

Intro to Optimization

Optimization and Pattern Recognition Fall, 2021

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Types of Optimization

Finding Extremums

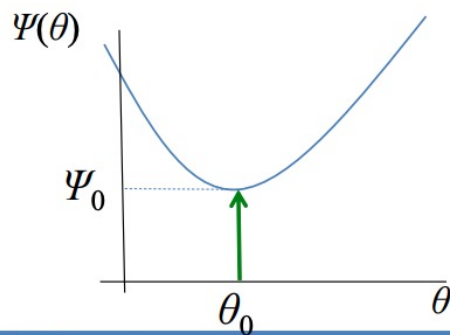
最適化とは：関数の値(最小値/最大値)を決める問題

$\Psi(\theta)$: 目的関数

θ : 説明変数

$\Psi(\theta)$ を θ で最小化 (あるいは最大化)

関数の最小値/最大値, それを与える θ を求める問題



$$\Psi_0 = \min_{\theta} \Psi(\theta)$$

$$\theta_0 = \operatorname{argmin}_{\theta} \Psi(\theta)$$

Determining Shape of Function

最適化とは：関数の形を決める問題

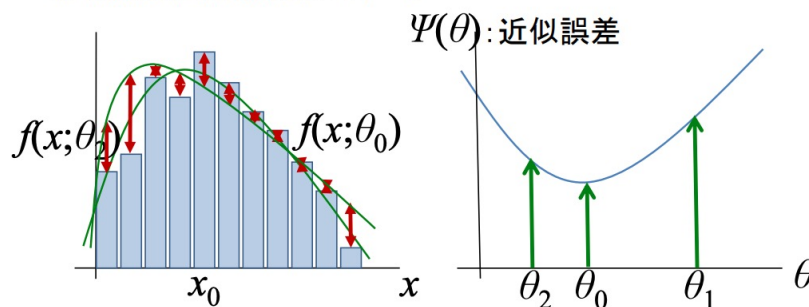
$\Psi(\theta)$: 目的関数

θ : 説明変数

$\Psi(\theta)$ を θ で最小化 (あるいは最大化)

関数 $f(x; \theta)$ の形状を求める問題

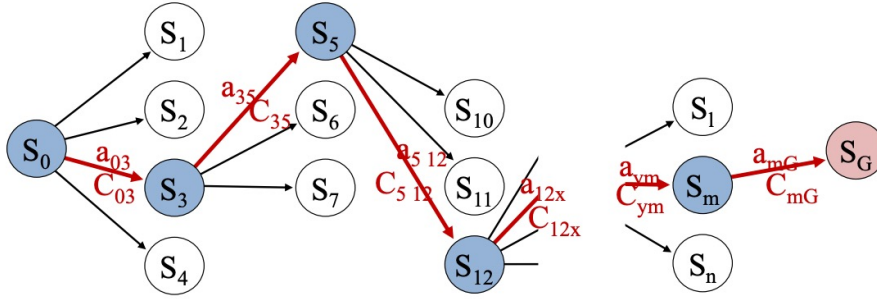
θ : 関数の形状を決めるパラメータ



Find the value of variable θ_0 such that the error function $\Psi(\theta)$ is minimized. Note that this optimization problem can be translated into the first type (optimizing by finding extremums) through the use of an error function. Many approximation problems make use of this method.

Determining Optimal Sequence in Sequential Decision Process

最適化とは：行動の列を決める問題（逐次決定過程）



$\theta = \{a_{03}, a_{35}, \dots, a_{mG}\}$: 行動の列,
 $A = \{03, 35, \dots, mG\}$: θ が決める
状態遷移の列,

$$\Psi(\theta) = \sum_{ij \in A} C_{ij} \quad : \theta \text{が決める遷移に沿って得られる費用／利得の総和}$$

異なる行動を選べば、費用／利得の総和も変わる

θ represents a sequence of actions, A represents the sequence of states determined by θ , and $\Psi(\theta)$ is the cost of taking a certain θ , computed as $\sum_{ij \in A} C_{ij}$.

Formal Definition

$\Psi(\theta)$: Objective (Target) Function

θ : Explanatory Variable determining Ψ

Optimization means to minimize (maximize) Ψ using θ .

Formal Characterization

Continuous Optimization

- **Nonlinear Programming Problem:**
Includes nonlinearity in objective function or constraints.
 - *Quadratic Programming Problem:*
Objective function is a quadratic function, constraints are linear
 - *Convex Programming Problem:*
Objective function is a convex function, area of constraint is convex
- **Linear Programming Problem:**
Objective function is linear, constraints are linear

Discrete Optimization

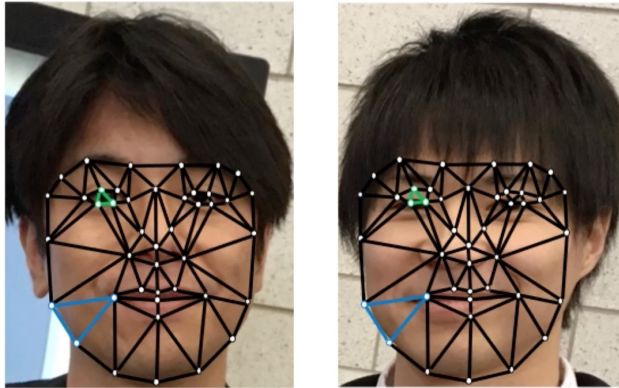
- **Combinatorial Optimization Problem**
- **Integer Programming Problem**
 - *0-1 Integer Programming Problem:*
Variables are either 0 or 1

