University of Nottingham

Formal Report

How does a violin produce sound?

Student Number:
14334056

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Contents

Abstract

This report aims to investigate how a violin produces sound and what makes a violin's sound quality superior to another. Firstly the fundamental frequency of the violin's D string was calculated as 260(±80)Hz.
Analysing the harmonic spectra of the violin, when plucked at it's midpoint the odd harmonics were excited the most. Comparing the differences between plucking and bowing of the violin, it was discovered that bowing produced a significantly richer harmonic spectrum. Lastly when comparing two different violins, an entry-level violin and a replica of a Stradivarius violin, the Stradivarius violin produced the most desirable sonic information. Arbitrary values for the rate of harmonic amplitude decay were calculated as: $2(\pm 2)\times 10^{-8}$ *for the entry-level violin and* $5.7(\pm 0.6)\times 10^{-9}$ *for Stradivarius' violin.*

1 Introduction

The primary objectives of this report is to investigate the different ways in which a violin can produce sound and what makes a violin's sound quality better when comparing two different violins.
Although it is unknown who created the first violin, it is known a four stringed instrument emerged during early 16th century similar to a conventional violin seen today. The most acoustically advanced violins began to be developed in the 17th and 18th centuries. The violins produced during this 'golden era' are what most violin's form factor we see today are based upon.[1]

The components of a traditional violin can be seen in figure 1.

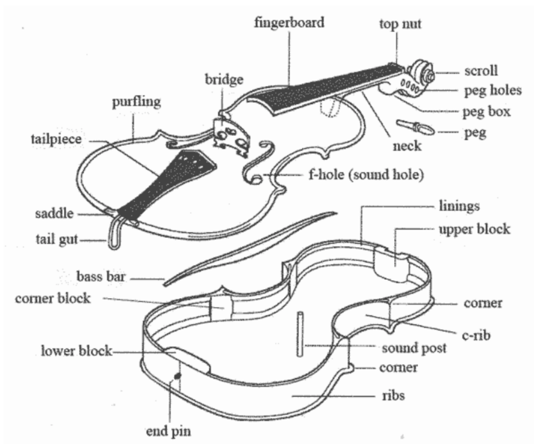


Figure 1: This diagram details all the parts of a traditional violin.[2]

The components that we are most interested in for this report are the bridge, the sound post, the bass bar and the fingerboard. Other parts of the violin might be mentioned such as the pegs and scroll but are insignificant to the objectives of the investigation.
The bridge of a violin is a piece of maple wood that attaches to the soundpost via the back plate and bass bar via the front plate components. The bridge acts as a 'gatekeeper' to transfer energy between the strings and the body of the violin. The bridge is one of the most scientifically controversial elements of violin and is still not fully understood to date. *C.M.Hutchins*

in her report on *'A History of a Violin Research'* discusses the evolution in understanding of the violin bridge over time in detail. This is however beyond the reach of this report.[1]
What can be agreed and is relevant to this investigation is the strings of the violin apply a force perpendicular to themselves on the bridge. This causes the bridge and front plate of the violin to resonate. As a result the soundpost and bass bar transfer energy to the air inside the body of the violin to produce sound. It should be noted that although the bridge transduces sound energy, it also emphasises certain frequency ranges of the violin - around 2300Hz - known as the 'bridge hill'. The theory of the bridge hill is beyond the scope of this report.[3]

During the golden era, one of the most notable violin producers was Antonio Stradivari. He was renowned for his exquisite craftsmanship and as a result violins that produced supposedly superior sound. There is still no exact theory behind what makes a Stradivari sound better to violin experts - whether it be the quality and age of the violin wood or potentially the shaping of the violin body.[4] Neither scientifically is there a measure of the performance quality of the violin. The best estimates from *E.Heller* are in the power of the violin and how this is transmitted via the bridge and the body of a violin.

This investigation will first look into how a violin produces by analysing frequency spectra of a student violin (referred to as the first violin in this report) for both plucking and bowing of the strings. From this we will then compare the frequency spectra of a replica of Stradivarius' violin (referred to as the second violin), with the student violin to put a measure on the quality of violin sound production. *M.Powell* carried out a similar investigation into the frequency spectrum of both a violin and viola in her report. *M.Powell* used a magnetic resonator to oscillate the violin strings. Although not the exact method used, this can be used as a source of comparison.[5]

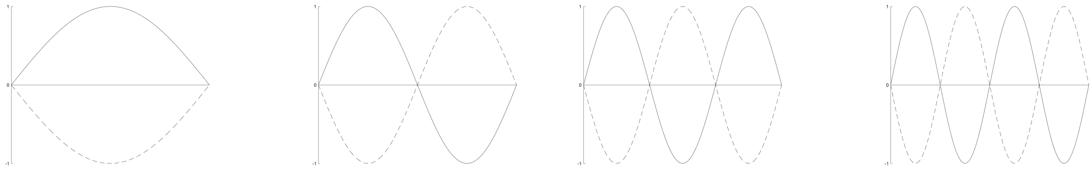
Putting a quantitative value on the sound quality of a violin will help violists gauge how much a violin is truly worth. Stradivarius' violins can cost up to £10 million at auction, however does this value correspond to a proportional increase sound quality .

It should be noted that although there is a difference in structure between the student violin and the Stradivarius replica, the replica will not be exactly comparable to that of an original Stradivari. Furthermore, the quality of the bow has been neglected from this investigation. Different bows can have different effects on the timbre of the violin however this is beyond the scope of this investigation.

2 Theory

2.1 The standing wave

The simplest way to model how a violin string produces sound is using standing wave theory. Consider a string tied at both its ends so there is tension through the string. The string is displaced by pulling it up from its equilibrium position, when let go it begins simple harmonic oscillation in the form of a standing wave. The first four harmonics of a standing wave are presented in figure 2.



(a) First harmonic. **(b)** Second harmonic. **(c)** Third harmonic. **(d)** Fourth harmonic.

Figure 2: These figures present the first four harmonics of a standing wave when in oscillation.

The points at which the wave displacement is a minimum are called nodes and where the wave is at maximum displacement are known as anti-nodes. Seen in figure 2 as the harmonic number increases the frequency of the wave increases. The first harmonic's frequency is a quarter of the fourth harmonic's and a half of the second harmonics. This alters the number of nodes and anti-nodes and their respective positioning on the standing wave.

