# Who Gets the Job: A Model of Delegated Recruiting with Multidimensional Applicants

Jacob Kohlhepp and Stepan Aleksanko

UCLA

September 4, 2020

# Motivation

- Moving talent across sectors and geographic areas is critical for economic growth.
- Distributional effects also have become important:
- A main pathway for reallocation is recruitment: the formal process of an agent searching for a person on behalf of a firm.
- Search effort is not observed, and the quality of the applicant is usually not contractible (for non-sales employees).
- How do agency problems generated by contract restrictions impact who gets hired (or, more specifically, who ends up in the applicant pool).
- How does the efficiency of delegation change with market conditions?

# Motivation - Interviews

I interviewed 3 recruiters regarding their compensation and practices.

**Two key insights emerged:**

1. Most recruiters across industries are paid according to a standard **bonus contract**, where they receive a fixed percentage of their suggested applicant's salary if the applicant is hired and is not fired and does not quit for some number of months.
2. Recruiters prefer "less risky" hires - they are not willing to trade the option value of a risky choice if there is a chance the person will not "fit" the company.

# Key Economic Force

- Recruitment is a delegated task, and the most common bonus-type contract seems to produce a **misalignment** between the recruiter and the firm.
- The firm keeps residual profits from an employee (beyond the market expectation), and as a result there is some option value to a risky hire.
- The recruiter has no such incentive, and only cares about maximizing the probability that a hire stays.
- The recruiter will focus on applicants which it "understands" better: this may mean excluding non-traditional applicants or relying on private information, which can bias the recruiter towards applicants of a certain gender, race, pedigree, etc.

# Literature

Four strands:

- **Delegated Search:** Ulbricht (2016), Foucart (2020), Lewis (2012)
- **Delegated Choice:** Armstrong and Vickers (2009), Frankel (2014), Frankel (2016)
- **Labor search and matching models with heterogeneity:**
  1. One dimension: Postel-Vinay and Robin (2002), Moscarini (2003), **Lazear (1998)**
  2. Multidimensional: Lindenlaub and Postel Vinay (2017)

# Contributions

1. Two-dimensional sequential delegated search.
2. First model of the delegation problem applied to recruiters specifically where objects have two dimensions.
3. Incorporate real-world contract shape within model, rather than specifying utility as random variables linked by correlation.

# Environment

**The Players**

1. One risk-neutral firm which wishes to hire a worker.
2. One risk-neutral recruiter operating a search technology.
3. The worker is not a player.

**The Game**

1. Firm proposes a contract consisting of upfront payment $\alpha$, and a bonus $\beta$ contingent on whether the worker remains at the firm.
2. The recruiter accepts or rejects the contract.
3. The recruiter sequentially searches for a worker and proposes one worker to the firm.
4. Ability ($a$) realizes and the worker exogenously separates from the firm if $a < 0$.
5. The contract realizes.

# Search Process

**Workers**

1. A worker is ex-ante described by $(\mu, \sigma)$.
2. Conditional on $(\mu, \sigma)$ ability $a$ is distributed $N(\mu, \sigma^2)$.

**Search Process**

1. Search is sequential in the style of McCall 1970.
2. Recruiter takes i.i.d. draws of $(\mu, \sigma)$ with joint distribution $G$.
3. Search has unit cost $c$.

**Payoffs**

- Firm ex-post profit is: $a - \beta \mathbb{I}\{a \geq 0\} - \alpha$
- Recruiter utility is $\alpha + \beta \mathbb{I}\{a \geq 0\}$ less the search costs.

