324 formula sheet

Maxwell's Equations: Everything we need when working in vacuum.

	vacuum	matter	or
1. Gauss's Law	$\nabla\cdot\vec{E}=\frac{1}{\epsilon_0}\rho$	$\nabla\cdot\vec{E}=\frac{1}{\epsilon}(\rho_f-\nabla\cdot\vec{P})$	$\nabla\cdot\vec{D}=\rho_f$ if $\vec{D}\equiv\epsilon_0\vec{E}+\vec{P}$
2. No mag monopoles	$\nabla\cdot\vec{B}=0$	1: $\nabla\cdot\vec{E}=\frac{\rho_{free}}{\epsilon}$	
3. Faraday's Law	$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	4: $\nabla\times\vec{B}=\mu\vec{J}_{free}+\mu\sigma\vec{E}+\mu\epsilon\frac{\partial\vec{E}}{\partial t}$	
4. Ampere's Law	$\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$	$\nabla\times\vec{B}=\mu_0(\vec{J}_f+\nabla\times\vec{M}+\frac{\partial\vec{P}}{\partial t})+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$	$\nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$ if $\vec{H}=\frac{1}{\mu_0}\vec{B}-\vec{M}$
0. Continuity	$\nabla\cdot\vec{J}=-\frac{\partial\rho}{\partial t}=-\nabla\cdot(\epsilon_0\frac{\partial\vec{E}}{\partial t})$	Vac, no charges or currents: $\nabla^2\vec{E}$	$=\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$ and $\nabla^2\vec{B}=\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$

Boundary Conditions: $\epsilon_1E_1^\perp=\epsilon_2E_2^\perp$ $\quad B_1^\perp=B_2^\perp$, $E_1^{\parallel}=E_2^{\parallel}$, $\frac{1}{\mu_1}B_1^{\parallel}=\frac{1}{\mu_2}B_2^{\parallel}$ $\quad \beta=\frac{\mu_1v_1}{\mu_2v_2}, E_{0R}=\frac{1-\beta}{1+\beta}E_{0I}$ and $E_{0T}=\frac{2}{1+\beta}E_{0I}$

Wave Equation: $\frac{\partial^2f}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2f}{\partial t^2}$ $\quad$ Standing Wave to sum of waves: $A\sin kz\cos kvt=\frac{A}{2}(\sin kz+kvt+\sin kz-kvt)$

Standing Wave: $kL=n\pi$ for $n=1,2,3...$ And frequency: $\nu=\frac{1}{T}=\frac{kv}{2\pi}=\frac{nv}{nL}$

String boundary conditions: Position cannot change across boundary, and neither can first derivative.

2 connected strings, $\omega_1=\omega_2$ must be, $v=\frac{\omega}{k}$, then $r=\frac{A_R}{A_I}=\frac{k_1-k_2}{k_1+k_2}=\frac{v_2-v_1}{v_1+v_2}$ and $t=\frac{A_T}{A_I}=\frac{2k_1}{k_1+k_2}=\frac{2v_2}{v_1+v_2}$

Monochromatic plane waves: $\vec{E}=E_0\hat{n}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ and $\vec{B}=\frac{E_0}{c}\hat{k}\times\hat{n}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ and $\vec{S}=c\epsilon_0E_0^2\cos^2(kz-\omega t+\delta)\hat{z}=cu\hat{z}$

Poynting Vector: $\vec{S}\equiv\frac{1}{\mu_0}(\vec{E}\times\vec{B})$ Poynting Theorem: $\frac{dW}{dt}=-\frac{d}{dt}\int_\nu u\,d\tau-\oint_S\vec{S}\cdot d\vec{a}$ Momentum: $\vec{g}=\frac{1}{c^2}\vec{S}$

Averages: $\langle u\rangle=\frac{1}{2}\epsilon_0E_0^2$, $\langle\vec{S}\rangle=\frac{1}{2}c\epsilon_0E_0^2\hat{z}$, $\langle\vec{g}\rangle=\frac{1}{2c}\epsilon_0E_0^2\hat{z}$, Intensity $I\equiv\langle S\rangle=\frac{1}{2}c\epsilon_0E_0^2$

