(1 - (1 - d^*)^2)\alpha_{BA} = (1 - (\theta - d^*)^2)\alpha_{BA} + (1 - d^*)(1 - \alpha_{BA})$$

which ultimately reduces to

$$d^* = \frac{1 - \theta^2 \alpha_{BA}}{1 - (2\theta - 1)\alpha_{BA}}$$

Note that $d^* = 1$ when $\alpha_{BA} = 0$, that $d^* = \frac{1+\theta}{2}$ when $\alpha_{BA} = 1$, and that it scales smoothly with α_{BA}.

Alice can then uniquely choose $\hat{d}$ to maximize utility,

$$u_A = \begin{cases} J_{BA}(\hat{d}, \theta)\alpha_{AB} + \hat{d}(1 - \alpha_{AB}) & \text{if Bob accepts} \\ J_{BA}(\hat{d}, \theta)\alpha_{AB} & \text{if Bob rejects} \end{cases}$$

First, note that it is never better for Alice to have her proposal rejected than accepted, since a proposal of $d = \theta$ dominates all rejected offers. Thus Alice's proposals are bounded above by d^*. We can then solve for $\hat{d}$ below this level by expanding the utility,

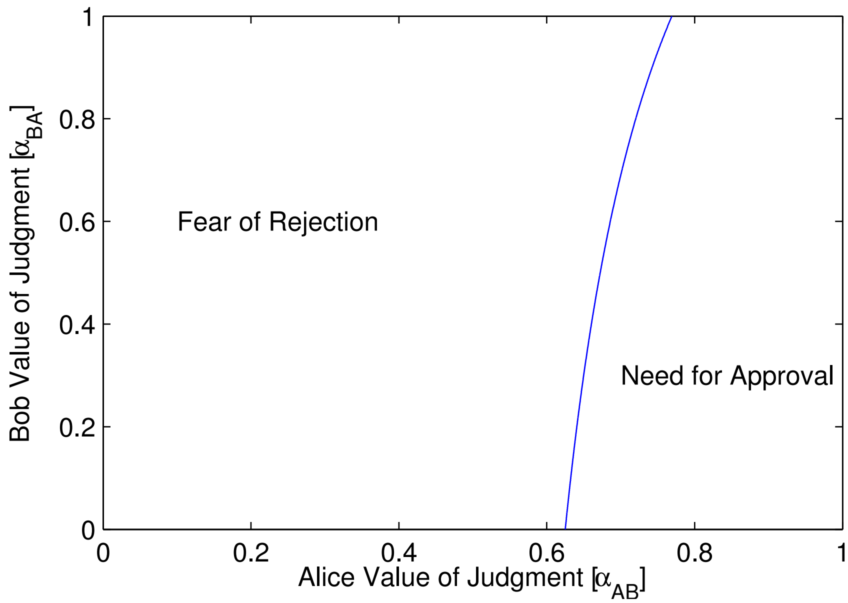

Figure 4: When Alice values Bob's judgement highly, "need for approval" determines the proposed division; when Alice doesn't value Bob's judgement, "fear of rejection" dominates. (Figure generated with $\theta = 0.7$.)

$$u_A(\hat{d}) = F(\theta, \hat{d})\alpha_{AB} + \hat{d}(1 - \alpha_{AB})$$

$$u_A(\hat{d}) = (1 - (\theta - \hat{d})^2)\alpha_{AB} + \hat{d}(1 - \alpha_{AB})$$

which we can maximize by solving for

$$\frac{du_A}{d\hat{d}}(-\alpha_{AB}\hat{d}^2 + (1 + (2\theta - 1)\alpha_{AB})\hat{d} + (1 - \theta^2)\alpha_{AB}) = 0$$

$$0 = -2\alpha_{AB}\hat{d} + (1 + (2\theta - 1)\alpha_{AB})$$

$$\hat{d} = \frac{1 + (2\theta - 1)\alpha_{AB}}{2\alpha_{AB}}$$

Note that $\hat{d} = \theta$ when $\alpha_{AB} = 1$, that it is unbounded when $\alpha_{AB} = 0$, and that it scales smoothly until from θ to 1 as α_{AB} decreases (once it reaches 1, no further increase is possible).

