Lecture 6: Finite Dynamic Games

Jacob Kohlhepp

Econ 101

July 26, 2021

Introduction

- In many economic situations, people move **sequentially**, taking into account prior actions.
- This introduces more strategic effects.
- To analyze these situations we need to broaden our tool set.
- After this lecture you should be able to solve finite dynamic games (those where $1 < T < \infty$).
- Next class we will learn to solve infinite repeated games (the same game is played over and over forever).

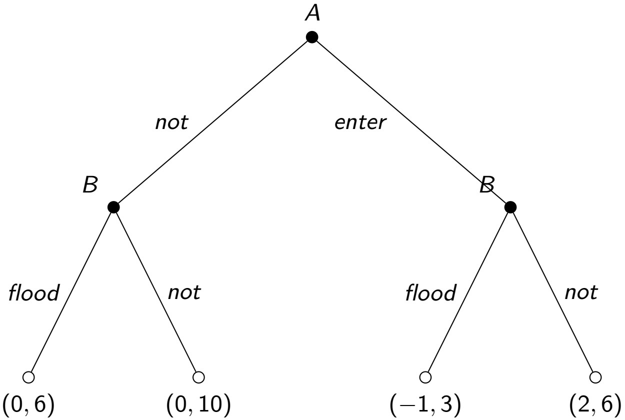
Entry Example

Consider the following game:

- At $t = 1$ Firm A decides whether to enter the market for designer jeans at fixed cost 4.
- Firm B is a long time producer of jeans. At $t = 2$, Firm B can choose to flood the market with jeans or not.
- If Firm A does not enter, it makes 0 profit regardless of what Firm B does.
- If Firm B floods, profit for all entrants is 3. If Firm B does not flood profit is 6 if Firm A enters and 10 otherwise. (this does not include entry cost).
- Notice that the strategy of firm B depends on the entry decision. It is not longer an action but a function!

Entry Example

This is a lot to track. We can summarize it in a game tree:



Finding Two NE

See handwritten notes.

Refining Nash Equilibrium

- One NE we derived consists of a **non-credible threat.**
- Given that Firm A has already entered, Firm B has no reason to follow through on flooding.
- We need a new equilibrium concept that removes such irrational equilibria.
- First we need a new definition.

Definition 1

A **proper subgame** is a part of the extensive form beginning at a decision node that is not part of an information set and including everything that branches down from it.

- Identify subgames in entry example.

Subgame Perfect Nash Equilibrium

- Armed with our definition of a subgame, we can define a new equilibrium concept:

Definition 2

A **subgame-perfect Nash Equilibrium** (SPNE) is a strategy profile $(s_1^*, ..., s_n^*)$ that is a Nash equilibrium in every proper subgame.

- All SPNE are NE, not all NE are SPNE!
- We can find SPNE using backwards induction (show how to find it in example - see handwritten notes).

The Pirate Riddle

There is a very famous and tricky riddle that you can solve using SPNE. We will use it to practice backwards induction.

> *Five pirates, numbered 1 through 5, must decide how to divide 100 gold coins. Their decision process is as follows. Starting with pirate 1, each pirate proposes a split consisting of a number of coins for each of the pirates on the ship. Then all pirates vote. If a strict majority approve, the allocation happens. If it does not the proposer is thrown off the ship and the remaining pirates repeat the process. Assume pirates value 2 coins more than 1, etc and that getting thrown off is worse than getting 0 coins. Assume pirates vote no when indifferent (they get a little bit of enjoyment from watching someone walk the plank). What is the maximum number of coins P1 can obtain and not get thrown off?*

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!
- Rolling back, P4 needs pirate 5's vote for a strict majority. There is no way to get it since P5 knows they get 100 coins if they throw off 4. Thus P4 can propose anything, and P5 always rejects.

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!
- Rolling back, P4 needs pirate 5's vote for a strict majority. There is no way to get it since P5 knows they get 100 coins if they throw off 4. Thus P4 can propose anything, and P5 always rejects.
- Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving him/her 0 coins. P3 proposes

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!
- Rolling back, P4 needs pirate 5's vote for a strict majority. There is no way to get it since P5 knows they get 100 coins if they throw off 4. Thus P4 can propose anything, and P5 always rejects.
- Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving him/her 0 coins. P3 proposes
- Roll back. Pirate 2 needs to get two votes. P4 and P5 are the cheapest to convince because they get 0 next round. So P2 gives P4 and P5 1, P3 0, and keeps 98.

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!
- Rolling back, P4 needs pirate 5's vote for a strict majority. There is no way to get it since P5 knows they get 100 coins if they throw off 4. Thus P4 can propose anything, and P5 always rejects.
- Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving him/her 0 coins. P3 proposes
- Roll back. Pirate 2 needs to get two votes. P4 and P5 are the cheapest to convince because they get 0 next round. So P2 gives P4 and P5 1, P3 0, and keeps 98.
- Roll back. P1 needs two other votes. P3 is the cheapest to convince. P4 and P5 are next cheapest, and P1 need only convince one. So P1 proposes 0 for P2 and P5, P3 1, and P4 2 and keeps 97!

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!
- Rolling back, P4 needs pirate 5's vote for a strict majority. There is no way to get it since P5 knows they get 100 coins if they throw off 4. Thus P4 can propose anything, and P5 always rejects.
- Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving him/her 0 coins. P3 proposes
- Roll back. Pirate 2 needs to get two votes. P4 and P5 are the cheapest to convince because they get 0 next round. So P2 gives P4 and P5 1, P3 0, and keeps 98.
- Roll back. P1 needs two other votes. P3 is the cheapest to convince. P4 and P5 are next cheapest, and P1 need only convince one. So P1 proposes 0 for P2 and P5, P3 1, and P4 2 and keeps 97!