Intensity stuff: $I_I=\frac{\epsilon_1v_1}{2}|E_{0I}|^2, I_R=\frac{\epsilon_1v_1}{2}|E_{0R}|^2, I_T=\frac{\epsilon_2v_2}{2}|E_{0T}|^2$, then $R=\frac{I_R}{I_I}=(\frac{1-\beta}{1+\beta})^2, T=\frac{I_T}{I_I}=\frac{\epsilon_2v_2}{\epsilon_1v_1}(\frac{2}{1+\beta})^2$ and $R+T=1$

Related: $,c=\frac{1}{\sqrt{\mu_0\epsilon_0}}, v1=\frac{1}{\sqrt{\mu_1\epsilon_1}}$ Brewster's Angle when $\frac{E_{0R}}{E_{0I}}=0$ And for the following, $n=\frac{n_2}{n_1}$ or $\frac{n_T}{n_I}$

Fresnel Coeff, S-polarized, $\hat{n}=\hat{y}$: $r_\perp=\frac{E_{0R}}{E_{0I}}=\frac{n_I\cos\theta_I-n_T\cos\theta_T}{n_I\cos\theta_I+n_T\cos\theta_T}=\frac{\cos\theta_I-\sqrt{n^2-sin\theta_I{}^2}}{\cos\theta_I+\sqrt{n^2-sin\theta_I{}^2}}$, $t_\perp=\frac{E_{0T}}{E_{0I}}=\frac{2\cos\theta_I}{\cos\theta_I+\sqrt{n^2-sin\theta_I{}^2}}=\frac{2n_I\cos\theta_I}{n_I\cos\theta_I+n_T\cos\theta_T}$

P-polarized, $\hat{n}\in\hat{x}+\hat{z}: r_{\parallel}=\frac{E_{0R}}{E_{0I}}=\frac{n_I\cos\theta_T-n_T\cos\theta_I}{n_I\cos\theta_T+n_T\cos\theta_I}=\frac{\sqrt{n^2-sin\theta_I{}^2}-n^2\cos\theta_I}{\sqrt{n^2-sin\theta_I{}^2}+n^2\cos\theta_I}$, $t_{\parallel}=\frac{E_{0T}}{E_{0I}}=\frac{2n\cos\theta_I}{n^2\cos\theta_I+\sqrt{n^2-sin\theta_I{}^2}}=\frac{2n_I\cos\theta_I}{n_I\cos\theta_T+n_T\cos\theta_I}$

Conductors: Conductivity σ. $e^{i(\beta z-\omega t)}, \beta=k_r+ik_i=\sqrt{\mu\epsilon\omega^2+i\mu\sigma\epsilon}=\omega\sqrt{\frac{\epsilon\mu}{2}}\left(\sqrt{1+(\frac{\sigma}{\epsilon\mu})^2}+1\right)^{\frac{1}{2}}+i\omega\sqrt{\frac{\epsilon\mu}{2}}\left(\sqrt{1+(\frac{\sigma}{\epsilon\mu})^2}-1\right)^{\frac{1}{2}}$

Skin Depth $d=\frac{1}{k_i}$ Boundary Conditions: $\epsilon_1E_1^\perp-\epsilon_2E_2^\perp=\rho_s$, $B_1^\perp-B_2^\perp=0$, $E_1^{\parallel}-E_2^{\parallel}=0$, $\frac{1}{\mu_1}B_1^{\parallel}-\frac{1}{\mu_2}B_2^{\parallel}=\vec{J}_s\times\hat{n}$

coeffecients don't change: $\frac{E_0R}{E_0I}=\frac{1-\beta}{1+\beta}, \frac{E_0t}{E_0I}=\frac{2}{1+\beta}$ Difference is β is complex, $\beta=\frac{\mu_1v_1}{\mu_2\omega}k_2$