

# Ch. 3: Productivity, Output and Employment

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# Chapter Outline

- Production Function
- Demand for Labor
- Supply of Labor
- Labor Market Equilibrium
- Unemployment
- Okun's Law

# Production Function

- How much the economy produces depends on factors of production
  - ▶ Capital ( $K$ )
  - ▶ Labor ( $N$ )
  - ▶ Others (raw materials, land, energy)
- Productivity of factors depends on technology and management

# Production Function (Cont'd)

- Production function

$$Y = A \times F(K, N),$$

where  $A$  = Total Factor Productivity (TFP)

= proxy for technology.

- Cobb-Douglas production function:  $Y = AK^\alpha N^{1-\alpha}$

Contains many things that affect the amount of output,

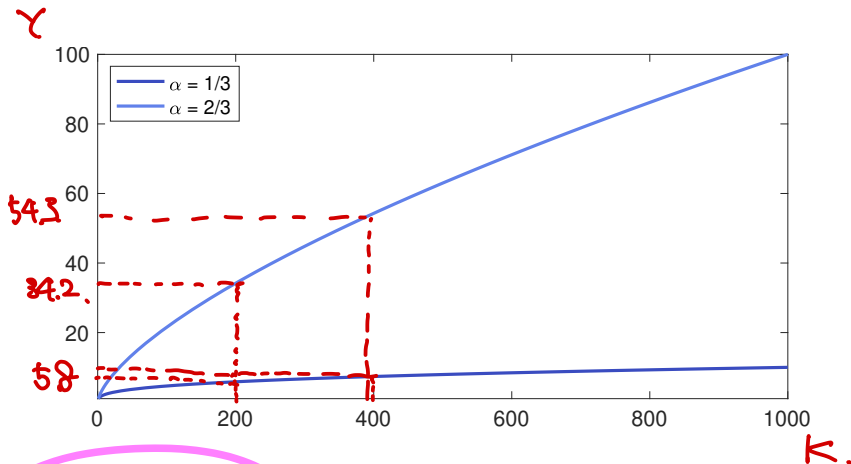
- What value of  $\alpha$  should be?

Holding  $K$  and  $N$  fixed,

$\uparrow$  in  $A \Rightarrow \uparrow$  in  $Y$ .

other than  $K$  and  $N$ .

## Illustration: Plot of $Y$ against $K$



- $Y = AK^\alpha N^{1-\alpha}$  with  $A = 1$  and  $N = 1$

# Marginal Product of Capital (MPK)

*≈ productivity of additional capital stock (= machines).*

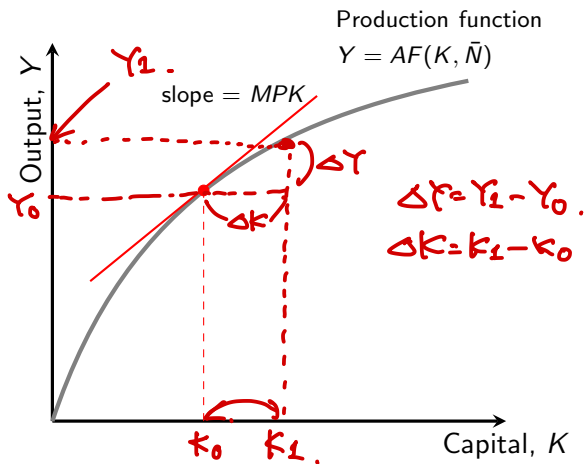
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•  $MPK = \frac{\Delta Y}{\Delta K}$

- Slope of production function

- Always positive, but diminishing MPK as  $K \uparrow$

- The same thing applies to marginal product of labor (MPN)



**Figure:** Production Function (holding  $N$  fixed)

$$\underline{\underline{Y}} = \textcircled{A} \cdot F(K, N)$$

↑  
total factor productivity.

e.g.

