

# Assignment 1

tldr: Perform linear regression of a noisy sinewave using a set of gaussian basis functions with learned location and scale parameters. Model parameters are learned with stochastic gradient descent. Use of automatic differentiation is required. Hint: note your limits!

**Problem Statement** Consider a set of scalars  $\{x_1, x_2, \dots, x_N\}$  drawn from  $\mathcal{U}(0, 1)$  and a corresponding set  $\{y_1, y_2, \dots, y_N\}$  where:

$$y_i = \sin(2\pi x_i) + \epsilon_i \tag{1}$$

and  $\epsilon_i$  is drawn from  $\mathcal{N}(0, \sigma_{\text{noise}})$ . Given the following functional form:

$$\hat{y}_i = \sum_{j=1}^M w_j \phi_j(x_i | \mu_j, \sigma_j) + b \tag{2}$$

with:

$$\phi(x | \mu, \sigma) = \exp \frac{-(x - \mu)^2}{\sigma^2} \tag{3}$$

find estimates  $\hat{b}$ ,  $\{\hat{\mu}_j\}$ ,  $\{\hat{\sigma}_j\}$ , and  $\{\hat{w}_j\}$  that minimize the loss function:

$$J(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2 \tag{4}$$

for all  $(x_i, y_i)$  pairs. Estimates for the parameters must be found using stochastic gradient descent. A framework that supports automatic differentiation must be used. Set  $N = 50, \sigma_{\text{noise}} = 0.1$ . Select  $M$  as appropriate. Produce two plots. First, show the data-points, a noiseless sinewave, and the manifold produced by the regression model. Second, show each of the  $M$  basis functions. Plots must be of suitable visual quality.

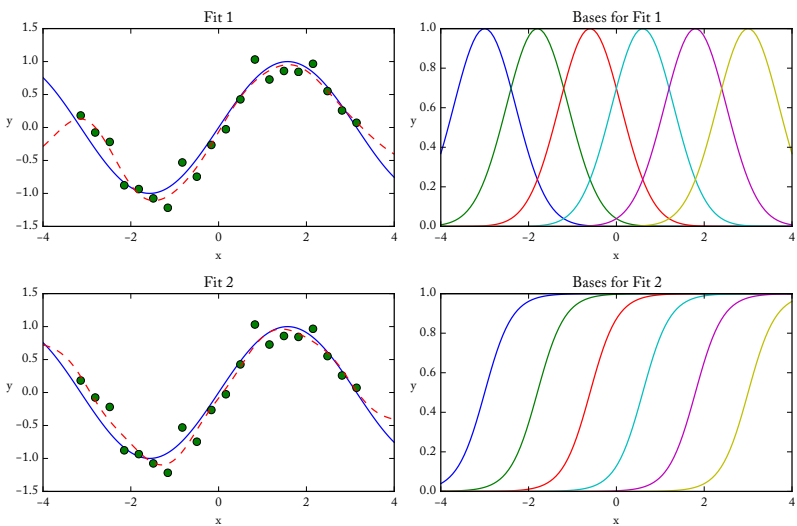


Figure 1: Example plots for models with equally spaced sigmoid and gaussian basis functions.