13-June 2023

To: Professor Keller

From: Jake Daniels

ME-341-07

Fluid Mechanics Water Fountain Design Project

Background

This project focuses on the analysis and design of a water fountain system. This project allowed me to utilize the concepts we explored in class by designing and solving questions about this water fountain. The aesthic of my design is inspired by the architecture of a 2,000 year old ancient Roman water fountain [6] in Kibyra, Turkey, that was recently fully restored. Like this ancient fountain, my design has a large structure supported by six massive columns. There is one vertical shooter in the center of the fountain, a ring with 6 laminar flow shooters, and ring with 12 fan oarsman nozzles. There are three total pumps. Two identical pumps and pipes deliver water to the fan oarsman nozzles and a third pump delivers water to the ring of laminar shooters. The vertical shooter has no pump and utilizes compression to release water vertically. All the pipes run 10 ft underground until reaching their fountain entry point. All the pipes are made of stainless steel. There are also three suction pipes, one for each pump. There is one drainage pipe that runs back to the control room as well. When water depth reaches 4 ft, a drainage gate is set to open and releases water into the drainage pipe.

Purpose

The objective of this project was to create a water fountain with logical parameters and solve for other parameters using methods taught in class. I used concepts such as dimensional analysis, iterative solving, and other various techniques to gather solutions to thee questions I proposed. I gained a more practical sense for many of these principles by attempting to create realistic scenarios and using realistic values. This project was very beneficial because it embodied Cal Poly's "Learn By Doing" motto and gave me experience applying these ideas we've spent ten weeks learning.

Conclusion

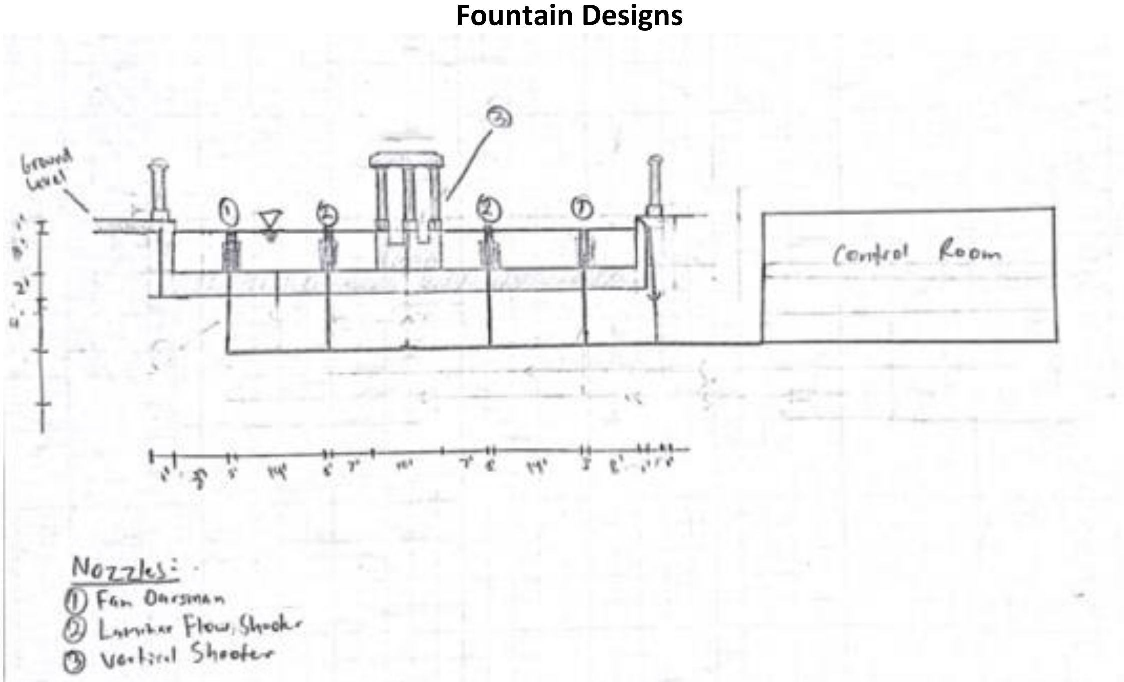
While this was a daunting project, I have a significantly better understanding of these topics than I previously had. Below is a table with the values I calculated. I realized that because this is on such a large scale, the head losses are very large. I was also surprised that the pipe diameter does not need to be very large. I went into this thinking I would use massive pipes, but, because the fountain is not pumping a massive amount of water, the diameters are very small. If you have any feedback for me, please let me know.

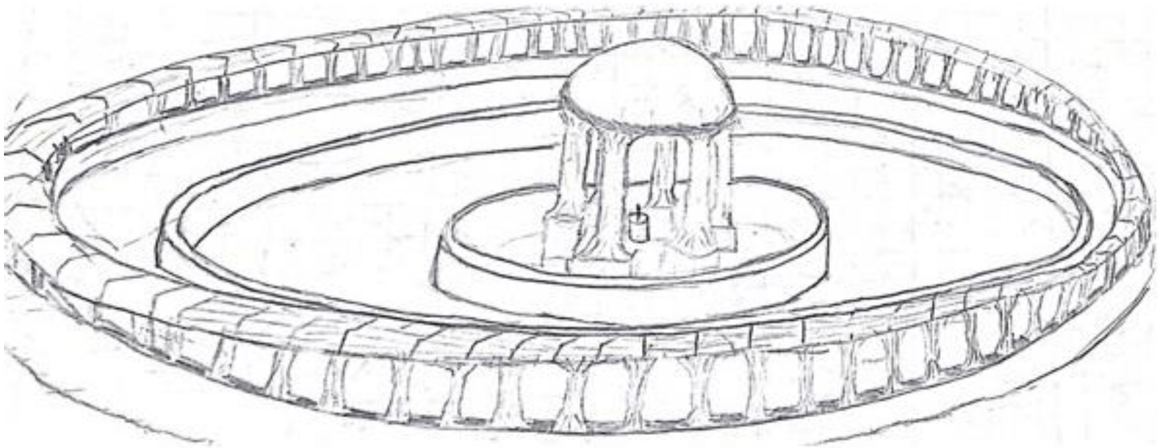
Other
I decided to use stainless steel pipes to avoid corrosion. I also avoided aluminum because I was unsure whether chlorine would be necessary in the fountain.

References:

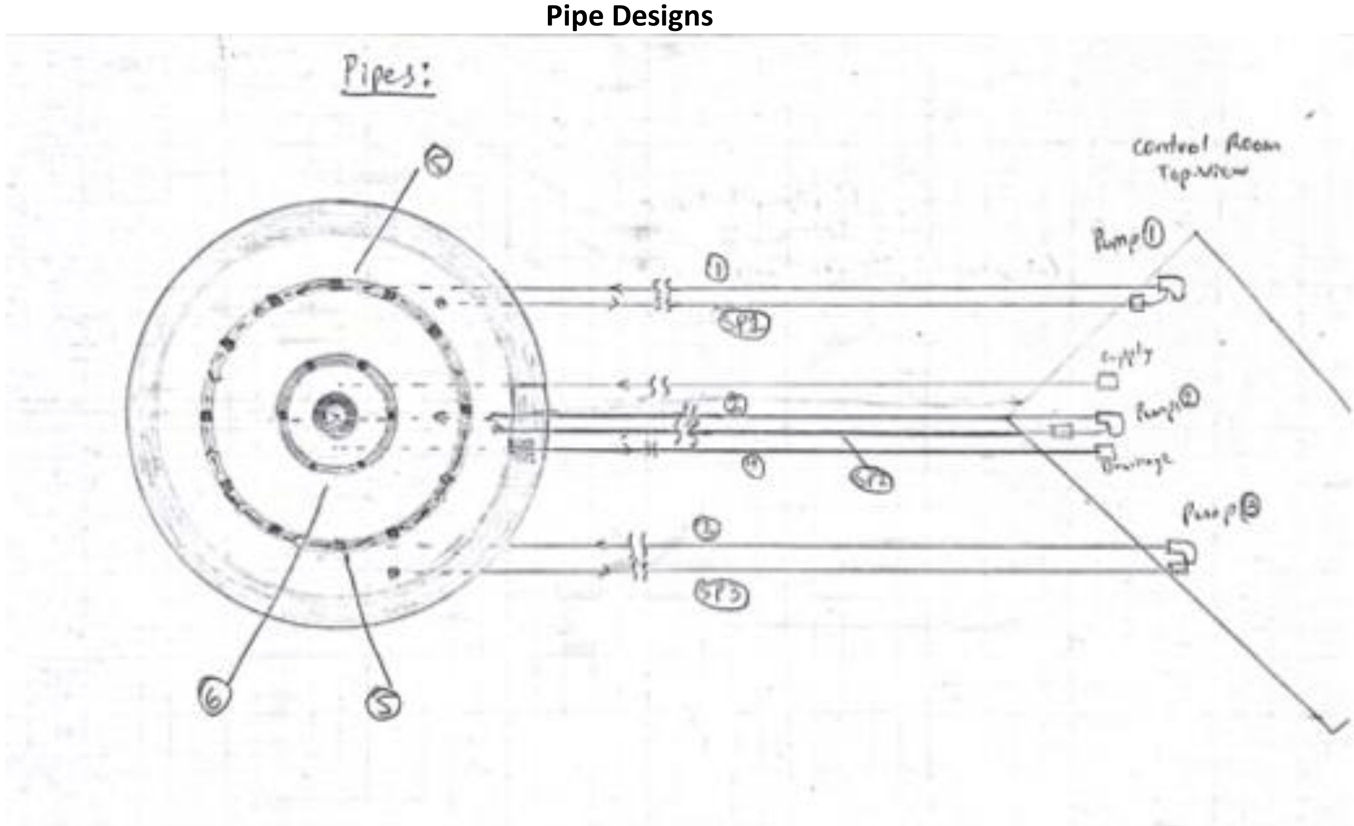
Reference Number	Name	Link
[1]	Fox and McDonald Textbook 9th Ed	NA
[2]	Engineering Tool Box Surfaces Roughness Table	https://www.engineeringtoolbox.com/surface-roughness-ventilation-ducts-d_209.html
[3]	Excel With Iterative Solver	excel
[4]	SSP Fountain Design Guide	PDF
[5]	Safe Rain Architectural Fountains Blog	Blog
[6]	Arkeonews, Aesthetic Inspiration	Fountain Inspiration

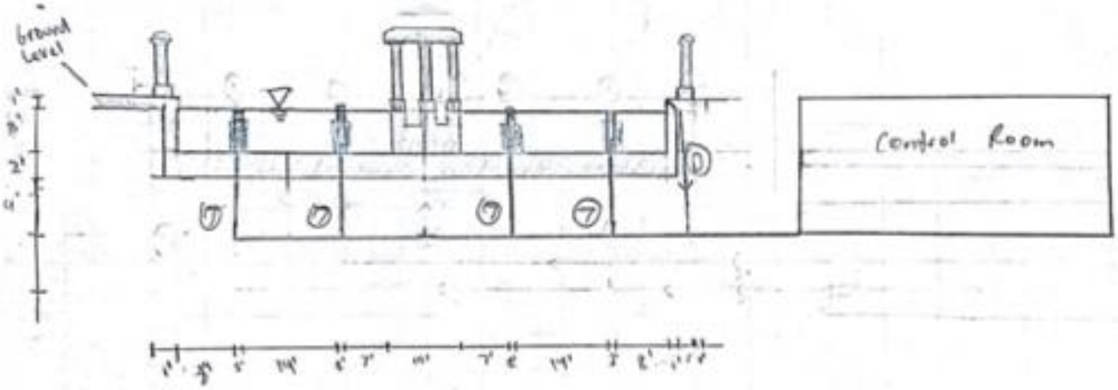
Fountain Designs





Pipe Designs

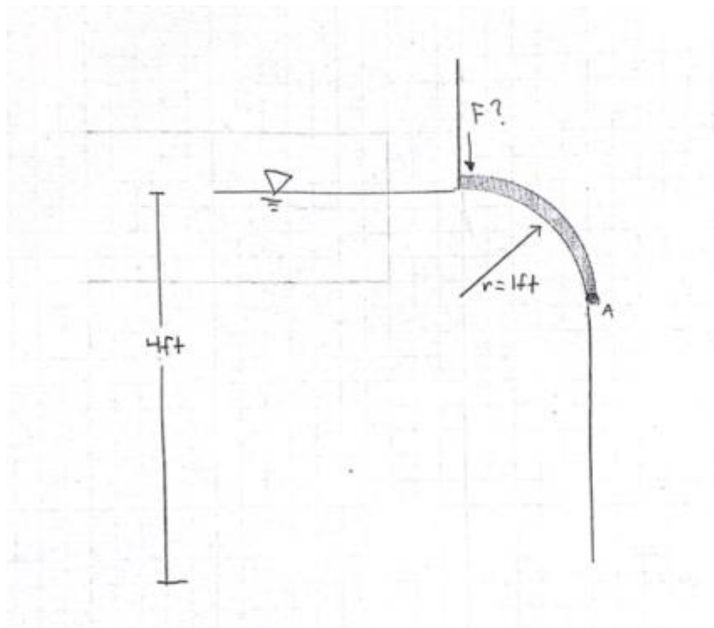




Pipe Details: Component Details

Problem 1: (hydrostatics ch 3)

There is a drainage pipe that helps keep the water in the fountain at the desired level. There is a quarter-circle gate of radius 1 ft that holds water in the fountain. There is a spring that holds the gate closed. The fountain has an acceptable depth threshold between 3 and 4 ft. When at 3 ft, the surface of the water is at the bottom of the gate, so no force from the water acts on it. If the water reaches 4 ft in depth, the gate will open and release water to the drainage pipe. The width of the gate is also 1 ft. that has a uniform weight of 5 lbf. While at maximum water height, how much force is required on the gate at the given location?


F?
r=1ft
A
4ft

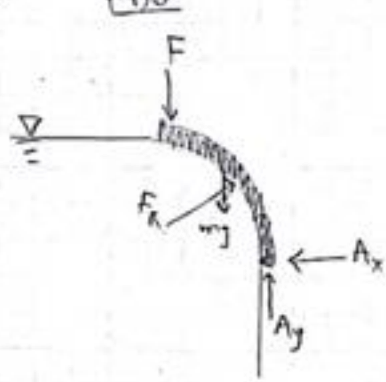
Given: Gate radius = 1ft, w = 1ft, $W_{gate} = mg = 51$ lbf

Find: Force needed to keep gate closed

Assume:

