ENEE 324 HW #4
Jacob Besteman-Street
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1. Suppose that a random variable (RV) Y has the following cumulative distribution function (CDF):

$$F_Y(y) = \begin{cases} 0 & ify < 1 \\ \frac{1}{8}(y-1)^3 & if 1 \leq y < 3 \\ 1 & ify \geq 3 \end{cases}$$

(a) Find and sketch the probability density function (PDF) of RV Y.
The PDF is the derivative of the CDF.

$$PDF = f_Y(y) = \begin{cases} 0 & ify < 1 \\ \frac{3}{8}(y-1)^2 & if 1 \leq y < 3 \\ 0 & ify \geq 3 \end{cases}$$

(b) Compute $P[0.5 < Y \leq 2.5]$

$$P[0.5 < Y \leq 2.5] = \int_{0.5}^{2.5} f_Y(y)dy = \int_{1}^{2.5} 38(y-1)^2 dy = \frac{1}{8}(y-1)^3 \text{ at } y = 2.5 = 0.421875$$

(c) Compute $E[Y]$ and $Var(Y)$.

$$E[Y] = \int_{-\infty}^{infty} y \cdot f_Y(y)dy = \int_{1}^{3} \frac{3y}{8}(y-1)^2 dy = \int_{1}^{3} \frac{3}{8}(y^3 - 2y^2 + y)dy = [\frac{3}{32}y^4 - \frac{1}{4}y^3 + \frac{3}{16}y^2]_1^3$$

$$= [\frac{243}{32} - \frac{27}{4} + \frac{27}{16}] - [\frac{3}{32} - \frac{1}{4} + \frac{3}{16}] = [\frac{243}{32} - \frac{216}{32} + \frac{54}{32}] - [\frac{3}{32} - \frac{8}{32} + \frac{6}{32}] = \frac{81}{32} - \frac{1}{32} = \frac{80}{32} = 2.5 = E[Y]$$

To find $Var(Y)$, need $E[Y^2]$.

$$E[Y^2] = \int_{-\infty}^{infty} y^2 \cdot f_Y(y)dy = \int_{1}^{3} \frac{3y^2}{8}(y-1)^2 dy = \int_{1}^{3} \frac{3}{8}(y^4 - 2y^3 + y^2)dy = [\frac{3}{40}y^5 - \frac{3}{16}y^4 + \frac{1}{8}y^3]_1^3$$

$$= [\frac{729}{40} - \frac{243}{16} + \frac{27}{8}] - [\frac{3}{40} - \frac{3}{16} + \frac{1}{8}] = [\frac{1458}{80} - \frac{1215}{80} + \frac{270}{80}] - [\frac{6}{80} - \frac{15}{80} + \frac{10}{80}] = \frac{513}{80} - \frac{1}{80} = \frac{512}{80} = 6.4 = E[Y^2]$$

$$Var[Y] = E[Y^2] - E[Y]^2 = 6.4 - 6.25 = 0.15$$