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CSC 225 Assignment #2

1. a) $1A$

$1A + n^2[1C + 1A] + 1C$

$n^2[1A + (n^2+1)/2[1A+1C] + 1C]$

$n^2[[n^2+1]/2(1A)] \times 2$

$\therefore 3 + 2n^2 + n^2(n^2+1+2) + n^2+1$

$\boxed{\therefore 2n^4 + 6n^2 + 3}$

b) $1A$

$1A + n(1C+1A) + 1C$

$n(1A)$

$n[(n/2)(1A+1C) + 1C)$

$n(n/2)(1A) \times 2$

$\therefore 3 + 2n + n + n^2 + n^2 + n$

$\boxed{\therefore 2n^2 + 4n + 3}$

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2.

a)

Base case $T(1) = 1$

$T(n) = 2T(n-1) + 1$

$\therefore T(n-1) = [2T(n-2) + 1] + 1$

$T(n-2) = 2T(n-3) + 1$

$\vdots$

$\therefore T(n-n+2) = 2T(n-n+1) + 1$

$\therefore T(n) = 2[2T(n-2) + 1] + 1$

$= 2^2 T(n-2) + 2 + 1$

$= 2^2 [2T(n-3) + 1] + 2^1 + 2^0$

$= 2^3 T(n-3) + 2^2 + 2^1 + 2^0$

$\therefore$ Following this pattern we can see

$$T(n) = 2^{n-1} + \frac{1 - 2^{n-1}}{-1} = \boxed{2^{n-1} - (1 - 2^{n-1})}$$

b) $T(n-1) = T(n-2) + 2(n-1)$

Base case $T(1) = 4$

$T(n-2) = T(n-3) + 2(n-2)$

$T(n-3) = T(n-4) + 2(n-3)$

$\vdots$

$T(n-n+2) = T(n-n+1) + 2(n-n+2)$

$\Rightarrow T(n-n+1) + 2n + 2(n-1) + (2n-2) + \ldots 2(n-n+2)$

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Following this pattern between 1 and n we can see the following equation emerging $\frac{n(n+1)}{2} \cdot 2$ (however missing case at 1)

$\therefore T(n) = 4 + 2\left(\frac{n(n+1)}{2} - 1\right)$

$\therefore T(n) = 4 + n(n+1) - 2$

$T(n) = 4 + n^2 + n - 2$

$$\boxed{T(n) = n^2 + n + 2}$$

3.

a) $\sum_{i=1}^{n} (i)(i)! = (n+1)! - 1$ for all $n \geq 1$

Base case:

$n = 1$ $\quad (1)(1)! = 2! - 1 \Rightarrow 1 = 1$ ✓

IH: $\sum_{i=1}^{k} (i)(i)! = (k+1)! - 1$ for $k \geq 1$

IS: Want $\sum_{i=1}^{k+1} (i)(i)! = (k+2)! - 1$

$\sum_{i=1}^{k+1} (i)(i)! = (k+1)! - 1 + (k+1)(k+1)!$

$(k+1)! \cancel{-1} + (k+1)(k+1)! = (k+2)! \cancel{-1}$

$(k+1)!\,(1 + k + 1) = (k+2)!$

$(k+1)!\,(k+2) = (k+2)!$

$(k+2)! = (k+2)!$ $\quad \therefore$ RHS = LHS

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$\therefore$ By induction

$$\sum_{i=1}^{n}(i)(i)! = (n+1)! - 1 \quad \text{for all} \quad n \geq 1$$

b. $1+6+11+16+\dots+(5n-4) = n(5n-3)/2$

for all $n \geq 1$

Base case:

$5(1)-4 = 1(5(1)-3)/2 = 1 = 1$ ✓

IH: $\sum_{i=1}^{k}(5(i)-4) = k(5k-3)/2$ for $k \geq 1$

IS: Want $\sum_{i=1}^{k+1}(5(i)-4) = (k+1)(5(k+1)-3)/2$

$$\sum_{i=1}^{k+1}(5(i)-4) = k(5k-3)/2 + 5(k+1)-4$$

$$\Rightarrow \frac{5k^2-3k}{2} + 5k+1 = \frac{5k^2-3k+10k+2}{2}$$

$$= \frac{5k^2+7k+2}{2} = \frac{(k+1)(5(k+1)-3)}{2}$$

$$\Downarrow$$

$$\frac{(k+1)(5k+2)}{2} = \frac{5k^2+2k+5k+2}{2}$$

$$= \frac{5k^2+7k+2}{2}$$

$\therefore$ RHS = LHS

$\therefore$ By induction

$$\sum_{i=1}^{n}(5(i)-4) = n(5n-3)/2 \quad \text{for all} \quad n \geq 1$$

4.

BC: a^n for ~~$q4(0,0)$~~ $(0,0)$

before entering the loop, pow is 1

IS: Assume at the start of itteration K, $pow = a^k$

Want to prove for $K+1$

Given that $pow = a^k$ after the k^{th} iteration by multiplying the result with the constant, we want to see $K+1^{th}$ iteration

$\therefore$ k^{th} iteration $pow = pow \times a$

$\therefore$ $K+1$ iteration $pow = pow \times a \times a \ldots$

which returns a^{k-1}

The loops ends after $k+1$ iterations when $i = K+1$

$\therefore$ From the inductive step we can that $q4(K+1, K+1) = a^{k+1}$

5. $(n+2)^4 \leq (n+n)^4$

a) $\therefore (n+2)^4 \leq (2n)^4$ for $n_0 \geq 2$

$\boxed{\therefore O(n)^4}$ where $c = 16$

b) $\dfrac{n^4 + n^2 + 1}{n^3 + 1} \leq \dfrac{n^4 + n^4 + n^4}{n^3}$ for $n_0 \geq 1$

$\therefore \dfrac{n^4 + n^2 + 1}{n^3} \leq \dfrac{3n^4}{n^3} = 3n$ where $c = 3$

$\boxed{\therefore O(n)}$

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C. $5n^3 + 3n^2 \log n$

$5n^3$ is the dominant term

since $5n^3 \leq 5n^3 + 3n^2 \log n$

$x \neq 3$ $\therefore$ The x value must be ≥ 4

$5n^3 + 3n^2 \log n \leq 5n^4$ for all $n_0 \geq 1$

$\therefore$ The smallest $x = 4$ where $c = 5$

$\boxed{\therefore O(n^4)}$

6.

From smallest to largest $O(\,)$

1. $\frac{1}{n}$
2. 2^{40}
3. $\log(\log n)$
4. $n^{0.01}$
5. $\sqrt{\log x}$
6. $2^{\log n}$
7. $\sqrt[2]{n}$
8. $25n^{0.5}$
9. $n\,14n$
10. $A \log_5 m$
11. $5n \log n$
12. $n^2 \log n$
13. $(n \log n)^2$
14. n^3
15. $7x^{(5/2)}$
16. 4^x
17. $2^{(2^x)}$
18. 5^{x^2}

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