Ch. 6: Long-Run Economic Growth

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Chapter Outline

- The Sources of Economic Growth
- Long-Rung Growth: The Solow Model
- Endogenous Growth Theory
- Government Policies to Raise Long-Run Living Standards

Output per Capita for Different Countries over Time

(1) Level of living standards
differs country to country
and time to time.

- Gap Minder – Trends (Real GDP per Capita, PPP$)
- http://bit.ly/2k2POr4

(2) Speed of economic growth
varies country to
country.

⇒ We want to know
why such differences
arise.

The Sources of Economic Growth

Contribution of labor growth.

- Production function

$$Y = AF(K, N)$$

level.

- Decompose into growth rate form → the growth accounting equation

TFP growth.

$$\frac{\Delta Y}{Y} = \boxed{\frac{\Delta A}{A}} + a_K \frac{\Delta K}{K} + a_N \frac{\Delta N}{N},$$

Growth rate.

where a_K and a_N are the elasticities of output with respect to capital and labor

Output growth rate.

Contribution of capital growth.

Typically we assume $a_K + a_N = 1$..

$$\frac{\Delta A}{A} = 10\% \qquad \frac{\Delta K}{K} = 3\% \qquad \frac{\Delta N}{N} = 5\%.$$

$$\frac{\Delta Y}{Y} = 10\% + a_K \times 3\% + a_N \times 5\%.$$

Suppose $a_K = 0.2$. and $a_N = 0.8$.

$$\Rightarrow \frac{\Delta Y}{Y} = 10\% + 0.2 \times 3\% + 0.8 \times 5\%.$$

$$= \underline{\underline{14.6\%}} \ ?$$

Usually we know $\frac{\Delta Y}{Y}$ $\frac{\Delta K}{K}$. $\frac{\Delta N}{N}$.

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - a_K \frac{\Delta K}{K} - a_N \frac{\Delta N}{N}.$$

We can back out productivity growth.

as residual.

Sources of Economic Growth in the US

Source of Growth	(1) 1929–1948	(2) 1948–1973	(3) 1973–1982	(4) 1929–1982	(5) 1982–2013
Labor	1.42	1.40	1.13	1.34	0.99
Capital	0.11	0.77	0.69	0.56	1.19
Productivity	1.01	1.53	-0.27	1.02	1.10
Output Growth	2.54	3.70	1.55	2.92	3.28

Table 6.3 Sources of Economic Growth in the US (% per year)

Note: Labor growth and capital growth are contributions of labor and capital ($a_N \frac{\Delta N}{N}$ and $a_K \frac{\Delta K}{K}$).

Productivity Growth and Labor Productivity Growth

- Labor productivity is defined as

$$\frac{Y}{N}$$

- We can relate productivity growth with labor productivity growth

∵ $a_K + a_N = 1.$

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + a_K \frac{\Delta K}{K} + a_N \frac{\Delta N}{N}$$

$$= \frac{\Delta A}{A} + a_K \frac{\Delta K}{K} + (1 - a_K)\frac{\Delta N}{N}$$

$$\underbrace{\frac{\Delta Y}{Y} - \frac{\Delta N}{N}}_{\text{growth rate of } \frac{Y}{N}} = \frac{\Delta A}{A} + a_K \underbrace{\left(\frac{\Delta K}{K} - \frac{\Delta N}{N}\right)}_{\substack{\text{growth rate of capital} \\ \text{per unit of labor}}}$$

growth rate of $\left(\frac{K}{N}\right)$

Productivity Growth in the US

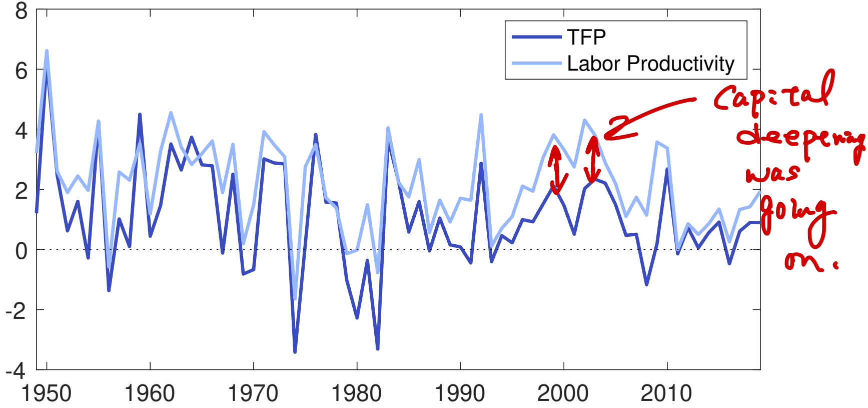

Fig. 6.2 Productivity Growth, 1949 – 2018

Source: FRED database, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org/series/MFPNFBS, https://fred.stlouisfed.org/series/OPHNFB.

