8 Kiyotaki-Moore Model

8.1 Introduction

Research Questions

- Why there are boom and bust?
- Why do we need to care about the financial market?
- Why do governments rescue banks in recession?
- This motivation comes from Japan's experience
  - Japan's bubble & burst associated with land prices.
  - Special role of land? Japan's banking system?

Readings

- Published version: Journal of Political Economy, 1997
  We study the full model in Section III

- Excellent summary in Japanese.
  `https://www.imes.boj.or.jp/research/papers/japanese/kk12-4-6.pdf`

- Unpublished version. Take a look at Figure 4.
  `https://www.minneapolisfed.org/economic-research/conferences/~/media/files/research/events/1993_09-17/Moore_CreditCycles.pdf`

8.2 Financial market imperfections

- The financial market equilibrium does not attain the efficient allocation due to several reasons.
- Firms want to borrow money from banks (or investors?), but the loan may be insufficient. This limitation is called credit constraint. (信用制約)
- There are everal sources of credit constraints.
- Imperfect information (不完全情報): two types
  1. Moral hazard: After the loan contract, the firm may not make enough effort. The firm's behavior after the contract is unobservable
  2. Adverse selection (逆選択): the investor may not distinguish good and bad firms. Information before the contract is unobservable.
- Incomplete contract (不完備契約)
  - Kiyotaki-Moore's assumption
  - information is perfect, but agents cannot write detailed contracts.
  - Renegotiation may be possible.

8.3 Incomplete contract

- Simple example
- A company (called farmer in Kiyotaki-Moore) wants to build a factory.

It needs land k as input.

- Initially, the company owns land e. It also borrows money b and purchases land. Suppose the land price is 1. Then, $k = e + b$.
- A bank (called gatherer in Kiyotaki-Moore) plans to invest in the company. The gross interest rate is fixed at $R = 1 + r$. (small open economy, 小国開放経済)
- The company's owner may fly with money. It may not repay Rb, but the land k is attached to the bank. It is called *collateral.* (担保) Under the legal contract, the company promises to given up land k in case of insolvency.
- Suppose that people expect the future land price as q. Then, in case of insolvency, the bank will get qk.
- If $Rb > qk$, the company's owner will ask renegotiation. "Please reduce the debt, otherwise I will ran away!"
- The bank expects it at the timing of contract. It lends only

$$Rb \leq qk \;\Leftrightarrow\; b \leq \frac{qK}{R} = \frac{q(e+b)}{R} \;\Leftrightarrow\; b \leq \frac{qe}{R-q}$$

- $R - q$ is called *user cost* or *down payment* (頭金・手付金). It is the net amount of money (per unit of capital) that the company needs to pay.
- qe is called net worth. (純資産) It is also interpreted as capital or equity (自己資本) on balance sheets in accounting.
- If this holds with equality, the financial market is inefficient.
- A key property is that the net worth depends on the future land price q. In bubble period, people expect an increase in q. Firms can invest more, production will increase in the future, then the market expects more return. Eventually, q goes up again. This credit cycle makes the financial market boom. The opposite direction is possible. It is financial crisis.

8.4 Model

- This is a very unique model. Non-standard settings (agents, preference, production, etc.) to obtain tractable equilibrium. The goal is to highlight the main mechanism.
- Two types of agents
  - Farmers: $\sum_{t=0}^{\infty} \beta^t x_t$
  - Gatherers (集金人): a kind of investor with low productivity
  - The utility function is linear (corner solution!)
  - Assume small enough β. Farmers borrow and produce as much as possible, in other words, farmers' credit constraints are always binding.
- Two types of physical goods
  - Land: fixed supply $\bar{K}$ in total with price q_t
  - Fruit: numeraire (Price is 1)

Land must be cultivated (開墾) for production.

$$\underbrace{k_{t-1}}_{\text{cultivated land}} \rightarrow \begin{cases} \lambda k_{t-1} & \text{kept cultivated next period} \\ (1-\lambda)k_{t-1} & \text{worn out. needs cultivation for future use.} \\ ak_{t-1} & \text{tradable fruit,} \\ ck_{t-1} & \text{non-tradable fruit.} \end{cases}$$

Farmers are more productive than Gatherers. Farmers sell all the tradable fruits given small β. They consume only ck_{t-1} in the equilibrium.

