**Homework #1**

**Due date: 03 September 2025**

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**Instructions:**

Download and run each of the Shiny app R scripts provided (SIR, SEIR, SI, SEIR with vital dynamics, SEIR with waning immunity).

Answer each question with both:

1) A screenshot of the plot(s) showing your simulation results, with parameter settings clearly visible.

2) A brief explanation (2–4 sentences) interpreting the results.

Note: Parameters are defined as follows: β = transmission rate, γ = recovery rate, σ = progression rate from exposed to infectious, μ = per capita birth/death rate, ω = rate of immunity loss.

Conventions: Time unit: days. All rates are per day unless explicitly stated. If a rate is given in years⁻¹, convert it to day⁻¹ before entering it in the app. For example, μ = 1/70 years⁻¹ ≈ 0.000039 day⁻¹; μ = 1/50 years⁻¹ ≈ 0.000055 day⁻¹. Also, ω = 1/365 days⁻¹ ≈ 0.00274 day⁻¹; ω = 1/182.5 days⁻¹ ≈ 0.00548 day⁻¹.

Initial conditions: Unless otherwise stated, use R(0) = 0 and S(0) = N − E(0) − I(0) − R(0).

Screenshots: Ensure your screenshot shows both the plot and the parameter/initial-condition panel.

**PART 1 – SIR Model**

Parameter Sensitivity:

Run the SIR model with N = 1,000,000, S(0) = 999,000, I(0) = 1,000, R(0) = 0, β = 0.3, γ = 0.1. Then double β to 0.6 and compare the epidemic curves. Describe how doubling β changes both (a) the time to peak and (b) the peak magnitude of I(t).

Flattening the Curve:

Keep all parameters as before, but reduce β by 50% (to 0.15). How does the final size of the epidemic (total number infected) change compared to the baseline? Explain why reducing β alters the epidemic size.

**PART 2 – SEIR Model**

Latent Period Effects:

Using the SEIR model, set σ = 0.5 day⁻¹ (1/σ = 2 days), γ = 0.1 day⁻¹ (1/γ = 10 days), β = 0.3. Compare this to a scenario with σ = 0.2 day⁻¹ (1/σ = 5 days). How does a longer latent period affect the epidemic curve of I(t)? Provide reasoning in terms of the average latent period (1/σ).

Peak Timing:

With N = 1,000,000, set E(0) = 1,000, I(0) = 10, R(0) = 0, and S(0) = N − E(0) − I(0) − R(0). Keep γ = 0.1, β = 0.3, and vary σ between 0.2 and 0.5 day⁻¹. Which value leads to the earliest peak in I(t)? Explain your answer in terms of how σ influences the timing of infectiousness.

**PART 3 – SI Model**

No Recovery Dynamics:

Set N = 1,000,000. Run the SI model with β = 0.3 and compare I(t) for I(0) = 1,000 vs. I(0) = 10,000 (with S(0) = N − I(0)). How does changing the initial infected count affect the time to reach 90% infection?

Long-Term Behavior:

Explain why in the SI model I(t) never decreases. Why is the SI model unrealistic for most real-world infectious diseases?

**PART 4 – SEIR with Vital Dynamics**

Endemic Equilibrium:

Using the SEIR vital dynamics model with μ converted to day⁻¹ (μ = 1/70 years⁻¹ ≈ 0.000039 day⁻¹), β = 0.3, σ = 0.25, γ = 0.1, simulate for 73,000 days (≈ 200 years). Use initial conditions such as E(0) = 0, I(0) = 10, R(0) = 0, S(0) = N − 10. Does the system reach an endemic steady state? Estimate the steady-state proportion infected.

Effect of Birth/Death Rates:

Increase μ to 1/50 years⁻¹ (≈ 0.000055 day⁻¹) while keeping other parameters fixed. How does the steady-state level of I(t) change and why (intuition about demographic turnover and replenishment of susceptibles)?

**PART 5 – SEIR with Waning Immunity**

Waning Immunity Duration:

With ω = 1/365 days⁻¹ (≈ 0.00274 day⁻¹), simulate for 1,825 days (≈ 5 years) with constant β = 0.3. Describe the long-term behavior of I(t) and explain why recurrent epidemics appear (immunity loss replenishes susceptibles).

Shorter Immunity Period:

Reduce the immunity duration to 6 months (ω = 1/182.5 days⁻¹ ≈ 0.00548 day⁻¹) while keeping all other parameters fixed as before. Compare the epidemic peak heights and the intervals between peaks to the 1-year immunity scenario.