Hayashi and Prescott (2002)

		Factors			
Period	Growth Rate	TFP factor	Capital intensity	Workweek length	Employment rate
1960–1973	7.2%	6.5%	2.3%	−0.8%	−0.7%
1973–1983	2.2%	0.8%	2.1%	−0.4%	−0.3%
1983–1991	3.6%	3.7%	0.2%	−0.5%	0.1%
1991–2000	0.5%	0.3%	1.4%	−0.9%	−0.4%

Table: Accounting for Japanse Growth per Person Aged 20-69

Source: Table 1 from Hayashi and Prescott (2002) “The 1990s in Japan: A Lost Decade” *Review of Economic Dynamics*, Vol. 5, 206–235.

Long-Run Growth: The Solow Model

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Two basic questions about economic growth:

(1) What's the relationship between the long-run standard of living and the saving rate, population growth rate, and rate of technological progress?

(2) How does economic growth change over time? Will it speed up, slow down, or stabilize?

The Solow Model

Basic Assumptions and Variables

- Population and work force grow at same rate n
- Economy is closed and no government, $G = 0$

$Y = C + I.$

NX=0.

$$C_t = Y_t - I_t \tag{1}$$

- Production function

level of technology. (can be time-varying)

$$Y_t = AF(K_t, N_t) \tag{2}$$

- No productivity growth (for now)

constant A.

- Transition equation for capital stock

# of machines available at t+1

$$I_t = K_{t+1} - \underbrace{(1-d)K_t}_{\text{undepreciated \# of machines}} \tag{3}$$

$K_{t+1} = I_t + (1-d)K_t.$

The Solow Model (Cont'd)

- Rewrite everything in per-worker terms:

output per worker

$$y_t = \frac{Y_t}{N_t} \qquad c_t = \frac{C_t}{N_t}$$

$$i_t = \frac{I_t}{N_t} \qquad k_t = \frac{K_t}{N_t} = \text{capital-labor ratio}$$

investment per worker.

- Rewrite eqs. (1)–(3)

$$C_t = Y_t - I_t \quad \Rightarrow \quad c_t = y_t - i_t$$

$$Y_t = AF(K_t, N_t) \quad \Rightarrow \quad y_t = f(k_t)$$

$$I_t = K_{t+1} - (1-d)K_t \quad \Rightarrow \quad i_t = \boxed{\frac{K_{t+1}}{N_{t+1}}}\frac{N_{t+1}}{N_t} - (1-d)\frac{K_t}{N_t}$$

$$= k_{t+1}(1+n) - (1-d)k_t$$

Implicit assumption. Production function has constant returns to scale property.

For Cobb-Douglas production function, $\alpha+\beta=1$.

$$Y_t = A \cdot F(K_t, N_t)$$

$$= A \cdot (K_t)^{\alpha} (N_t)^{1-\alpha}. \qquad (\because \alpha+\beta=1)$$

Divide the both sides by N_t.

$$\frac{Y_t}{N_t} = A \cdot \left(\frac{K_t}{N_t}\right)^{\alpha} \cdot \left(\frac{N_t}{N_t}\right)^{1-\alpha}.$$

$$y_t = \underline{\underline{A}} \cdot (k_t)^{\alpha} \cdot 1. \quad \Rightarrow \quad y_t = f(k_t).$$