1. Incompressible
2. Static
3. std atm
4. Rigid Rot
5. frictionless hinge
6. Thin surface

FBD:



$$\frac{dP}{dy} = -\rho g$$

$$\int_{P_0}^{P_1} = -\int_{y_0}^{y_1} \rho g\, dy$$

$$P_1 - \cancel{P_0} = -\rho g(y_1 - \cancel{y_0})$$

$$\boxed{P_1 = -\rho g y_1}$$

$$F = PA$$

$$dF = P\,dA$$

$$F_y = -\rho g \underbrace{y A_y}_{\text{Volume}}$$

$$F_y = \rho g(r^2 w - \tfrac{1}{4}\pi r^2 w)$$

$\rho = 1.94$ slug/ft^3 [1] A.1

$g = 32.2$ ft/s^2

$r = 1$ ft

$W = 1$ ft

$$F_y = (1.94\ slug/ft^3)(32.2\ ft/s^2)\left((1ft)^2(1ft) - \frac{1}{4}\pi(1ft)^2(1ft)\right)$$

$$\boxed{F_y = 13.40575\ lbf}$$

$$F_H = PA$$

$$F_H = (62.4\ lbf/ft^3)(0.5ft)(1ft \times 1ft)$$

$$\boxed{F_H = 31.2\ lbf}$$

$$\Sigma M_A^{+} : 0 = \underbrace{-mg(1ft)(0.5\ ft)}_{\text{distributed load}} - F(1ft) + F_y(\bar{x}) + F_H(\bar{y})$$

$$\bar{x}A = \Sigma x_i A_i$$

$$\bar{x}\left(r^2 - \frac{1}{4}\pi r^2\right) = r^2\left(\frac{r}{2}\right) - \frac{1}{4}\pi r^2\left(r - \frac{4r}{3\pi}\right)$$

$$\bar{x} = \frac{(1)^2\left(\frac{1}{2}\right) - \frac{1}{4}\pi(1)^2\left(1 - \frac{4(1)}{3\pi}\right)}{(1)^2 - \frac{1}{4}\pi(1)^2}$$

$$\boxed{\bar{x} = 0.22337\ ft}$$

$$\bar{y} = y + \frac{I_x}{Ay}$$

$$\bar{y} = \frac{r}{2} + \frac{\frac{1}{12}wr^3}{wr\frac{r}{2}}$$

$$\bar{y} = \left(r - \frac{2}{3}r\right)$$

$$\bar{y} = \frac{1}{3}\ ft$$

$$\Sigma M_A^{+} = -(5\ lbf)(1ft)(0.5ft) - F(1ft) + (13.40575\ lbf)(0.22337ft) + (31.2\ lbf)\left(\frac{1}{3}ft\right) = 0$$

$$\boxed{F = 10.894\ lbf}$$

Conclusion 1:

When the water is all the way up to the top of the gate, the force from the water acting on the gate is at its highest. Because of this, we can calculate the force needed to keep it closed. After solving this, we found that there must be a 10.9 lbf acting downwards to prevent the water from pushing the gate open. This is not terribly difficult to implement because the force is relatively low.

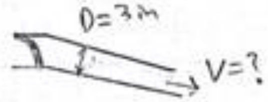
Problem 2: (Viscosity, ch 2)

The drainage pipe is 3 inches in diameter and runs downhill with an angle of 5 degrees so that gravity can move the water. The gate that allows water a Volumetric Flow Rate of 1 ft^3/s into the drainage pipe. The flow is turbulent and has a Reynold's Number of 475,000 Use this information to calculate the viscosity of the water in the drainage pipe. Then use Table A.7 in textbook [1] to check temperature and determine if this value makes sense.

Given: $D = 3\,in$, $\theta_{incline} = 5°$, $\dot{V}_{water} = 1\ ft^3/s$
$Re = 475{,}000$, Seamless steel pipes

Assume:
Newtonian
Steady
Incompressible
Internal
std atm

Schematic



velocity of water in pipe:

$\dot{V} = A \cdot velocity$

$1\,ft^3/s = \pi\left(\frac{D}{2}\right)^2 \cdot V$

$1\,ft^3/s = \pi\left(\frac{3\,in \times \frac{1\,ft}{12\,in}}{2}\right)^2 \cdot V$

$V_{horizontal} = 20.372\ ft/s$

$V_{pipe} = \sqrt{V_{horizontal}^2 + V_{vertical}^2}$

$V_{vertical} = V_{horizontal}(\tan(\theta))$

$V_{vertical} = (20.372\,ft/s)(\tan(5°))$

$V_{vertical} = 1.7823\ ft/s$

$V_{pipe} = \sqrt{(20.372\,ft/s)^2 + (1.7823\,ft/s)^2}$

$V_{pipe} = 20.45\ ft/s$

$\frac{\varepsilon}{D} = \frac{3.28 \times 10^{-7}}{0.25} = 1.312e{-6}$ [2]

$D = 0.25\ ft$

$Re = 475{,}000$

$\rho = 1.94\ slug/ft^3$

[2]

$Re = \frac{\rho V L}{\mu}$

$\mu = \frac{\rho V L}{Re}$

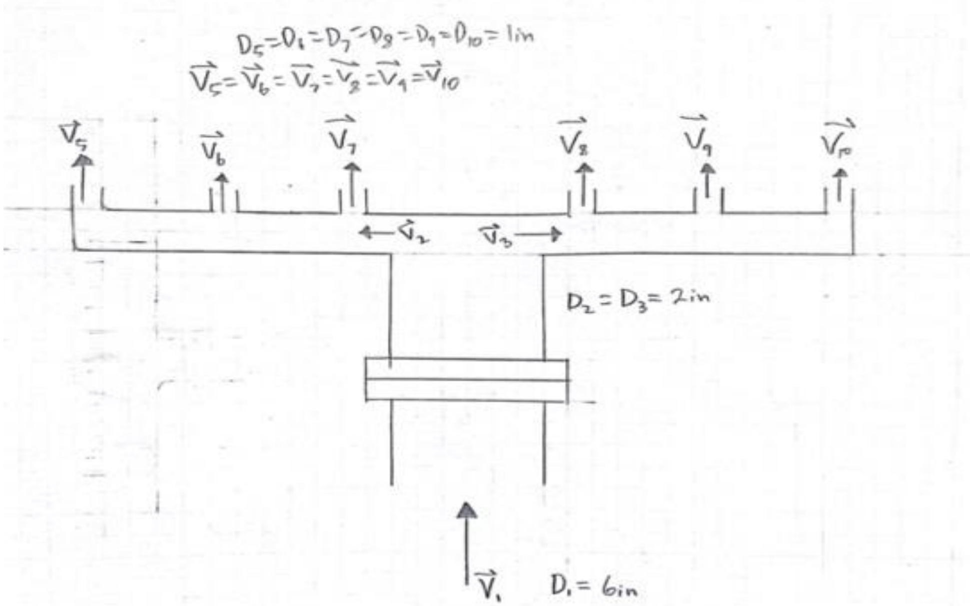
$\mu = \frac{(1.94\,slug/ft^3)(20.45\,ft/s)(0.25\,ft)}{475{,}000} =$

$$\boxed{\mu = 2.0881 \times 10^{-5}\ lbf\cdot s/ft^2}$$

$$\boxed{T \approx 68\ ^\circ F}$$ [1] Table, A.7

Conclusion 3:
This calculation utilizes a basic Reynolds number equation. After finding the velocity entering the pipe, which is horizontal, we had to calculate the vertical velocity. After getting the total velocity, we used the rest of our values to solve for viscosity, giving us a good value of 2.0881E-05. After checking Table A.7, we discovered that the water temperature is 68 degrees F, which makes sense for an outdoor fountain.

Problem 3: (mass flow at pipe junction ch 4)
The junction in the pipe that delivers water at a steady flow to 6 of the fan oarsman nozzles is shown below. The diameter of the half-circle fan oarsman discharge pipe is 54 ft and the pipe has an internal diameter of 2 inches. To work properly, the nozzle must have water pressure of 7 psi and a volumetric flow rate of 10 gpm through its 1 in diameter. All pipes are made of stainless steel. The main pipe from pump to the first 90 degree elbow is straight L = 360 ft. Find the necessary velocity in the half-circle pipe. Then use this to find the necessary velocity in the main pipe.



Calculation 3:

Given: $D_1 = 6in$, $D_2 = D_3 = 2in$, $\dot{V} = 10gpm$, $L_{long\,pipe} = 360ft$,
$L_{half\text{-}circle\,pipe} = 54ft$, $P_{nozzle} = 17psi$, $D_{nozzle} = 1in$
steel pipes.