# Intuitive Example

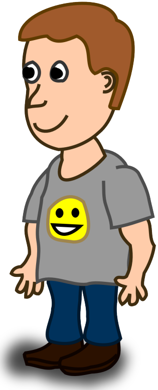


Figure: Mr. Self-Taught

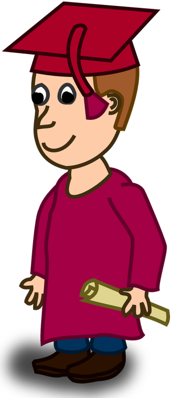


Figure: Mr. Ivy League

# Comments

- $(\mu, \sigma)$: interpret as belief about ability after viewing applicant's resume or LinkedIn profile.
  - $\sigma$ captures precision of belief, and could differ for different recruiters looking at the same person based on personal information (understanding candidates of a certain type/race/gender/pedigree).
- No assumptions about the joint distribution of $(\mu, \sigma)$.
- Will consider only firm-proposing Perfect Bayesian Nash Equilibrium (second-best) and the benchmark where the firm performs search directly (first-best).
- We will be concerned with the payment $(\beta)$ and **acceptance regions.**

# Acceptance Regions

**Definition 1**

An acceptance region, denoted $\mathcal{D}_i$, is the set of applicant types $(\mu, \sigma)$ which entity $i$ would select if they operated the search technology.

# First-Best Problem

In the first-best benchmark, the firm searches directly, and it solves:

$$V = -c + \int \max\{E[a|\mu = u], V\} dF(\mu)$$

where $V$ is the value function.

# First-Best Result

**Lemma 2**

*In the first-best benchmark, where the firm operates the search technology directly, the acceptance region is given by:*

$$\mathcal{D}_F = \{\mu, \sigma | \mu \geq \mu^*\}$$

*where* $\mu^*$ *solves:* $c = \int_{\mu \geq \mu^*} (1 - G_\mu(\mu)) d\mu$

# The Firm-Optimal Contract

$$\max_{\alpha,\beta,\mathcal{D}_R} E[a|(\mu,\sigma) \in \mathcal{D}_R] - \alpha - \beta E[\mathbb{I}\{a > 0\}|(\mu,\sigma) \in \mathcal{D}_R] \qquad \text{(OBJ)}$$

s.t.

$$\mathcal{D}_R = \{\mu,\sigma|\beta E_a[\mathbb{I}\{a > 0\}|(\mu,\sigma)] - U \geq 0\} \qquad \text{(IC)}$$

$$\alpha + E[U|(\mu,\sigma) \in \mathcal{D}_R] \geq 0 \qquad \text{(IR)}$$

where $U$ is the value function of the recruiter (less $\alpha$) during the sequential search problem, defined as:

$$U = -c + \int \max\{\beta E_a[\mathbb{I}\{a > 0\}|(\mu,\sigma)], U\} dG(\mu,\sigma) \qquad \text{(VAL)}$$

# Solving the Firm's Problem

**Lemma 3**

*Given $\beta$, define $M(u)$ as the CDF of $u := \beta\Phi\left(\frac{\mu}{\sigma}\right)$. In any incentive compatible contract, the recruiter's acceptance region is given by:*

$$\mathcal{D}_R = \{\mu, \sigma | \mu/\sigma \geq \Phi^{-1}(u^*/\beta)\}$$

*where $u^*$ solves:*

$$c = \int_{u \geq u^*} (1 - M(u))du$$

# Solving the Firm's Problem

### Proposition 1

*Define* $x := \Phi^{-1}(u^*/\beta)$*. The firm-optimal contract can be solved by first solving the unconstrained maximization problem:*