The Per-Worker Production Function

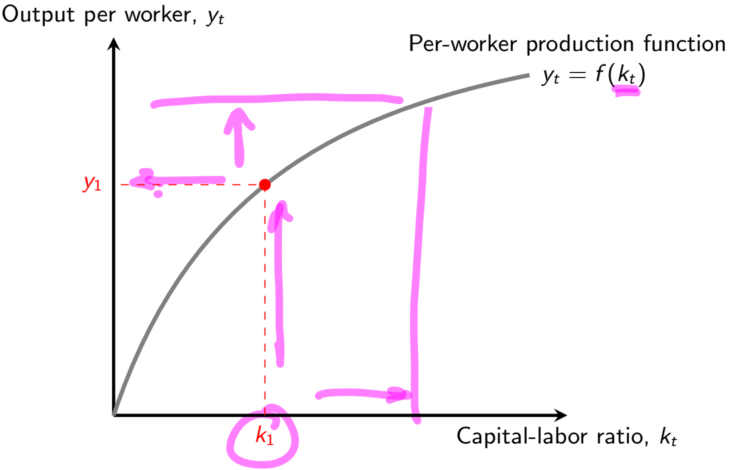


Figure: The Per-Worker Production Function

The Solow Model (Cont'd)

- Steady state (SS): all variables stay constant over time

$$y_t = y, \quad c_t = c, \quad k_t = k \quad \forall t$$

- Two key equations at the steady state:

$$i = (n + d)k \tag{4}$$
$$c = f(k) - (n + d)k \tag{5}$$

- In a steady state, gross investment must

(i) replace depreciated capital (dk)

(ii) expand so the capital stock grows as the economy grows (nk)

$$i_t = k_{t+1}(1+n) - (1-d)k_t.$$

$$k_{t+1} = k_t = k.$$

$$i = k(1+n) - (1-d)k.$$

$$i = (n+d)k.$$

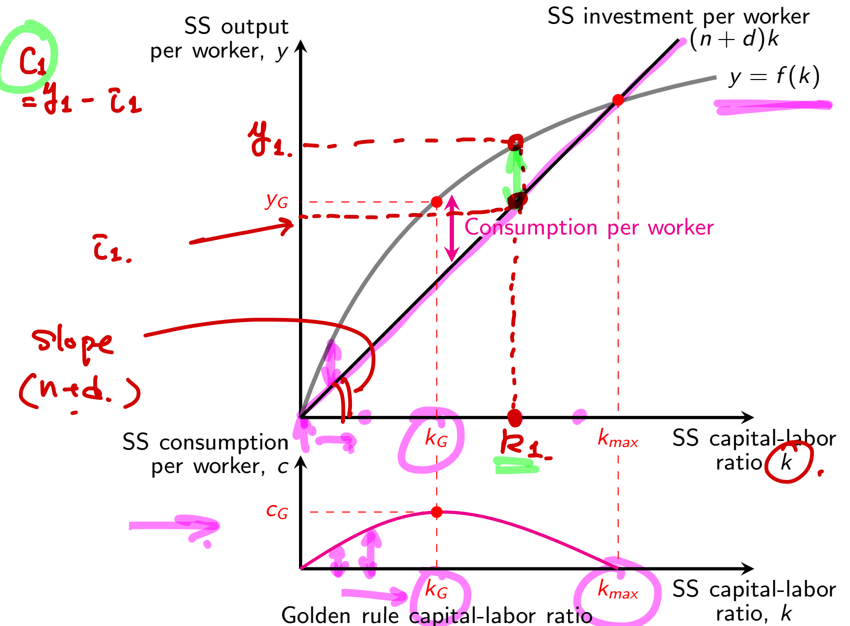
$$c_t = y_t - i_t.$$

$$c = y - i.$$

$$c = \underbrace{f(k)}_{\text{SS output per worker.}} - \underbrace{(n+d)k}_{\text{SS investment per worker.}}.$$

There can be many different steady states.

Figure: Steady-State Per Worker Consumption and the Capital-Labor Ratio



Moving Toward the Steady State

- Suppose saving is proportional to current income:

$$S_t = sY_t,$$

where $0 < s < 1$ is the saving rate

- In a per-worker term, per-worker saving is $sf(k_t)$.
- In the steady state, saving equals to investment

S = I.

$$\underline{sf(k)} = \underline{(n+d)k}$$

Steady-State Saving per worker.

SS investment per worker.