Interpreting the Pirate Game

- Brief comment on SPNE vs outcome (see lecture recording).
- Despite the fact that all pirates are identical with equal voting power, the first player gets 97 out of 100 coins.
- There is a first mover advantage: going first gives a higher payoff.
- This advantage exists in many games, including bargaining and in Stackleberg model (see problem set).
- Because voting is invovled, this also illustrates **agenda setting power.**
- The ability to decide what gets considered first is very valuable.
- Examples: Congress, corporate board meetings, etc.

Interpreting the Pirate Game, Meta Discussion

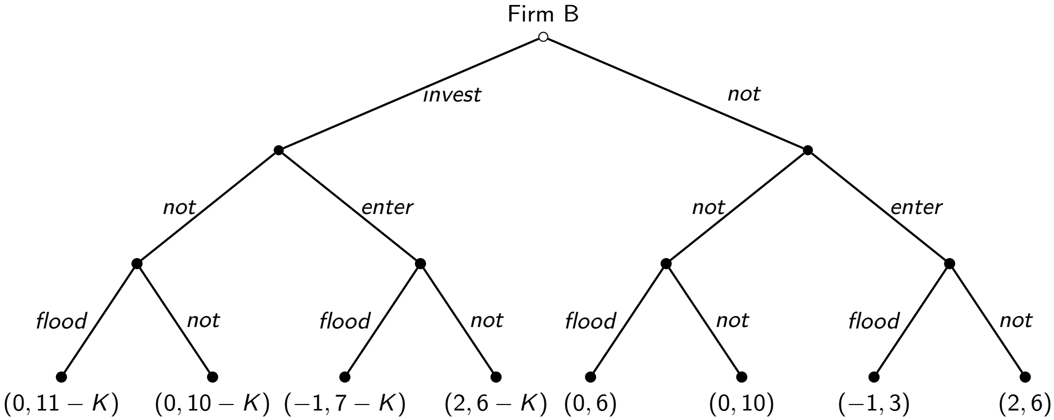
- Models are just stylized stories that help us understand the world.
- They can be unreasonable! Assuming this degree of rationality and forward looking behavior is a little silly (especially for pirates).
- Indeed, Kalah (from ancient African game mancala), is solved: the first player can always win if they play optimally!
- But people play as if the game is not determined! Why?
- Extreme example: Chess. Finite moves, not solved!

Source: The Spruce Crafts

Another Example: Investment as Commitment

- We now apply SPNE again to illustrate the value of commitment.
- Consider the entry game from earlier. Remember that the only SPNE was that firm A enters and firm B does not flood.
- Add a stage to the game before the entry decision: Firm B can invest in increased manufacturing capacity at cost K.
- If it does so, the payoff from flooding the market becomes 1 more than not flooding regardless of whether firm A enters.
- Find the SPNE based on K.

Investment As Commitment



Interpreting Investment as Commitment

- See handwritten notes for solution.
- The upfront investment **credibly commits** firm B to flood the market.
- This induces firm A to stay out.
- Thus this is a form of entry deterrence.

Hoteling with Location Choice

Recall the Hoteling product differentiation model from several classes ago.

- In a simultaneous game where two hot dog stands set prices given fixed locations, we derived that prices were:

$$p_A^* = \frac{t}{3}(b-a)(2L+a+b) \qquad p_B^* = \frac{t}{3}(b-a)(4L-a-b)$$

- This is in a static NE without location choice. Suppose now we add a first stage where firms simultaneously choose locations along the pier.

- That is firm A and B choose locations a, b along the pier at $t = 1$, then compete in prices given locations at $t = 2$.

- Now we can utilize backwards induction and SPNE to understand equilibrium behavior.

Hoteling with Location Choice: Solution

- In any SPNE strategies must be NE in every subgame.
- Therefore the solution to the second stage is done: it is the same as in the game without location choice:

$$p_A^* = \frac{t}{3}(b-a)(2L+a+b) \qquad p_b^* = \frac{t}{3}(b-a)(4L-a-b)^2$$

- The payoffs in the second stage with fixed locations are given by profit, which is:

$$\pi_B^* = \frac{t}{18}(b-a)(2L+a+b) \qquad \pi_B^* = \frac{t}{18}(b-a)(4L-a-b)^2$$

- Now we can solve the location choice part of the game. (See handwritten notes).

Hoteling with Location Choice: Solution

- SPNE locations are:

$$a^* = 0, \qquad b^* = L$$

Definition 3

The **principle of maximum differentiation** refers to the idea that firms want to make their products as different as possible in order to minimize price competition.

- It can be shown that the socially optimal locations are $a^{**} = 0.25L$, $b^{**} = 0.75L$.[1]
- There is too much differentiation in equilibrium.

[1] Showing this requires showing that all that matters for social surplus is transport cost. It is a little involved so you are not required to know it.

Interpretation of Hoteling with Location Choice

- In reality we often see instances where competing firms cluster together (gas stations, burger joints, etc).

- This is inconsistent with the previoous model.

- However if we think price is fixed in the second round we get that $a^* = b^* = 0.5L$.

Definition 4

The principle of **minimal differentiation** (Hoteling's Law) refers to the idea that absent price competition firms often want to make their products as similar as possible.

- This is also inefficient (by same argument as before). We have too little differentiation.

- Fixed prices represents electoral competition well: Downs applied the reduced Hoteling model to describe why political parties in the US tend to gravitate towards the center on issues (Median Voter Theorem).