Assignment #2

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1 Question 1

Design an algorithm (in pseudocode) to determine whether a digraph has a unique topological ordering. Your algorithm should return the ordering if a unique one exists or indicate that no unique order exists. You may use the algorithm in lecture 4 as a template for your algorithm.

2 Question 2

Consider the following directed graph, G, with vertices labelled 1 to 6.

$$G = (\{1,2,3,4,5,6\}, \{(1,2),(1,6),(2,3),(2,4),(3,1),(3,5),(4,5),(5,6),(6,3)\})$$

(a) Graph Drawing and Adjacency Matrix

Draw graph G and give its adjacency matrix representation, labeling it G_0.

(b) Floyd-Warshall Algorithm

Run the Floyd-Warshall transitive closure algorithm on G, resulting in the new graph G^*. Show the contents of the adjacency matrix after each iteration of the outside loop of the algorithm (that is, show G_k for each $k = 1, \ldots, 6$).

3 Question 3

Suppose we are given an unweighted, directed graph G with n vertices (labelled 1 to n), and let M be the $n \times n$ adjacency matrix for G (that is, $M[i,j] = 1$ if directed edge (i,j) is in G and 0 otherwise). Let the product of M with itself (M^2) be defined, for $1 \leq i,j \leq n$, as follows:

$$M^2[i,j] = (M[i,1] \cdot M[1,j]) + (M[i,2] \cdot M[2,j]) + \ldots + (M[i,n] \cdot M[n,j])$$

where "$\cdot$" is the Boolean AND operator and "+" is the Boolean OR operator.

Given this definition, what does $M^2[i,j] = 1$ imply about vertices i and j? What if $M^2[i,j] = 0$? Show the work you do to come to these conclusions.

4 Question 4

Rewrite the DFS procedure for undirected graphs iteratively using a stack (Provide pseudocode).

Requirements:

- Classify edges as *discovery edges* or *back edges*.
- Function signature: `IterativeDFS`(V, E, u), where V is the set of nodes, E is the set of edges, and u is the starting vertex.