Moving Toward the Steady State (Cont'd)

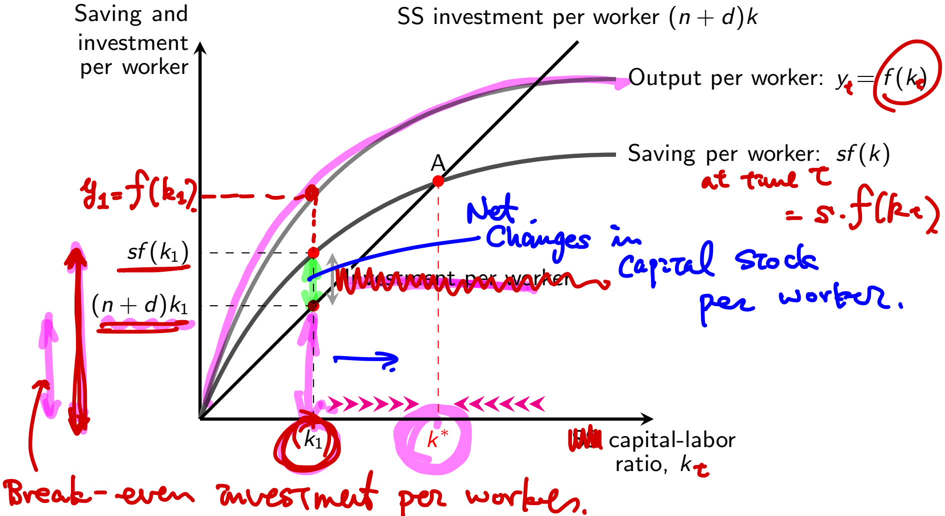
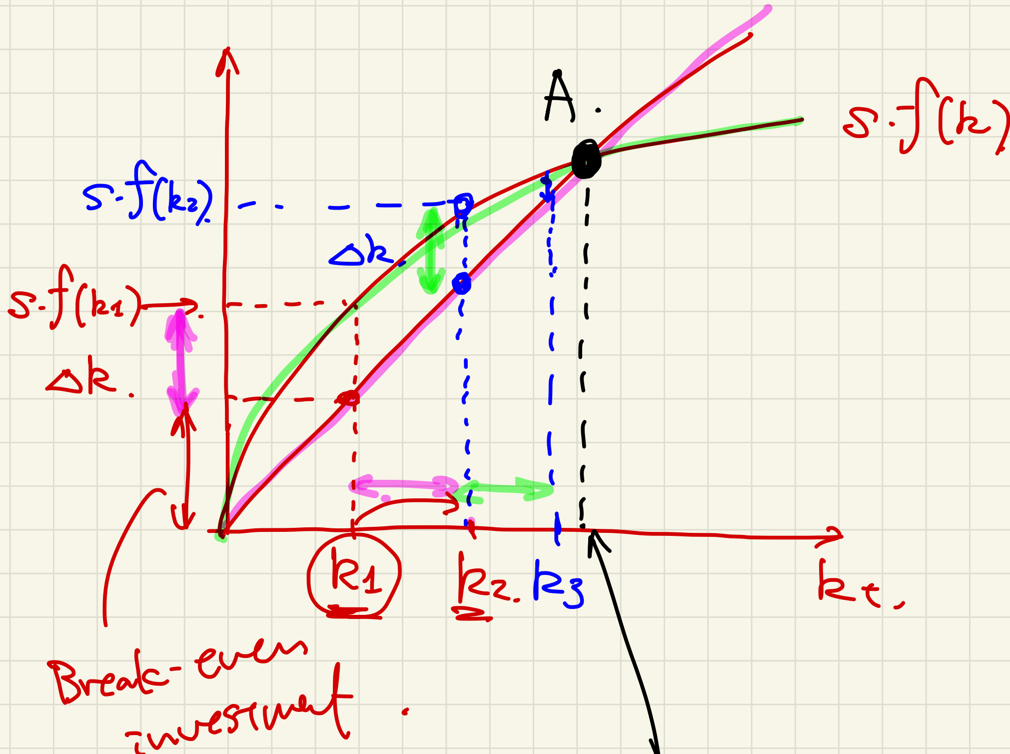


