Experimental Validation of Wave Propagation through a Thin-film Sheet

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Introduction

All objects when subjected to constant vibrations will experience wave propagation in the form of longitudinal and transverse waves. Understanding a materials response to these oscillations provide insight into its applicability. Consider a string hanging from a rotating servo motor; the result is a one-dimensional oscillation that causes a wave to pulse across the string. In the case of a two-dimensional thin material, a transverse wave will travel across each point on the surface at different rates. This wave is prominent for larger surface areas, for a significant amount of time is required for a wave to propagate through the material. The theoretical basis for the propagation of a wave on a string is identified as $v = \sqrt{\tau/\mu}$, where τ is the tension applied on the string, and μ represents linear density [1].

Developing an accurate mathematical model that represents the rate at which a wave propagates on a thin-film material is essential for space applications. A company called GeoShade wants to design an attitude controller that adjusts the orientation of a solar sail, and understanding wave propagation of the material is essential in this design. An experiment is conducted on multiple thin-film sized sheets made up of different materials to validate this model.

Methods

The theoretical wave speed, $v_{theoretical}$, is calculated using,

$$v_{theoretical} = \sqrt{g(L-y)} \tag{1}$$

where g is the gravity acting on the thin-film sheet, L is the length of the sheet, and y is the arbitrary distance from the top end to the free end of the sheet. Eq. 1 is derived from the relation between the tension, linear density and the length of the thin-film sheet [1].

To determine the transverse wave speed, two thin-filmed materials are used, an aluminum sheet (AL) and an emergency blanket (EM). The sheets are hung vertically inside the test frame as shown in

Figure 1. The top of the sheet is tapped to an Aluminum rod, which was clamped to a Dynamixel AX-12 servo motor [2].

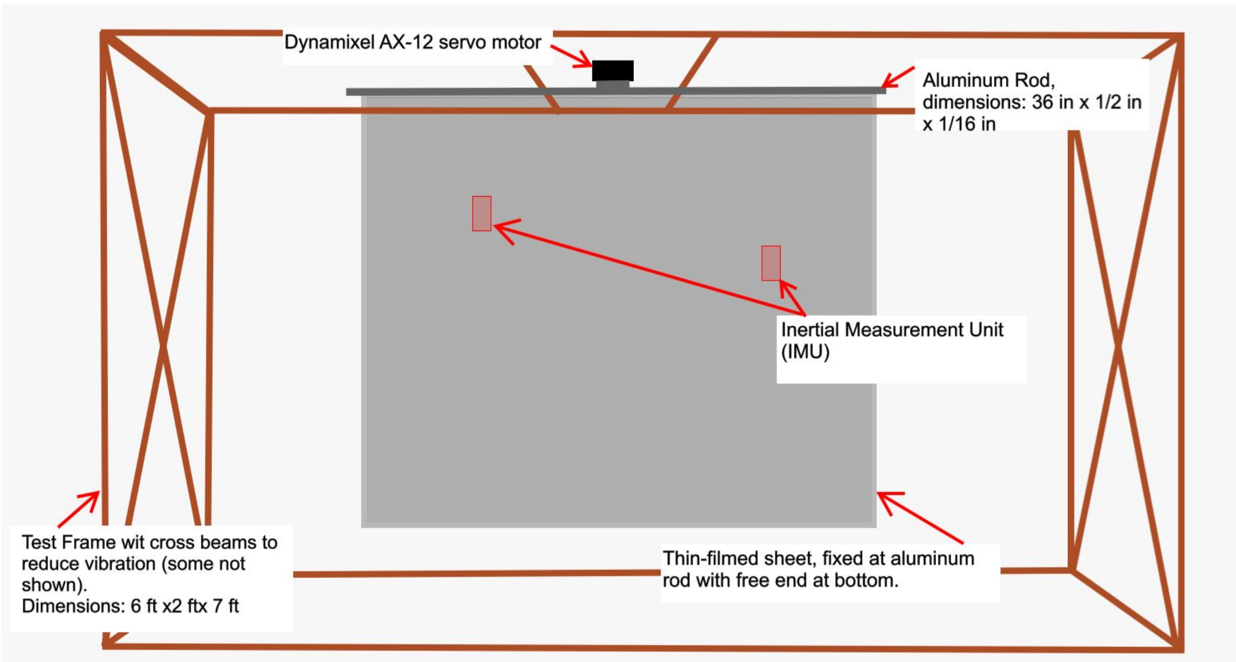

Figure 1: Blueprint of the test setup and orientation of the thin-film material. Dimensions are not to scale.