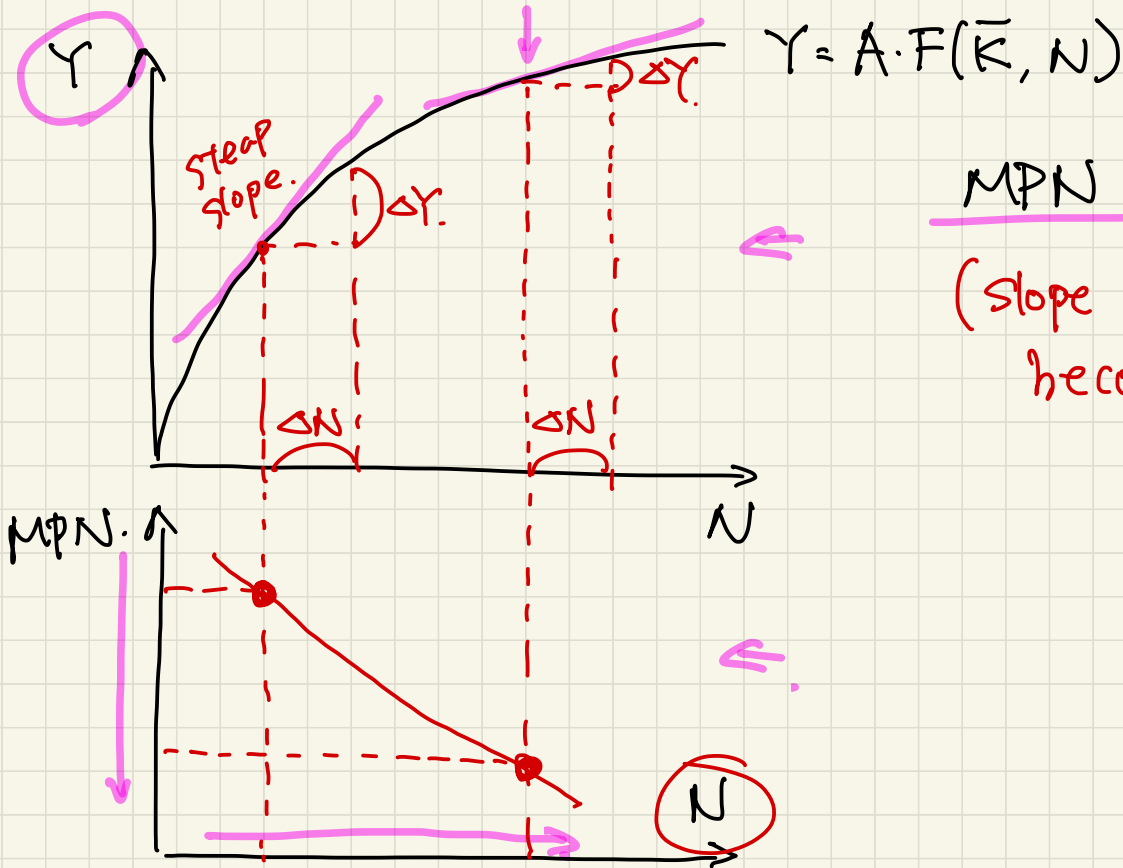
$$Y = A \cdot (K)^\alpha (N)^{1-\alpha} \quad \leftarrow \text{Cobb-Douglas Production function.}$$

Marginal Product =  $\Delta$  in output when you  $\uparrow$  production inputs ( $K$  or  $N$ ).

MPK or MPN.

Marginal Product of Labor.

→ Hold  $K$  fixed.



$MPN \downarrow$  as  $N \uparrow$ .  
(slope of production function becomes flatter as  $N \uparrow$ )



$$Y = A (K)^\alpha (N)^{1-\alpha}.$$

$$\begin{aligned} MPN &\equiv \frac{\partial Y}{\partial N} = (1-\alpha) \cdot A \cdot (K)^\alpha (N)^{-\alpha} \\ &= (1-\alpha) \cdot A \cdot \left(\frac{K}{N}\right)^\alpha \end{aligned}$$

As  $N \uparrow$ ,  $K/N \downarrow \Rightarrow MPN \downarrow$ .