The fundamental frequency of a string with a constant tension can be determined using elementary mechanics. By hanging a mass from the string's centre and measuring the deflection various properties of the string can be determined. As we add more mass the violin string, the string's displacement should increase with string tension. The figure below shows the force diagram used to determine the string's tension.

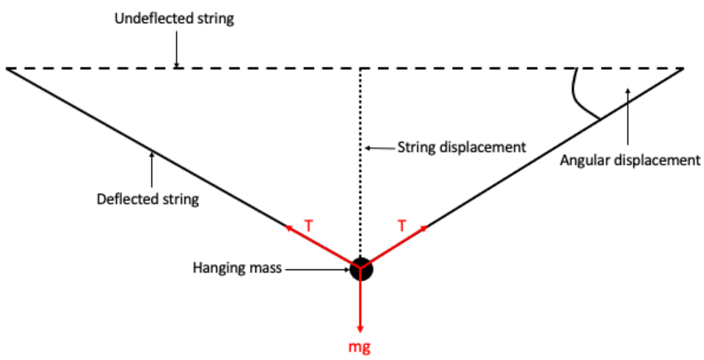


Figure 3: This diagram shows the set up used to determine the fundamental frequency of the violin's D string. The labels and arrows in red represent the forces acting in the system.

Assuming the mass is in equilibrium

$$2T \sin\theta = mg \rightarrow T = \frac{mg}{2\sin\theta} \tag{1}$$

where T is the tension of the violin string, m is the magnitude of hanging mass from the string, θ is the angle between the deflected string and the undeflected string and g is gravity at the Earth's surface.

Since θ is small, $\sin\theta \approx \tan\theta$. Using Pythagoras on the geometry of the string

$$d = \frac{l\tan\theta}{2} \approx \frac{l\sin\theta}{2} \rightarrow \sin\theta = \frac{2d}{l} \tag{2}$$

where l is the length of the string and d is the displacement of the string. Therefore subbing $\sin\theta$ determined in equation 2 into equation 1 gives tension.

$$T = \frac{mgl}{4d} \tag{3}$$

Knowing the tension of the string, the wave speed of the string can be determined using the equation below

$$c = \sqrt{\frac{T}{\mu}} \tag{4}$$

where c is the string's wave speed and μ is the linear density of the string.
The fundamental frequency, f_0, is given by

$$f_0 = \frac{c}{2l} \tag{5}$$

Plotting a graph of string displacement against mass gives a gradient equal to

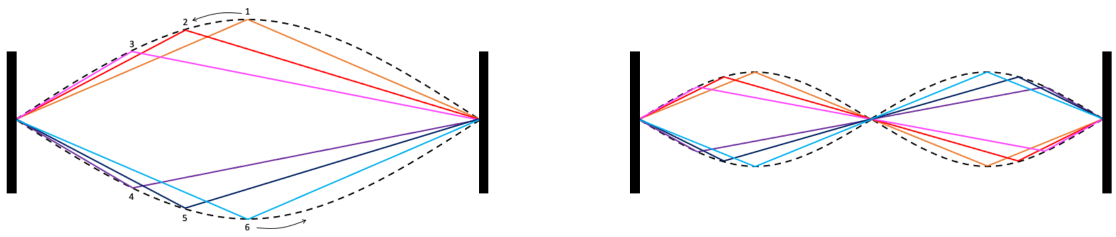
$$\frac{gl}{4T} \tag{6}$$

hence the tension of the string can be calculated. The value for tension can then be propagated through equation 4 and subsequently equation 5 to determine the fundamental frequency of the string.

When a string is plucked it would be unlikely to see purely the motion of any singular harmonic presented in figure 2. Typically multiple harmonic frequencies become excited at once and there is a combination of modes. A superposition of different harmonics form the sound spectrum of a plucked violin string. However, not all the modes will be excited equally, some may be excited more than others. As discussed in section 1 the bridge amplifies specific frequencies known as the 'bridge hill'. Also certain harmonics may be more excited depending on plucking position and different times after excitation.
When the string is initially plucked, both low frequency and high frequency harmonics are excited. However, very quickly the amplitudes of the higher harmonics decay quickly. Positionally, it is expected that when the string is plucked at the centre of the string the odd harmonics would be the most excited. This because odd harmonics have anti-nodes at the centre of the violin string, as seen in figure 2. The even harmonics are expected to have minimal displacement when the string is plucked at its midpoint. This is because there is a node at midpoint of the even harmonics. When force is applied to the string at the mid-point, the amplitude given to the string will dissipate at the node.

2.2 Helmholtz Motion

Helmholtz discovered the motion of a bowed string by watching the oscillation pattern of a small white dot painted on a string. Something similar to what he observed is presented in figure 4a.



(a) This figure shows snapshots of the progression in Helmholtz corners of the fundamental frequency in the violin string when up bowed.

(b) This figure shows snapshots of the progression in Helmholtz corners of the second harmonic in the violin string when up bowed.

Figure 4: These figures present the mechanics of Helmholtz motion of a violin string. Each coloured kink represents a different Helmholtz corner at a snapshot in time.

From each snapshot in figure 4a it can be seen that the string takes the shape of a kink, known as a Helmholtz corner. This is similar to the shape of a string just before a pluck is released. The corners then travel counter clockwise in the shape of a parabola to complete one full cycle per period. This movement is known as 'Helmholtz Motion'.
The Helmholtz motion for the second harmonic is presented in figure 4b. The kinks move at the same speed as the kink of the fundamental, but because there is two of them the cycle repeats twice as quickly. This same pattern of reflection is observed as harmonic frequencies increase.
The abrupt reflection of the kink at the bridge causes a sharp reversal of the force on the bridge. Hence the force on the bridge forms a saw-tooth force-time graph seen in figure 5. This saw-tooth force akin to bowing is rich in high frequency harmonics.

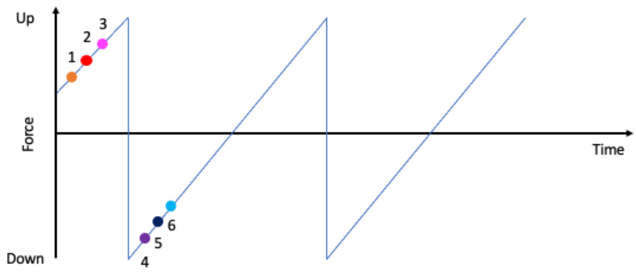


Figure 5: This figure shows the perpendicular force on the bridge as a function of time.The numbering on the waveform aligns the position of the string (see figure 4a) with time.