Figure: Determining the Capital-Labor Ratio in the Steady State



At A,
investment = break-even investment

$k_2 = k_1 + \Delta k$

$k_3 = k_2 + \Delta k$

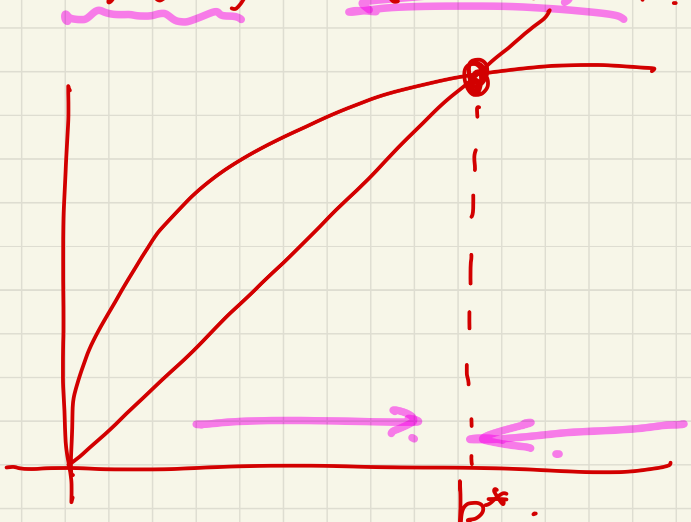
k^*: steady state capital stock per worker.

$k_t < k^*$ (left-hand side of k^*)

$\rightarrow \quad sf(k_t) > (n+d)\,k_t. \Rightarrow k\uparrow.$

$k_t > k^*$ (RHS of k^*)

$\rightarrow \quad sf(k_t) < (n+d)\,k_t. \Rightarrow k\downarrow.$



Summary of the Solow Growth Model

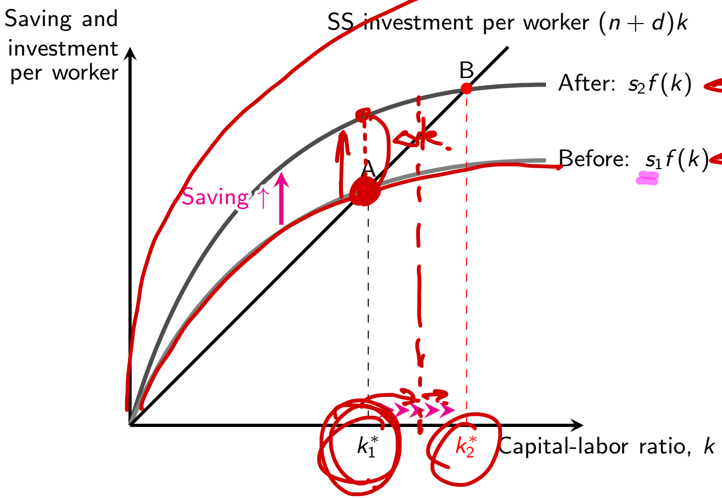
- The only possible steady-state capital-labor ratio is k^*, such that $sf(k^*) = (n + d)k^*$
- The steady-state k^* determines steady-state values of y^* and c^*
- With no productivity growth, the economy reaches a steady state, where capital-labor ratio (k), output per worker (y), and consumption per worker (c) stay constant

