Lecture 4: Oligopoly

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Introduction

- We discussed monopoly, which is when one firm has full supply-side market powr.
- In Econ 11 we discussed perfect competition, which is where no firms have market power.
- Now we discuss the intermediate case: when a handful of firms have some but not full supply-side market power.
- We will draw on our new tools from static game theory to solve games by finding Nash Equilibria in continuous strategies.

What is Oligpoly?

Definition 1

An oligipoly is a market with relatively few firms but more than one.[a]

[a] Source: N&S Chapter 15

- In other words, not a monopoly but not perfect competition.
- This is more interesting than monopoly, because there is **strategic interaction**.
- Firms can impact the price, but not fully.
- **Example:** Mass media (Disney, Comcast, Viacom, News Corp), Smartphone software (Android, Apple iOS), automakers, airlines.
- A whole sub-field of economics is devoted to the study of strategic firm interaction: **industrial organization.**

Road Map

This Class:

- Duopoly with price competition (Cournot)

- Duopoly with quantity competition (Bertrand)

Next Class:

- Duopoly with Spatial Competition/Product Differentiation

In a few classes, because we need some dynamic game theory tools:

- Repeated Duopoly (Collusion)

Duopoly with price competition (Bertrand)

Like in all game theory problems, we layout the game:

- **Players.** Two identical firms, numbered $1, 2$.
- **Actions.** Firms choose prices *continuously*: $0 \leq p_i < \infty$
- **Payoffs.**
  1. Market demand is given by $D(p)$, which we assume slopes down.
  2. When firms set the same price demand is split evenly.
  3. When prices are different the lower price gets all demand.
  4. We can write this mathematically this way:

$$D_i(p_i, p_{-i}) = \begin{cases} 0 \text{ if } p_i > p_{-i} \\ \frac{1}{2} D(p_i) \text{ if } p_i = p_{-i} \\ D(p_i) \text{ if } p_i < p_{-i} \end{cases}$$

  5. Marginal cost is constant and equal to c (so fixed per unit cost of production).

Solving Bertrand: Deriving Profit

$$\Pi_i(p_i, p_{-i}) = \begin{cases} 0 \text{ if } p_i > p_{-i} \\ \frac{1}{2}D(p_i)[p_i - c] \text{ if } p_i = p_{-i} \\ D(p_i)[p_i - c] \text{ if } p_i < p_{-i} \end{cases}$$

Solving Bertrand: Nash Equilibrium

- If you read N&S Ch. 15, it almost seems like the authors guess the equilibrium.
- This is no mistake: sometimes it is easier to guess an equilibrium and then verify it satisfies the NE conditions.
- So we guess: $p_1 = p_2 = c$ is the only equilibrium.
- To prove this, we need to first show it is an NE. Then we need to show there are no other NE.

Solving Bertrand: Nash Equilibrium

First, we show it is an NE.

- Recall the definition of an NE: every player must have no profitable deviation.
- To prove this is an NE, we just need to check that given the other player is playing $p_{-i} = c$, player i does not gain by playing something other than $p_i = c$.
- First note that when $p_1 = p_2 = c$ profit is 0 because average cost is equal to average revenue.
- Suppose one player deviates to $p_i < c$ while other stays. Does the deviator gain?
- No: the deviate gets the full market demand, but now price is less than cost, so profit is negative.
- What if one player deviates to $p_i > c$? Do they gain?
- No: the deviator loses all demand to the other player, and makes 0 profit (the same as not deviating).

Solving Bertrand: Nash Equilibrium

Second, show it is the unique NE.

- Suppose there is another NE (proof by contradiction).
- For clarity, just assume that the low price firm is 1: $p_1 \leq p_2$.
- **Case 1**: $c > p_1$. In this case, firm 1 is making negative profit. This cannot be an NE because the firm could just set $p_1 = c$ and at least make 0 profit.
- **Case 2**: $c < p_1$. Now firm 1 is making positive profit. But firm 2 is making 0 profit, and firm 2 could deviate to a price between c and p_1 and make positive profits.
- **Case 3**: $c = p_1 < p_2$: Now firm 1 earns 0 profit as does firm 2. But firm 1 could slightly raise price and make positive profit.
- Therefore, $p_1 = p_2 = c$ is the unique Nash Equilibrium!

