324 formula sheet

Bayes Rule:

$$P(A\cap B)P(B) = P(B\cap A)P(A) \text{ therefore } P(A\cap B) = \frac{P(B\cap A)P(A)}{P(B)} \text{ and } P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$p_{X,Y}(x,y) = p_{X|Y}(X|Y)p_Y(y) = p_{Y|X}(y|x)p_X(x) \text{ therefore } p_{X|Y}(X|Y) = p_{Y|X}(y|x)\frac{p_X(x)}{p_Y(y)}$$

Laws of Total Probability: If $B = \{B_1, ...B_m\}$ is an event space, then

$$P(X) = \sum_{i=1}^{m} P(X\cap B_i) \text{ and } P(X) = \sum_{i=1}^{m} P(X|B_i) \text{ and } p_X(x) = \sum_{i=1}^{m} p_{X|B_i}(x)P(B_i) \text{ and } p_X(x) = \sum_{y\in S_Y} p_{X|Y}(x|y)p_Y(y)$$

Expectation and variance: $E[X] = \sum_{x\epsilon S_x} x\cdot p_X(x)$ and $Var(X) = E[(X-E[X])^2] = E[X^2] - E[X]^2$
$E[aX+b] = aE[X]+b$ and $Var(aX+b) = a^2\cdot Var(X)$ if $a,b\in R$

Conditional Expectation: $E[X|B] = \sum_{x\epsilon B} x\cdot p_{X|B}(x)$ and $E[E[X|Y]] = E[X]$ and $Var(X|B) == E[X^2|B] - E[X|B]^2$

De Morgan's Law $(A\cup B)^c = A^c\cap B^c$ and $(A\cap B)^c = A^c\cup B^c$

RV Type	PMF	Exp	Var
Bernoulli	$p_X(x) = \begin{cases} 1-p & x=0 \\ p & x=1 \\ 0 & otherwise \end{cases}$	p	$p(1-p)$
Binomial	$p_X(x) = \begin{cases} \begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x} & x=0,1,...,n \\ 0 & otherwise \end{cases}$	np	$np(1-p)$
Geometric	$p_X(x) = \begin{cases} (1-p)^{x-1}p & x=0,1,... \\ 0 & otherwise \end{cases}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Pascal	$p_X(x) = \begin{cases} \begin{pmatrix} x-1 \\ k-1 \end{pmatrix} p^k(1-p)^{x-k} & x=k,k+1,... \\ 0 & otherwise \end{cases}$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$

Marginal PMF and independence

$$p_X(x) = \sum_{y\in S_Y} p_{X,Y}(x,y) \text{ and } p_{X,Y}(x,y) = p_X(x)\times p_Y(y)$$

Correlation	Covariance	Correlation Coefficient
$\gamma_{X,Y} = E[X\cdot Y] = 0$ if Orthogonal	$\sigma_{X,Y} = E[(X-E[X])(Y-E[Y])] = 0$ if uncorrelated	$\rho_{X,Y} = \frac{Cov(X,Y)}{Var(X)\cdot Var(Y)}$

If $W = g(X,Y)$ then $E[W] = \sum_{(x,y)\in S_{X,Y}} g(x,y)p_{X,Y}(x,y)$ and $Var(X+Y) = Var(X)+Var(Y)+2\sigma_{X,Y}$

$$E[W] = \sum_{(x,y)\in S_{X,Y}} g(x,y)p_{X,Y}(x,y) \text{ and } E[W|B] = \sum_{(x,y)\in S_{X,Y}} g(x,y)p_{X,Y|B}(x,y)$$

$$p_{X,Y|B}(x,y) = P(X=x, Y=y|B) = \begin{cases} \frac{p_{X,Y}(x,y)}{P(B)} & \text{if } (x,y)\in B \\ 0 & otherwise \end{cases} \quad \text{given } B\subset R^2$$