Find: a) velocity in half-circle pipe
b) velocity in long pipe

Assume:
Steady flow
Incompressible
uniform
std atm

$$\dot{V}_4 = VA$$

$$10gpm = 0.02280104\ ft^3/s$$

$$V_4 = \frac{0.02280104\ ft^3/s}{\pi\left(\frac{\frac{1}{12}}{2}\right)^2}$$

$$V_4 = 4.1805\ ft/s$$

$$\Sigma_{cs} = \vec{V} \cdot \vec{A} = 0$$

$$\vec{V}_4 \cdot \vec{A}_4 + \vec{V}_5 \cdot \vec{A}_5 + \vec{V}_6 \cdot \vec{A}_6 + \vec{V}_2 \cdot \vec{A}_2 = 0$$

$$\vec{V}_4 \cdot \vec{A}_4 = V_4 A_4$$

$$\vec{V}_5 \cdot \vec{A}_5 = V_5 A_5$$

$$\vec{V}_6 \cdot \vec{A}_6 = V_6 A_6$$

$$\vec{V}_2 \cdot \vec{A}_2 = -V_2 A_2$$

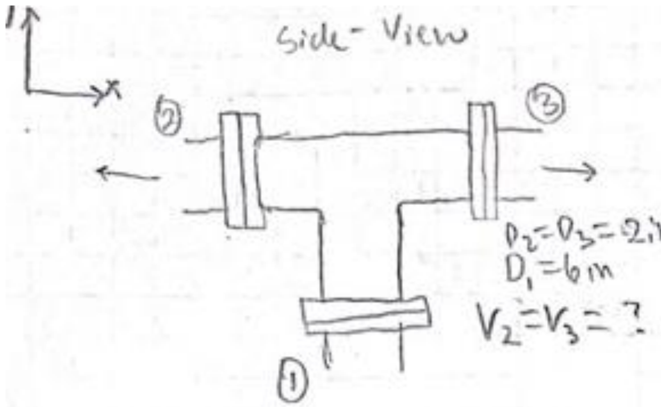
$$V_4 A_4 + V_5 A_5 + V_6 A_6 - V_2 A_2 = 0$$

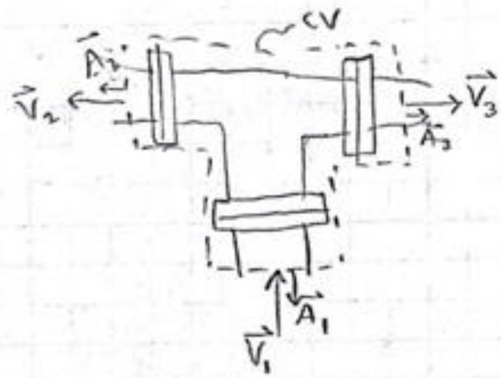
$$\vec{V}_4 A_4 = V_5 A_5 = V_6 A_6$$

$$3\vec{V}_4 A_4 = V_2 A_2$$

$$3(4.1805\ ft/s)\left(\pi\left(\frac{\frac{1}{12}}{2}\right)^2\right) = V_2\left(\pi\left(\frac{\frac{2}{12}}{2}\right)^2\right)$$

$$\boxed{V_2 = V_3 = 3.13536182\ ft/s}$$





$$\Sigma_{cs} = \vec{V}\cdot\vec{A} = 0$$

$$\vec{V_1}\cdot\vec{A_1} + \vec{V_2}\cdot\vec{A_2} + \vec{V_3}\cdot\vec{A_3} = 0$$

$$\vec{V_1}\cdot\vec{A_1} = -V_1A_1$$

$$\vec{V_2}\cdot\vec{A_2} = V_2A_2$$

$$\vec{V_3}\cdot\vec{A_3} = V_3A_3$$

$$V_1A_1 = V_2A_2 + V_3A_3$$

$$V_1A_1 = 2(V_2A_2)$$

$$V_1\left(\pi\left(\frac{\frac{6}{12}}{2}\right)^2\right) = 2(3.13536182\,ft/s)\left(\pi\left(\frac{\frac{2}{12}}{2}\right)^2\right.$$

$$\boxed{V_1 = 0.6967\ ft/s}$$

Conclusion 3:
After these two calculations we can see that the velocity from the pump is rather slow. This is because of the relatively large pipe we chose. Had this pipe been smaller, the velocity would be much closer to the velocity in the half-circle pipe.

Problem 4:
Using the information and diagram given in Problem 2, calculate the work done by Pump 1 to supply enough water to the fan oarsman nozzles. Use head loss calculations to find this. Velocities calculated in Problem 2 will be used.

Calculation 4:

<u>Given</u>: $\vec{V}_1 = 0.6967$ ft/s, $\vec{V}_2 = \vec{V}_3 = 3.1353618$ ft/s

$L_{long\ pipe} = 360$ ft, $L_{half-pipe} = 54$ ft, $D_1 = 6$ in, $D_2 = D_3 = 2$ in

$\Delta z = 7.5$ ft

<u>Find</u>: Work done by pump

<u>Assume</u>:
steady flow
incompressible
streamline flow
std atm
empirical losses
$\alpha = 1$

$$\Delta h_{total} = \Delta h_{pipe} + \Delta h_{elbow} + \Delta h_{vertical} + \Delta h_{half\text{-}circle} + \Delta h_{nozzle}$$

$$\Delta h_{pipe} = \frac{f \cdot L \cdot V_{pipe}^2}{2 D_{pipe}}$$

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$\frac{\varepsilon}{D} = 1.84 \times 10^{-4}$ ft [2]