Same thing applies to  $MPK \equiv \frac{\partial Y}{\partial K}$ .

As  $K \uparrow \Rightarrow MPK \downarrow$ .

# Supply Shocks



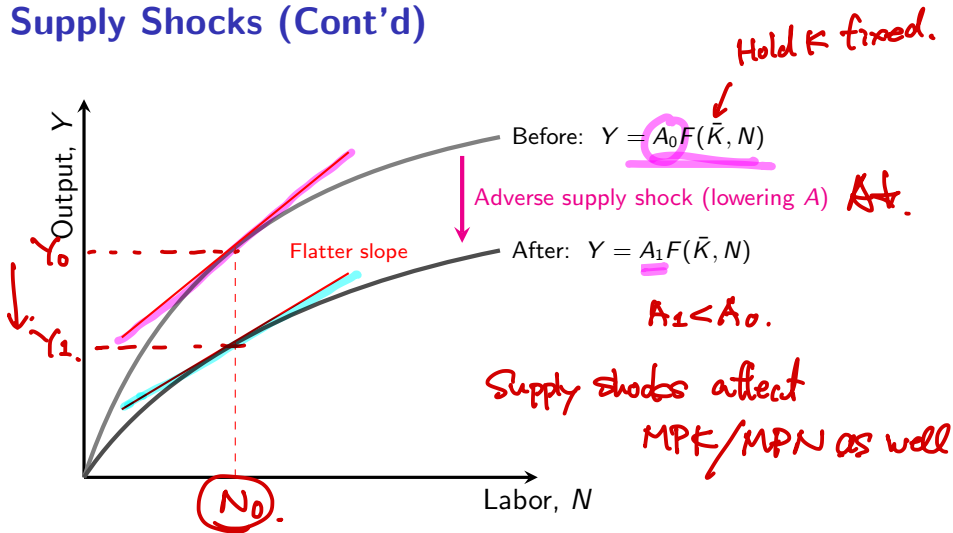
Hand-drawn diagram showing arrows pointing to the variables in the production function equation. A large pink arrow points to the variable  $A$ , and two smaller pink arrows point to the variables  $K$  and  $N$ .

$$Y = A(K)^\alpha (N)^{1-\alpha}$$

Total Factor Productivity / level of Tech.

- A change in an economy's production function (productivity shock)
- May be positive (increasing output) or negative (decreasing output)
- Examples: weather, inventions and innovations, government regulations, oil prices

## Supply Shocks (Cont'd)



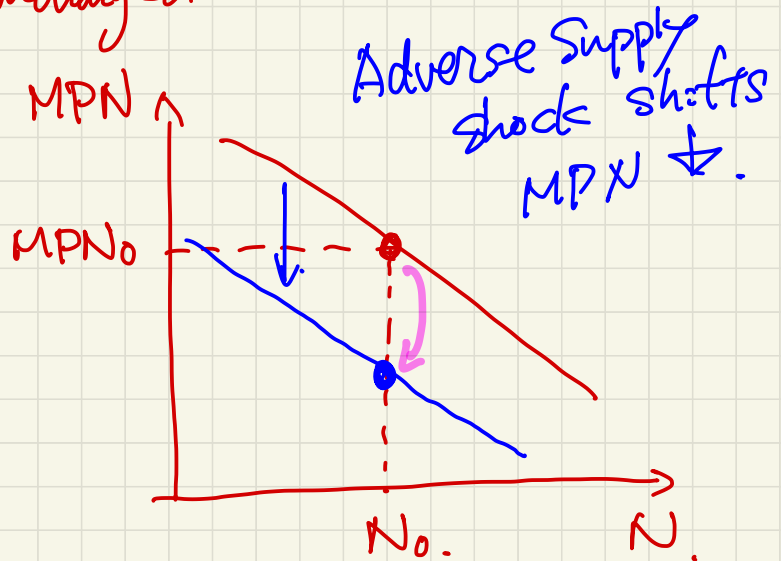
**Figure:** An Adverse Supply Shock Lowering the MPN (holding  $K$  fixed)

