2016 年度春学期 経済数学 I（解析学基礎）（月 2）

×【第 10 回: 極値問題】(担当: 瀧澤 武信) 提出期限: 6 月 27 日 (月) 17:00 (出題日 : 6 月 20 日 (月))

○【第 10 回: 極値問題】(担当: 瀧澤 武信) 提出期限: 7 月 4 日 (月) 17:00 (出題日 : 6 月 27 日 (月))

問題

関数の極値 (Extremum) を求めよ.　　(1+1*3 点)

1. $f(x) = x^2 \sin x \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$

2. $f(x) = \sin(x^2) \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$

3. $f(x) = (\sin x)^2 \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$

解答例

関数の極値 (Extremum) を求める.

1. $f(x) = x^2 \sin x \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$ のとき, 極値なし.

2. $f(x) = \sin(x^2) \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$ のとき, $f(0) = 0$: 極小値, $f(\pm\sqrt{\frac{\pi}{2}}) = 1$: 極大値.

3. $f(x) = (\sin x)^2 \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$ のとき, $f(0) = 0$: 極小値.

解説

1.

$$
\begin{aligned}
f(x) &= x^2 \sin x \quad (-\frac{\pi}{2} < x < \frac{\pi}{2}) \text{ のとき,} \\
f'(x) &= 2x \sin x + x^2 \cos x \\
f''(x) &= 2 \sin x + 2x \cos x + 2x \cos x - x^2 \sin x \\
&= (2 - x^2) \sin x + 4x \cos x
\end{aligned}
$$

$$
\begin{aligned}
f'(x) = 0 &\leftrightarrow 2x \sin x + x^2 \cos x = x(2 \sin x + x \cos x) = 0 \\
&\Leftrightarrow x = 0 \text{ または } -2 \ \sin x = x \cos x \leftrightarrow x = -2\frac{\sin x}{\cos x} = -2 \tan x \\
&\Leftrightarrow x = 0 \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})
\end{aligned}
$$

$$
\begin{aligned}
f''(0) &= 0 \\
f'''(x) &= -2x \sin x + (2 - x^2) \cos x + 4 \cos x - 4x \sin x \\
&= -6x \sin x + (6 - x^2) \cos x
\end{aligned}
$$

$f'''(0) = 6 > 0 \Longrightarrow f(0)$: 増加点 $\Longrightarrow y = f(x)$: 極値なし.

2.

$$f(x) = \sin(x^2) \quad (-\frac{\pi}{2} < x < \frac{\pi}{2}) \quad \text{のとき},$$
$$f'(x) = \cos(x^2) \cdot 2x$$
$$f''(x) = -\sin(x^2) \cdot 2x \cdot 2x + \cos(x^2) \cdot 2$$
$$= -4x^2 \sin(x^2) + 2\cos(x^2)$$

$f'(x) = 0 \leftrightarrow x = 0$ または $x^2 = \frac{\pi}{2} + n\pi$

$$x^2 = \frac{\pi}{2} \approx 1.57 \leftrightarrow x = \pm\sqrt{\frac{\pi}{2}} \approx \pm 1.3$$
$$x^2 = \frac{3\pi}{2} \approx 4.71 \leftrightarrow x = \pm\sqrt{\frac{3\pi}{2}} \approx \pm 2.2$$
$$1.3 < \frac{\pi}{2} < 2.2 \quad \text{より}$$
$$x = \pm\sqrt{\frac{\pi}{2}} \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$$

$f''(0) = 2 > 0 \Longrightarrow f(0) = 0$: 極小値,

$$f''(\pm\sqrt{\frac{\pi}{2}}) = -4 \cdot \frac{\pi}{2}\sin(\frac{\pi}{2}) + 2\cos(\frac{\pi}{2})$$
$$= -2\pi \cdot 1 + 2 \cdot 0$$
$$= -2\pi < 0 \Longrightarrow f(\pm\sqrt{\frac{\pi}{2}}) = \sin(\frac{\pi}{2}) = 1 : \text{極大値}.$$

3.

$$f(x) = (\sin x)^2 \quad (-\frac{\pi}{2} < x < \frac{\pi}{2}) \quad \text{のとき},$$
$$f'(x) = 2\sin x \cos x$$
$$f''(x) = 2\cos x \cos x - 2\sin x \sin x$$
$$= 2[(\cos x)^2 - (\sin x)^2]$$

$$f'(x) = 0 \Leftrightarrow x = 0 \quad (-\frac{\pi}{2} < x < \frac{\pi}{2})$$

$f''(0) = 2(1-0) = 2 > 0 \Longrightarrow f(0) = 0$: 極小値.