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解析学入門 演習問題 解答例

次の関数を微分せよ.

(1) $f(x) = x^2(1-x) \Longrightarrow f'(x) = 2x - 3x^2 \ (= -3x^2 + 2x)$

(2) $f(x) = 1 - x^2 \Longrightarrow f'(x) = -2x$

(3) $f(x) = (1-x)^2 \Longrightarrow f'(x) = -2 + 2x \ (= 2x - 2)$

(4) $f(x) = x^2 \log x \quad (x > 0) \Longrightarrow f'(x) = x(2\log x + 1)$

(5) $f(x) = \log(x^2) \quad (x > 0) \Longrightarrow f'(x) = \dfrac{2}{x}$

(6) $f(x) = (\log x)^2 \quad (x > 0) \Longrightarrow f'(x) = \dfrac{2\log x}{x}$

(7) $f(x) = x^2 e^x \Longrightarrow f'(x) = xe^x(x+2)$

(8) $f(x) = e^{x^2} \Longrightarrow f'(x) = 2xe^{x^2}$

(9) $f(x) = (e^x)^2 \Longrightarrow f'(x) = 2e^{2x}$

(10) $f(x) = e^{(\log x)^2+1} \quad (x > 0) \Longrightarrow f'(x) = \dfrac{2e^{(\log x)^2+1}\log x}{x}$

演習問題解説

(1) $f'(x)=(x^2)'\cdot(1-x)\ +x^2\cdot(1-x)'=2x\cdot(1-x)+x^2\cdot(-1)$
$=2x-2x^2-x^2=2x-3x^2$

(注) $f(x)=x^2-x^3\Rightarrow f'(x)=2x-3x^2$

(2) $f(x)=1-x^2\Longrightarrow \begin{cases} y=1-u \\ u=x^2 \end{cases}$
$\Rightarrow \dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}=(-1)\cdot(x^2)'=-2x$
(注)$f'(x)=(1)'-(x^2)'=0-2x=-2x$

(3) $f(x)=(1-x)^2\Longrightarrow \begin{cases} y=u^2 \\ u=1-x \end{cases}$
$\Rightarrow \dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dx}=2u\cdot(1-x)'=2(1-x)\cdot(-1)=-2+2x$

(注) $f(x)=1-2x+x^2\Rightarrow f'(x)=-2+2x$

(4) $f(x)=x^2\log x \qquad (x>0)\Longrightarrow$
$f'(x)=2x\cdot\log x\ +\ x^2\cdot\frac{1}{x}=x(2\log x\ +\ 1)$

(5) $f(x)=\log(x^2) \qquad (x>0)\Longrightarrow f'(x)=\dfrac{1}{x^2}\cdot(2x)=\dfrac{2}{x}$

(注 1) $f(x)=2\log x\Rightarrow f'(x)=\dfrac{2}{x}$
(注 2) x の条件は $x\neq 0$ でもよい.

(6) $f(x)=(\log x)^2 \qquad (x>0)\Longrightarrow f'(x)=2(\log x)\cdot\dfrac{1}{x}=\dfrac{2\log x}{x}$

(7) $f(x)=x^2e^x\Longrightarrow f'(x)=2xe^x+x^2e^x=xe^x(x+2)$

(8) $f(x)=e^{x^2}\Longrightarrow f'(x)=e^{x^2}\cdot 2x=2xe^{x^2}$

(9) $f(x)=(e^x)^2\Longrightarrow f'(x)=2e^x\cdot e^x=2e^{2x}$

(注) $f(x)=e^{2x}\Rightarrow f'(x)=e^{2x}\cdot 2=2e^{2x}$

(10) $f(x)=e^{(\log x)^2+1} \qquad (x>0)$
$\Longrightarrow f'(x)=e^{(\log x)^2+1}\cdot[(\log x)^2\ +\ 1]'=e^{(\log x)^2+1}\cdot 2(\log x)\cdot\dfrac{1}{x}$
$=\dfrac{2e^{(\log x)^2+1}\ \log x}{x}$