MPN. in the case of Cobb-Douglas production  $f^N$ .

$$MPN = (1-\alpha) \cdot A \cdot \left(\frac{K}{N}\right)^\alpha.$$

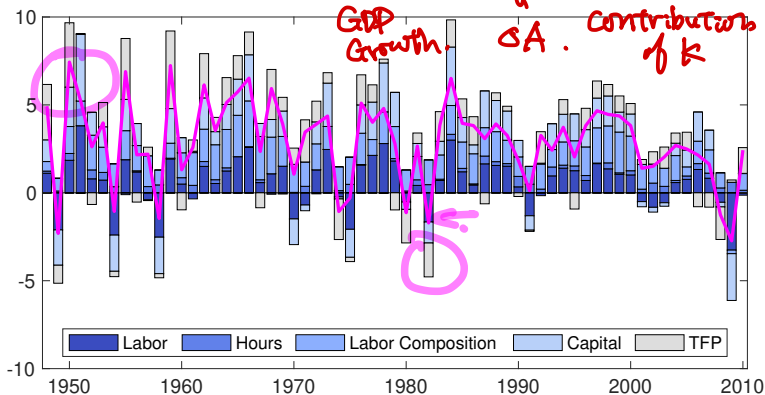
Holding other thing unchanged.

$A \downarrow \Rightarrow MPN \downarrow$ .  $MPN \uparrow$



# US Growth Accounting

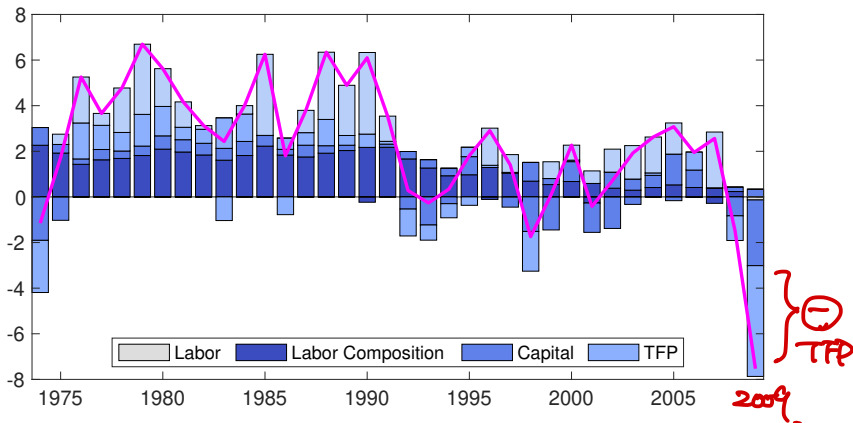
$$\underbrace{\frac{\Delta Y}{Y}}_{\text{GDP Growth}} = \underbrace{\frac{\Delta \text{TFP}}{\text{TFP}}}_{\text{SA}} + \underbrace{\alpha \frac{\Delta K}{K}}_{\text{Contribution of } K} + \underbrace{(1-\alpha) \frac{\Delta L}{L}}_{\text{Contribution of } L}$$



**Figure:** Contributions to US GDP Growth Rate (%)

Source: World KLEMS Data.

# Japan's Growth Accounting



**Figure:** Contributions to Japan's GDP Growth Rate (%)

Source: [World KLEMS Data](#).

# Demand for Labor

- Assumptions

- ▶ Hold capital stock fixed (short-run analysis) ( $K$  doesn't change).
- ▶ Labor market is competitive
- ▶ Firms maximize profits
- ▶ Workers are all alike

- Labor demand is determined by

$$\frac{W}{P} = MPN$$

Take the real wage as given.

