Arithmetic mean

Suppose we have a data of size n: $x_1, x_2, \ldots, x_n$. Then the arithmetic mean, $\overline{x}_n$, of the data is

$$\overline{x}_n = \sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + x_2 + \cdots + x_n).$$

To give an examle, consider a data of size three, say $x_1 = 1, x_2 = -3, x_3 = 8$. Then its arithmetic mean is $\overline{x}_3 = \frac{1}{3}(1 + (-3) + 8) = 2$.

The arithmetic mean of the data $x_1, x_2, \ldots, x_n$ can be regarded as the center of gravity of the data, because we have an equality

$$\sum_{i=1}^{n}(x_i - \overline{x}_n) = 0$$

which is shown as $\sum_{i=1}^{n}(x_i - \overline{x}_n) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \overline{x}_n = n \times \overline{x}_n - n \times \overline{x}_n = 0$.

$\boxed{\text{Exercise 01}}$ Solve a linear equation $\displaystyle\sum_{i=1}^{n}(x_i - c) = 0$ for c, and verify that its solution is the arithmetic mean $\overline{x}_n$.

Answer: Since $0 = \displaystyle\sum_{i=1}^{n}(x_i - c) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} c = n \times \overline{x}_n - n \times c = n(\overline{x}_n - c)$, we get the solution $c = \overline{x}_n$.

$\boxed{\text{Exercise 02}}$ Calculate the arithmetic mean of the following data, and express it in the form of an irreducible fraction or an integer.

(1) $-3, 2, 5, 8$.

(2) $-9, -5, 0, 1, 8$.

(3) $1, 3, 6, 8, 11, 14, 21$

Answer:

(1) $\overline{x}_4 = ((-3) + 2 + 5 + 8)/4 = 12/4 = 3$.

(2) $\overline{x}_5 = ((-9) + (-5) + 0 + 1 + 8)/5 = (-5)/5 = -1$.

(3) $\overline{x}_6 = (1 + 3 + 6 + 8 + 11 + 14 + 21)/7 = 64/7$.