$$Re = \frac{\rho V D}{\mu}$$

$\rho = 1.94$ slug/ft^3

$V = 0.6967$ ft/s

$D = 6$ inch or 0.5 ft

$\mu = 2.10E\text{-}05$ lbfs/ft^2 [1] Table A7

$$Re = \frac{(1.94)(0.6967 ft/s)(0.5 ft)}{(2.10E\text{-}05)}$$

$Re = 32183.144$

$\dot{V} = VA = (0.6967 ft/s)\left(\pi\left(\frac{0.5}{2}\right)^2\right)$

$\dot{V} = 0.1367967$ ft^3/s

$f = 0.02356626$ [3] iterative calc

Excel Tab 1

$$\Delta h_{pipe} = \frac{(0.02356626)(360 ft)(0.6967 ft/s)^2}{2(0.5 ft)(32.2}$$

$\boxed{\Delta h_{pipe} = 0.1278877 ft}$

$$\Delta h_{elbow} = \frac{K V_{pipe}^2}{2g}$$

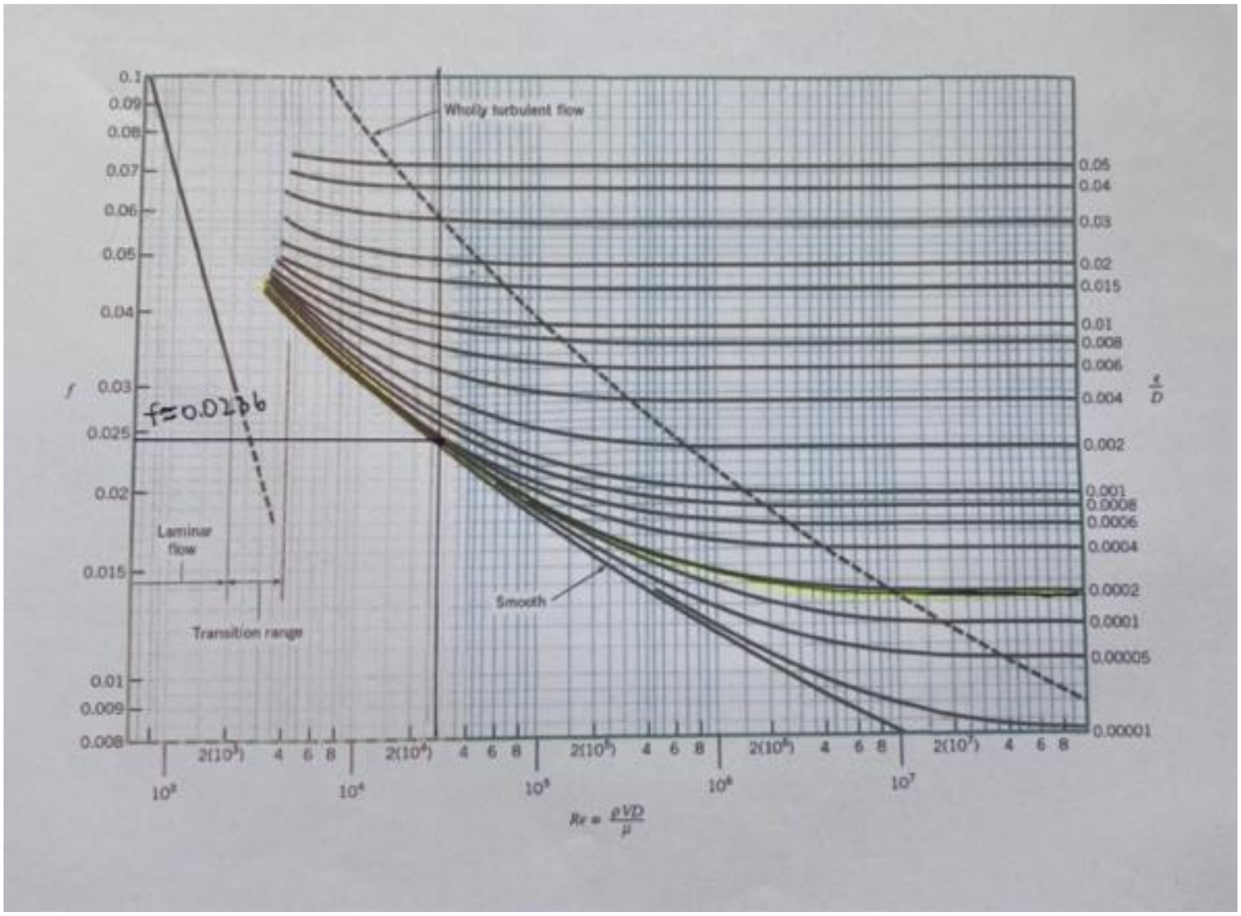
$K = 0.3$ [1] Table 8.4

$V_{pipe} = ?$

At elbow, $V = 0.6967$ ft/s

$$\Delta h_{pipe} = \frac{0.3(0.6967 ft/s)^2}{2}$$

$\boxed{\Delta h_{elbow} = 0.07281 ft/s}$



Wholly turbulent flow
f=0.0236
Laminar flow
Transition range
Smooth
f
ε/D
Re = ρVD/μ
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001

$$\boxed{\Delta_{vertical} = 7.5\ ft}$$

$$\Delta h_{half\text{-}circle} = f\frac{V^2}{2g}\frac{L}{D}$$

$$V_{half\text{-}circle} = 3.13536182\ ft/s$$

$$D_{half\text{-}circle} = 2in = \frac{1}{6}ft$$

$$L_{half\text{-}circle} = \frac{5}{12}(2\pi(27ft))$$

$$L_{half\text{-}circle} = 70.68583471\ ft$$

$$\Delta h_{half\text{-}circle} = 0.02356626\left(\frac{(3.13536182\ ft/s)^2}{2(32.2\ ft/s^2)}\right)\left(\frac{70.68583471\ ft}{(\frac{1}{6}ft)}\right)$$

$$\boxed{\Delta h_{half\text{-}circle} = 1.52568\ ft}$$

$$\Delta h_{nozzle} = K\frac{V^2}{2}$$

$$K = 0.6$$

$$V = 4.1805\ ft/s$$

$$\Delta h_{nozzle} = \frac{(0.6)(4.1805\ ft/s)^2}{2}$$

$$\boxed{\Delta h_{nozzle} = 5.242974\ ft}$$

$$\Delta h_{Tee\text{-}Dividing} = \frac{0.9(0.6967\ ft/s)^2}{2}$$

$$\boxed{\Delta h_{Tee\text{-}Dividing} = 0.21843\ ft}$$

$$\Delta h_{nozzle\text{-}TeeDivide} = \frac{0.9(3.13536182\ ft/s)^2}{2}$$

$$\boxed{\Delta h_{nozzle\text{-}TeeDivide} = 4.42372\ ft}$$

$$\Delta h_{total} = \Delta h_{major} + \Delta h_{minor}$$

$$\Delta h_{major} = \Delta h_{pipe} + \Delta h_{vertical} + \Delta h_{half\text{-}circle}$$

$$\Delta h_{major} = 0.1278877\ ft + 7.5\ ft + 1.52568\ ft$$

$$\boxed{\Delta h_{major} = 9.1535677\ ft}$$

$$\Delta h_{minor} = \Delta h_{elbow} + \Delta h_{nozzle} + \Delta h_{Tee} + \Delta h_{nozzle\text{-}Tee}$$

$$\Delta h_{minor} = 0.07281 \text{ ft} + 5.242974 \text{ ft} + 0.2184 \text{ ft} + 4.42372 \text{ ft}$$

$$\boxed{\Delta h_{minor} = 9.957904 \text{ ft}}$$

$$\Delta h_{total} = \Delta h_{major} + \Delta h_{minor}$$

$$\Delta h_{total} = 9.1535677 \text{ ft} + 9.957904 \text{ ft}$$

$$\boxed{\Delta h_{total} = 19.11147 \text{ ft}}$$

$$\Delta h = \frac{\Delta P}{\rho}$$

$$19.11147 \text{ ft} = \frac{\Delta P}{1.94 \text{ slug/ft}^3}$$

$$\Delta P = 37.0762551 \text{ psf}$$

$$\dot{W} = \dot{\forall} \Delta P$$

$$\dot{W} = (0.6967 \text{ ft/s})\left(\pi\left(\frac{0.5}{2}\right)^2 \text{ ft}^2\right)(37.0762551 \text{ psf})$$

$$\boxed{\dot{W} = 5.072 \text{ lbf}\cdot\text{ft/s}}$$

Conclusion 4:
The head loss in this pipe system is roughly equal between major and minor losses. However, because we neglected velocity change in the main pipe, the losses here are very small. Most of the losses came from the nozzle and the vertical pipe. The resulting work (power) is very small.

Problem 5: Conservation of mass ch 5, incompressible flow:
The flow field for discharge pipe 2 is $v = (x^2 + y^2 + z^2)\mathrm{i} + (\mathrm{xyz} + x^2)\mathrm{j} + (-2\mathrm{xz} - \frac{xz^2}{2})\mathrm{k}$
What assumptions can be concluded about this flow field?
Find the local, convective, and total acceleration. If x =2 m, y =1 m, and z = 1 m, what is the pressure gradient?
Calculation 5:

Given:

$V = (x^2+y^2+z^2)\hat{i} + (xyz + x^2)\hat{j} + (-x^2z - \frac{xz^2}{2})\hat{k}$

Find: assumptions, accelerations, and pressure gradient when $x=2m$, $y=1m$, $z=1m$.