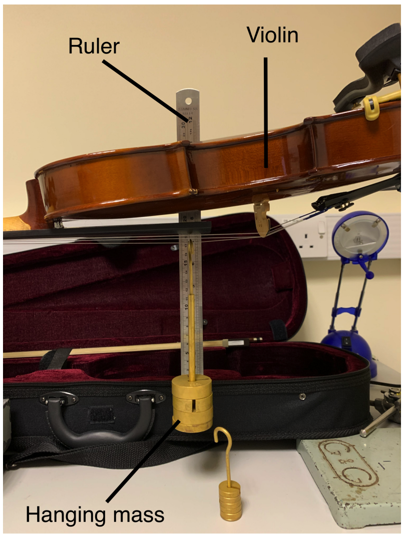
3 Experimental method

To determine the deflection in the string, the violin was set up as pictured in figure 6a. It should be noted that the violin was tuned frequently throughout the experiment. This was to ensure the string's tension was the same for all measurements. The violin string had to be tuned more frequently than normal practice due to the extra tension being applied to the string with the hanging masses.

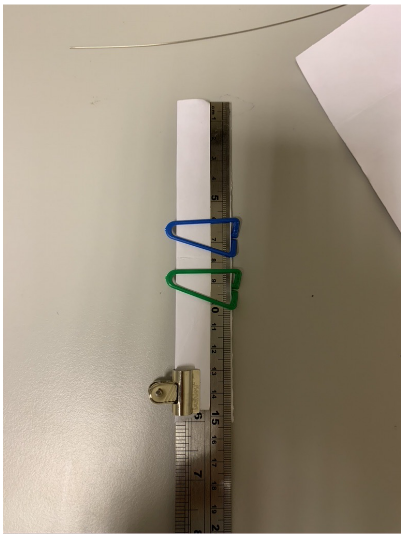
The violin was mounted upside down using a clamp and stand. This allowed for weights to be hung from the D string. A ruler was positioned behind the violin body perpendicular to the table and in line with the string deflection point.

Using this arrangement, weights were incrementally added to the violin string. For each increment, the deflection was recorded by looking at the maximum displacement point of the string against the ruler. This displacement value recorded by the ruler was taken away from the string's original position to give a value for string deflection. The parallax on the measurement was minimised by taking the deflection reading at eye level with the violin strings.
Despite attempts to reduce the parallax on the deflection readings, to make it clearer paper was added to the ruler as seen in figure 6b. Instead of visually determining the deflection, the displacement of the string was measured by placing the ruler directly behind the D string. Using a pencil, the deflection of the D string was marked for each total hanging mass. The difference between the marking of the undeflected string and the displaced string markings were measured on the ruler to give the string displacement.

The extra string tension meant precautions had to be taken when recording measurements with the violin. Weights were added to the string slowly to ensure no quick changes in string tension whilst keeping the violin at arms length. This reduced the likelihood of the string snapping and it causing harm.



(a) This figure shows how the violin was initially setup to measure the various string deflections for different mass.



(b) This figure shows the modification made to the ruler to measure the string deflection with greater accuracy. Clips were attached to the paper to keep it in place on the ruler.

Figure 6: These figures present how the equipment was set up and utilised to measure the deflection in the violin string.

This data was then propagated using the equations presented in section 2.1. Once propagated the data was processed using MATLAB's cftool to produce a linear line of best fit and a corresponding uncertainty on the gradient. This gradient was then used to calculate the fundamental frequency of the D string.

After the fundamental frequency of the D string was measured, the sound spectrum of the violin was investigated. This was done using two different pickup devices, a Piezoelectric pickup and a microphone. The violin was orientated into a regular playing position and held by a clamp and stand.
The first pickup used was the Piezo. The Piezo was manufactured by Musedo. The Piezo was clamped onto the bridge of the violin and connected to the *MDO3000 Series Tektronix* oscilloscope using a BNC connector. The configuration of the apparatus can be seen in figure 7. Before use of the violin, the strings were tuned using the tuner.

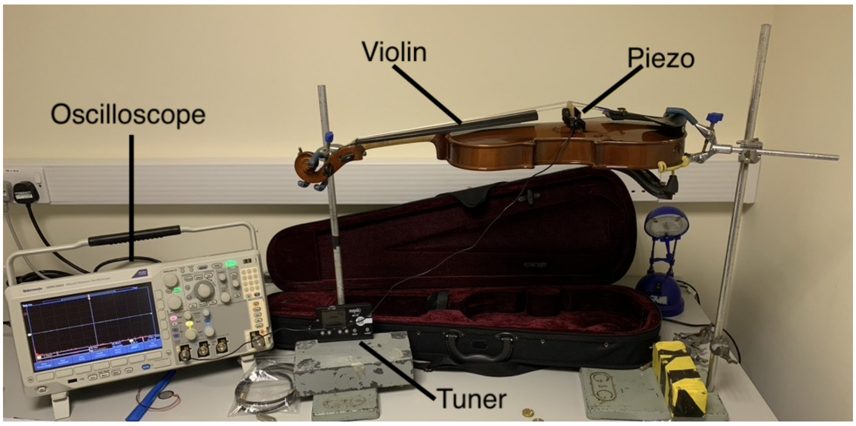


Figure 7: This figure shows the experimental setup when using the Piezo to pickup the violin's harmonic spectrum.

Firstly, the harmonic spectrum when the D string is plucked was recorded. In this case the D string was plucked at its midpoint. In plucking the string, the Piezo transmitted a temporal sound signal to the oscilloscope. The Tektronix had a Fourier transform function that converted the temporal sound signal from the Piezo into a frequency spectrum with corresponding amplitudes. Data from the Fourier transform was extracted from the oscilloscope using a USB drive and plotted in MATLAB.
After plucking the D string, the violin was bowed. Before use of the bow, it is important to apply rosin to the bow hair to allow the bow to stick and slip on the violin strings. The D string was down bowed (pulling the bow towards the player) by its self in one continuous motion for approximately two seconds. As when the string was plucked, the temporal signal from the Piezo was analysed using a Fourier transform in the oscilloscope. This data was subsequently plotted in MATLAB.

The second pick up used was a microphone. The microphone used was a *Behringer ECM8000*. The microphone was plugged into a *Xenyx 802* mixer that was connected to the oscilloscope via a BNC connector. The violin was setup in a similar fashion to that in figure 7, with the Piezo removed. The method that was carried out for the plucking and bowing of the violin with the Piezo was also used with the microphone pickup. The data was similarly extracted from the oscilloscope and plotted in MATLAB.