The motor is mounted rigidly to the top of the test frame. The servo motor is programmed to oscillate the Aluminum rod about its center of mass to create a wave pattern down the sheets. To determine the wave velocity of the sheet, the linear acceleration in the direction normal to the sheet surface is measured using the LSM9DS1 Inertial Measurement Unit (IMU). Assuming that there is no external damping, the distance of the IMU to the top of the bar is less than one wavelength, and the phase shift is less than $2\boldsymbol{\pi}$, the following equations are used to calculate wave propagation. The experimental velocity, v_{exp}, can be found using,

$$v_{exp} = L/\Delta t \quad (2)$$

where L represents the distance of the IMU location from the Aluminum rod, and Δt represents the time phase shift of the IMU in reference to the angular position of the servo motor. The angular position, φ, of the motor is modeled by,

$$\varphi = A sin(\omega_s t) \quad (3)$$

where t represents time, ω_s is the angular frequency of the forcing input, and A represents the amplitude of the wave. The IMU measures the linear acceleration, x'', which has the form of

$$x'' = -A^2 sin(\omega_s t + \alpha) \quad (4)$$

where α represents the phase change of the IMU with respect to the reference input of the servo motor given in Eq. 3. The phase shift alpha is the only point of interest, therefore the other components are neglected by normalizing Eq. 4 and Eq. 3. The time phase shift is determined graphically by superimposing the normalized Eq. 3 and Eq. 4 as in Figure 3..

Each sheet is approximately 1m wide by 1.5m long. The weight of the aluminum rod is 0.3lbs. Each sheet is connected to an aluminum rod using tape. Two IMUs are attached using electrical tape on either side of the sheet at varying heights as shown in Figure 2.

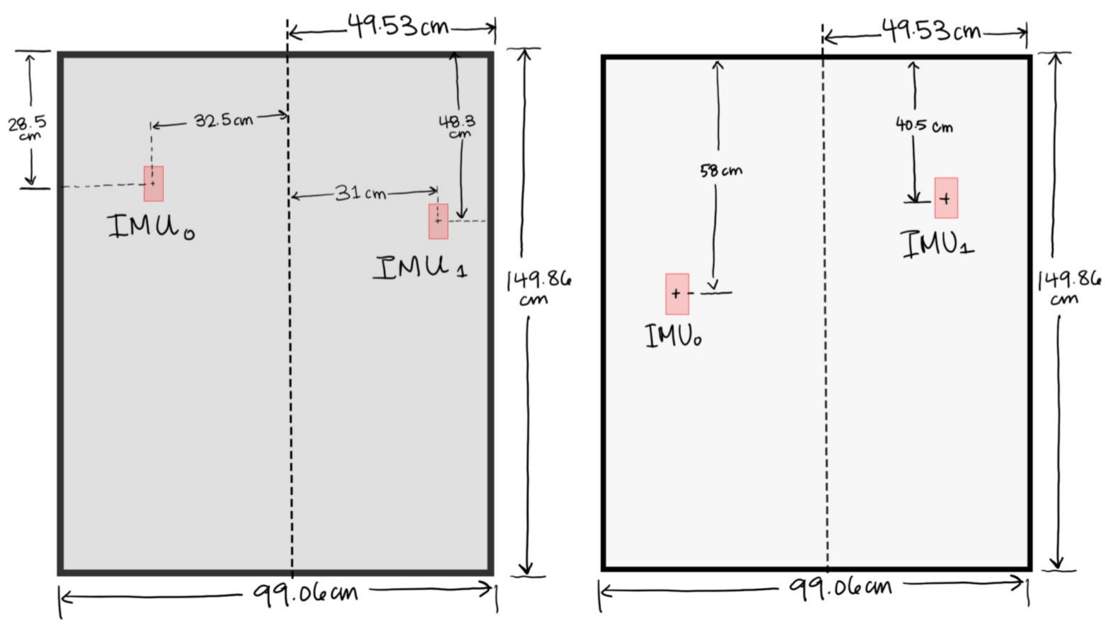

Figure 2: Arrangement of IMU0 and IMU1 on the two sheets of thin-filmed material. The left image is the Aluminum sheet and the right image is the emergency blanket. Dimensions are not to scale.

