Lecture 3: Static Game Theory

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Introduction

- For the rest of this class, we will study the **strategic interactions** of economic actors.
- To do this we need a new tool.
- This new tool is called **game theory.**

What is Game Theory?

Definition 1

Game theory is the study of mathematical models of strategic interaction among rational decision-makers.[a]

[a] Myerson (1991)

- Used in many fields: computer science, economics, political science, psychology.
- Came to exist as a field around 1928.
- Came to prominence in the 1950s with John Nash.
- Became a dominant force in economics by the 1970s.
- 11 game theorists have won Econ. Nobel Prize, including Shapley (UCLA)
- Many legitimate critiques (we will not cover), but still very useful tool.

Game Theory in This Class

- This lecture will cover static game theory: games where actions occur simultaneously in one period.
- In a few weeks we will cover dynamic game theory: games where actions can occur sequentially.
- The main application of both tools will be **oligopoly games**.
- However the concepts themselves are fair game for the final/midterm.
- For example, I may ask you to solve a game that is not oligopoly using Nash Equilibrium/dominant strategies/etc.

Defining a Game

A game consists of a list of players, their possible strategies, and the resulting payoffs.

- **Players.** A set of decision-makers, assumed to be rational. Indexed by $i = 1, ..., n$. Often we call the set of players which are not i as $-i$.

- **Strategies.** The set of "courses of action" available to each player. Denote each strategy as s_i, the full set as S_i. Denote s_{-i} as the strategies of all players but i.
  - When the game is static this is equivalent to actions.

  - When the game is not static players may take different actions based on the past actions of others.

- **Payoffs.** The utility obtained by all players given the chosen strategies. Denoted by a function $u_i(s_1, ..., s_n)$ which maps strategies to a number.

Example: The Prisoner's Dilemma

We now describe one of the most famous games in game theory. Tell story.

- **Players.** Two players, $1, 2$.
- **Strategies.** $S_i = \{silent, betray\}$. Each player can either stay silent or betray the other.
- **Payoffs.**

$$u_i = \begin{cases} 1 \text{ if } s_i = s_{-i} = silent \\ 3 \text{ if } s_i = betray,\ s_{-i} = silent \\ 0 \text{ if } s_i = silent,\ s_{-i} = betray \\ 1 \text{ if } s_i = s_{-i} = betray \end{cases}$$

Note: we can usually apply many interpretations to the same game.

Example: The Prisoner's Dilemma

We can summarize all of this in the normal-form of the game. See hand-written notes.

Solving the Game: Best Response

- We can now describe or write down a game.
- To understand what is likely to happen when rational players play the game, we need some more concepts.
- First idea: What is the best thing for a player to do if that player **knows** the strategy of the other player?

Definition 2

Strategy s_i is a **best-response** for player i to other players strategies s_{-i} if:

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \text{ for all } s_i' \in S_i$$

- In words: a best-response is a utility maximizing action given a strategy of others.
- We denote a best response to the strategy of others $BR_i(s_{-i})$.

Example: The Prisoner's Dilemma

Let's find the best-responses in the prisoner's dilemma. See handwritten notes.

It is often helpful to underline the best-response of each player to each other player's action.

Solving the Game: Dominant Strategy

- We want to understand what strategies players are **likely** to play.
- We start with a very strong concept called a dominant strategy.

Definition 3

Strategy s_i is a **dominant strategy** for player i if it is a best-response to all possible combinations of strategies of others.

- In words: an action/strategy is dominant if it is best regardless of what others do.
- Issue: there might not be a dominant strategy.
- If one does exist, we sense people are likely to play it.
- Closely related concept: dominated strategy. This is a strategy which yields a lower payoff for every strategy of others than another strategy.

Example: The Prisoner's Dilemma

Let's find the dominant strategy of each player in the prisoner's dilemma. See handwritten notes.

Solving the Game: Nash Equilibrium

- We want to understand what outcomes are likely to occur.
- We now use a concept of equilibrium, developed by Nash.
- This will be the main concept we use in this class for solving static games.