$u = x^2+y^2+z^2$

$v = xyz + x^2$

$w = -2xz - \frac{xz^2}{2}$

a) incompressible if $\nabla \cdot \vec{V} = 0$

$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = \nabla \cdot V$

$\frac{du}{dx} = 2x$

$\frac{dv}{dy} = xz$

$\frac{dw}{dz} = -2x - xz$

$2x + xz - 2x - xz = 0$

$\boxed{\text{Incompressible}}$

$\boxed{\text{Independent from time, so flow field is steady}}$

if $\vec{\omega} = 0$, flow field is irrotational:

$\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$

$\vec{\omega} = \frac{1}{2}\left(\frac{dw}{dy} - \frac{dV}{dz}\right) + \frac{1}{2}\left(\frac{du}{dz} - \frac{dw}{dx}\right) + \frac{1}{2}\left(\frac{dV}{dx} - \frac{du}{dy}\right)$

$\vec{\omega} = \frac{1}{2}(0 - xy) + \frac{1}{2}(2z - (-2xz - xz)) + \frac{1}{2}((yz + 2x) - (2y))$

$\vec{\omega} = \frac{1}{2}(-xy + 2z + 2xz + xz + yz + 2x - 2y)$

$\boxed{\vec{\omega} = \frac{1}{2}(-xy + 3xz + yz + 2z + 2x - 2y)}$

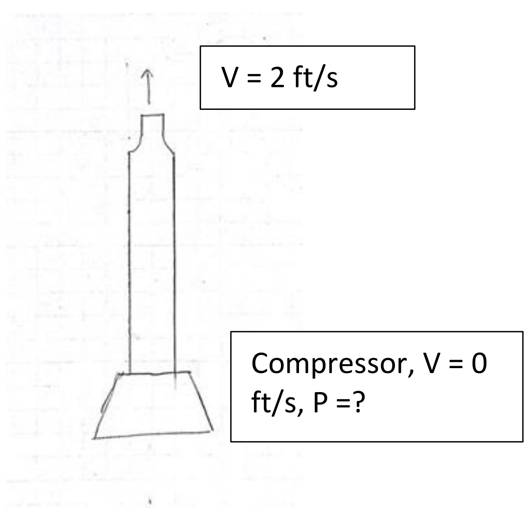
$\boxed{\text{Flow field } \vec{\omega} \neq 0 \text{ so it is not irrotational}}$

There is no time so there is no acceleration.

Pressure gradient is also dependent on time, so it is constant here.

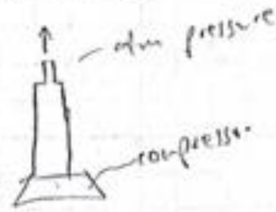
Conclusion 5:
After calculating, I found that this flow is incompressible, matching assumptions we have made elsewhere. However, I also found that the flow is rotational, which makes sense because it is water in a discharge pipe flowing slightly downwards towards the control room. There is no time variable, so it is steady and there is no acceleration. We cannot calculate pressure gradient as it is dependent on time.

Problem 6: In the center of the fountain, there is one vertical shooter. There is a compressed air tank attached that pushes water through the nozzles at 5 ft/s. The pipe connecting the compressor and the nozzle is 2 ft long and has a diameter of 2 inches. The nozzle's entrance is 1 inch in diameter. Find the pressure needed at the compressor to ensure the vertical shooter exits at the desired velocity. How does changing the diameter of the nozzle affect the required pressure?



Given: $V_2 = 2$ ft/s
$V_1 = 0$ ft/s
$z_2 = 2$ ft
$z_1 = 0$ ft
$\rho = 1.94$ slug/ft^3

schematic



Assume:
1) incompressible
2) steady
3) uniform
4) std atm
5) inviscid
6) streamline flow
7) frictionless flow

a) $$\frac{P_1}{\rho} + \frac{\cancel{V_1^2}}{2} + gz_1 = \frac{\cancel{P_2}}{\rho} + \frac{V_2^2}{2} + gz_2$$
$V_1 = 0$ ft/s $\qquad$ 0 (g)

$$\frac{P_1}{1.94 \text{ slug/ft}^3} + (32.2 \text{ ft/s}^2)(0 \text{ ft}) = \frac{(2 \text{ ft/s})^2}{2} + (32.2 \text{ ft/s}^2)(2 \text{ ft})$$

$$\boxed{P_1 = 128.816 \text{ psf}}$$

b) Increasing diameter:
assuming $\dot{V}$ is the same:
V_2 decreases, resulting in
pressure @ point 1 also decreasing

Conclusion 6:
We can use Bernoulli's equation because we know the water flowing through this system is steady, incompressible, inviscid, and along a streamline. This means that it is constant. We found that the pressure initially must be 128.816 psf for the nozzle exit velocity to be 2 ft/s. This is significant pressure and would also require significant work to achieve.

Problem 7:
Consider one of the laminar flow shooter nozzles that is receiving water from pump 2. The nozzle has a diameter of 1/2 inch and length of 30 inches. If we want the exit velocity of the water to be 0.5 ft/s, determine the relationship between the parameters D, V, rho, mu, and L

using Buckingham Pi. Density and viscosity can be assumed as 1.94 slugs/ft^3 and 2.10E-05 lbfs/ft^2 respectively.

GIVEN: M L T

D	V	ρ	μ	L
L	$\frac{L}{T}$	$\frac{M}{L^3}$	$\frac{M}{LT}$	L

$$\pi_1 = D^a V^b \rho^c \mu^1$$

$$\pi_1 = L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^1$$

$$\pi_1 = M^0 L^0 T^0 =$$

$M: 0 = \quad c + 1 = 0$

$L: 0 = a + b - 3c - 1$

$T: 0 = -b - 1$

$c = -1$
$b = -1$
$a = -1$

$$\boxed{\pi_1 = \frac{D\mu}{V\rho}}$$

$$\pi_1 = \frac{1}{\left(\frac{L}{T}\right)} \frac{M}{LT} \frac{L^3}{M L} \checkmark$$

$$\pi_2 = D^a V^b \rho^c L^1$$

$$\pi_2 = M^0 L^0 T^0 = L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c L^1$$

$M: 0 = c = 0$

$L: 0 = a + b - 3c + 1 = 0$

$T: 0 = -b = 0$

$b = 0$
$c = 0$
$a = -1$

$$\boxed{\pi_2 = \frac{L}{D}}$$

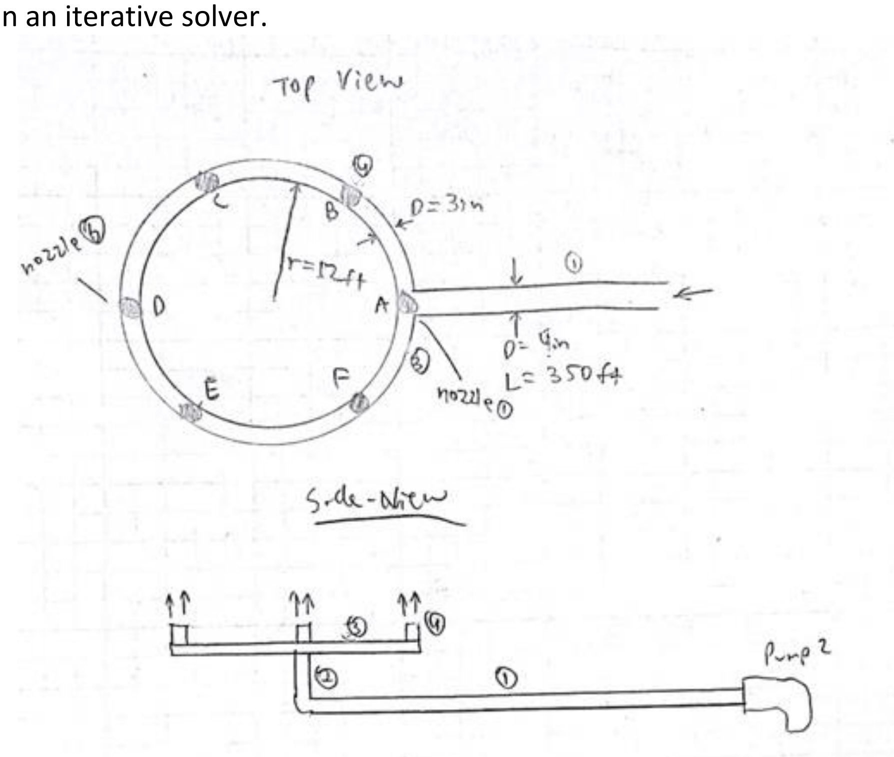
$$\pi_1 = f(\pi_2)$$

$$\boxed{\frac{D\mu}{V\rho} = f\left(\frac{L}{D}\right)}$$

Conclusion 7:

I was able to create a relationship between these 5 dimensionless parameters. Both equations come out to be dimensionless as you can see. The first is similar to Reynolds number, but slightly different. The second is a relationship between the length and diameter of the pipe.