Using the microphone as the pickup device, the violin was plucked at different positions along the fingerboard to investigate the effect it had on the harmonic spectrum. This was done by plucking near the bridge (approximately 10% of the string length away from the bridge) and plucking closer to the pegs (approximately 20% of the string length away from the pegs). We can also use the spectrum when the string was plucked at its midpoint for later analysis. The different sound spectra picked up by the microphone were analysed using a Fourier transform and plotted in MATLAB.

Next we explored the harmonic spectrum produced by a second violin - a Stradivarius replica. The second violin was set up in the same manner as seen in 7. The violin was tuned before use. Using the microphone pickup, the second violin was bowed and plucked using the same technique as the first violin. The violin was both long bowed for approximately two seconds and short bowed for approximately half a second. The data recorded by the microphone was analysed using a Fourier transform and then plotted in MATLAB. It should be noted that while measuring the harmonic spectra of the first violin, short bowing data was also taken for later comparison.

4 Results

4.1 Determining the fundamental frequency of the D string.

Figure 8 presents the plot used to determine the fundamental frequency of the D string on the first violin. Error bars of $\pm$ 2mm were estimated for the deflection of the string. This was because despite making pencil markings on the ruler, there was still a noticeable uncertainty in measuring the deflection with the ruler. The data was analysed using MATLAB's cftool to plot figure 8. It should be noted that no weighting was used as all data points had the same associated error.

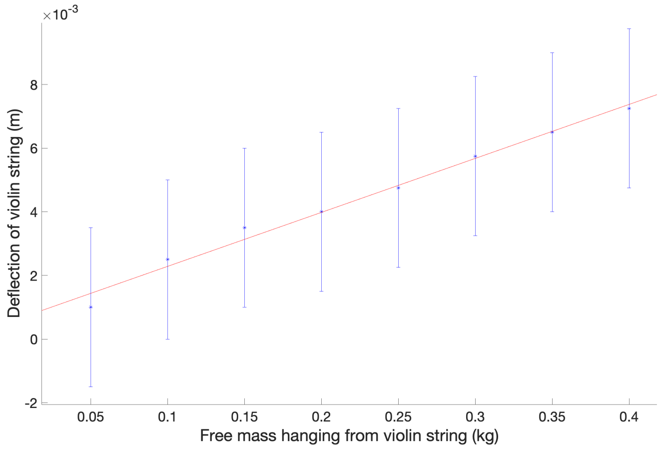
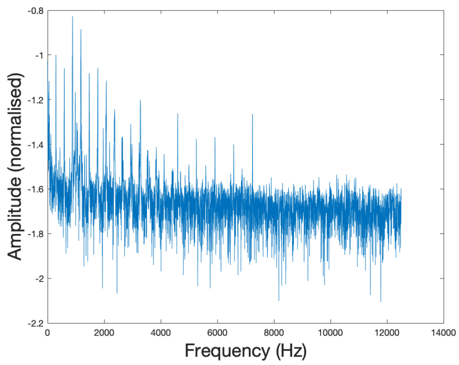


Figure 8: This figure shows a graph of the string deflection against magnitude of mass hanging from the violin string. The plot was analysed in MATLAB using cftool to fit a linear line of best fit. The gradient was calculated as $0.017(\pm 0.001)\ mkg^{-1}$.

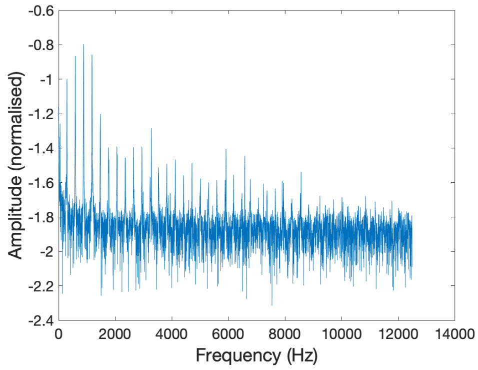
The gradient of figure 8 was calculated to be $0.017(\pm0.001)\ mkg^{-1}$. The gradient value was propagated through equation 6 to determine the tension of string. This could be then used to determine the D string's fundamental frequency of $260(\pm80)Hz$.
The error on the gradient of figure 8 was calculated using equation 7. The error formula in equation 9 was then used propagate errors in the gradient and string length through equation 6 to determine the error in tension. The error in the tension of the violin string and mass per unit length can then be propagated through equation 4 using the propagation equation again. Finally, propagating the error on the string's wave speed and its length using equation 5 gives the error on the fundamental frequency of the violin's D string.

4.2 Harmonic frequencies of the violin

Using the Piezo pickup, the harmonic spectra measured for the first violin are presented in figure 9.



(a) This figure presents the Fourier transform of the temporal signal for the first violin. The string was plucked at its midpoint picked up using the Piezo. The amplitudes have been normalised to the first harmonic.

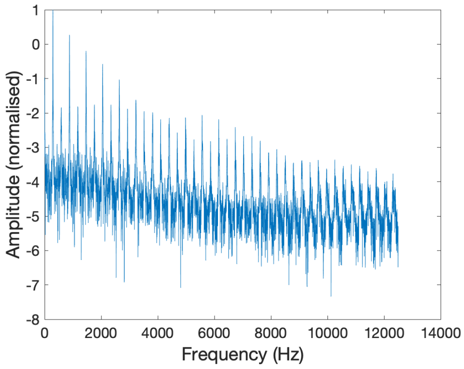


(b) This figure presents the Fourier transform of the temporal signal for the first violin. The string was long bowed ($\approx$ 2 seconds) picked up using the Piezo. The amplitudes have been normalised to the first harmonic.

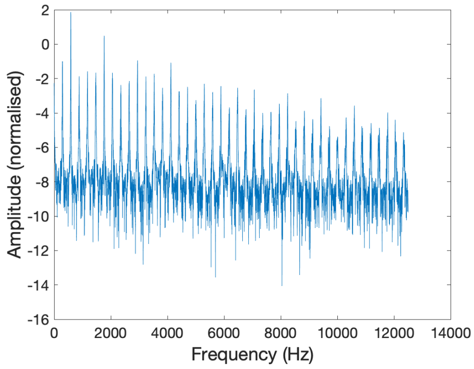
Figure 9: These figures present the Fourier transform of the temporal signal for the first violin when using the Piezo pickup. The amplitudes have been normalised to the first harmonic.