Thus, this game has a unique equilibrium in which Alice proposes $\hat{d} = min(\frac{1+(2\theta-1)\alpha_{AB}}{2\alpha_{AB}}, d^*)$ and Bob always accepts. There are some properties in the game which are worth noting. The obvious and unsurprising one is that

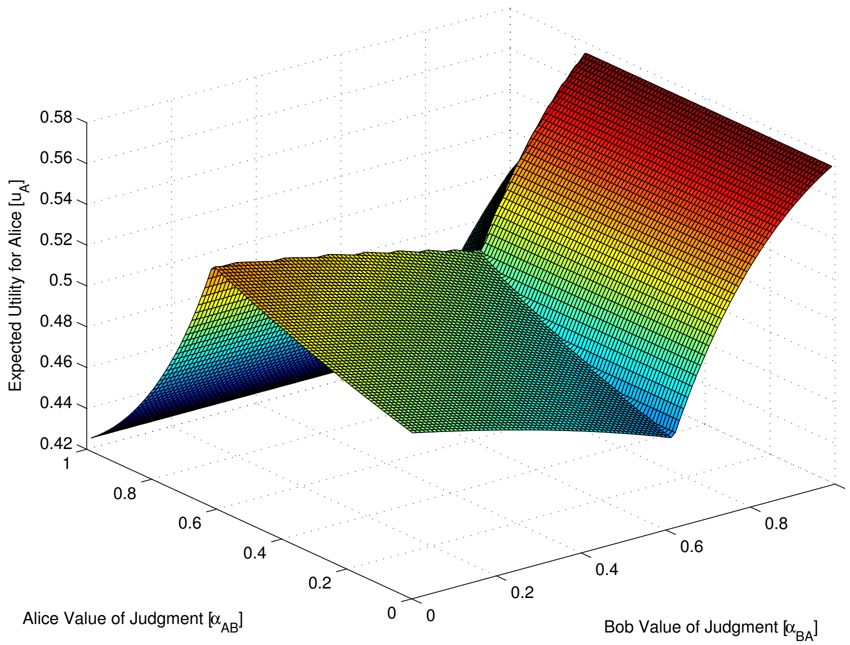

Figure 5: Expected payoff for Alice in the coin-flip Ultimatum Game, plotted against judgement valuation for Alice (α_{AB}) and Bob (α_{BA}) with $\theta = 0.7$. High valuation by Bob and moderate valuation by Alice produces the highest objective payoff.

when Alice cares more about Bob's judgement, she offers Bob a larger slice of the pie. The second is a counterintuitive result which is nonetheless quite familiar in human experience: the more that Bob cares about pie, the less pie he receives. In effect, Bob's desire for pie gives Alice bargaining power.

If we solve for when $d^* = \hat{d}$, we can determine which player's values dominate in determining the outcome of the game. Bob's value of judgement dominates when

$$\alpha_{AB} \leq Eqv(\alpha_{BA}) = \frac{1 + (1 - 2\theta)\alpha_{BA}}{(1 - 4\theta + 2\theta^2)\alpha_{BA} + 3 - 2\theta}.$$

This is not generally invertible, as the range covers only a portion of valid values for α_{AB}. For low values of α_{AB}, "fear of rejection" dominates and Alice offers the least that Bob will accept. For high values of α_{AB}, Alice's "need for approval" dominates instead, and Alice offers whatever is necessary to satisfy that need. In the intermediate range, either behavior may dominate, depending on their relative values.

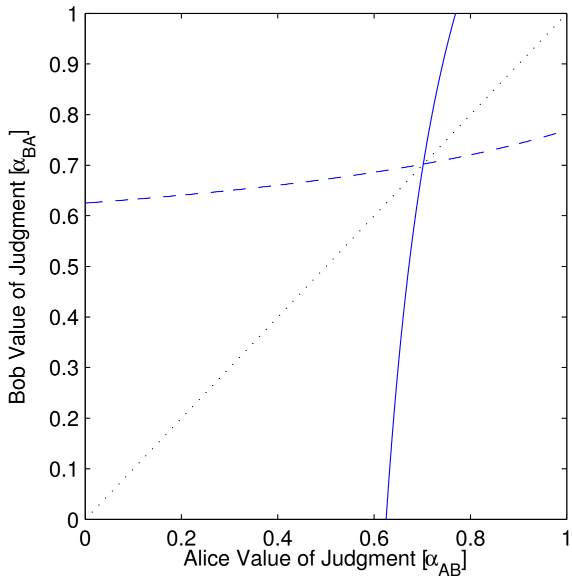

Figure 6: For a fair split $\theta = 0.7$, the solid line shows the dominance boundary between α_{AB} and α_{BA} when Alice is the proposer and the dashed line shows the same when Bob is the proposer. The intersection of these curves, at approximately $\alpha_{AB} = \alpha_{BA} = 0.7$, is the unique rationalizable valuation of judgement in the coin-flip Ultimatum Game.