Problem 8: (minor/major losses, moody chart, and branch flow with iterative solution)(solve for flow rate, D or V)
Pipe 7 is laid horizontally for 350ft with a 4 inch diameter before there is a 90 degree elbow and a vertical Pipe of 7.5 ft connects to a junction. At this junction, water flows through a circular pipe connecting to all 6 laminar flow shooter pipes. The circular pipe's radius, the distance from the center of the fountain to the pipe, is 12 ft. The circular pipe's diameter is 3 inches. We know from Problem 7 that each nozzle's diameter is 1/2 inch, has a length of 30 inches, and has an exit velocity of 0.5 ft/s. Head loss of the entire system is 875 ft. The pump releases water at an initial 10 ft/s, and in Pipe 3, the circular pipe, it is 4 ft/s. Assuming the vertical pipe is the same diameter as Pipe 1, calculate the velocity of the water through it using major and minor calculations in an iterative solver.



Given: all stainless steel, $g = 32.2\ ft/s^2$

Pipe 1:	Pipe 2:	Pipe 3:	Nozzle
$L = 350\ ft$	$L = 7.5\ ft$	$L = 2\pi r$	$L = 2.5\ ft$
$D = 4\ in$	$D = 4\ in$	$r = 12\ ft$	$D = 0.5\ in$
$V = 10\ ft/s$	$\Delta z = 7.5\ ft$	$\Delta z = 0$	$\Delta z = 2.5\ ft$
$\Delta z = 0$	$V = ?$	$V = 4\ ft/s$	$V = 0.5\ ft/s$

Find: velocity through pipe 2

Assume:
incompressible
laminar @ nozzle
uniform
steady
$\alpha = 1$
empirical losses
streamline flow

$$\left(\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1\right) - \left(\frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + g z_2\right) = h_{lT} = h_l + h_{lm}$$

$$\downarrow \qquad = f \frac{L}{D}\frac{V^2}{2} + \frac{KV^2}{2}$$

V in circular pipe is constant

$$h_l = f \frac{L}{D}\frac{V^2}{2}$$

$$h_l = h_{l\,pipe①} + h_{l\,pipe②} + h_{l\,ABCD} + h_{l\,AFED}$$

$$h_{lm} = \cancel{h_{lm\,pipe①}}^{0} + h_{lm\,elbow} + \cancel{h_{lm\,pipe②}}^{0} + h_{lm\,T} + h_{lm\,nozzles}$$

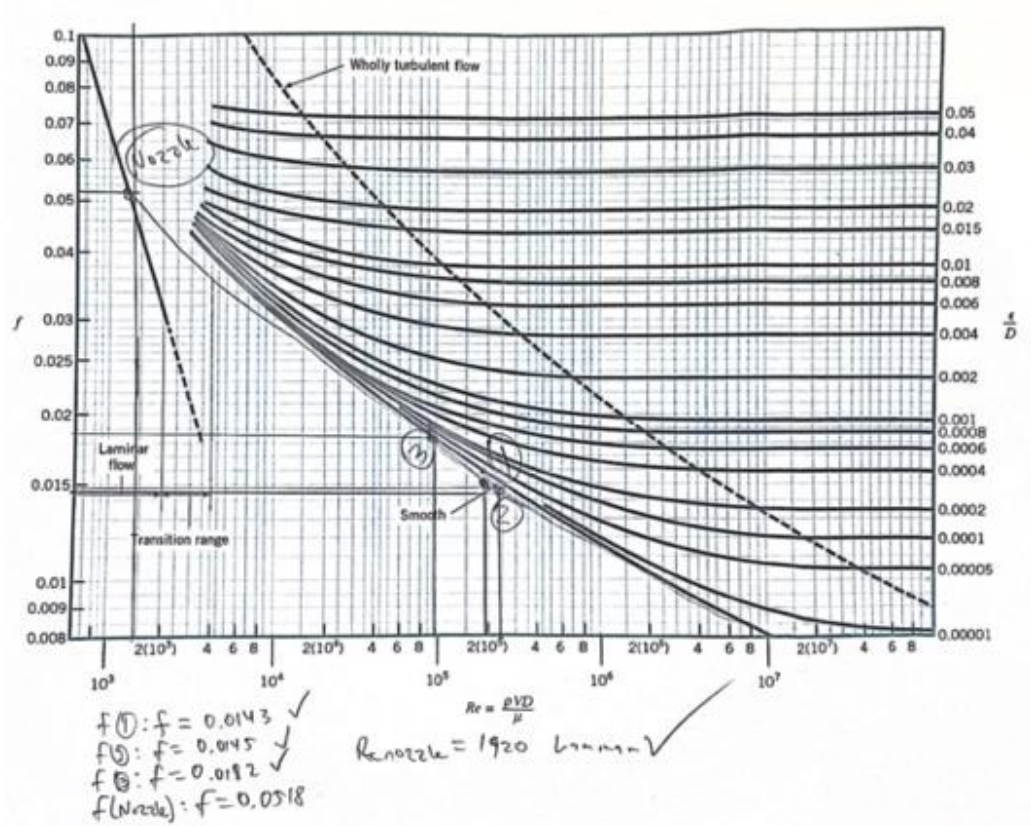
$$h = 875\ ft = h_l + h_{lm}$$

Divide Pipe sections and Loss type:

Iterative Solver

Iterative Solver Excel Sheet:
Losses Tab

V = 9.40708 ft/s



Conclusion 8:
This iterative solver was able to solve for the velocity in the second pipe. The calculated velocity is 9.40708 ft/s. This is slightly slower than the initial velocity from the pump. Most of the head loss came from the first pipe's major loss. This makes sense because this pipe is 350 ft long, so naturally there is significant loss. Major loss in the circular pipe was the next largest contributor. Also, Reynolds' number for the nozzle is less than 2000, so it is laminar flow, which checks out.

Problem 9:
Use Buckingham Pi to get a set of dimensionless groups for the given parameters: Diameter of the pipe, volumetric flow rate, density, viscosity, and pressure drop. Get two Buckingham Pi equations and use them to during similitude. Create a model with air as a fluid through a pipe with a diameter 3 times smaller than the full one. If volumetric flow in the model is 2 ft^3/s, what is it in the full pipe? If pressure drop in the model is 42 lbf/ft^2, what is it in the full pipe?
Calculation 9:

Given: $D, \dot{\forall}, \rho, \mu, \Delta P,$

$\frac{D_f}{D_m} = 3, \; \dot{\forall}_m = 2 ft^3/s, \; \Delta P_m = 42 lbf/ft^2$

Find: a) Buckingham Pi equations (2)

b) $\dot{\forall}_f$

c) ΔP_f

$L, T, M, \quad r=3$

D	$\dot{\forall}$	ρ	μ	ΔP	$n=5$
L	$\frac{L^3}{T}$	$\frac{M}{L^3}$	$\frac{M}{LT}$	$\frac{ML}{T^2}$	$r=3$

$\pi_1 = D^a \dot{\forall}^b \mu^c \rho^1$

$\pi_1 = M^0 L^0 t^0 = L^a \left(\frac{L^3}{T}\right)^b \left(\frac{M}{LT}\right)^c \left(\frac{M}{L^3}\right)^1$

$M: 0 = c+1 = 0 \rightarrow \boxed{c=-1}$

$L: 0 = a + 3b - c - 3$

$T: 0 = -b - c$

$b = -c$

$\boxed{b=1}$

$a = c + 3 - 3b$

$a = -1 + 3 - 3$

$\boxed{a=-1}$

$\pi_1 = D^{-1} \dot{\forall}^1 \mu^{-1} \rho^1$

$$\boxed{\pi_1 = \frac{\dot{\forall} \rho}{D \mu}}$$

$\frac{ML}{T}$

$$\pi_1 = \frac{\left(\frac{\cancel{L^3}}{\cancel{T}}\right)\left(\frac{\cancel{M}}{\cancel{L^3}}\right)}{(L)\left(\frac{\cancel{M}}{LT}\right)} \quad \checkmark$$

$$\Pi_2 = D^a \dot{\forall}^b \mu^c \Delta P^1$$

$$M^0 L^0 T^0 = L^a \left(\frac{L^3}{T}\right)^b \left(\frac{M}{LT}\right)^c \left(\frac{ML}{T^2}\right)^1$$