The data presented in figure 9 was plotted using MATLAB. It should be noted that the spectra in this report are normalised to the amplitude of the first harmonic. By taking the maximum amplitude within a 100 Hz range around the expected fundamental frequency of the D string the amplitude of the first harmonic was derived. This was then used to weight the amplitude spectrum for the corresponding data set. This process was executed using a MATLAB script that is insignificant to the results of this investigation. The normalised amplitudes allow easier comparison between harmonic spectra.

Using the microphone pickup, the harmonic spectra is presented in figure 10.



(a) This figure presents the Fourier transform of the temporal signal for the first violin. The string was plucked at it's midpoint picked up using the microphone. The amplitudes have been normalised to the first harmonic.

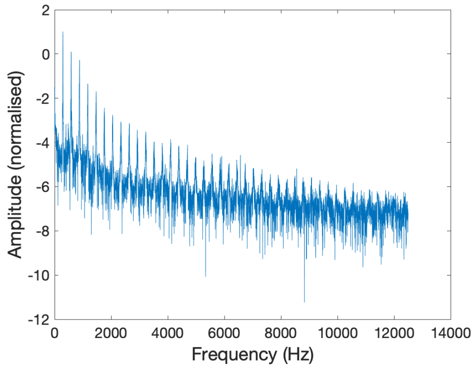


(b) This figure presents the Fourier transform of the temporal signal for the first violin. The string was long bowed ($\approx$ 2 seconds) picked up using the microphone. The amplitudes have been normalised to the first harmonic.

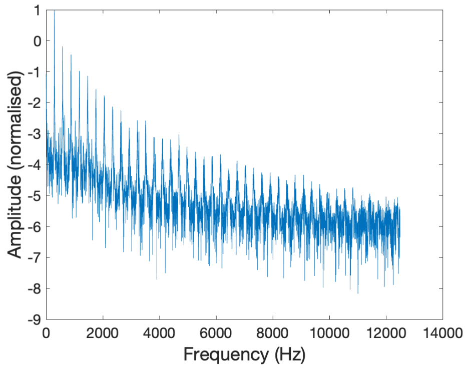
Figure 10: These figures present the Fourier transform of the temporal signal for the first violin when using the microphone pickup. The amplitudes have been normalised to the first harmonic.

Similarly to figure 9, the data in figure 10 was plotted using MATLAB. Again the data was normalised to the amplitude of the first harmonic using the same method.

Figure 11 presents the harmonic spectra for when the violin was plucked at different positions along the fingerboard.



(a) This figure presents the Fourier transform of the temporal signal when the string was plucked approximately 10% of string length away from the violin bridge. The microphone pick up was used.



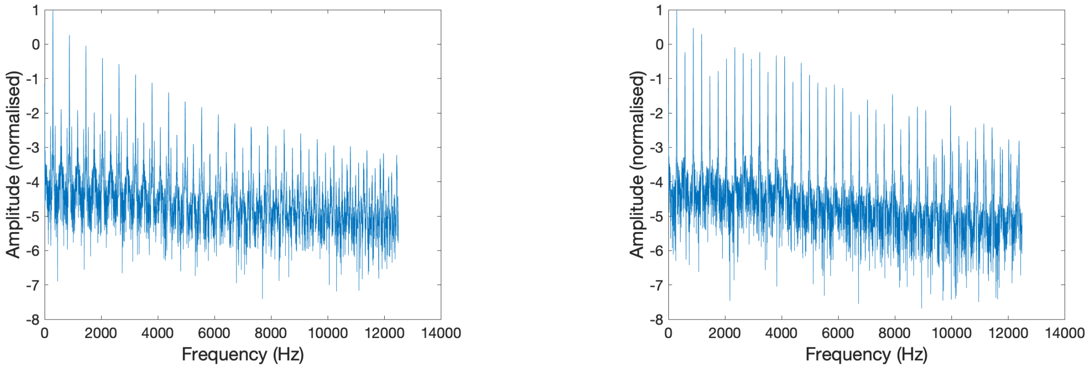
(b) This figure presents the Fourier transform of the temporal signal when the string was plucked approximately 20% of string length away from the violin pegs. The microphone pick up was used.

Figure 11: These figures present the Fourier transform of the temporal signal when the violin was plucked at different positions along the fingerboard of the violin. The microphone pickup was used.

The plots in figure 11 were plotted in MATLAB. The amplitude was normalised to the first harmonic.

4.3 Comparing two violins

The plots presented in figure 12 show the harmonic spectra for when the D string of the second violin was plucked at its midpoint and long bowed.



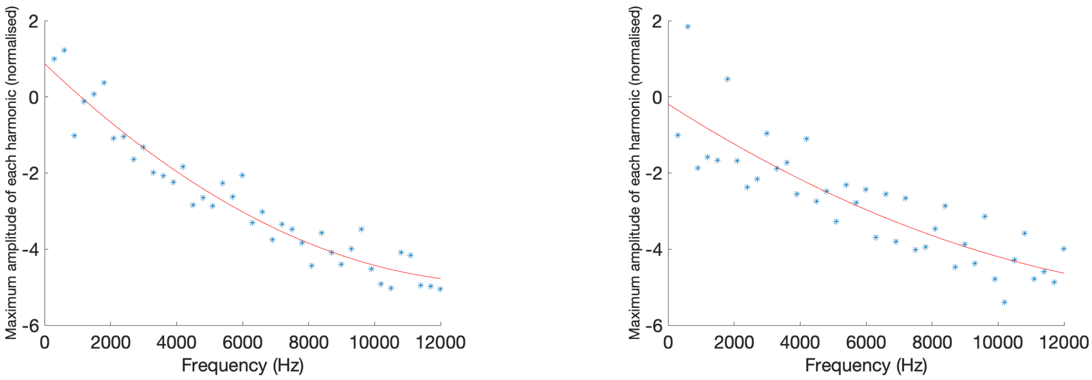
(a) This figure presents the Fourier transform of the temporal signal for the second violin. The string was plucked at its midpoint picked up using the microphone. The amplitudes have been normalised to the first harmonic.

(b) This figure presents the Fourier transform of the temporal signal for the second violin. The string was long bowed picked up using the microphone. The amplitudes have been normalised to the first harmonic.

Figure 12: These figures present the Fourier transform of the temporal signal for the second violin when using the microphone. The amplitudes have been normalised to the first harmonic.