Fundamental Determinants of Long-Run Living Standards

- The saving rate
- Population growth
- Productivity growth

The Role of the Saving Rate

$y = f(k)$

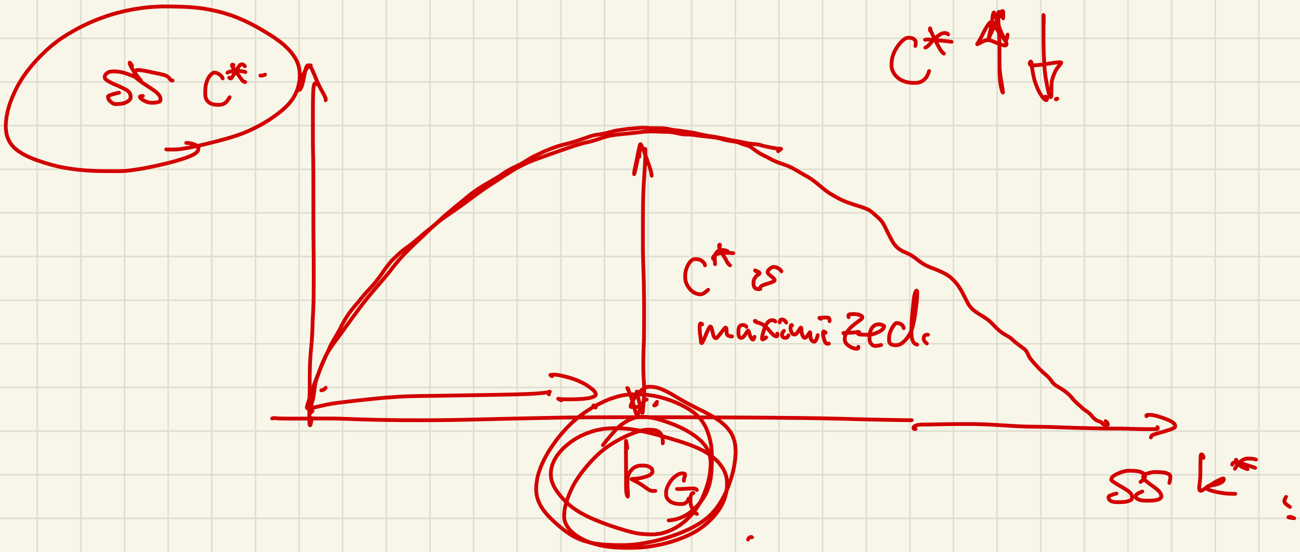


saving per worker

Figure: The Effect of an Increased Saving Rate on k^*

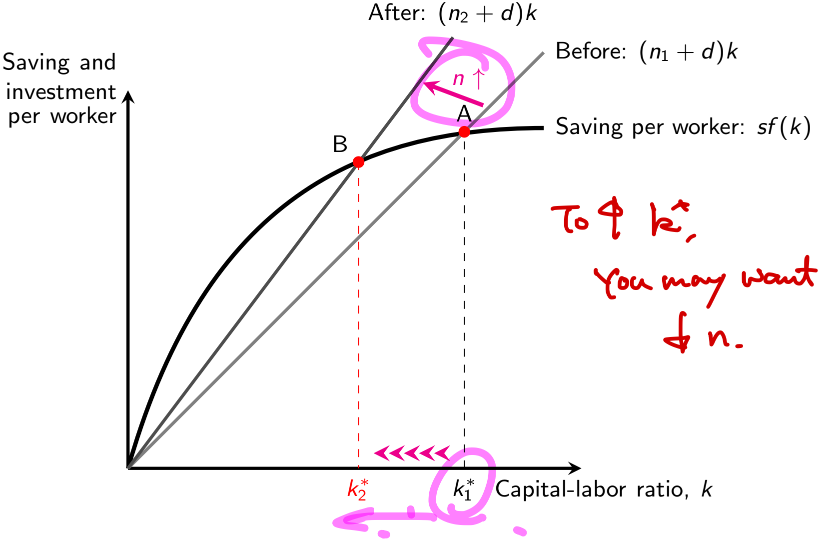
As the saving rate ↑, k^* ↑. y^* ↑.

c^* ↑↓



Optimal saving rate. that will support. the Golden-Rule Steady State.

The Role of Population Growth



To ↑ k^*,
You may want to ↓ n.

Figure: The Effect of a Higher Population Growth Rate on k^*

The Role of Productivity Growth

$y_t = f(k_t)$

$= \uparrow A (k_t)^{\alpha}.$

- The key factor in economic growth is productivity improvement
- Productivity improvement raises output per worker for a given level of the capital-labor ratio

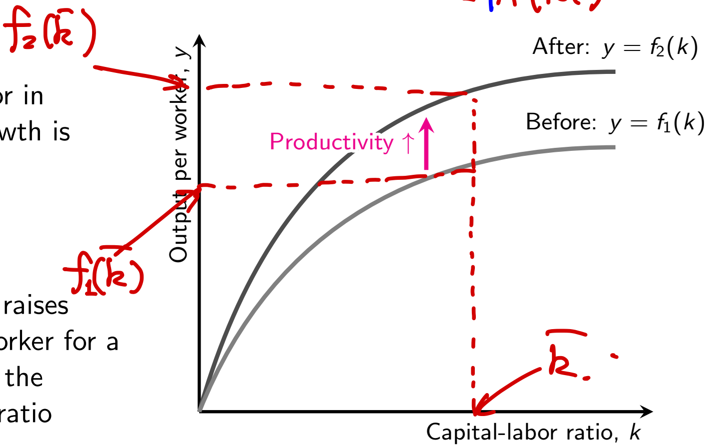


Figure: An Improvement in Productivity

The Role of Productivity Growth (Cont'd)