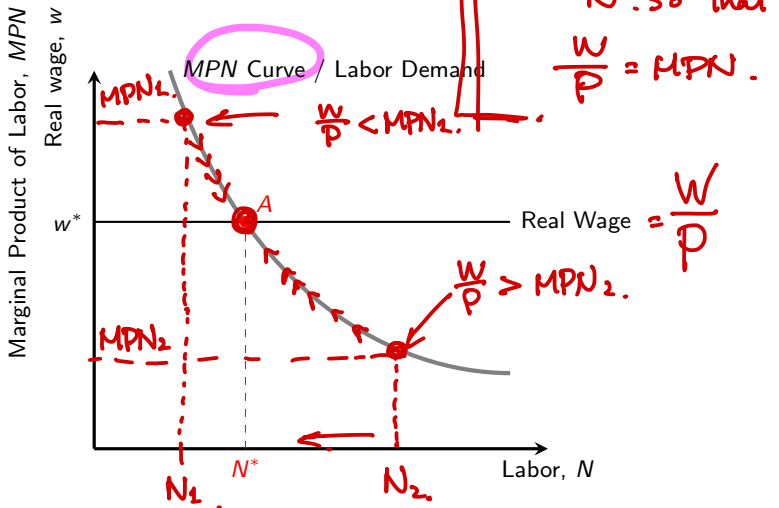
Marginal benefit of having additional worker.

Marginal cost of having additional worker.

- Aggregate labor demand

- ▶ Adding up firms' labor demand
- ▶ Factors that shift firms' labor demand cause shifts in aggregate labor demand

## Demand for Labor (Cont'd)



**Figure:** The Determination of Demand Curve



## Shifts in Labor Demand

Recall

$$MPN = (1-\alpha) \cdot A \cdot \left(\frac{K}{N}\right)^{\alpha}$$

$\uparrow K \leftrightarrow \uparrow A$   
 $\Rightarrow MPN \uparrow$

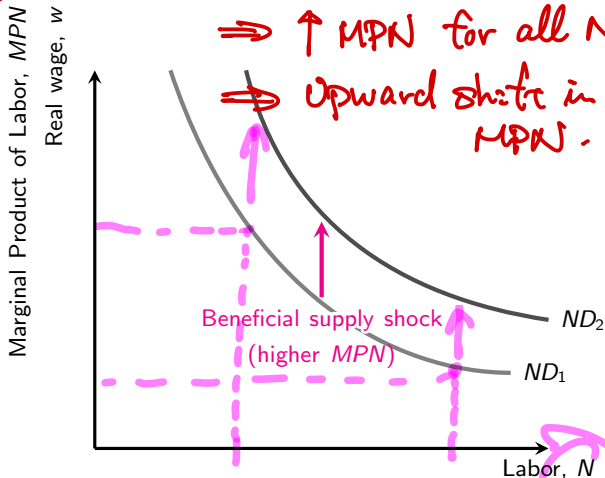
Labor Demand Shifters:

- Supply shocks
- Size of capital stock

Beneficial supply shock  
( $\uparrow$  in  $A$ ).

$\Rightarrow \uparrow MPN$  for all  $N$ .

$\Rightarrow$  Upward shift in  $MPN$ .





**Figure:** The Effect of a Beneficial Supply Shock

# Supply of Labor

- Supply of labor is determined by individuals
- Aggregate supply of labor is the sum of individuals labor supply
- Labor supply of individuals depends on labor-leisure choice
  - ▶ Utility depends on consumption and leisure ← ∴ 24H/day.
  - ▶ Need to compare costs and benefits of working another day → earn more.
  - ▶ Keep working additional days until benefits equal costs

less leisure.

# Supply of Labor (Cont'd)

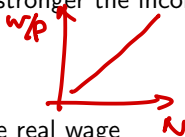
- How does an increase in the real wage affects the labor supply decision?
- **Substitution effect:** Higher real wage encourages work, since reward for working is higher 
- **Income effect:** Higher real wage increases income for same amount of work time, so person can afford more leisure, so will supply less labor 

# Supply of Labor (Cont'd)

- A one-day rise in the real wage
  - ▶ A temporary real wage increase has just a pure substitution effect, since the effect on wealth is negligible
- Winning the lottery
  - ▶ A pure income effect
  - ▶ Doesn't have a substitution effect, because it does not affect the reward for working
  - ▶ Since a person becomes wealthier, s/he will both consume more goods and take more leisure