Application Example: Facial Recognition

θ : Model for Facial Features

$\Psi(\theta)$: Pixel Error

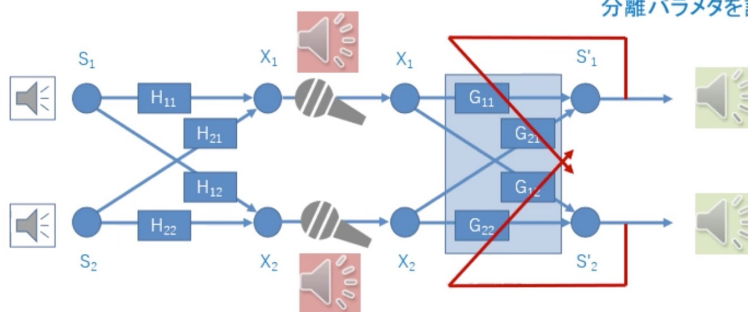
Begin by defining a model θ for facial features. The one below takes several prominent points of the face and connects them to form triangles. Then, we can define the error function $\Psi(\theta)$ to be the squared sum of pixel differences; that is, if a triangle on the training set captures black pixels but when applied to the testing set captures beige pixels, that is computed as an error. Then, the θ that minimizes $\Psi(\theta)$ provides us with a model for the test data.



Application Example: Audio Source Separation

Two audio sources, two microphones, 4 paths (transformation functions) for the audio to travel. We can reverse this filter by feeding it back into the loop until each output audio sounds closer and closer to a normal human being.

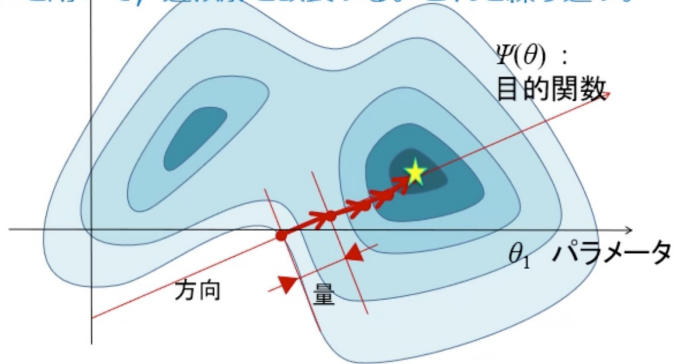
連想記憶(DNN)利用に基づく
分離フィルタの更新



Solving Continuous Optimization: Gradient Method

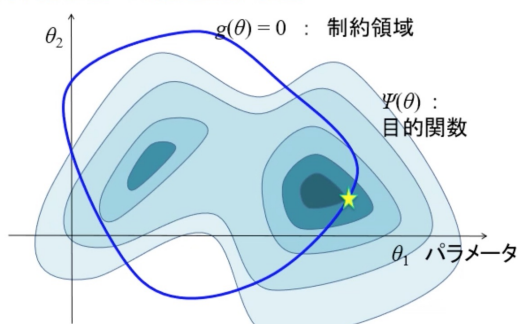
There is no analytic solution to continuous optimization, but one method is to approximate an initial value and study the gradient in the neighborhood of this value to arrive at the optimum.

勾配法：まずある近似値を定め、この近似解の近傍の関数の勾配（等）を用いて、近似解を改良する。これを繰り返す。

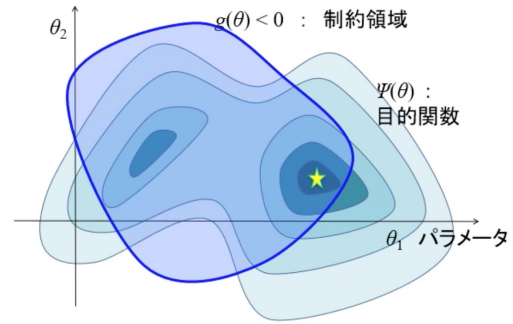


We can add equality and inequality constraints of the parameters θ_1, θ_2 to the above:

等式制約付きの最適化問題

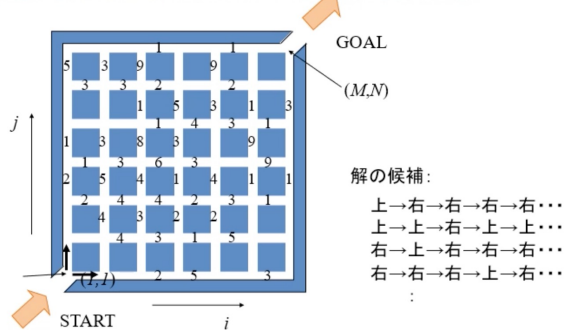


不等式制約付きの最適化問題



Discrete Optimization Problems:

離散最適化の応用例：最適経路探索



離散最適化の応用例：最適経路探索

