ENEE 381 HW #3
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I recently injured my wrist, which has made my already poor handwriting down-right illegible. While it recovers, I'm using LATEXto write up homework assignments. If that is unacceptable, let me know.

Thanks,
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- **Problem 9.14:** Calculate the *exact* reflection and transmission coeffecients, *without* assuming that $\mu_1 = \mu_2 = \mu_0$. Confirm that $R + T = 1$.

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} \tag{1}$$

$$E_{0R} = \frac{1 - \beta}{1 + \beta} E_{0I} \text{ and } E_{0T} = \frac{2}{1 + \beta} E_{0I} \tag{2}$$

$$I_I = \frac{\epsilon_1 v_1}{2} |E_{0I}|^2 \text{ and } I_R = \frac{\epsilon_1 v_1}{2} |E_{0R}|^2 \text{ and } I_T = \frac{\epsilon_2 v_2}{2} |E_{0T}|^2 \tag{3}$$

Plugging the equations from (2) in for E_{0R} and E_{0T} in (3), we get:

$$R = \frac{I_R}{I_I} = \frac{\frac{\epsilon_1 v_1}{2} (\frac{1-\beta}{1+\beta})^2 E_{0I}^2}{\frac{\epsilon_1 v_1}{2} E_{0I}^2} = (\frac{1-\beta}{1+\beta})^2 = \frac{1 - 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}}{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}} \tag{4}$$

and

$$T = \frac{I_T}{I_I} = \frac{\frac{\epsilon_2 v_2}{2} (\frac{2}{1+\beta})^2 E_{0I}^2}{\frac{\epsilon_1 v_1}{2} E_{0I}^2} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} (\frac{2}{1+\beta})^2 = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} \frac{4}{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}} \tag{5}$$

Show that $R + T = 1$:

$$R + T = \frac{1 - 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2} + 4\frac{\epsilon_2 v_2}{\epsilon_1 v_1}}{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}} \tag{6}$$

For that to equal 1, we must show that

$$4\frac{\epsilon_2 v_2}{\epsilon_1 v_1} - 2\frac{\mu_1 v_1}{\mu_2 v_2} = 2\frac{\mu_1 v_1}{\mu_2 v_2} \text{ or that } \frac{\epsilon_2 v_2}{\epsilon_1 v_1} = \frac{\mu_1 v_1}{\mu_2 v_2} \tag{7}$$

For that we need the equations for v_1 and v_2:

$$v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}} \text{ and } v_2 = \frac{1}{\sqrt{\mu_2 \epsilon_2}} \tag{8}$$

Then rearrange Equation (7) into:

$$\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1} = \frac{v_1^2}{v_2^2} \tag{9}$$

And substitute for v_1 and v_2 to get:

$$\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1} = \frac{\frac{1}{\sqrt{\mu_1 \epsilon_1}}^2}{\frac{1}{\sqrt{\mu_2 \epsilon_2}}^2} = \frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1} \tag{10}$$

This proves the first version of Equation (7), which means we can substitute into Equation (6):

$$R + T = \frac{1 - 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2} + 4\frac{\epsilon_2 v_2}{\epsilon_1 v_1}}{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}} = \frac{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}}{1 + 2\frac{\mu_1 v_1}{\mu_2 v_2} + \frac{\mu_1^2 v_1^2}{\mu_2^2 v_2^2}} = 1 \tag{11}$$

- **Problem 9.15:** Prove that for a normal incident wave, the reflected and transmitted waves have the same polarization.

At normal incidence, there is no component of the E or B field that is perpendicular to the boundary. It is all parallel. Therefore, we are only concerned with the parallel boundary conditions. Specifically:

$$(\tilde{E}_{0I} + \tilde{E}_{0R})_{x,y} = (\tilde{E}_{0T})_{x,y} \tag{12}$$

$$\frac{1}{\mu_1}(\tilde{B}_{0I} + \tilde{B}_{0R})_{x,y} = \frac{1}{\mu_1}(\tilde{B}_{0T})_{x,y} \tag{13}$$

Substituting $\tilde{B} = \frac{1}{v}\tilde{E}$ gives

$$\frac{1}{\mu_1 v_1}(\tilde{E}_{0I} + \tilde{E}_{0R})_{x,y} = \frac{1}{\mu_1 v_1}(\tilde{E}_{0T})_{x,y} \tag{14}$$

If the polarization of the incident wave is strictly in the $\hat{x}$ direction, then the x, y components of Equation (12) are:

$$\tilde{E}_{0I} + \tilde{E}_{0R}\cos\theta_R = \tilde{E}_{0T}\cos\theta_T \tag{15}$$

$$\tilde{E}_{0R}\sin\theta_R = \tilde{E}_{0T}\sin\theta_T \tag{16}$$

Likewise, the x, y components of Equation (14) are, respectively:

$$\frac{1}{\mu_1 v_1}\tilde{E}_{0I} + \frac{1}{\mu_1 v_1}\tilde{E}_{0R}\cos\theta_R = \frac{1}{\mu_1 v_1}\tilde{E}_{0T}\cos\theta_T \tag{17}$$

$$\frac{1}{\mu_1 v_1}\tilde{E}_{0R}\sin\theta_R = \frac{1}{\mu_1 v_1}\tilde{E}_{0T}\sin\theta_T \tag{18}$$

We will consider just the y components:

$$\tilde{E}_{0R}\sin\theta_R = \tilde{E}_{0T}\sin\theta_T \tag{19}$$

$$\frac{\tilde{E}_{0R}}{\mu_1 v_1}\sin\theta_R = \frac{\tilde{E}_{0T}}{\mu_1 v_1}\sin\theta_T \tag{20}$$

There are only two circumstances under which both Equation can be true: Either $\mu_1 v_1 = \mu_2 v_2$, in which case both sides of the boundary are electro-magnetically identical and there *is no* boundary,
or $sin\theta_R = sin\theta_T = 0$ which means that $\theta_R = \theta_T = 0$.