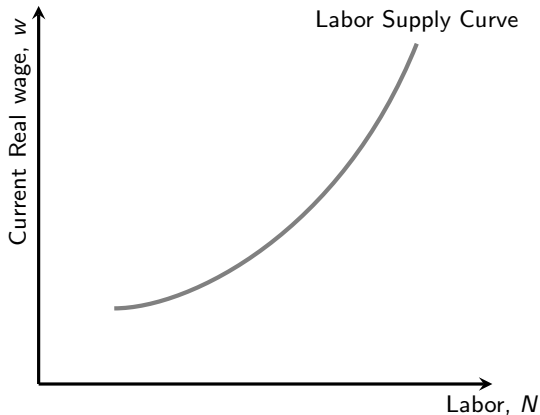
# Supply of Labor (Cont'd)

- A long-term increase in the real wage
  - ▶ The substitution effect AND the income effect
  - ▶ The reward to working is greater (substitution effect toward more work)
  - ▶ With higher wage, a person does not need to work as much (income effect toward less work)
  - ▶ The longer the high wage is expected to last, the stronger the income effect
- Empirical evidence on real wages and labor supply
  - ▶ Labor supply increases with a temporary rise in the real wage
  - ▶ Labor supply falls with a permanent increase in the real wage



# Labor Supply Curve

- Labor supply curve relates quantity of labor supplied to real wage
- Upward-sloping

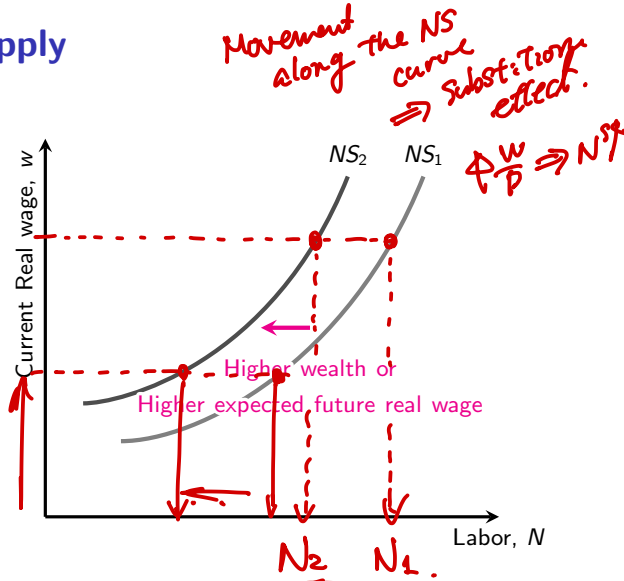


**Figure:** The Labor Supply Curve of an Individual Worker

# Shifts in Labor Supply

Labor Supply Shifters:

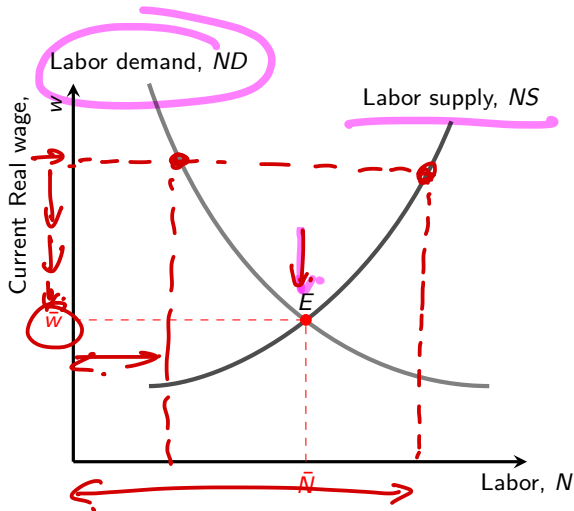
- Wealth
- Expected future real wage



**Figure:** The Effect on Labor Supply of an Increase in Wealth

# Labor Market Equilibrium

- Classical model of the labor market – real wage adjusts quickly
- Determines full-employment level of employment and market-clearing real wage
- Problem with classical model: can't study unemployment



### Figure: Labor Market Equilibrium

unemployment Figure: Labor Market Equilibrium  
 ↳ Search and Matching model of labor market.