Definition 4

A **Nash equilibrium** is a strategy for every player $(s_1^*, ..., s_n^*)$ such that for each player $i = 1, 2, .., n$ s_i^* is a best-response to the strategies of others s_{-i}^*

- In words: everyone is playing a strategy which is a best-response to everyone else's strategy.
- Intuition: if we gave any individual player the option to deviate while holding everyone else's action fixed, no one would deviate.

Example: The Prisoner's Dilemma

Let's find a Nash Equilibrium in the prisoner's dilemma. See handwritten notes.

Comments on Nash Equilibrium

- With finite players and finite actions, Nash proved that every game has at least one Nash equilibrium.
- However sometimes the only NE is in mixed strategies (more on this next).
- If there is a dominant strategy for a player, it must be part of the NE.
- If every player has a dominant strategy, then there is a unique Nash Equilibrium.
- Actually if we iteratively remove strictly dominated strategies and we find a unique strategy at the end this is the unique Nash Equilibrium.

New Example: Penalty Kick

Consider the following situation:

- **Players:** Goalkeeper trying to defend goal, kicker trying to score.
- **Actions:** $S_g = \{kickleft, kickright\}$, $S_k = \{diveleft, diveright\}$.
- **Payoffs:** Goalkeeper gets 1 if dives same way as kick, 0 if not. Kicker gets 1 if kicks the way the goalkeeper does not dive, 0 if not.

Write down the normal form of this game with payoffs yourself. Check for a Nash Equilibrium in pure strategies.

Solving a Game: Mixed Nash Equilibrium

- We observe that in real soccer games the goalkeeper must essentially try and predict which way the ball gets kicked.
- Knowing this, kickers tend to randomize. If they are predictable, this will make them score less.
- This is a mixed strategy Nash Equilibrium: players are playing random combinations of pure strategies. Example: $1/3$ left, $2/3$ right.

Definition 5

A **mixed strategy** is a probability distribution over the possible actions.

- In words: Mixed strategies assign probabilities to each action. A pure strategy is just a mixed strategy where one action has probability 1 and all others have probability 0.
- It is as if the players have a random number generator and they decide a probability with which to play each action.

New Example: Penalty Kick

Let's find the mixed strategy Nash Equilibrium of the penalty kick game. See handwritten notes. Please take it easy on yourself if this is hard. Finding mixed NE is much more difficult than pure NE.

Hint: A trick to find mixed NE is that when players are playing strictly mixed strategies (all actions played with probability strictly less than 1) players must be indifferent between each pure action. This gives an **indifference condition** for each player which we can use to solve for the probabilities.

Continuous Actions

- So far we have focuses on discrete actions.
- Often we are concerned with continuous choices, like setting prices and quantities.
- Most of the same concepts apply to continuous actions, we just need to use calculus to get best-responses.

Example: Lawn Maintenance (Positive Externalities)

- **Players:** 2 neighbors ($i = 1, 2$)
- **Actions:** Number of hours to spend on maintenance: $S_i = [0, \infty)$. Note this is a continuous choice, sot here are infinite possible strategies.
- **Payoffs:** The benefit per hour:

$$10 - l_i + \frac{l_{-i}}{2}$$

  the cost is 4 per hour.

Write the complete payoff function yourself. We will derive and plot best-response functions and then find the Nash Equilibrium. See handwritten notes.

Source: N&S 8.4

Interpretation of lawn Maintenance game

- Question: Is the Nash Equilibrium socially optimal or pareto efficient?

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- Question: Is the Nash Equilibrium socially optimal or pareto efficient?
- Is there a number of lawn maintenance hours that would leave both better off?
- What number of hours would a social planner maximizing the sum of utilities choose? See handwritten notes.

Game Theory Concepts To Know for the Midterm/Final

- Dominant Strategies
- Iterated Deletion of Dominant Strategies
- Best-Response
- Nash Equilibrium
- Continuous Action Best-Response/Nash Equilibrium