Lecture 7: Infinite Repeated Games

Jacob Kohlhepp

Econ 101

July 26, 2021

Introduction

- Last Lecture: sequential play introduces interesting forces in economic models.
- Today: what if the same game is played infinitely many times?
- Because players know they will always see each other again, this introduces new forces.

Applications

- How do OPEC countries sustain high oil prices (above the Cournot oligopoly level).
- How do employers motivate employees without performance pay?
- How do people sustain economic relationships in situations or places where there is no rule of law?
- How do we model reputation or relationships?

Some Additional Tools: Discounting

- Main solution concept remains **subgame-perfect Nash Equilibrium.**

- But because payoffs are infinite sums, they will be unbounded unless they are discounted.

Definition 1

The discount factor represents the value of 1 util tomorrow if it is provided today. It is always between 0 and 1, and with it we can write the present utility from a stream of utilities $\{u_t\}$ as:

$$U = \sum_{t=0}^{\infty} \delta^t u_t = u_0 + \delta u_1 + \delta^2 u_2 + \ldots$$

- Interpretations:
  1. The time value of money means that cash today is more valuable than cash tomorrow because we can invest it. Therefore the interest rate drives the discount rate.
  2. δ can represent the probability players meet again (uncertainty about the future).

Some Additional Tools: Infinite Sums

Suppose we want to evaluate $\sum_{t=0}^{\infty} \delta^t u$, where u is some constant payoff. It is useful to remember that:

$$\sum_{t=0}^{\infty} \delta^t u = \frac{u}{1-\delta}$$

This will help us calculate the utility of receiving a payoff of u every period forever, including today. If we want to evaluate the payoff of recieving u every period forever (except for today) that would be:

$$\sum_{t=1}^{\infty} \delta^t u = \frac{\delta u}{1-\delta}$$

Some Additional Tools: Stage Game

Definition 2

The **stage game** is the subgame that is played each time period.

Suppose we consider a situation where two players play the prisoner's dilemma game every period for infinitely many periods. The stage game looks like:

		Player 1	
		Silent	*Betray*
Player 2	*Silent*	$(2, 2)$	$(0, 3)$
	Betray	$(3, 0)$	$(1, 1)$

where the only NE is $(Betray, Betray)$ in the stage game.

Some Additional Tools: Trigger Strategies

- Now we wish to find an SPNE of the repeated prisoner's dilemma. To do this we will consider the following types of strategies.

Definition 3

A **trigger strategy** is one where a player initially cooperates until the other player deviates from cooperation, after which the player punishes the other player for a certain period of time.

- The most severe types of trigger strategies are called grim trigger strategies: they involve cooperating until one player defects. Then both players "punish" each other for the rest of eternity.
- "Punishment" usually refers to playing one of the static Nash Equilibria.
- In the case of the prisoner's dilemma it involves playing $(Betray, Betray)$.

Solving the Infinitely Repeated Prisoner's Dilemma

Suppose two players with the same discount factor δ play the prisoner's dilemma infinitely many times.

- We wish to find an SPNE of the game where cooperation is sustained.
- In PD, cooperation refers to sustaining the strategy $(Silent, Silent)$ which yields a higher payoff than the static NE of $(Betray, Betray)$
- To do this, we need to come up with a strategy that supports $(Silent, Silent)$.
- Let's try grim trigger: each player plays *Silent* until they observe the other player *Betray*.
- After the first *Betray* both players play *Betray* forever.

Solving the Infinitely Repeated Prisoner's Dilemma

- We now need to check that grim trigger is indeed a PD.
- Two conditions:
  1. No player wants to Betray given no betrayal has occurred yet.
  2. If a betrayal occurs, no player wants to play *Silent*.
- These are the only requirements for SPNE because of the following principle.

Definition 4

The **one-shot deviation principle** states that a strategy profile is a subgame-perfect Nash equilibrium if and only if no player can increase their payoff by changing a single decision in a single period.

- Because players are symmetric we only need to check for one player. There are two conditions because there are effectively only two proper subgames.

Solving the Infinitely Repeated Prisoner's Dilemma

- We will now derive the two inequalities that these two conditions imply, and check when they are satisfied. (See handwritten notes).

Solving the Infinitely Repeated Prisoner's Dilemma

- We will now derive the two inequalities that these two conditions imply, and check when they are satisfied. (See handwritten notes).
- It turns out that these two conditions reduce to just one condition, which is that the gain from cooperating (future payoffs) is greater than the one-time gain from betrayal:
- This holds whenever $\delta > 1/2$, that is whenever both players are sufficiently patient or forward looking.
- This is surprising, because in any finite repeated prisoner's dilemma game (when we repeat the game $T < \infty$ times) the only SPNE is to both betray. (It is a good exercise to derive this yourself).

Folk Theorem

The finding from the prisoner's dilemma, that we can support most outcomes in an infinitely repeated game given players are sufficiently patient, is general.

Theorem 5

The Folk Theorem *states that in an infinitely repeated game with discounting, we can support any strategy which has strictly higher payoff than the minmax payoff*[a] *if δ is close enough to 1.*

[a] minmax payoff is the maximum payoff a player can garuntee him/herself given the other player is trying to give them the lowest payoff.

Intuition: With enough patience, virtually anything is possible with an infinite future.

Comments

When I took this class as an undergraduate, this section of the class felt like dark magic. I hope these comments prove helpful.

- Grim trigger is not the only way to sustain cooperation. There are many others, including tit-for-tat (see problem set).
- Infinitely repeated games are hard to understand because they infinite sequences of occurrences.
- More complicated infinitely repeated games that involve signals and other things are actual the cutting edge of economic theory.
- Many theoretical political science textbooks make great use of simple infinitely repeated games.
- Grim trigger is a useful way to understand the Cold Warpolicy of mutually assured destruction (MAD).

Main Application: Collusion in Quantities

- N oil producing countries with common discount factor δ and unit production cost of c.
- $t = 1, 2, \ldots$ (infinite periods).
- Every period the countries simultaneously compete in quantities (they play a Cournot stage game).
- Inverse market demand is given by $p = 50 - \frac{Q}{100}$
- Find the stage game NE, and the single period profit from this strategy.
- Find the maximum achievable profit (monopoly solution) and the quantity that achieves it.
- Use grim trigger with the stage NE as a punishment to support the maximum achievable profit.

See handwritten notes for solution.

Collusion Solution

- The condition needed to support the monopoly quantity is:
- Notice that this inequality becomes harder to support when N (the number of countries) rises and when δ falls.
- Interpretation: collusion is easier when there are fewer players and players are more patient.
- N&S call this **tacit collusion** because there is no explicit association between countries - repeated interaction organically supports collusion.
- I prefer to think of this as a relational cartel: oil producing countries forming an alliance based on future profits.
- This actually occurs in real life! OPEC keeps prices artificially high by jointly restricting supply.