The recorded data is collected by an Arduino Due [3] and sent to MATLAB for analysis. The Dynamixel is programmed to oscillate at 0.43 hz, 0.87 hz, and 1.32 hz. Two replications are done for each oscillating frequency. The sampling rate is 43.5 Hz.

Results

The theoretical velocity was calculated using Eq. 1 to get 3.84 $\pm$ 0.13 m/s. The uncertainty is propagation of error from the sheet measurements.

The IMU data is filtered using the *fftfilt* [4] function. The filter design parameter is an 80th order lowpass filter. Since the highest oscillation frequency is 1.32Hz, the corner frequency is set to 3Hz. Figure 3 shows the normalized filtered IMU sinusoidal response superimposed on the servo motor reference input. The time phase shift of the aluminum sheet and emergency blanket material are shown in the Appendix 1. The digital filter creates a temporal shift factor to the data as an inherent characteristic of the filter to accumulate data history. Due to the same filter application for both the reference sinusoidal and the response of the IMU, the temporal shift factor is the same, i.e. the time phase shift will not be affected by the filtering process.

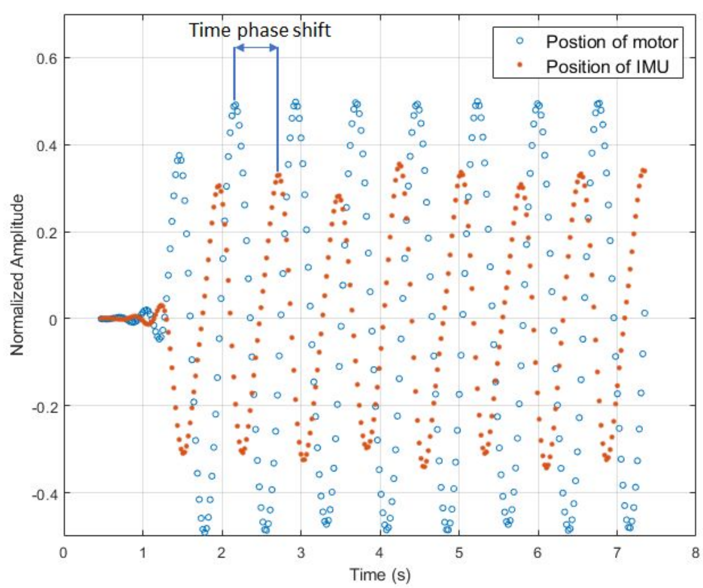

Figure 3: The normalized position of the IMU superimposed on the position of the motor.

The experimental velocity is calculated using Eq. 2 with the uncertainty propagated from the standard error of the time phase shift and the distance measurements. The results are shown in Table 1.

Table 1: Experimentally determined velocity for AL and EB from Equation 2.

Velocity							
		Aluminum sheet			Emergency Blanket		
	Frequency (Hz)	0.43	0.87	1.32	0.43	0.87	1.32
IMU 0	mean (m/s)	0.94	1.7	2.8	1.3	1.50	1.2
	uncertainty	0.22	0.40	0.48	0.27	0.20	0.16
IMU 1	mean (m/s)	0.39	0.89	0.7	0.34	0.51	0.74
	uncertainty	0.052	0.093	0.07	0.064	0.067	0.091

Discussion

From the results of the experimentally calculated velocity a correlation between the theoretical velocity value and the experimental velocity value was not established. The theoretical velocity is a function of tension and linear density in the direction that the wave travels. However, our resultant velocity seems to indicate that the velocity is related to oscillation frequency as shown in Figure 4. The transverse velocity of a one dimensional wave is calculated as,

$$v = \lambda/T \tag{5}$$

where λ is the wavelength and T is the period of the wave. From the experimental velocity results, the averaged velocity is 2 to 3 times less than the theoretical velocity of 3.84 ± 0.13 m/s; it is evident that Equation 5 does not fully define the transverse velocity for our system as the velocity is influenced by the frequency.