# Labor Market Equilibrium

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theoretical / hypothetical construct.

- Full-employment output  
= potential output  
= level of output when labor market is in equilibrium

$$\bar{Y} = \underline{A} F(K, \bar{N})$$

full-employment level.

- Affected by changes in full employment level or production function
- How does an adverse supply shock affect potential output?

$$\begin{aligned}\bar{Y}_1 &= A_1 F(K, \bar{N}_0) \\ \bar{Y}_2 &= A_2 F(K, \bar{N}_0)\end{aligned}$$

$$A_2 < A_1$$

Holding  $K, \bar{N}_0$  fixed.  
 $\bar{Y}_2 < \bar{Y}_1$

# Effects of a Temporary Adverse Supply Shock

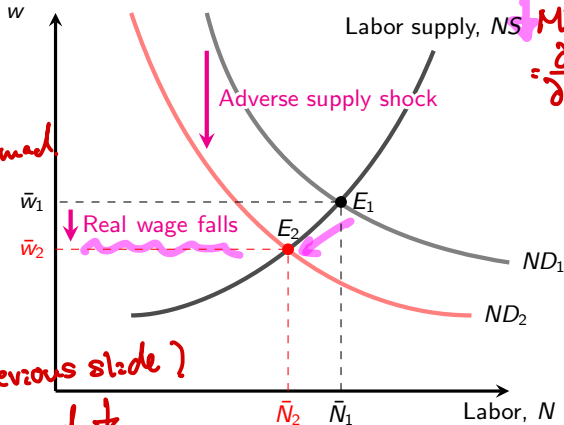
$$Y = A(K)^\alpha (N)^{1-\alpha}$$

$$\begin{aligned} \text{MPN} &= \frac{\partial Y}{\partial N} = A(1-\alpha) \left(\frac{K}{N}\right)^\alpha \end{aligned}$$

$A \downarrow$

$\Rightarrow \text{MPN} \downarrow$

$\Rightarrow$  Shift labor demand down.



$A \downarrow$

(1).  $\bar{Y} \downarrow$  (previous slide)

(2) Labor demand  $\downarrow$

$\rightarrow \bar{N} \downarrow \& \bar{w} \downarrow$

$\rightarrow \bar{Y} \downarrow$

# Relative Price of Energy and Recessions

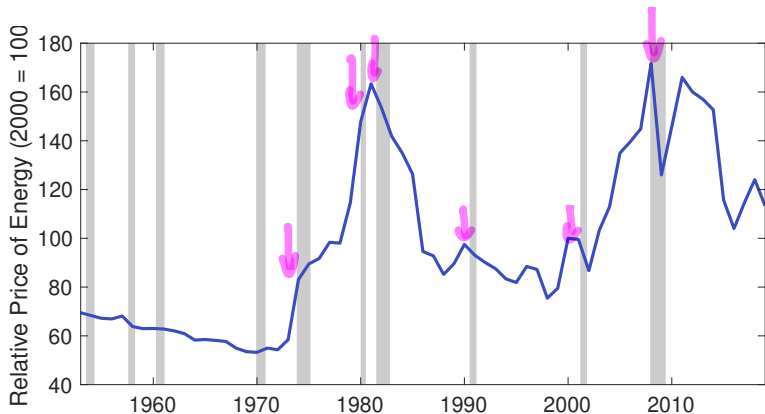


Figure: Relative Price of Energy

Energy price  $\uparrow \Rightarrow$  Higher input costs  $\Rightarrow$  Adverse supply shock

Source: FRED database, Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org/series/PPIENG>; <https://fred.stlouisfed.org/series/GDPDEF>.

# Unemployment

In labor market equilibrium,  
there is NO unemployment.  
( $\because ND = NS$ ).