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金利の計算（解答例）

$f(x)=\left(1+\dfrac{1}{x}\right)^x$ のとき，$\displaystyle\lim_{x\to+\infty} f(x)$ が存在することを仮定し，
その値を e で表す．このとき，

(1)

$$\begin{aligned}
\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x &= \lim_{t\to+\infty}\left(1-\frac{1}{t}\right)^{-t} \quad (t \overset{\leftarrow}{=} -x)\\
&= \lim_{t\to+\infty}\left[\frac{1}{\left(1-\frac{1}{t}\right)}\right]^t\\
&= \lim_{t\to+\infty}\left(\frac{t}{t-1}\right)^t\\
&= \lim_{r\to+\infty}\left(\frac{r+1}{r}\right)^{r+1} \quad (r \overset{\leftarrow}{=} t-1)\\
&= \lim_{r\to+\infty}\left[\left(1+\frac{1}{r}\right)^r\left(1+\frac{1}{r}\right)\right]\\
&= e
\end{aligned}$$

(2)

$$\begin{aligned}
\lim_{x\to+\infty} c\left(1+\frac{1}{x}\right)^{ax} &= c\lim_{x\to+\infty}\left[\left(1+\frac{1}{x}\right)^x\right]^a\\
&= ce^a
\end{aligned}$$

(3)

$$A \overset{\leftarrow}{=} \lim_{x\to+\infty} c\left(1+\frac{a}{x}\right)^x \text{とする．}$$

(i)
$$\begin{cases}
a=0 \Longrightarrow \displaystyle\lim_{x\to+\infty} c\left(1+0\right)^x = c\\
a\neq 0 \Longrightarrow A = c\displaystyle\lim_{x\to+\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{\frac{x}{a}\cdot a}
\end{cases}$$

(ii)
$$\begin{cases}
a>0 \Longrightarrow \frac{x}{a}\to+\infty\\
A = c\left[\displaystyle\lim_{\frac{x}{a}\to+\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{\frac{x}{a}}\right]^a = ce^a
\end{cases}$$

(iii)
$$\begin{cases}
a<0 \Longrightarrow \frac{x}{a}\to-\infty\\
A = c\left[\displaystyle\lim_{\frac{x}{a}\to-\infty}\left(1+\frac{1}{\frac{x}{a}}\right)^{\frac{x}{a}}\right]^a = ce^a\\
\qquad\qquad\qquad\qquad\text{((1) 参照)}
\end{cases}$$

(i) では $c=ce^0$ であるから，$\displaystyle\lim_{x\to+\infty} c\left(1+\frac{a}{x}\right)^x = ce^a$

(4)

$$\lim_{x\to+\infty} c\left(1+\frac{a}{x}\right)^{bx} = c\,(1+\alpha) \text{ とする } (b \neq 0\ ,\ c \neq 0).$$

$$(\text{左辺}) = c\cdot e^{ab} \Longrightarrow e^{ab} = 1+\alpha\ \ (c\neq 0)$$

$$ab = \log(1+\alpha) \Longrightarrow a = \frac{\log(1+\alpha)}{b}\ \ (b\neq 0)$$

(5)

$$\lim_{x\to+\infty} 10000\left(1+\frac{a}{x}\right)^{x} = 10000\,(1+0.1) \text{ とする}.$$

$$(\text{左辺}) = 10000\cdot e^{a} \Longrightarrow e^{a} = 1+0.1$$

$$a = \log 1.1$$

(6)

$$y = \lim_{x\to+\infty} 10000\left(1+\frac{a}{x}\right)^{\frac{182.5}{365}x} = 10000\,e^{\frac{182.5}{365}a} = 10000\,(e^{a})^{\frac{182.5}{365}}$$

$$a = \log 1.1 \Longrightarrow e^{\log 1.1} = 1.1$$

$$y = 10000\times 1.1^{\frac{1}{2}} = 10000\sqrt{1.1}$$