4 Choosing Valuation of Judgement

We have seen that when a player values the judgement of the other player, offers become more fair. We have not yet shown why objectively rational players should base their decisions on the judgement of other players.

We have already seen part of the reason. When Bob values Alice's judgement, it effectively raises his reservation value, allowing him to reject low offers. When Alice knows α_{BA} at the start of the game, Bob has effectively committed to a strategy of rejecting offers less than d^*. This turns the question on its head, since it is now unclear why Bob should value the objective payoff at all!

To see how "reasonable" valuations of judgement might arise, let us consider a slight elaboration of the Ultimatum Game. Instead of including valuation of judgement in the type of the game, Alice and Bob will first independently choose values for α_{AB} and α_{BA} respectively, maximizing objective payoff. Then a coin is flipped to determine whether Alice or Bob will make the proposal and the game is played as before.

The coin-flip makes this similar an evolutionary game of the Ultimatum Game, but Alice and Bob's strategies are not constrained, and we need to consider only a single round of the game. Intuitively, the symmetry added by the coin-flip gives each player incentive to choose a moderate valuation of judgement.

We can find the unique rationalizable equilibrium by iterated domination (Figure 6). Starting the process with Alice, we can eliminate all values of $\alpha_{AB} \leq Eqv(0)$, because they will lead to less gain if Bob is the proposer regardless of Bob's choice of α_{BA}, and all $\alpha_{AB} \geq Eqv(1)$ because they will lead to less gain if Alice is the proposer regardless of Bob's choice of α_{BA}.

Given this narrowing, Bob can similarly eliminate values of α_{BA} which correspond with values of α_{AB} which Alice will not choose, and so on until a unique rationalizable solution is found where $\alpha_{BA} = \alpha_{AB} = Eqv(\alpha_{AB})$.

Thus, if players choose valuation without knowing which will propose, they will choose identical moderate valuations of each other's judgement. In effect, valuing one another's judgement allows players to guarantee they receive some objective payoff at the price of yielding some opportunity for exploitation.

5 Uncertainty Leads to Rejections

Previously, we have assumed that θ, $F(\theta, d)$, $J_{ij}(A_j, \theta_i)$, and α_{ij} were all common knowledge and found a unique equilibrium where Bob never rejects an offer. In experiments, however, humans do reject some offers.

We can generate the same effect in this model by adding some noise to Alice's observation of how much Bob values her judgement. Instead of being common knowledge, α_{BA} is known to Bob and observed as $\gamma_{BA} = \alpha_{BA} + N(0, \epsilon^2)$ by Alice, where $N(0, \epsilon^2)$ is a normally distributed random variable with mean zero and variance ϵ^2.

We will also assume that it is common knowledge that the game is in the "fear of rejection" range, so that Bob's valuation of judgement dominates. This will simplify our analysis without affecting the phenomena we are interested in.

Analysis of the Ultimatum Game is as before, except that Alice no longer knows α_{BA} and therefor does not know d^* either. Instead of choosing $\hat{d}$, Alice will choose an estimate $\hat{\alpha}_{BA}$ of α_{BA}, from which $\hat{d}$ can be derived. Rewriting u_A around the estimate $\hat{\alpha}_{BA}$ yields the equation

$$u_A = J_{BA}(\hat{d}(\hat{\alpha}_{BA}), \theta)\alpha_{AB} + E(\hat{\alpha}_{BA} \geq \alpha_{BA}|\gamma_{BA})\hat{d}(\hat{\alpha}_{BA})(1 - \alpha_{AB}).$$

Due to our assumption that it is a "fear of rejection" game, we can safely assume that this is maximized when $\hat{\alpha}_{BA}$ is within ϵ of the observation γ_{BA}, such that the expectation can be written as

$$E(\hat{\alpha}_{BA} \geq \alpha_{BA}|\gamma_{BA}) = \frac{1}{2}(1 + erf(\frac{\hat{\alpha}_{BA} - \gamma_{BA}}{\epsilon\sqrt{2}})).$$