- What is the strong assumption on the labor market model discussed so far?
- In reality, not everyone who would like to work has a job

Another approach  
to study unemployment  
→ search and matching  
in labor market.

# Why There are Always Unemployed People?

- Frictional unemployment
  - ▶ Search activity of firms and workers due to heterogeneity
  - ▶ Matching process takes time
- Structural unemployment
  - ▶ The long-term and chronic unemployment that exists even when the economy is not in a recession
  - ▶ Lack of skills prevents some workers from finding long-term employment
  - ▶ Reallocation of workers out of shrinking industries or depressed regions

# How is the Unemployment Rate Measured?

- Categories:

- ▶ Employed
- ▶ Unemployed
- ▶ Not in the labor force

$$\text{Unemployment Rate} = \frac{\text{Unemployed}}{\underbrace{\text{Employed} + \text{Unemployed}}_{\text{Labor Force}}}$$

$\neq \text{Pop.}$

- Labor force = Employed + Unemployed

# Employment Status

	Number (thousands)	Labor Force Share (%)	Adult Pop. Share (%)
Employed Workers	(1) 133,403	85.25	51.32
Unemployed Workers	(2) 23,078	14.74	8.88
Labor Force	(3) = (1) + (2) 156,481	100.00	60.21
Not in Labor Force	(4) 103,415		39.79
Adult Population	(5) 259,896		100.00

**Table:** Employment Status of the US Adult Population, April 2020

- Unemployment rate?  $\frac{(2)}{(3)} = 14.74\%$
- Employment ratio?  $\frac{(1)}{(5)} = 51.32\%$
- Labor participation rate?  $\frac{(3)}{(5)} = 60.21\%$

Source: Bureau of Labor Statistics, [Employment Situation Summary, Table A](#)

# Natural Rate of Unemployment

theoretical / hypothetical construct we do NOT observe in the data.

- Natural rate of unemployment ( $\bar{u}$ ): When output and employment are at full-employment levels

$$\bar{u} \geq 0$$

$$\bar{u} = \text{frictional} + \text{structural unemployment}$$

- Cyclical unemployment: Difference between actual unemployment rate and natural rate of unemployment ( $u - \bar{u}$ )

$$\text{cyclical} = \underbrace{u - \bar{u}}_{\text{Actual.}}$$

$$\boxed{\text{Actual } u} = \text{cyclical} + \bar{u} \\ = \text{cyclical} + \text{structural} + \text{frictional.}$$



# Okun's Law

- Relationship between output (relative to full-employment output) and cyclical unemployment

In the boom,  $LHS = \ominus$ ,  $RHS = \ominus$ ,  $u < \bar{u}$ .

$$\frac{\bar{Y} - Y}{\bar{Y}} = 2(u - \bar{u})$$

$Y > \bar{Y}$       cyclical unemployed.

- Alternative formulation if average growth rate of full-employment output is 3%:

When  $\Delta u = 0$   
 $\rightarrow \frac{\Delta Y}{Y} = 3.$

$$\frac{\Delta Y}{Y} = 3 - 2\Delta u$$

$\uparrow$  GDP growth rate.       $\uparrow$   $\Delta u$  in unemployment rate.

$\Delta u > 0$ .  $\rightarrow$  the state of the economy is worsened.

e.g.  $\Delta u = 1$ .

$$\frac{\Delta f}{f} = 3 - 2 \times 1 = 1.$$

$\Delta u < 0 \rightarrow$  labor market condition improves.

$\rightarrow$  Expect the higher-than-average  
GDP growth rate. "3%".

$$\Delta u = -1.$$

$$\frac{\Delta f}{f} = 3 - 2 \times (-1) = \underline{\underline{5}}.$$

## Okun's Law (Cont'd)



**Figure:** Relating Output and Unemployment

Estimated relationship:  $\widehat{\Delta Y/Y} = 3.2 - 1.7\Delta u$

Source: FRED database, Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org/series/GDPC1>; <https://fred.stlouisfed.org/series/UNRATE>.