

1. Suppose that X is a discrete random variable (rv) with range $S_X \subset N := \{1, 2, 3, \dots\}$. Prove

$$E[X] = \sum_{i \in N} P[X \geq i]$$

2. Suppose that X and Y are two discrete rvs, and $S_X = S_Y = \{1, -1\}$

3. Let X and Y be two discrete rv with joint PMF

$$p_{X,Y}(x,y) = \begin{cases} 0.1 & x = 1, 2, \dots, 10, y = 1, 2, \dots, 10, \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the PMF of $W = \min(X, Y)$?

There are 100 possible combinations for $x = 1, 2, \dots, 10, y = 1, 2, \dots, 10$. Only one has a minimum value of 10. Nineteen have a minimum value of 1. And so on in between.

$$p_W(w) = \begin{cases} 0.19 & w = 1 \\ 0.17 & w = 2 \\ 0.15 & w = 3 \\ 0.13 & w = 4 \\ 0.11 & w = 5 \\ 0.9 & w = 6 \\ 0.7 & w = 7 \\ 0.5 & w = 8 \\ 0.3 & w = 9 \\ 0.1 & w = 10 \\ 0 & \text{otherwise} \end{cases}$$

- (b) What is the PMF of $Z = \max(X, Y)$?