PROBLEM SET 3

DUE JANUARY 31

(1) Let A and B be two (not necessarily finite) subsets of $\mathbb{N}$. Prove that there exists an injective function $g : A \to B$ if and only if there exists a surjective function $h : B \to A$.
(2) 1.7.19
(3) Find the inverse f^{-1} of the function f from Exercise 1.7.22.
(4) 1.7.24 (ii) and (iv)
(5) 1.7.25
(6) 1.7.38
(7) 1.8.12
(8) Prove that the set of positive rational numbers $\mathbb{Q}$ is countably infinite following the steps. The set $\mathbb{Q}_+$ is the set of positive rational numbers.
    (a) Construct an injection $g : \mathbb{N} \to \mathbb{Q}_+$.
    (b) Construct an injection $h : \mathbb{Q}_+ \to \mathbb{N} \times \mathbb{N}$.
    (c) Use the above, Exercise 1.8.12, and the fact $|\mathbb{N}| = |\mathbb{N} \times \mathbb{N}|$ (proved in a video), to conclude that $\mathbb{Q}_+$ is countably infinite.
    (d) We similarly show that the set of negative rational numbers is countable. Use Fact 1.8.25, to deduce that $\mathbb{Q}$ is countably infinite.

    For other proofs that $\mathbb{Q}$ is countably infinite, see Example 1.8.13.
(9) Give an example of a bijection $f : (0, 1) \to \mathbb{R}$. Explain, how this completes the proof that $\mathbb{R}$ is not a contable set (using the fact from one of the videos that there does not exists a bijection $\mathbb{N} \to (0, 1)$).
(10) (Bonus question) Give an example of a bijection $f : [0, 1) \to (0, 1)$.