$M: 0 = c + 1 = 0$

$L: 0 = a + 3b - c + 1$

$T: 0 = -b - c - 2$

$\boxed{c = -1}$

$b = 1 - 2$

$\boxed{b = -1}$

$a = c - 1 - 3b$

$a = -1 - 1 + 3$

$\boxed{a = 1}$

$$\Pi_2 = D^1 \dot{\forall}^{-1} \mu^{-1} \Delta P^1$$

$$\boxed{\Pi_2 = \frac{D \Delta P}{\dot{\forall} \mu}}$$

$$\Pi_2 = \frac{(\cancel{L})\left(\frac{M\cancel{L}}{\cancel{T^2}}\right)}{\left(\frac{\cancel{L^3}}{\cancel{T}}\right)\left(\frac{\cancel{M}}{\cancel{LT}}\right)} \quad \checkmark$$

$$\Pi_1 = f(\Pi_2)$$

$$\boxed{\frac{\dot{\forall} \rho}{D \mu} = f\left(\frac{D \Delta P}{\dot{\forall} \mu}\right)}$$

Similitude:

Full scale:

$D_f, \dot{V}_f, \rho_f, \mu_f, \Delta P_f$

Model:

$D_m, \dot{V}_m, \rho_m, \mu_m, \Delta P_m$

$$\Pi_{1f} = \frac{\dot{V}_f \rho_f}{D_f \mu_f}$$

$$\Pi_{1m} = \frac{\dot{V}_m \rho_m}{D_m \mu_m}$$

$$\frac{\dot{V}_f \rho_f}{D_f \mu_f} = \frac{\dot{V}_m \rho_m}{D_m \mu_m}$$

$$\frac{\dot{V}_f}{\dot{V}_m} = \frac{\rho_m D_f \mu_f}{\rho_f D_m \mu_m}$$

Π_2:

$$\Pi_{2f} = \frac{D_f \Delta P_f}{\dot{V}_f \mu_f}$$

$$\Pi_{2m} = \frac{D_m \Delta P_m}{\dot{V}_m \mu_m}$$

$$\frac{\Delta P_f}{\Delta P_m} = \frac{\dot{V}_f \, \rho_m \, \mu_f}{\dot{V}_m \, D_f \, \mu_m}$$

$$\frac{D_f}{D_m} = \frac{3}{1} = 3$$

$$\Delta P_m = 42 \text{ lbf}$$

$$\frac{\dot{V}_f}{\dot{V}_m} = \frac{\rho_m D_f \mu_f}{\rho_f D_m \mu_m}$$

$\rho_m = \rho_{air} = 0.00234 \text{ slug/ft}^3$ [1] A.9

$\mu_m = \mu_{air} = 3.79E{-}07 \text{ lbf}\cdot\text{s/ft}^2$

$\rho_f = \rho_{water} = 1.94 \text{ slug/ft}^3$

$\mu_{water} = 2.10E{-}05 \text{ lbf}\cdot\text{s/ft}^2$

$\dot{V}_m = 2 \text{ ft}^3/\text{s}$

$$\frac{\dot{V}_f}{2 \text{ ft}^3/\text{s}} = \frac{(0.00234 \text{ slug/ft}^3)}{(1.94 \text{ slug/ft}^3)} (3) \frac{(2.10E{-}05 \text{ lbf}\cdot\text{s/ft}^2)}{(3.79E{-}07 \text{ lbf}\cdot\text{s/ft}^2)}$$

$$\boxed{\dot{V}_f = 0.401 \text{ ft}^3/\text{s}}$$

$$\frac{\Delta P_f}{42 \text{ lbf/ft}^2} = \frac{(0.401 \text{ ft}^3/\text{s})\left(\frac{1}{3}\right)(2.10E{-}05 \text{ lbf}\cdot\text{s/ft}^2)}{(1.94 \text{ slug/ft}^3)(3.79E{-}07 \text{ lbf}\cdot\text{s/ft}^2)}$$

$$\boxed{\Delta P_f = 160.344 \text{ lbf/ft}^2}$$

Conclusion 9:

Using Buckingham Pi, I found the relationship between these variables. Pi1 is similar to Reynolds number, but volumetric flow rate is in its place, changing it slightly. Pi2 relates these with pressure drop instead of density. Using these equations and the given information about the air model, I found the volumetric flow rate of the water in the actual pipe to be 0.401 ft^3/s and the pressure drop to be 160.344 lbf/ft^2. This shows how using similitude is beneficial for testing these parameters on a small scale before implementing on the actual project.

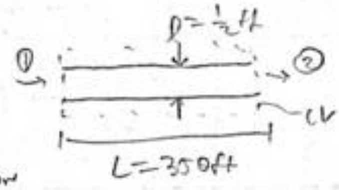
Problem 10: (fully developed flow derivation and analysis choosing an appropriate case)
Fully developed laminar water flows through suction pipe 1. We know that the pipe length is 350 ft and has a diameter of 6 inches. Because this water must eventually be pumped into the fan oarsman nozzles, we know the volumetric flow rate is 60 gpm. Assume density and viscosity are the same as in the fountain. Derive an expression for the velocity profile in the fully developed pipe and solve for the pressure drop and average velocity.

Calculation 10:

Given: $L = 350\ ft$, $D = 6\ inches$, $\dot{V} = 60\ gpm$

Find: velocity expression, ΔP, and average velocity

Assume:
- incompressible
- Std atm
- steady
- laminar
- fully developed flow
- horizontal tube



$$V_{avg} = \frac{\dot{V}}{A} \qquad [1]\ 8.13d$$

$$A = \frac{\pi}{4}(D)^2$$

$$A = \frac{\pi}{4}\left(\frac{1}{2}\right)^2$$

$$A = 0.19635\ ft^2$$

$\dot{V}$ must be $\underline{60\ gpm}$ to deliver enough water to the fan oarsman nozzles.

$$V_{avg} = \frac{60\ gpm}{0.19635\ ft^2} \times \frac{1\ min}{60\ s} \cdot \frac{ft^3}{7.48\ g}$$

$$\boxed{V_{avg} = 0.681\ ft/s}$$

$$\Delta P = \frac{128\ \mu L \dot{V}}{\pi D^4} \qquad [1]\ 8.13c$$

$$\mu = 2.10E\text{-}05\ _{lb \cdot s/ft^2}\ [1]$$

$$\Delta P = \frac{128(2.10E\text{-}05)(350\ ft)(0.681\ ft/s)}{\pi\left(\frac{1}{2}\right)^4 ft^4}$$

$$\Delta P = 3.262\ lb/ft^2$$

$$\boxed{\Delta P = 3.262\ psf}$$

velocity profile:

$$u = \frac{-R^2}{4\mu}\frac{\Delta P}{L}\left[1-\left(\frac{r}{R}\right)^2\right] \quad [1]\ 8.12$$

R is the radius of the pipe, r is the distance from the center to a designated point in the pipe.

$$u = -\frac{(0.25)^2(3.262\ psf)}{4(2.10E-05)(350\ ft)}\left[1-\left(\frac{r}{0.25}\right)^2\right]$$

$$\boxed{u = 2.367\times10^5(0.0625\ ft^2 - r^2)}$$

Conclusion 10:

I was able to use the given and prior information to solve for the average velocity and pressure drop through this discharge pipe. Using these values, I set up a velocity profile equation that now relies on r, the distance from the centerline. When an r value is plugged in, the velocity at that point is calculated. Also, if r=0, the velocity is at its greatest. This means the centerline of the pipe has the maximum velocity through this pipe.