$$\max_x E[\mu|\mu/\sigma \geq x] - \frac{c}{Pr(\mu/\sigma \geq x)}$$

*and then obtaining* $\beta, u^*$ *from:*

$$c = \int_{u \geq u^*} (1 - M(u))du$$

*and* $\alpha$ *from:*

$$\alpha = -u^*$$

# Visualizing Misalignment

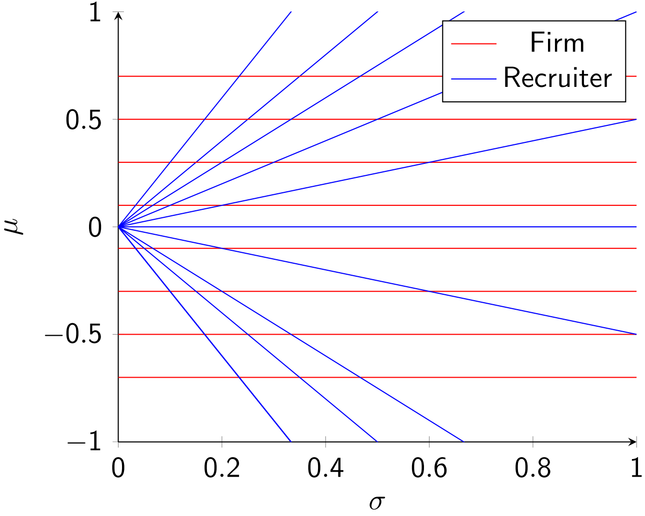
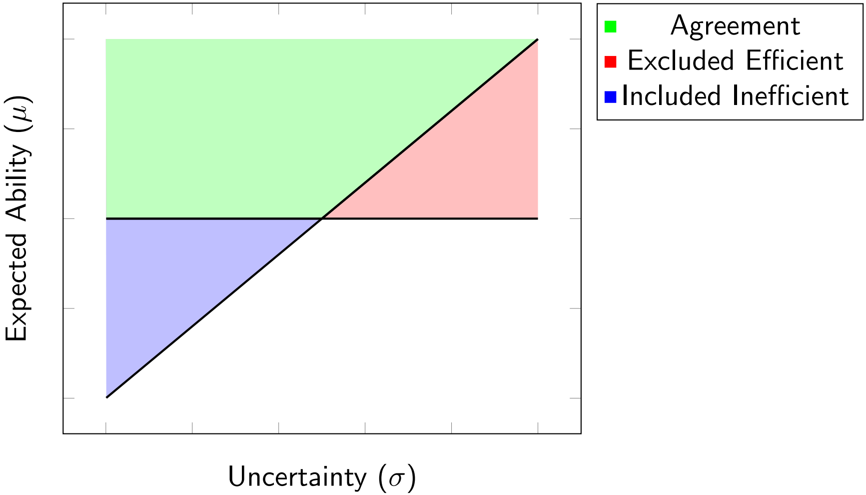



Figure: Indifference Curves

# Visualizing Acceptance Regions: $\mu > 0$



# Uncertainty Distribution of Selected Applicants

**Proposition 2**

*If $\mu$ and $\sigma$ are independent, $\sigma|\sigma \in \mathcal{D}_F$ first-order stochastically dominates $\sigma|\sigma \in \mathcal{D}_R$.*

Proof

**Intuition:** The recruiter will tend to over select "safe-bets" and under-select "high-potential risks." The relationship between $\mu, \mu/\sigma$ is key to the problem.

**Remark:** This is unrealistic. In reality, it is likely that $Cov(\mu, \sigma) < 0$ or $Cov(\mu, \sigma) > 0$.

# Parametric Assumption

### Assumption 1

*The joint cumulative distribution function of $(\mu, \tilde{\mu})$ is given by:*

$$H_{\mu,\tilde{\mu}}(\mu, \tilde{\mu}) = (1 - e^{-\lambda_1 \mu})(1 - e^{-\lambda_2 \tilde{\mu}})[1 + \rho e^{-\lambda_1 \mu - \lambda_2 \tilde{\mu}}]$$

*where we require:* $c \leq 1/\lambda_1$.

1. Can think of $\mu/\sigma$ as standardized expected ability.
2. Marginals are exponential.
3. $\rho$ controls correlation between $\mu, \tilde{\mu}$ and also roughly the correlation between $\mu, 1/\sigma$.
4. Restriction on $\lambda_1$ is so that the problem is not trivial.
5. Developed by Gumbel in the 1950s.

Note

# First-Best

**Proposition 3**

*Under Assumption 1, the first-best acceptance region is given by:*

$$\mathcal{D}_F = \{\mu, \sigma | \mu \geq \mu^*\}$$

*where $\mu^*$ has the closed-form solution:*

$$\mu^* := -\frac{log(c\lambda_1)}{\lambda_1}$$

Proof

# Equilibrium

## Theorem 4

*Under Assumption 1, the firm's problem has a unique solution with the following characteristics.*

1. **Acceptance Region:**

$$\mathcal{D}_R = \{\mu, \sigma | \mu/\sigma \geq x^*\}$$

2. **Bonus Payment $\beta$:**

$$\beta = c\left\{e^{\lambda_2^2/2}[1 - \Phi(x^* + \lambda_2)]\right\}^{-1}$$

$$x^* := \begin{cases} \frac{1}{2\lambda_2} log\left(\frac{\rho\lambda_2}{2\lambda_1 c}\right) & if \ \frac{\rho\lambda_2}{2\lambda_1 c} > 1 \\ 0 & else \end{cases}$$

Proof Sketch

# Agency Loss/Alignment

**Definition 5**

Agency loss is defined as the difference between firm profit in the first-best benchmark and firm profit in equilibrium.

