Lecture 4: Performance Pay

Compensation in Organizations

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Discussion: Loyalka et. al. (2019)

The Principal-Agent Model

Players

- There is a firm (the principal) who is risk neutral (exponential utility with parameter $r = 0$).
- There is a worker (the agent) who is risk averse (exponential utility with parameter $r \geq 0$).

Actions

- Firm chooses a linear wage which depends on effort ($w(e)$) or output ($w(y)$)
- After seeing the wage, the worker either accepts or rejects the job.
- If they accept, worker chooses effort e at an increasing, convex cost $c(e)$

The Principal-Agent Model

Output

- Output is effort (e) plus noise/luck (ϵ): $y = e + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$
- This implies output is normal with mean e and variance σ^2

Payoffs

- If accepted, firm's payoff π is expected output minus expected wages: $E[y - w|e]$
- If accepted, worker's payoff is expected utility of the wage minus effort cost: $E[u(w) - c(e)|e]$
- If rejected, worker has "outside option" of $\bar{u}$ and firm has "outside option" of 0.[1]

1. We will assume throughout that the firm prefers to hire the worker ex-ante.

Timing

See the board!

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Recap: Effort-Based Pay

- Suppose the firm can pay based on the worker's effort.
- Then wage is a linear function of effort: $w(e) = \alpha + \beta e$
- We now go to the board to solve!

Recap: Effort-Based Pay

Theorem 1

When wages depend directly on effort, effort is e^* *which solves* $c'(e^*) = 1$ *and* $\beta^* = 1, \alpha^* = \bar{u} + c(e^*) - 1$

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Performance-Based Pay

- Suppose the firm can pay based ONLY on output y
- Then wage is a linear function of output: $w(y) = \alpha + \beta y$
- We now go to the board to solve!

Performance-Based Pay

Theorem 2

When wages depend only on output, effort is e_p which solves

$$c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

and $\beta_p = c'(e_p), \alpha_p = \bar{u} - \beta_p e_p + r\beta^2\sigma^2/2 + c(e_p)$.

- Notice that $\frac{1}{1+r\sigma^2 c''(e_p)} < 1$.
- Therefore we are getting less than surplus maximizing effort: $e_p < e^*$
- Performance pay generates inefficiency relative to effort based pay!

Performance-Based Pay: Explicit Cost Function

- Suppose that $c(e) = e^2/2$
- Let's work it out on the board!

Performance-Based Pay: Explicit Cost Function

- Suppose that $c(e) = e^2/2$
- Let's work it out on the board!
- Under this quadratic effort cost:

$$e_p = \beta_p = \frac{1}{1 + r\sigma^2}$$

$$\alpha_p = \frac{r\sigma^2 - 1}{2}\left(\frac{1}{1 + r\sigma^2}\right)^2 - \bar{u}$$

Interpreting Results

- β is the average amount of money paid to the worker per unit of effort.
- β represents the strength of incentives (question: why?)
- We say incentives are high-powered when β is high (close to 1)
- Because $\beta = c'(e)$ we have that:

$$\beta_p = c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

Interpreting Results

- Because $\beta = c'(e)$ we have that:

$$\beta_p = c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

- The strength of incentives rises when:
  - risk-aversion decreases $\downarrow r$ (question: what if $r = 0$?)
  - noise/luck becomes less important $\downarrow \sigma^2$ (question: what if $\sigma^2 = 0$?)
  - the marginal-marginal cost of effort decreases $\downarrow c''(e_p)$
    - a high $c''(e_p)$ means working one more hour after already working 8 hours is much harder than working one more hour after working 1 hour.

Discussion: Drago and Garvey (1998)