塙宏 : $f(x) = \left(1+\frac{1}{x}\right)^x$ ⚅ ♯ ♀, $\lim_{x\to+\infty} f(x)$ ☞ 圷僞♝ 〗.

(封呈)

(1) $$\underline{x \in N = \{1, 2, 3, \ldots\}} \text{ ⚅ ♯ ♀}$$

$n = x$ ♯♝ 〗.

$$\begin{aligned}\left(1+\frac{1}{n}\right)^n &= {}_n\mathrm{C}_0 + {}_n\mathrm{C}_1\frac{1}{n} + {}_n\mathrm{C}_2\left(\frac{1}{n}\right)^2 + {}_n\mathrm{C}_3\left(\frac{1}{n}\right)^3 + \cdots + {}_n\mathrm{C}_n\left(\frac{1}{n}\right)^n \\ &= 1 + \frac{n}{1!}\times\frac{1}{n} + \frac{n(n-1)}{2!}\times\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\times\frac{1}{n^3} + \cdots + \frac{1}{n^n} \\ &= 1 + \frac{1}{1!} + \frac{1}{2!}\left(1-\frac{1}{n}\right) + \frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + \cdots + \frac{1}{n^n}\end{aligned}$$

♜♤☞♨♬,

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots = \sum_{k=0}^{+\infty}\frac{1}{k!}(\text{仳努})$$

仳努√俅☞犴俦俨◣ 〗塘♭◣ 〗☜〕, ♢⚅”塘”√嚅员⚀√仫」〕〝⚀◥. 啰噬塘⚅努墺⚅丗俦☞圷僞♝〗♯♀, 仳努☞圷僞♝〗♯◥○.

♗♗♭, 啰噬塘

$$S_p \overset{\leftarrows}{=} 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{p!}$$

メ偏僟♝〗.

$$S_{p+1} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{p!} + \frac{1}{(p+1)!}$$

♭◣ 〗☜〕,

(A) $$S_{p+1} - S_p = \frac{1}{(p+1)!} > 0$$

〔♨♬,

$$\forall p : S_p < S_{p+1} \Longrightarrow S_p : \underline{\text{叟呀卯μg}}$$

《♤,

$$T_p \overset{\leftarrows}{=} 1 + \frac{1}{1!} + \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{(p-1)\cdot p}$$

♯☎♔♯,

(B) $$\begin{aligned}S_p \le T_p &= 1 + \frac{1}{1} + \left(\frac{1}{1}-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + \left(\frac{1}{p-1}-\frac{1}{p}\right) \\ &= 3 - \frac{1}{p}\end{aligned}$$

(哐傑√ $p = 1,\ 2,\ 3$ ⚅ ♯ ♀)

〔♨♬,

$$S_p \le T_p = 3 - \frac{1}{p} < 3 \Longrightarrow S_p : \underline{\text{剉⚀坰lm}}$$

⊙Weierstrass 𝌆啜埗：垇lm⚀叟呀努墽√写卻♝〗
♗ 〝メ徹☆✔,

$$(A)\&(B) \Longrightarrow \lim_{p\to+\infty} S_p = \sum_{k=0}^{+\infty}\frac{1}{k!} = \lim_{n\to+\infty}\big(1+\frac{1}{n}\big)^n$$

√斤傿♝〗. ◇𝌆叻メ e ♭嘟❙♝.

(2) $\underline{x \in R, x > 0}$ ⚅♯♀

⚀⚅〔○ ⚀尫努 $x > 0$ 『, 唻♩⚅兇匥努⚅区 $\big[n, n+1\big)$ ♭侼「♗♯☞♭♀〗. ♝⚀❙♦,

$$\forall x \in R, x > 0 \Rightarrow \exists n \in N, n \leq x < n+1$$

♝〗♯, 仟努メ冄〝✔,

$$\frac{1}{n} \geq \frac{1}{x} > \frac{1}{n+1}$$

♝⚀❙♦,

$$1+\frac{1}{n} \geq 1+\frac{1}{x} > 1+\frac{1}{n+1}$$

$$\big(1+\frac{1}{n}\big)^{n+1} \geq \big(1+\frac{1}{x}\big)^{n+1} > \big(1+\frac{1}{n+1}\big)^{n+1}$$

$$\big(1+\frac{1}{n}\big)^n\big(1+\frac{1}{n}\big) \geq \big(1+\frac{1}{x}\big)^x\big(1+\frac{1}{x}\big)^\beta > \big(1+\frac{1}{n+1}\big)^{n+1}$$

$$\text{叐♜}, \beta \stackrel{\leftarrow}{=} (n+1) - x$$

♗♗♭,$0 < \beta \leq 1$ ♭◣ 〖,《♤,$x \to +\infty$ ⚅♯♀ $n \to +\infty, (n+1) \to +\infty$ ♭◣ 〗☜〕, 且噜♭ $x \to +\infty$ ♯♝〗♯,

$$\lim_{n\to+\infty}\big(1+\frac{1}{n}\big)^n\big(1+\frac{1}{n}\big) \geq \lim_{x\to+\infty}\big(1+\frac{1}{x}\big)^x\big(1+\frac{1}{x}\big)^\beta \geq \lim_{(n+1)\to+\infty}\big(1+\frac{1}{n+1}\big)^{n+1}$$

♜♤☞♨♬,

$$e \geq \lim_{x\to+\infty}\big(1+\frac{1}{x}\big)^x \geq e \Longrightarrow \lim_{x\to+\infty}\big(1+\frac{1}{x}\big)^x = e$$