**Theorem 6**

*Under Assumption 1, agency loss declines with $\rho$. Additionally, incentives become stronger, in the sense that:*

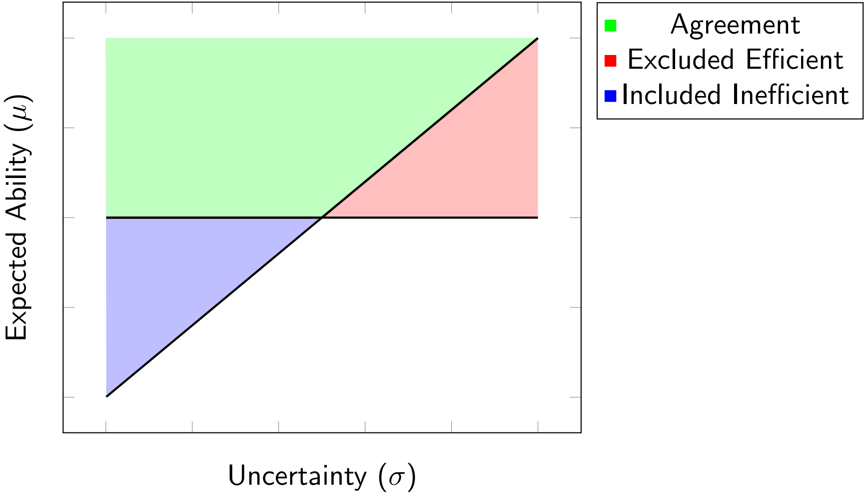
1. *The equilibrium expected number of searches increases.*
2. *Equilibrium bonus payment $\beta$ increases.*

Proof

**Intuition:** $\rho$ in a sense measures how good of an instrument the bonus contract is.

**Application:** In industries where quality job applicants (high $\mu$) tend to have precise signals of ability (low $\sigma$), recruiters should see more use.

# Recall: Regions of the Applicant Space



# Acceptance Regions

**Theorem 7**

*Under Assumption 1, when $\rho$ increases:*

1. *The* **Recruiter Acceptance Region** *decreases in probability.*
2. *The* **Agreement Region** *increases in probability.*
3. *The* **Included Inefficient Region** *decreases in probability.*
4. *The* **Excluded Efficient Region** *decreases in probability.*

**Intuition:** As alignment rises, the recruiter responds by becoming more selective (searching "harder"). Additionally, the set of mutually acceptable candidates rises, because the recruiter finds more "risky bets" acceptable and finds more "safe bets" unacceptable.

# Summary

- Firms delegate applicant search to recruiters, and use a particular bonus contract.
- **Non-parametric Result:** When applicant ability and applicant uncertainty are independent, the contract induces recruiters to select too many low-risk low-reward candidates.
- **Parametric Results in $\rho$:**
  1. Agency loss decreases in the correlation between ability and standard ability.
  2. The probability measure of the agreement region rises in $\rho$.
- Further Work:
  1. Comparative statics in $c, \lambda_1, \lambda_2$.
  2. Another non-parametric proposition for the non-independent case.
  3. Extension with no $\alpha$ (just bonus).