The plots were created using the same method as the first violin seen in figure 10 in MATLAB.

Deriving from figures 10b and 12a, the figures presented in figure 13 show the rate of decay of harmonic amplitudes when the first violin was short and long bowed.



(a) This figure shows the decay in amplitude of the first violin when it's bowed for approximately one second. The arbitrary decay rate was $3.0(\pm0.7)\times10^{-8}$.

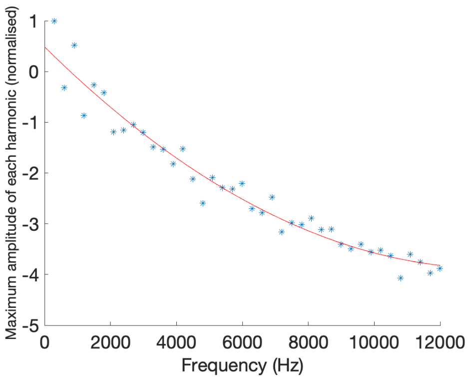
(b) This figure shows the decay in amplitude of the first violin when it's bowed for approximately five seconds. The arbitrary decay rate was $2(\pm2)\times10^{-8}$.

Figure 13: These figures present the plot of the maximum amplitudes of each harmonic frequency for the first violin. The plots were analysed using MATLAB cftool and produced a polynomial line of best fit. An arbitrary decay rate for the line of best was determined using the second order coefficient.

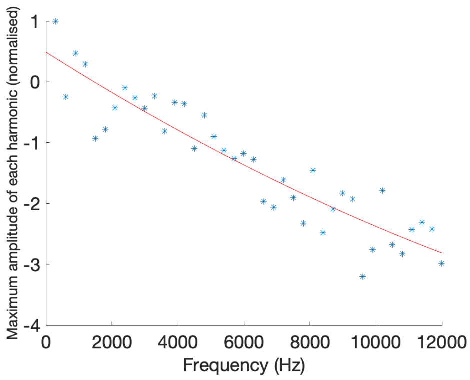
The points plotted represent the maximum amplitude of each integer valued harmonic.

MATLAB's cftool was then used to plot a line of best through the points. It should be noted that a second order polynomial fit was used over an exponential fit. The polynomial fit gave the most suitable line of best fit for both violin's amplitude decay. The polynomial's second order coefficient was used as an arbitrary measure of the rate of decay of amplitudes - extracted from cftool.

The amplitude decay for the second violin is presented in figure 14 below.



(a) This figure shows the decay in amplitude of the second violin when it's bowed for approximately one second. The arbitrary decay rate was $2.4(\pm 0.4) \times 10^{-8}$.



(b) This figure shows the decay in amplitude of the second violin when it's bowed for approximately five seconds. The arbitrary decay rate was $5.7(\pm 0.6) \times 10^{-9}$.

Figure 14: These figures present the plot of the maximum amplitudes of each harmonic frequency for the second violin. The plots were analysed using MATLAB cftool and produced a polynomial line of best fit. An arbitrary decay rate for the line of best was determined using the second order coefficient.

The exact method that was carried out to produce plots for the first violin was carried out for plots 14a and 14b.
The amplitude decay constants derived from both figures 13 and 14 are presented below in table 1.

	Amplitude Decay Constant (Arbitrary)	
Violin	Short Bowed	Long Bowed
1	$3.0(\pm 0.7) \times 10^{-8}$	$2(\pm 2) \times 10^{-8}$
2	$2.4(\pm 0.4) \times 10^{-8}$	$5.7(\pm 0.6) \times 10^{-9}$

Table 1: This table presents the arbitrary decay constants for both violins when they are long and short bowed.

The errors presented in this table were derived from MATLAB's 95% confidence bounds on the coefficients of the polynomial fit. Subsequently, the confidence bounds were converted into standard error values.

4.4 Analysis of Uncertainties

Data was fitted using MATLAB's cftool which created a line of best fit to our data points in figures 8, 13 and 14. The cftool produced parameter values for the lines of best fit with error estimates from cftool's 95% confidence bounds. These confidence bounds can then be used to calculate the standard error using the following equation

$$S = \frac{B_{Upper} - B_{Lower}}{4} \tag{7}$$

where S is the standard error and B_{Upper} and B_{Lower} are the upper and lower bounds.[6] Seen in equation 8 is the general error propagation equation

$${\sigma_f}^2 = \sum_{i=1}^{N} (\frac{\partial f}{\partial x_i})^2 {\sigma_i}^2 \tag{8}$$

where the partial derivative of function with respect to one of it's variables all squared multiplied by the variables associated error squared (summed for all variables) gives the squared error on the function.[7] The error propagation equation should be used when combining errors of different measurements used in a calculation. In our instance the error propagation is trivial and the general equation is simplified to equation 9

$$(\frac{\delta f}{f})^2 = (\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2 + (\frac{\delta z}{z})^2 \tag{9}$$

where f is the function of the error we are trying to calculate, δf is f's associated error and x y z are variables of f with associated errors δx δy δz respectively.

5 Discussion of Results

5.1 Discussion of the fundamental frequency of the D string

Firstly, it was experimentally established that the fundamental frequency of the first violin's D string was $260(\pm 80)Hz$. The typical fundamental frequency of a D string is $293.7Hz$.[8] The graph and results plotted in figure 3 line up with the theoretical predictions made; mass increase is directly proportional to string deflection. The error bars were estimated to be two millimetres. The plot produced a good linear regression which intersected all error bars. This suggests that the error bars fully compensated for the uncertainty in measuring the deflection of the violin string. When considering the scatter of points from the line of best fit is negligible, a smaller error estimation of the string displacement would have been more appropriate. However, this has likely had inconsequential effect on the result of the investigation. The small scatter that did arise was likely a product of the random error in measuring the difference between displacement markings on the ruler. The expected D string frequency lies within one standard deviation of the experimental frequency value. This consequently means that our estimate for the fundamental frequency of a violin D string lies within good confidence of previously determined values.

Despite this, there is still a discrepancy of approximately 30Hz between our experimental frequency and expected frequency. The minimal scatter in our data points implies that this could be due to a systematic error. As can be seen in figure 6a, the weights were not hung in exactly the middle of the violin string. The theory presented in figure 3 assumes the mass acts at the midpoint of the string so that tension is equal on either side. This might have had a significant impact on the measured frequency value although this is hard to quantify. To improve the accuracy of our frequency value, the masses should be hung at exactly half the string length.