Interpreting Bertrand

- What is interesting about the solution $p_1 = p_2 = c$?

Interpreting Bertrand

- What is interesting about the solution $p_1 = p_2 = c$?
- It is exactly the perfect competition outcome!
- This is the Bertrand paradox: two firms yields maximum competition.
- On the one hand this is general: we did not specify demand, and this is true even with more firms.
- On the other hand it is knife-edge (sensitive). It falls apart when:
  1. There is product differentiation (next class).
  2. Prices are discrete (see problem set).
  3. We switch to quantity competition (next slide).

Duopoly with quantity competition (Cournot)

- **Players.** n firms.

- **Actions.** Firms choose quantities *continuously*: $0 \leq q_i < \infty$

- **Payoffs.**
  1. Total market quantity is $Q = \sum_i^2 q_i$.
  2. Inverse demand (price as a function of quantity) is based on total market quantity $P(Q)$.
  3. Cost of production given by $C_i(q_i)$. Often we will have identical costs so $C_i(q_i) = C(q_i)$.
  4. Profit will be revenue less costs, like in the monopoly problem.

- Exercise: Write down the payoff for firm i from producing quantity q_i.

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$$P(Q)q_i - C_i(q_i)$$

Solving Cournot

The general problem (without specify the cost function or the inverse demand) is:

$$\max_{q_i} P(Q)q_i - C_i(q_i)$$

We take the first-order condition holding fixed the quantity of the other firms:

$$P'(Q)q_i + P(Q) - C'(q_i) = 0$$

This is the solution for any number of firms! Notice that taking the derivative and setting equal to zero is equivalent to finding the best-response when strategies are continuous.

Solving Cournot: Natural Spring Duopoly

This example follows Example 15.1 from N&S Ch. 15.

- Suppose $N = 2$.
- Suppose $C_i(q_i) = cq_i$, so cost functions are symmetric.
- Suppose Inverse demand is $P(Q) = a - Q$, which is a linear demand (remember this from monopoly?)
- Suppose the firms compete in quantities. What is the Nash Equilibrium quantities and price?

For the solution, see handwritten notes.

Adding a Twist: Natural Spring with a Cartel

Definition 2

A **cartel** is an association of firms that works together to keep prices above the competitive level.

- What happens in the previous example when we assume the two firms can perfectly cooperate?
- We model perfect cooperation as the two firms acting in unison to maximize total profit.
- Find the quantities that would be produced if the firms could form a cartel.
- **Challenge.** Compare this quantity to the monopoly quantity.
- For the solution see handwritten notes.

Comparing Three First-order Conditions

Perfect Competition: The firm acts as if its actions do not impact price.

$$P(Q) - C_i'(q) = 0$$

Cournot Oligopoly: The firm accounts for the fact that it can impact price.

$$P(Q) + P'(Q)q_i - C'(q_i) = 0$$

Cartel: The cartel through cooperation has full market power, and accounts for the externality that production of one firm has on the price all other firm's charge.

$$P(Q) + P'(Q)Q - C_i'(q_i) = 0$$

Profits are:

$$\pi_{perfect} \leq \pi_{oligpoly} \leq \pi_{cartel}$$

Interpreting Cournot Equilibrium

- For any non-infinite number of firms, Cournot/quantity competition results in a price above perfect competition, and thus more profit for firms than under perfect competition.
- When $n \to \infty$, it can be shown under general conditions that the price converges to the marginal cost.
- As a result, as more firms enter a market, competition increases and we approach perfect competition!
- Showing this is an exercise in the practice problems for this week.

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- Models are tools, but not the truth.

  "All models are wrong, but some are useful."
  - George Box

Concepts for the Midterm

The following concepts can potentially be on the midterm:

- Risk and Uncertainty
- Monopoly/Monopsony
- Static Game Theory (any problem using the tools we discussed)
- Oligopoly

Other notes:

1. Midterm is open book but not open contact.
2. Midterm will be 1 hour.
3. Academic dishonesty will not be tolerated.
4. If you do not do the practice problems, it is highly unlikely you will do well on the test.
5. Grading is either 30-70% or 100% final, whichever results in a better score.

Reminder: Lecture After Midterm

There will be a 1 hour lecture after the midterm.

The content of this lecture will NOT be on the midterm.

It will FOR SURE be on the final. Choose attendance strategies accordingly!