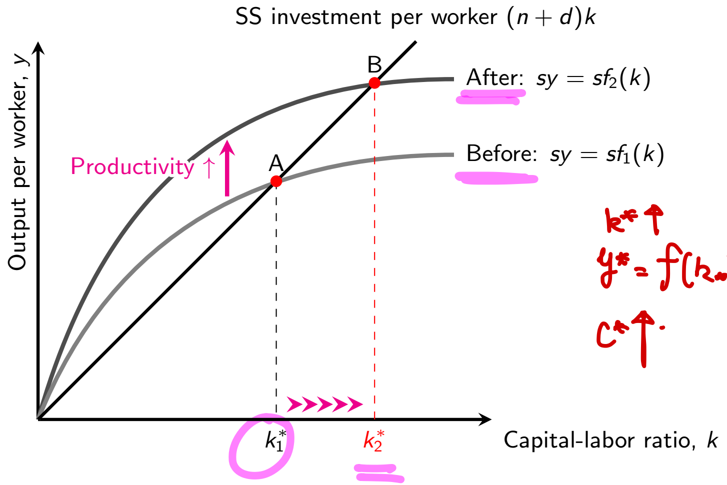


Figure: The Effect of a Productivity Improvement on k^*

$k^* \uparrow$

$y^* = f(k^*) \uparrow\uparrow$

$c^* \uparrow$

The Role of Productivity Growth (Cont'd)

- In equilibrium, productivity improvement increases k, y, and c.
- Productivity growth is the dominant factor for higher living standards!

Economic Growth of China

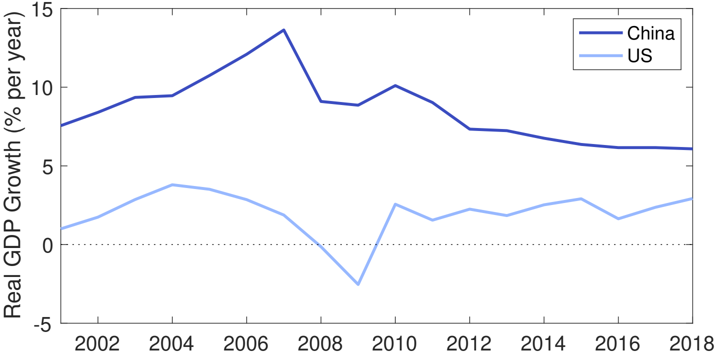


Fig. 6.10 Real GDP growth in China and the United States, 2001–2017

Source: FRED database, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org/series/NYGDPPCAPKDCHN, https://fred.stlouisfed.org/series/GDPC1.

Factors behind China's Economic Growth

- Huge increase in capital investment
- Fast productivity growth (in part from changing to a market economy)
- Increased trade

Will China be able to Keep Growing Rapidly?

- Rapid growth because of
  - use of underemployed resources
  - using advanced technology developed elsewhere
  - making transition from centrally-planned economy to market economy
- Such gains may not last
- It may take China a long time to catch up with the rest of the developed world

Endogenous Growth Theory ($\longleftrightarrow$ Exogenous growth in productivity in the Solow model)

- Tries to explain the sources of productivity growth
- Aggregate production function

$$Y = AK$$

In the Solow model.

$$Y = A(K)^{\alpha}(N)^{1-\alpha}.$$

$\downarrow$

MPK is diminishing

- Constant MPK
  - Human capital (knowledge, skills, training of individuals)
  - Research and development
  - Increases in capital and output generate increased technical knowledge, which offsets decline in MPK from having more capital

Endogenous Growth Theory (Cont'd)

- Suppose saving is a constant fraction of output

$$S = sY = sAK$$

- Investment is given by

$$I = \Delta K + dK$$

- Saving = Investment gives us

$$sAK = \underline{\Delta K + dK}$$

- Dividing both sides by K yields

$$\frac{\Delta K}{K} = \underline{sA - d} \quad \Leftarrow$$

growth rate of capital stock.

Endogenous Growth Theory (Cont'd)

$Y = AK.$

- Since output is proportional to capital, we have

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K}$$

- Thus,

$$\frac{\Delta Y}{Y} = sA - d$$

- The saving rate affects the long-run growth **rate** (not true in the Solow model)

Summary of Endogenous Growth Theory

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- It attempts to explain, rather than assume the economy's growth rate
- The growth rate depends on many things, such as the saving rate, that can be affected by government policies

Government Policies to Raise Long-Run Living Standards

- Affecting the saving rate
- Lowering population growth (in developing countries)
- Stimulating the rate of productivity growth

Policies to Raise the Rate of Productivity Growth

- Improving infrastructure (highways, bridges, utilities, dams, airports, and so on)?
- Building human capital (education policies, worker training programs, and health programs)
- Encouraging research and development (support scientific research, government research facilities, grants, and so on)