5.2 Discussion of different pickup devices used to analyse the violin's harmonic spectrum

There is a distinct difference between the harmonic spectra when the violin's sound spectrum was measured using the Piezo as opposed to the microphone.
When the violin is plucked and bowed, the harmonic spectrum of the microphone is significantly richer than the Piezo. The harmonic spectrum of the microphone has a larger range of excited harmonics with distinct peaks. These harmonics fulfil the 12,000 Hz range measured. Comparing this to the Piezo, the harmonic range is only around 8,000 Hz and the spectrum as a whole is significantly noisier. This is likely due to the way in which the Piezo pickup functions. It relies on being attached to the bridge of the violin and physically picking up the vibrations transmitted by the strings via the bridge. Discussed in section 1, the bridge is a crucial part of how the violin produces it's sound. By attaching the Piezo to the bridge, it dampens the force transmitted to the bridge by the strings. The perpendicular force on the bridge shown in figure 5, would not be as distinct if had been dampened. This is why the harmonic frequency spectrum was not as clear and full when using the Piezo. The effects of this can easily be seen in figure 9b where there is a rapid decay in relative amplitude as frequency increases.
The microphone doesn't require being attached to the violin so has no effect on the it's vibration.
As a result, it was decided the microphone would be the most appropriate pickup to continue the investigation with.

Carrying out this investigation again, it would be interesting to see how the harmonic spectrum of the microphone is affected if the Piezo is left attached to the bridge. It would be expected that the microphone would pick up a similar harmonic spectrum to the Piezo.

5.3 Discussion of plucking a violin string

Comparing the harmonic spectra when the violin's string is plucked at different positions provides insight into how the plucking method of playing produces sound. Comparing figure 10a to both figures in figure 11, it is clear that when the string is plucked in the middle the richest harmonic spectrum is produced. The rate of decay of relative amplitude is much slower and the number of excited harmonics is much greater in figure 10a compared to figures 11a and 11b. In figure 10a it can be seen that generally the odd harmonics are the most excited modes and the even harmonics are the least excited. This agrees with what was expected from the theoretical understanding of standing waves discussed in section 2.1. Considering the plucks towards the pegs and the bridge shown in 11, these harmonics are less agreeable with the theory. It would be expected that when the string is plucked approximately 10% of string length away from the bridge, the most excited modes would be fourth harmonic. This is due to the positioning of the anti-nodes shown in figure 2d. Likewise when the string is plucked nearer to the pegs, the 3rd harmonic was expected to be excited the most. Instead both plucking positions present spectra where the first harmonic has the greatest amplitude and the following harmonic's amplitudes decay sequentially. The most probable explanation for this is that when

a string is plucked, generally the frequencies in the low harmonics are amplified.
Foremost, the superposition of harmonic modes and their different amplitudes when the violin is plucked at its midpoint produces the most appealing harmonic spectrum.
What can be taken away is that violists should aim to pluck the violin string nearest to it's midpoint as possible. When the string is plucked at its midpoint, it produces a richer spectrum of sound with more harmonics that decay slower than if the string is plucked either end.

5.4 Discussion of the differences between plucking and bowing of a violin string

There are also some clear differences in harmonic spectra when the violin is plucked and bowed shown in figure 10. The harmonic amplitudes decay less rapidly when the string is bowed not plucked, higher harmonics have greater amplitudes when the string is bowed and there isn't a clear relation between which harmonic numbers are the most excited when bowed not like plucking. This is succinct with the theoretical understanding. When the string is bowed, it undergoes Helmholtz motion discussed in section 2.2. As seen in figure 4a, the nature of the motion excites the string in multiple locations along its length causing excitation in all harmonics. As a result, the violin's strings transmit a force via the bridge rich in high harmonics. This agrees with figure 10b, the amplitudes of the harmonics are still relatively high at 12,000 Hz and there isn't a pattern of varying harmonic amplitudes suggesting all harmonics are excited. Comparing this to when the string is plucked (see figure 10a), although multiple modes are still excited, lower frequencies are the most excited. This is because the higher harmonic's amplitudes decay more rapidly when the string is plucked.
Contributing to the slower decay in harmonic's amplitude when the violin is bowed is the constant application of force to the string. When the bow is dragged across the string, for the duration the bow is dragged there is constant tension applied to the string. This means that the string's harmonics are constantly being excited. In comparison to a pluck, the string is only under an external force for a fraction of a second. This why steeper decay gradients of harmonics amplitudes are seen in figure 10a.

Comparing the data presented in figure 10 to a similar investigation carried out by *M.Powell*, a spectrum similar to when a string is plucked in this investigation is produced (see appendix). Although the first harmonic in both investigations have the highest amplitude, Powell's investigation observes a much steeper drop in amplitude for the subsequent harmonics in comparison to figure 11. This is likely due to Powell using a magnetic resonator to oscillate the violin strings - renown for exciting predominantly lower frequencies. Not as much can be gathered from comparing this investigation to Powell's as was initially anticipated due to the difference in experimental methods. However, it is seen that the first harmonic of the violin is the most excited consistently across all the data sets in this report.

It should be noted that when the violin was plucked, the oscilloscope picked up all the sonic information until the pluck sound rung out. When the violin was bowed the sound took significantly longer to ring out. This meant only the richest of sonic information was measured when bowing. This may potentially skew the data to exaggerate how slowly the bowed string's amplitudes decay and how quickly the plucked string's decay.
Next time it would be useful to take a sample of sonic information for a short period after the

string is plucked. This would record the richest sound from the pluck. It would be expected in this scenario the plucked string would present a similar harmonic spectrum to when a string is bowed. This is because in this short time frame, high frequency harmonics will not have yet decayed.

It is inferred that there are stark differences between bowing and plucking. For a violist both serve a different purpose to the listener. A pluck has shorter sound form and is clear in it's fundamental sound. Bowing however provides the violist the opportunity to excite the listener's ear differently. Higher harmonics associated with bowing provide more sonic information to ear, giving a fuller sound.