We can derive $\hat{d}$ from $\hat{\alpha}_{BA}$ just like d^* is derived from α_{BA},

$$\hat{d}(\hat{\alpha}_{BA}) = \frac{1 - \theta^2\hat{\alpha}_{BA}}{1 - (2\theta - 1)\hat{\alpha}_{BA}}$$

The equation as a whole can then be expanded to

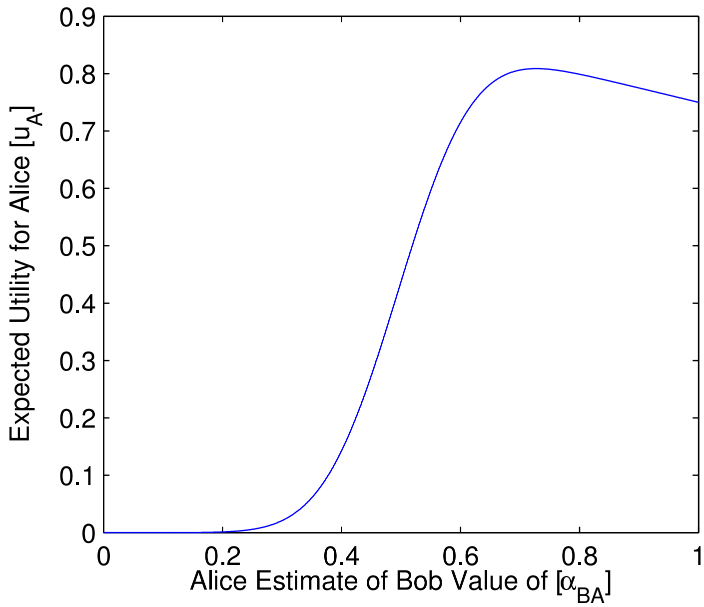

Figure 7: Expected utility for Alice u_A from acting on an estimate of $al\hat{p}ha_{BA}$, given a fair division $\theta = 0.5$, observation $\gamma_{BA} = 0.5$ and Gaussian noise with standard deviation $\epsilon = 0.1$. Alice obtains maximum expected utility from a conservative estimate with a small but significant chance of rejection.

$$u_A = (1-(\theta-\tfrac{1-\theta^2\hat{\alpha}_{BA}}{1-(2\theta-1)\hat{\alpha}_{BA}})^2)\alpha_{AB}+\tfrac{1}{2}(1+erf(\tfrac{\hat{\alpha}_{BA}-\gamma_{BA}}{\epsilon\sqrt{2}}))\tfrac{1-\theta^2\hat{\alpha}_{BA}}{1-(2\theta-1)\hat{\alpha}_{BA}}(1-\alpha_{AB}).$$

To see how this behaves, assume for a moment that $\alpha_{AB} = 0$, such that only the expectation portion of the equation is used.

$$(u_A|\alpha_{AB} = 0) = \frac{1}{2}(1 + erf(\frac{\hat{\alpha}_{BA} - \gamma_{BA}}{\epsilon\sqrt{2}}))\frac{1 - \theta^2\hat{\alpha}_{BA}}{1 - (2\theta - 1)\hat{\alpha}_{BA}}$$

Although a closed-form solution of this equation is not very revealing, graphing it illustrates its behavior adequately (Figure 7). As can be seen, there is a clear maximum a couple standard deviations above the received estimate — although Alice plays it safe, there is still a small chance that she will have mis-estimated Bob's actual bounds and be rejected.

6 Discussion

We have seen that the Valuable Judgement Model allows fair behavior to be derived from the option to value another player's judgement. The Valuable Judgement Model relies on two relatively weak assumptions: a shared notion of fairness and honest judgement signals.

Given the option of incorporating judgements into a subjective utility function, rational players will use it to raise their effective reservation values, maximizing their objective payoffs. When players do not know in advance which player will be proposing, the unique rationalizable choice is a moderate valuation of judgement.

There are many open avenues for exploration using this model. For example, I have not made strong claims to duplicate experimental results because there is too little constraint on the fairness and judgement functions at present, a gap which might be filled in with experimental evidence. It is also unclear at present what happens when common knowledge assumptions are weakened, introducing more uncertainty. Finally, the Valuable Judgement Model needs to be tested against other anomalies, such as the Dictator Game.

Most intriguing of all, emotional relations between humans are often persistent, so the values and judgements of one encounter might spill over to another one. Analysis of how heterogeneous sequences of games are affected by persistent values and judgements might shed much light on the dynamics of human relationships.

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