PROBLEM SET 4

DUE FEBRUARY 7

(1) Show that if A and B are countably infinite sets, then there exists a bijection between their power sets $P(A)$ and $P(B)$.
(2) 1.8.31
(3) Prove that a union $\bigcup_{n\in\mathbb{N}} A_n$ where sets A_n have the cardinality of the set of real numbers, has the cardinality of the set of real numbers.
(4) Prove that the set of irrational numbers has the same cardinality as the set of real numbers. Hint: consider a function $f : \mathbb{R} \to \mathbb{R} \setminus \mathbb{Q}$ given by
$$f(x) = \begin{cases} \arctan x & \text{when } \arctan x \in \mathbb{R} \setminus \mathbb{Q} \\ \arctan x + 10\sqrt{2} & \text{when } \arctan x \in \mathbb{Q} \end{cases}$$
and show that f is a well-defined injection. You can use the fact that $\sqrt{2}$ is not rational.
(5) 1.6.2
(6) 1.6.14. It refers to the relation described right above it.
(7) Let A be a set, and $P(A)$ its power set. For $x, y \in P(A)$ let $x \sim y$ if x and y have the same cardinality. Prove that $\sim$ is an equivalence relation. Compute the equivalence classes when $A = \{1, 2, 3\}$.
(8) 1.6.15. It refers to the relation described right above it.