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CSC 225 Assignment #1

1. UNDERGRADUATE

a) 2×A 2×D 2×E 2×R 2×U

1×G 1×T 1×N

There are 13 letters total

$\therefore \dfrac{13!}{2! \times 2! \times 2! \times 2! \times 2!} = 194,594,400$

$\boxed{\therefore \text{ There are } 194,594,400 \text{ possible arrangements}}$

b) In this case we will consider the two A's one unit, therfore we will say there are 12 letters total

$\therefore \dfrac{12!}{2! \times 2! \times 2! \times 2!} = 29,937,600$

$\boxed{\therefore \text{ There are } 29,937,600 \text{ possible arrangements}}$

c) This simply the total possible arragements minus the possibilities in b)

$\therefore 194,594,400 - 29,937,600 = 164,656,800$

$\boxed{\therefore \text{ There are } 164,656,800 \text{ possible arrangements}}$

d) Vowels => A, E, U

We will consider all the vowels as their own "letter", ∴ 7 letters total

∴ $\frac{7!}{2! \times 2!} = 1260$

$\boxed{\text{∴ There are 1260 possible arrangments}}$

2.

a) To pick 3 clubs there are $\binom{13}{3}$ possible ways to draw

The other two cards cannot be clubs. ∴ 52 − 13 clubs = 39 cards left

∴ There are $\binom{39}{2}$ ways to draw the other two cards

∴ $\binom{13}{3} \times \binom{39}{2} = \frac{13!}{3!(10)!} \times \frac{39!}{2!(37!)}$

$= 286 \times 741 = 211,926$

$\boxed{\text{∴ There are 211,926 ways}}$

b) To pick 2 hearts there are $\binom{13}{2}$ possible ways to draw.

For the other 3 cards there are $\binom{50}{3}$ ways to draw.

∴ $\binom{13}{2} \times \binom{50}{3} = 78 \times 19,600 = \boxed{1,528,800 \text{ ways}}$

c) To pick 3 clubs there are $\binom{13}{3}$ possible ways to draw and to pick 2 hearts there are $\binom{13}{2}$ possible ways to draw

$$\therefore \binom{13}{3} \times \binom{13}{2} = 286 \times 78 = 22{,}308$$

$\boxed{\therefore \text{ There are } 22{,}308 \text{ possible ways}}$

3.

a) $n = 12 \quad K = 5$

$$\therefore \binom{12}{5} 1 \cdot 1 = \boxed{792}$$

b) $$\binom{12}{5} (-4)^7 (3)^5 = 792 \times (-4)^7 (3)^5$$

$$= \boxed{-3{,}153{,}199{,}104}$$

c) $$\binom{12}{5} (12)^7 (-2)^5 = 792 (12)^7 (-2)^5$$

$$\boxed{= -9.08\ldots \times 10^{11}}$$

4.

a) Using combinations with repitition formula

$$\binom{4+16-1}{16} = \binom{19}{16} = 969$$

$\boxed{\therefore 969 \text{ possible ways}}$

b) Here, we have already distributed a total of 8 → $x_1, x_2 \geq 1$ & $x_3, x_4 \geq 3$

$$\therefore \binom{4+8-1}{8} = \binom{11}{8} = 165$$

$\boxed{\therefore 165 \text{ possible ways}}$

c) In this case each variable has been given a −1 value, ∴ we need to add an additional 4 to 16.

$$\therefore \binom{4+20-1}{20} = \binom{23}{20} = 1771$$

$\boxed{\therefore 1771 \text{ possible ways}}$

d) Here there are 3 possible cases:

Case 1: $x_4 = 5 \rightarrow \binom{3+8-1}{8} = \binom{10}{8} = 45$

Case 2: $x_4 = 6 \rightarrow \binom{3+7-1}{7} = \binom{9}{7} = 36$

Case 3: $x_4 = 7 \rightarrow \binom{3+6-1}{6} = \binom{8}{6} = 28$

∴ Case 1 + case 2 + case 3 = 109

$\boxed{\therefore 109 \text{ possible ways}}$

5. The minimum Ali can run is 35 (5 weeks x 7 days) times during the 5 week period.

If Ali runs her max of 50 and since she runs at least once a day, there are only 50-35, or 15 runs which she can distribute over the 35 days.

Set$_1$ $\underbrace{1 <= d_1 < d_2 < \ldots < d_{35} <= 50}_{35 \text{ distinct numbers}}$

Set$_2$ $\underbrace{d_1 + 19, d_2 + 19, \ldots, d_{35} + 19 <= 69}_{35 \text{ distinct numbers}}$

∴ For none of the $d_i + 19$ numbers to exist as one of the d_i numbers we would need 70 distinct numbers in total.

∴ An element of set$_2$ must exist in set$_1$

∴ There must be a period of consecutive days where Ali runs 19 times