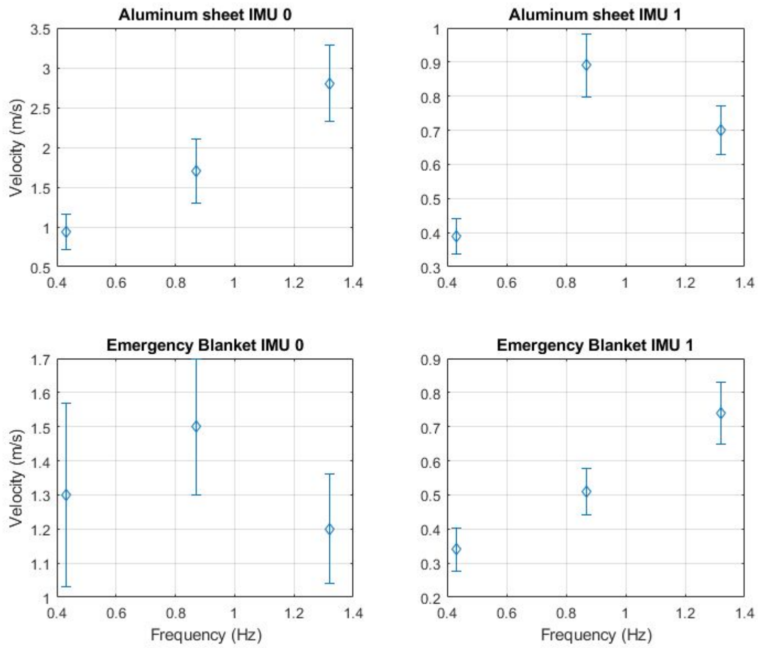

Figure 4: Relation of oscillating frequency to the experimentally determined transverse velocity.

We also expected the velocity to be the same for the IMU's that are on the same sheet and under the same forcing frequency. The discrepancy between the two IMU's is as large as 70% in the case of the EM at 0.43 Hz forcing frequency. There are two probable causes for this discrepancy: the wavelength assumption is invalid, or the damping is significantly higher than anticipated. From our observation, the forcing frequency is low such that the wavelength is larger than the distance of the IMU to the rod. However, the damping caused by air drag is significant such that the wave motion dies out when it reaches the bottom of the sheet. With significant damping, the Eq. 2 is invalid because it does not account for acceleration.

Conclusion

After carrying out this experiment, we can deduce that in order to display an accurate representation of the wave velocity, significant errors that factor in our procedure should be reduced. One method would be to carry out the experiment in a well ventilated room to minimize air drag. Lastly, in order to obtain more useful data, conducting further tests will be needed for establishing a correlation between theoretical and experimental velocity.

Acknowledgments

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References

[1] Elmore, W. , C., & Heald, M. , A. (2012). *Physics of Waves*. Courier Corporation.

[2] *Dynamixel AX-12A Robot Actuator from Robotis - Trossen Robotics*. https://www.trossenrobotics.com/dynamixel-ax-12-robot-actuator.aspx. Accessed 14 Mar. 2020.

[3] *Arduino Due | Arduino Official Store*. https://store.arduino.cc/usa/due. Accessed 14 Mar. 2020.

[4] *FFT-Based FIR Filtering Using Overlap-Add Method - MATLAB Fftfilt*. https://www.mathworks.com/help/signal/ref/fftfilt.html. Accessed 14 Mar. 2020.

Appendix A: Time phase shift data

The time phase shift data is represented below. Uncertainty is ± 2 standard deviation from all the trials.

Table 2: Time phase shift data from the IMUs.

		Time Phase Shift					
		Aluminum sheet			**Emergency Blanket**		
	Frequency (Hz)	0.43	0.87	1.32	0.43	0.87	1.32
IMU 0	mean (s)	0.30	0.17	0.10	0.45	0.39	0.47
	uncertainty	0.05	0.02	0.03	0.09	0.04	0.04
IMU 1	mean(s)	1.2	0.54	0.672	1.2	0.79	0.55
	uncertainty	0.1	0.3	0.03	0.2	0.03	0.02