PROBLEM SET 7

DUE FEBRUARY 28

(1) 3.2.9

(2) Show rigorously that $\sup\{1 - \frac{1}{n} \mid n \in \mathbb{N}\} = 1$.

(3) Let $S = \{1 - \frac{(-1)^n}{n} \mid n \in \mathbb{N}\}$. Find $\sup S$ and $\inf S$.

(4) Show that every bounded open interval $(a, b) \subseteq \mathbb{R}$ can be described as $\{x \in \mathbb{R} \mid |x - x_0| < \epsilon\}$ for some value $x_0 \in \mathbb{R}$ and some $\epsilon > 0$. What are the values of x_0 and ϵ in terms of a and b? And conversely, given x_0 and ϵ, what are the endpoints of the interval $\{x \in \mathbb{R} \mid |x - x_0| < \epsilon\}$?

(5) 3.4.3

(6) Let $I_n = [a_n, b_n]$ for $n \in \mathbb{N}$ be a nested collection of intervals. Suppose that $\inf\{b_n - a_n \mid n \in \mathbb{N}\} = 0$. Show that the number $\xi \in \bigcap_{n=1}^{\infty} I_n$ is unique.

(7) 3.6.5

(8) Show that the sequence $a_n = (-1)^n$ is divergent.

(Bonus) Use the nested interval theorem to obtain a new proof of the fact that $\mathbb{R}$ is uncountable.