

Maxwell's Equations: Everything we need when working in vacuum.

	vacuum	matter	or
1. Gauss's Law	$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$	$\nabla \cdot \vec{E} = \frac{1}{\epsilon} (\rho_f - \nabla \cdot \vec{P})$	$\nabla \cdot \vec{D} = \rho_f$ if $\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$
2. No mag monopoles	$\nabla \cdot \vec{B} = 0$	1: $\nabla \cdot \vec{E} = \frac{\rho_{free}}{\epsilon}$	
3. Faraday's Law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	4: $\nabla \times \vec{B} = \mu \vec{J}_{free} + \mu \sigma \vec{E} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$	
4. Ampere's Law	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{B} = \mu_0 (\vec{J}_f + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}) + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$ if $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$
0. Continuity	$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = -\nabla \cdot (\epsilon_0 \frac{\partial \vec{E}}{\partial t})$	Vac, no charges or currents: $\nabla^2 \vec{E}$	$= \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$ and $\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$
Boundary Conditions:	$\epsilon_1 E_1^\perp = \epsilon_2 E_2^\perp$ $B_1^\perp = B_2^\perp$, $E_1^\parallel = E_2^\parallel$, $\frac{1}{\mu_1} B_1^\parallel = \frac{1}{\mu_2} B_2^\parallel$	$\beta = \frac{\mu_1 v_1}{\mu_2 v_2}$, $E_{0R} = \frac{1-\beta}{1+\beta} E_{0I}$ and $E_{0T} = \frac{2}{1+\beta} E_{0I}$	
Wave Equation:	$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$	Standing Wave to sum of waves: $A \sin kz \cos kv t = \frac{A}{2} (\sin kz + kv t + \sin kz - kv t)$	String
Standing Wave:	$kL = n\pi$ for	$n = 1, 2, 3 \dots$ And frequency:	$v = \frac{1}{T} = \frac{kv}{2\pi} = \frac{nv}{nL}$
boundary conditions: Position cannot change across boundary, and neither can first derivative.			
2 connected strings, $\omega_1 = \omega_2$ must be, $v = \frac{\omega}{k}$, then $r = \frac{A_R}{A_I} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{v_2 - v_1}{v_1 + v_2}$ and $t = \frac{A_T}{A_I} = \frac{2k_1}{k_1 + k_2} = \frac{2v_2}{v_1 + v_2}$			
Monochromatic plane waves: $\vec{E} = E_0 \hat{n} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$ and $\vec{B} = \frac{E_0}{c} \hat{k} \times \hat{n} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$ and $\vec{S} = c \epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta) \hat{z} = cu \hat{z}$			
Poynting Vector: $\vec{S} \equiv \frac{1}{\mu_0} (\vec{E} \times \vec{B})$ Poynting Theorem: $\frac{dW}{dt} = -\frac{d}{dt} \int_V u d\tau - \oint_S \vec{S} \cdot d\vec{a}$ Momentum: $\vec{g} = \frac{1}{c^2} \vec{S}$			
Averages: $\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$, $\langle \vec{S} \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \hat{z}$, $\langle \vec{g} \rangle = \frac{1}{2c} \epsilon_0 E_0^2 \hat{z}$, Intensity $I \equiv \langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2$			
Intensity stuff: $I_I = \frac{\epsilon_1 v_1}{2} E_{0I} ^2$, $I_R = \frac{\epsilon_1 v_1}{2} E_{0R} ^2$, $I_T = \frac{\epsilon_2 v_2}{2} E_{0T} ^2$, then $R = \frac{I_R}{I_I} = (\frac{1-\beta}{1+\beta})^2$, $T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1} (\frac{2}{1+\beta})^2$ and $R + T = 1$			
Related: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $v_1 = \frac{1}{\sqrt{\mu_1 \epsilon_1}}$ Brewster's Angle when $\frac{E_{0R}}{E_{0I}} = 0$ And for the following, $n = \frac{n_2}{n_1}$ or $\frac{n_T}{n_I}$			
Fresnel Coeff, S-polarized, $\hat{n} = \hat{y}$: $r_\perp = \frac{E_{0R}}{E_{0I}} = \frac{n_I \cos \theta_I - n_T \cos \theta_T}{n_I \cos \theta_I + n_T \cos \theta_T} = \frac{\cos \theta_I - \sqrt{n^2 - \sin^2 \theta_I}}{\cos \theta_I + \sqrt{n^2 - \sin^2 \theta_I}}$, $t_\perp = \frac{E_{0T}}{E_{0I}} = \frac{2 \cos \theta_I}{\cos \theta_I + \sqrt{n^2 - \sin^2 \theta_I}} = \frac{2 n_I \cos \theta_I}{n_I \cos \theta_I + n_T \cos \theta_T}$			
P-polarized, $\hat{n} \in \hat{x} + \hat{z}$: $r_\parallel = \frac{E_{0R}}{E_{0I}} = \frac{n_I \cos \theta_T - n_T \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I} = \frac{\sqrt{n^2 - \sin^2 \theta_I} - n^2 \cos \theta_I}{\sqrt{n^2 - \sin^2 \theta_I} + n^2 \cos \theta_I}$, $t_\parallel = \frac{E_{0T}}{E_{0I}} = \frac{2 n \cos \theta_I}{n^2 \cos \theta_I + \sqrt{n^2 - \sin^2 \theta_I}} = \frac{2 n_I \cos \theta_I}{n_I \cos \theta_T + n_T \cos \theta_I}$			
<u>Conductors</u> : Conductivity σ . $e^{i(\beta z - \omega t)}$, $\beta = k_r + i k_i = \sqrt{\mu \epsilon \omega^2 + i \mu \sigma \epsilon} = \omega \sqrt{\frac{\epsilon \mu}{2}} \left(\sqrt{1 + (\frac{\sigma}{\epsilon \mu})^2} + 1 \right)^{\frac{1}{2}} + i \omega \sqrt{\frac{\epsilon \mu}{2}} \left(\sqrt{1 + (\frac{\sigma}{\epsilon \mu})^2} - 1 \right)^{\frac{1}{2}}$			
Skin Depth $d = \frac{1}{k_i}$ Boundary Conditions: $\epsilon_1 E_1^\perp - \epsilon_2 E_2^\perp = \rho_s$, $B_1^\perp - B_2^\perp = 0$, $E_1^\parallel - E_2^\parallel = 0$, $\frac{1}{\mu_1} B_1^\parallel - \frac{1}{\mu_2} B_2^\parallel = \vec{J}_s \times \hat{n}$			
coefficients don't change: $\frac{E_{0R}}{E_{0I}} = \frac{1-\beta}{1+\beta}$, $\frac{E_{0T}}{E_{0I}} = \frac{2}{1+\beta}$ Difference is β is complex, $\beta = \frac{\mu_1 v_1}{\mu_2 \omega} k_2$			