5.5 Comparison of two different violins

Comparing the harmonic spectra of the Stradivarius replica (see figure 12) and the student violin (see figure 10) the difference is not stark. Both the first and second violin spectra produce harmonic ranges up to 12,000 Hz. Both violin's spectra observe the same pattern for when the string is plucked at half its midpoint. The harmonic peaks of the second violin are marginally more distinct than the first violin; however the key difference between Stradivarius' replica and the student violin is the peak harmonic amplitude decay rate.
A clearer depiction for the comparison of amplitude decay rates is seen in figures 13 and 14. In both figures, the polynomial fit produces a suitable line of best fit to model the scatter with the smallest residuals. From the polynomial fit, the coefficients from the second order term were used as a measure of decay rate in harmonic amplitudes. The errors on decay rate were calculated from MATLAB's 95% confidence bounds. These were converted into standard errors using equation 7.
From table 1, it is seen the second violin has a smaller decay rate when long bowed and short bowed when compared respectively to the first violin. This quantitatively shows that the second violin's harmonic spectra decay less rapidly compared to the first. It should also be noted that for both violins the decay rates are smaller when long bowed compared to short bowed. This agrees with the theoretical perspective of bowing. When the string is long bowed, the string is excited for longer meaning the energy in the strings takes longer to dissipate.

Comparing the violins on solely their decay rates, it can be assumed the replica violin produces a more desirable sound compared to the student violin. This is because it produces a sound spectrum in which higher harmonics take longer to decay. The more sonic information that the violin can produce from these high harmonics, the more the sound appeals to the listener's ear. This agrees with the general consensus of violists. Stradivarius' violins are renowned for their superior quality in sound, especially when compared to an entry-level instrument.
To date there isn't a quantitative measure for the sound quality of a violin. It is thought that the power of a violin is an indication of quality but this doesn't directly relate to the timbre of the violin. By analysing the decay rate of amplitudes as frequency increases and giving this an arbitrary value, it gives violists a measure to compare violins. It is clear the Stradivarius violin produces superior sound quality but these violins can cost extortionate amounts. Using the decay rates, it is easier for violists to quantify the trade off between quality and price.

This investigation struggled to scientifically explore how the ear interprets sonic information and how this relates to an instrument's timbre. Although it is clear that Stradivarius' violin produces a full harmonic spectrum rich in sonic information, there is no explanation why high harmonics are more appealing to the ear. To extend this project, investigating how the ear analyses different frequencies could provide explanation for the timbre of the different violins and why the Stradivarius is hailed by violists.

The way in which the violin is held intrinsically effects the how it vibrates. Throughout the duration of the experiment the violin was held using two clamp and stands. One clamping the body of the violin on the front and back plate and one clamped at the scroll. By clamping the top and back plates, it is likely that the force transmitted to the plates via the sound post and bass bar were dampened. The effect this had on the investigation is unknown. However the dampening likely would have been a systematic error that caused the amplitudes of harmonics to decrease. This likely had negligible effect in determining the key findings. Next time it would be useful to have a trained violist play the violin for the investigation.

6 Conclusions

In this multi-faceted investigation into how a violin produces sound, different key results were produced. The fundamental frequency of the D string on an elementary violin was calculated to be 260($\pm$80) Hz. The expected fundamental frequency was 293.7 Hz.
Analysing the harmonic spectra for plucking and bowing of two different violins, there were many key findings. Firstly, when a violin string is plucked at it's midpoint it predominantly excites the odd harmonic modes. Looking at the differences between plucking and bowing, evidence is found that bowing produces a harmonic spectrum richer in higher frequencies where harmonic amplitudes decay slowly compared to plucking.
Finally comparing two violins, one of supposed high quality and one at entry level it can be seen that the high quality violin produced the most desirable sonic data. When long bowed the arbitrary amplitude decay constant for the student violin was $2(\pm 2) \times 10^{-8}$ and for Stradivarius' violin $5.7(\pm 0.6) \times 10^{-9}$.
Over the whole set of experiments, the results constituently matched those which were expected.

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A Appendix

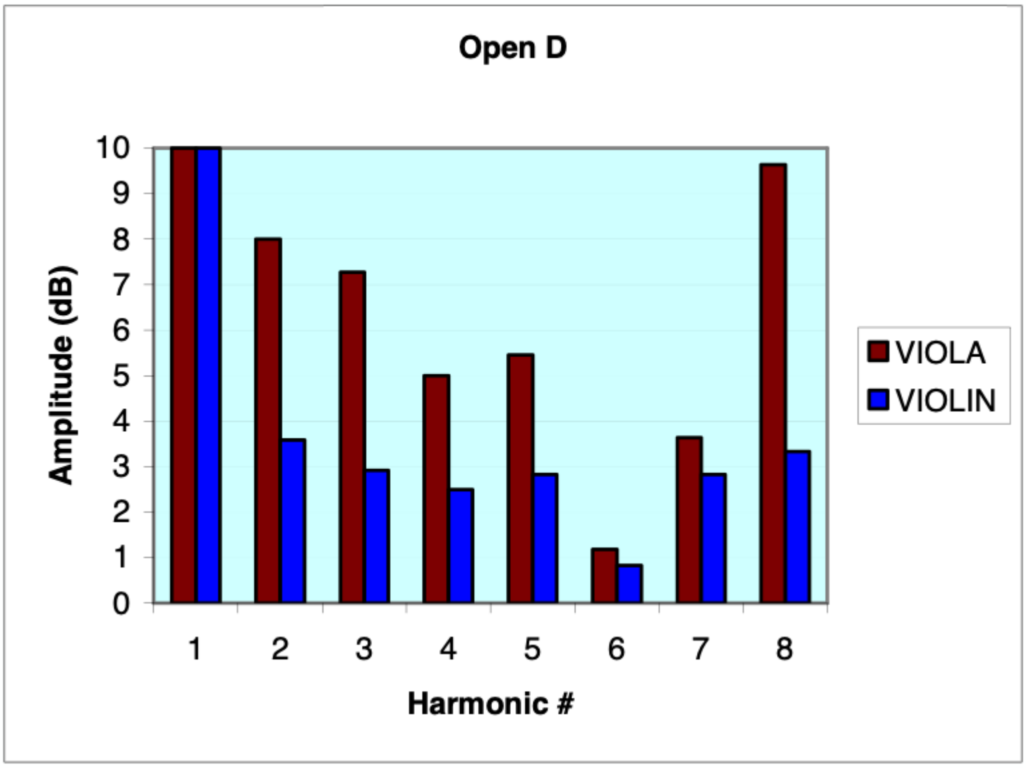

Figure 15: This figure shows the harmonic spectrum of a violin's open D string obtained by *M.Powell*. Powell used a magnetic resonator to excite the violin string. The fundamental frequency has been normalised to 10dB and is used to weight the other harmonic amplitudes.