PROBLEM SET 8

DUE MARCH 7

(1) Suppose $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.
   (a) Show that $\lim_{n\to\infty}(x_n + y_n) = x + y$.
   (b) Show that if for all $n \in \mathbb{N}$ $y_n \neq 0$ and $y \neq 0$, then $\lim_{n\to\infty}(\frac{1}{y_n}) = \frac{1}{y}$.

(2) Let $m \in \mathbb{N}$. Prove that $(x_n)_{n\in\mathbb{N}}$ converges if and only if $(x_{m+n})_{n\in\mathbb{N}}$ converges. Moreover, show that $\lim_{n\to\infty} x_n = \lim_{n\to\infty} x_{m+n}$.

(3) Show that $(x_n)_{n\in\mathbb{N}}$ converges to L if and only if every subsequence of $(x_n)_{n\in\mathbb{N}}$ converges to L.

(4) Prove directly from the definition that $a_n = \frac{n+2}{2n+1}$ is Cauchy.

(5) 3.6.13 (Note that part (i) is used in the proof of 3.6.14, so you should not use 3.6.14 here). Deduce that every convergent sequence is bounded.

(6) Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of strictly positive real numbers and suppose that $(a_n) \to a$.
   (a) Show that $a \geq 0$.
   (b) Show that $(\sqrt{a_n}) \to \sqrt{a}$.

(7) 3.6.18

(8) 3.6.21