Lecture 8: Incomplete Information

Jacob Kohlhepp

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Introduction

- This is the last main section of the class!
- We will now relax the assumption of "perfect information."
- We will allow economic actors to not know everything about a game.
- Example: A firm may not know the productivity/skill of a worker they hired.
- Example: A seller may not know how much a buyer values an item.
- Notice that we implicitly assumed complete information in Econ 11.

A New Tool: Player Types

- One useful way to think about incomplete information is **player types.**

Definition 1

Aplayer's **type** (t) describes all of the actions and information availble to a player.

- A player knows their own type, but may be uncertain (or have beliefs about) other player's types.
- This fits well with labor market examples: we can think of models with a high skill and low skill type of worker, where high skill workers may have different outputs but also different payoffs then low skill.
- This also covers things like poker.
  1. Set the type space (the set of possible types) to be all possible two card hands.
  2. Then which cards you have determines what payoffs you got from the flop, turn and river.
  3. Which cards you have also determine your belief about other player's cards.
  4. Your cards are your private information because only you know them for certain.

A Simple Example: Wall Street vs Reddit

Consider the following game inspired by the GameStop situation.

- There are two players: a Wall Street trader (W) and a anonymous Reddit trader (R).
- W can either short or leave while R can either buy or not.
- Now for some incomplete information: R is either rational (with prob. p) or a rage trader (with prob. 1-p).
- Payoffs are as follows:

t_r = rational

		R: B	R: N
W: S		$(-10, -1)$	$(1, 0)$
W: L		$(0, 0)$	$(0, 1)$

t_r = rage

		R: B	R: N
W: S		$(-10, 10)$	$(1, 0)$
W: L		$(0, 1)$	$(0, 0)$

A New Solution Concept

- Notice that we have introduced types, so there is another element involved in the game.
- Wall Street trader will trade differently depending on if he faces a rage Redditor or a rational Redditor.

Definition 2

A **Bayesian Nash Equilibrium** (BNE) is a strategy profile $\{s_1^*(t_1), s_2^*(t_2), ...s_n^*(t_n)\}$ such that each strategy for each type of each player is a best response given beliefs about the other's player's types ($Pr(t_{-i} = t)$):

$$\sum_j Pr(t_{-i} = t_j) U_i(s_i^*(t_i), s_{-i}^*(t_j)) \geq \sum_j Pr(t_{-i} = t_j) U_i(s_i'(t_i), s_{-i}^*(t_j))$$

for all types, all players, and all strategies s_i'.

- Lots of math. Bottomline: every player plays a best response given their type assuming all other players play a best response knowing their type.

Applying BNE to GameStop

See handwritten notes.

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See handwritten notes.

- In terms of technique this really just involved a few more steps.
- In terms of interpretation: Wall Street plays against an average of the two types.
- We can ask: would the rational Redditor prefer if Wall Street new their type?
- Would the rage Redditor prefer if Wall Street new their type?
- I highly recommend reviewing the tragedy of the commons example in N&S (Example 8.6)

Sequential Games with Incomplete Information

- The idea of types and incomplete information becomes more interesting and powerful when we add sequential moves.
- Why? Because then the actions of other players tell us something about their types.
- This is idea is called **signaling**, because observable actions serve as a signal of private information.
- Example: Going to college tells an employer you have some level of analytical ability/intelligence (this is the main example we will look at).
- Warning: this part of the class is considerably more difficult then the rest of the class.
- It is okay to struggle with this, and I will put at most one signaling-type question on the final.

Spence Job-market Signaling

- We now consider a model developed by Spence, for which he won the Nobel Prize.
- Two players: a firm and a worker.
- Worker is either high-skill ($t = h$) with prob. p or low-skill ($t = L$) with prob. $1 - p$.
- The firm cannot observe worker type.
- Profit from hiring low-skill is 0 and high-skill is $\pi > 0$
- At $t = 1$ the worker can acquire education at cost c_H if high type and c_L if low-type.
- Critically, the cost of education is higher for low-types $c_L > c_H$
- At $t = 2$ after observing education the firm either offers a job or not at the wage w.[1]

[1] For simplicity we assume $\pi - w > 0$ and that wage is not chosen by the firm.

Spence Job-market Signaling

- We know draw the game tree. See handwritten notes.
- In the drawing, note the information sets.

Another Tool: Bayes' Rule

- In order to understand how a belief should be updated based on new information (education choice) we need bByes' Rule.

Theorem 3

Given two events A and B[a] Bayes' Rule states that:

$$Pr(A|B) = \frac{Pr(A\&B)}{Pr(B)} = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

[a] On a shared probability space.

- $Pr(A|B)$ reads the probability of A given B.
- Generally A is the type of other player, and B is some action taken by other player.
- In equilibrium, sometimes different types play different strategies which makes the actions of others a signal of true type.

Final Equilibrium Concept: Perfect Bayesian Equilibrium

- Like when we added sequential moves with Nash Equilibrium, we need to refine BNE.
- Specifically we must say how beliefs change in response to actions.

Definition 4

A strategy profile $\{s_1^*(t_1), ..., s_n^*(t_n)\}$ and beliefs are a **Perfect Bayesian Nash Equilibrium** if:

1. at each information set players maximize utility given beliefs.
2. at each information

- We now proceed to derive a separating and a pooling equilibrium of the job-market signaling game.

Interpreting Job-market Signaling

- The game highlights how education can be useful even if it imparts no useful skills.
- Note that the separating equilibrium is inefficient: education is wasteful, and we could make everyone better off if we could costlessly signal type.
- The results of the model extend to cases where education also imparts skill.
- This model was crucial in separating the human capital vs the signaling value of education.
- This led to the realization that we need to be careful when we estimate the returns to education.
- More skilled/productive people may self-select into college due to the signaling value.
- If we do not account for this somehow, estimates of the returns will be biased upward.
- Solution: Use exogenous variation in schooling years to get human capital part.

Signaling Elsewhere in Economics

- The idea that actions contain information about economic agents is a powerful idea.
- Signaling takes this further: economic agents will sometimes take costly action purely because it conveys information to others.
- Signaling has also made a powerful impact in antitrust settings.
- Predatory pricing: where a firm prices low as a costly signal of low production cost to keep other firms from entering
- The recommended problem set illustrates this through a simple model.
- It also is a nice way to practice solving a PBNE.