- Bond: gross interest rate R
- Budget constraints
  - Only a part of farmers (π is the ratio) have the cultivation opportunity.
  - Fruits are necessary fertilizer (肥料) for cultivation.
  - Think about lucky π farmers. It owns k_{t-1} land in total and λk_{t-1} cultivated land. They decide to hold k_t cultivated land.*1 Then, the lucky farmers purchase $k_t - k_{t-1}$ land and use $\phi(k_t - \lambda k_{t-1})$ fruits as fertilizer. Their budget constraint is

    $$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + Rb_{t-1} = ak_{t-1} + b_t$$

    Note that consumption $x_t = ck_{t-1}$ is eliminated.
  - Unlucky $(1-\pi_t)$ farmers sell all uncultivated land.
- Land k_t is used as collateral. Credit constraint:

  $$Rb_t \leq q_{t+1}k_t.$$

  Note again that q_{t+1} is the future price.
- By small enough β, farmers with investment opportunity borrow as much as possible.

  $$b_t = \frac{q_{t+1}k_t}{R}$$

- Substitute b_t from budget constraint, we have

  $$\underbrace{(q_t + \phi - \frac{q_{t+1}}{R})}_{\text{user cost}} k_t = \underbrace{(a + q_t\lambda + \phi\lambda)k_{t-1} - Rb_{t-1}}_{\text{net worth}}$$

8.5 Equilibrium

- K_t and B_t are total amount of cultivated land and bond owned by all farmers.
- We want to get a system of three equations for K_t, B_t, and q_t.
- π shock is i.i.d. The aggregate variables of lucky farmers are always πK_t and πB_t, and those of unlucky are $(1-\pi)K_t$ and $(1-\pi)B_t$.
- Land Demand

*1 All land is cultivated in equilibrium. Suppose, on contrary that a farmer holds useless land. The farmer wants to sell it if $q_t > 0$. To hold equilibrium, somebody must demand it. It means, this person has an incentive to cultivate it. By symmetry of agents, it contradicts the first assumption.

- Lucky farmers

$$\pi K_t = (\frac{1}{q_t + \phi - \frac{q_{t+1}}{R}})[(a+\phi\lambda)\pi K_{t-1} - R\pi B_{t-1}]$$

- Unlucky farmers

$$(1-\pi)K_t = \lambda(1-\pi)K_{t-1}$$

- Total

$$K_t = \lambda(1-\pi)K_{t-1} + (\frac{1}{q_t + \phi - \frac{q_{t+1}}{R}})[(a+\phi\lambda)\pi K_{t-1} - R\pi B_{t-1}]$$

- Bond demand
  - By the budget constraint of lucky farmers (before substitution of the credit constraint),

$$(1-\pi)B_t = R(1-\pi)B_{t-1} + q_t[(1-\pi)K_t - (1-\pi)K_{t-1}] + \phi[(1-\pi)K_t - \lambda(1-\pi)K_{t-1}] - a(1-\pi)K_{t-1}$$

    We can eliminate $(1-\pi)$.
- Land supply
  - The last equation is

$$q_t - \frac{q_{t+1}}{R} = K_t - V$$

    where V is a constant.
  - It's not important, so let me skip the details.
  - In short, it is a supply function of land. Gatherers hold $\overline{K} - K_t$ land. They can also produce fruit with relatively inefficient technology. The left-hand side, $q_t - \frac{q_{t+1}}{R}$, is user cost. It is a kind of price. If the user cost increases, they have less incentive for production and sell more land K_t.
- Summary of the system

$$K_t = \lambda(1-\pi)K_{t-1} + (\frac{1}{q_t + \phi - \frac{q_{t+1}}{R}})[(a+\phi\lambda)\pi K_{t-1} - R\pi B_{t-1}]$$
$$B_t = RB_{t-1} + q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) - aK_{t-1}$$
$$q_t - \frac{q_{t+1}}{R} = K_t - V$$

8.6 Simulation

- Numerical Solution
  MIT shock: zero probability shock in deterministic model. In dyanre, deterministic simulation with changing productivity a_t only at the initial period.
- Dynare also calculates the log-linearized system

$$\begin{bmatrix} \tilde{K}_t \\ \tilde{B}_t \\ \tilde{q}_t \end{bmatrix} = Q \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} Q^{-1} \begin{bmatrix} \tilde{K}_{t-1} \\ \tilde{B}_{t-1} \\ \tilde{q}_{t-1} \end{bmatrix}$$

  Two eigenvalues are imaginary numbers! They make the cycles.
- Simulation result: By the amplification, K_t and q_t increase first, but eventually B_t becomes large enough and makes a downward pressure. (The return R is constant, while q_t is endogenous)