Lecture 8: Incomplete Information

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Introduction

- This is the last main section of the class!
- We will now relax the assumption of "perfect information."
- We will allow economic actors to not know everything about a game.
- Example: A firm may not know the productivity/skill of a worker they hired.
- Example: A seller may not know how much a buyer values an item.
- Notice that we implicitly assumed complete information in Econ 11.

A New Tool: Player Types

• One useful way to think about incomplete information is player types.

Definition 1

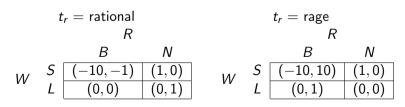
Aplayer's type (t) describes all of the actions and information availble to a player.

- A player knows their own type, but may be uncertain (or have beliefs about) other player's types.
- This fits well with labor market examples: we can think of models with a high skill and low skill type of worker, where high skill workers may have different outputs but also different payoffs then low skill.
- This also covers things like poker.
 - 1. Set the type space (the set of possible types) to be all possible two card hands.
 - 2. Then which cards you have determines what payoffs you got from the flop, turn and river.
 - 3. Which cards you have also determine your belief about other player's cards.
 - 4. Your cards are your private information because only you know them for certain.

A Simple Example: Wall Street vs Reddit

Consider the following game inspired by the GameStop situation.

- There are two players: a Wall Street trader (W) and a anonymous Reddit trader (R).
- W can either short or leave while R can either buy or not.
- Now for some incomplete information: R is either rational (with prob. p) or a rage trader (with prob. 1-p).
- Payoffs are as follows:



A New Solution Concept

- Notice that we have introduced types, so there is another element involved in the game.
- Wall Street trader will trade differently depending on if he faces a rage Redditor or a rational Redditor.

Definition 2

A Bayesian Nash Equilibrium (BNE) is a strategy profile $\{s_1^*(t_1), s_2^*(t_2), ... s_n^*(t_n)\}$ such that each strategy for each type of each player is a best response given beliefs about the other's player's types $(Pr(t_{-i}=t))$:

$$\sum_{j} Pr(t_{-i} = t_j) U_i(s_i^*(t_i), s_{-i}^*(t_j)) \geq \sum_{j} Pr(t_{-i} = t_j) U_i(s_i'(t_i), s_{-i}^*(t_j))$$

for all types, all players, and all strategies s'_i .

• Lots of math. Bottomline: every player plays a best response given their type assuming all other players play a best response knowing their type.

Applying BNE to GameStop

See handwritten notes.

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See handwritten notes.

- In terms of technique this really just involved a few more steps.
- In terms of interpretation: Wall Street plays against an average of the two types.
- We can ask: would the rational Redditor prefer if Wall Street new their type?
- Would the rage Redditor prefer if Wall Street new their type?
- I highly recommend reviewing the tragedy of the commons example in N&S (Example 8.6)

Sequential Games with Incomplete Information

- The idea of types and incomplete information becomes more interesting and powerful when we add sequential moves.
- Why? Because then the actions of other players tell us something about their types.
- This is idea is called signaling, because observable actions serve as a signal of private information.
- Example: Going to college tells an employer you have some level of analytical ability/intelligence (this is the main example we will look at).
- Warning: this part of the class is considerably more difficult then the rest of the class.
- It is okay to struggle with this, and I will put at most one signaling-type question on the final.

Spence Job-market Signaling

- We now consider a model developed by Spence, for which he won the Nobel Prize.
- Two players: a firm and a worker.
- Worker is either high-skill (t = h) with prob. p or low-skill (t = L) with prob. 1 p.
- The firm cannot observe worker type.
- ullet Profit from hiring low-skill is 0 and high-skill is $\pi>0$
- At t=1 the worker can acquire education at cost c_H if high type and c_L if low-type.
- ullet Critically, the cost of education is higher for low-types $c_L>c_H$
- At t=2 after observing education the firm either offers a job or not at the wage w.¹

¹For simplicity we assume $\pi - w > 0$ and that wage is not chosen by the firm.

Spence Job-market Signaling

- We know draw the game tree. See handwritten notes.
- In the drawing, note the information sets.

Another Tool: Bayes' Rule

• In order to understand how a belief should be updated based on new information (education choice) we need bByes' Rule.

Theorem 3

Given two events A and B^a Bayes' Rule states that:

$$Pr(A|B) = \frac{Pr(A\&B)}{Pr(B)} = \frac{Pr(B|A)Pr(A)}{Pr(B)}$$

^aOn a shared probability space.

- Pr(A|B) reads the probability of A given B.
- ullet Generally A is the type of other player, and B is some action taken by other player.
- In equilibrium, sometimes different types play different strategies which makes the actions of others a signal of true type.

Final Equilibrium Concept: Perfect Bayesian Equilibrium

- Like when we added sequential moves with Nash Equilibrium, we need to refine BNE.
- Specifically we must say how beliefs change in response to actions.

Definition 4

A strategy profile $\{s_1^*(t_1),...,s_n^*(t_n)\}$ and beliefs are a **Perfect Bayesian Nash Equilibrium** if:

- 1. at each information set players maximize utility given beliefs.
- 2. at each information
- We now proceed to derive a separating and a pooling equilibrium of the job-market signaling game.

Interpreting Job-market Signaling

- The game highlights how education can be useful even if it imparts no useful skills.
- Note that the separating equilibrium is inefficient: education is wasteful, and we could make everyone better off if we could costlessly signal type.
- The results of the model extend to cases where education also imparts skill.
- This model was crucial in separating the human capital vs the signaling value of education.
- This led to the realization that we need to be careful when we estimate the returns to education.
- More skilled/productive people may self-select into college due to the signaling value.
- If we do not account for this somehow, estimates of the returns will be biased upward.
- Solution: Use exogenous variation in schooling years to get human capital part.

Signaling Elsewhere in Economics

- The idea that actions contain information about economic agents is a powerful idea.
- Signaling takes this further: economic agents will sometimes take costly action purely because it conveys information to others.
- Signaling has also made a powerful impact in antitrust settings.
- Predatory pricing: where a firm prices low as a costly signal of low production cost to keep other firms from entering
- The recommended problem set illustrates this through a simple model.
- It also is a nice way to practice solving a PBNE.