# The End

# Proof of Proposition 2

**Proof.** $Pr(\sigma \leq y | \sigma \in D_F) = G_\sigma(y)$ by independence. Then:

$$\begin{aligned} Pr(\sigma \leq y | \sigma \in \mathcal{D}_R) &= \int Pr(\sigma \leq y | \sigma \leq x\mu^*) dG_\mu(\mu) \\ &= \int \mathbb{I}\{x^*\mu \leq y\} + \mathbb{I}\{x^*\mu \geq y\} G_\sigma(y) dG_\mu(\mu) \\ &= G_\mu(y/x^*) + (1 - G_\mu(y/x^*)) G_\sigma(y) \end{aligned}$$

This final term shows that the CDF conditional on the recruiter's acceptance region is a weighted average of 1 and $G_\sigma(y)$ which is always weakly greater than $G_\sigma(y)$. This is equivalent to first-order stochastic dominance of $\sigma | \sigma \in D_R$ by $\sigma || \sigma \in D_F$.■

Back

# Remark: Distributional Assumption

This assumption is about the joint distribution of $\mu, \tilde{\mu}$. To see how this connects to the joint distribution of $\mu, 1/\sigma$, note that:

$$E[\sigma^{-1}|\mu] = E[\tilde{\mu}/\mu|\mu] = \mu^{-1}E[\tilde{\mu}|\mu] = \frac{1+\rho(1/2 - e^{-\lambda_1\mu})}{\mu\lambda_2}$$

When $e^{-\lambda_1\mu} = 1/2$, $\mu = \mu_{median}$.

$\mu > \mu_{median}$: higher $\rho$ increases $E[\sigma^{-1}|\mu]$

$\mu < \mu_{median}$: higher $\rho$ decreases $E[\sigma^{-1}|\mu]$

Intuitively, higher $\rho$ makes high ability individuals have more precise signals. Thus $\rho$ also measures the association between $\mu, 1/\sigma$.

Back

## Proof of Proposition 3

**Proof.** From Lemma 1, we know the general form of the acceptance region, what remains is to find $\mu^*$. Next, note that under Assumption 1, the marginal distribution of $\mu$ is exponential, so the equation characterizing $\mu^*$ from Lemma 1 can be re-written as:

$$c = \int_{\mu^*}^{\infty} e^{-\lambda_1 \mu} d\mu = \frac{e^{-\lambda_1 \mu^*}}{\lambda_1}$$

Re-arrangement yields:

$$\mu^* = -\frac{log(c\lambda_1)}{\lambda_1}$$

which is the result. Note that the first-best solution does not depend on $\rho$.■

Back

# Proof of Proposition 3

Under the assumption, we have that:

$$Pr(\mu/\sigma > x) = e^{-\lambda_2 x}$$

$$E[\mu|\mu/\sigma > x] = \lambda_1^{-1}\left(1 + \rho/2 - \rho/2e^{-\lambda_2 x}\right)$$

From Lemma 3, the problem is characterized in terms of a single choice variable, $x$. We can now make the problem explicit:

$$\max_x \lambda_1^{-1}\left(1 + \rho/2 - \rho/2e^{-\lambda_2 x}\right) - ce^{\lambda_2 x}$$

$$\frac{\rho\lambda_2}{2\lambda_1} e^{-\lambda_2 x} - ce^{\lambda_2 x} = 0 \qquad \text{(FOC)}$$

$x^* = 0$ if $\rho\lambda_2/(2\lambda_1) < 1$. Otherwise, SOC are satisfied and:

$$x^*(\rho, c, \lambda_1, \lambda_2) = \frac{1}{2\lambda_2} log\left(\frac{\rho\lambda_2}{2\lambda_1 c}\right)$$

Back

# Proof of Theorem 1

**Proof.** Recall that agency loss is profit in the first-best less profit in equilibrium. $\rho$ does not appear in first-best profit (because $\rho$ impacts the joint distribution but not the marginal distributions), so it is only necessary to understand how equilibrium profit changes with $\rho$. Equilibrium profit is:

$$\Pi^*(\rho, x^*) = E[\mu|\tilde{\mu} \geq x^*] - \frac{c}{Pr(\tilde{\mu} \geq x^*)}$$

$$\Pi^*(\rho, x^*) = \lambda_1^{-1}(1 + \rho/2 - \rho/2e^{-\lambda_2 x^*}) - ce^{\lambda_2 x^*}$$

Using the Envelope Theorem, we have that:

$$\frac{d\Pi(x^*, \rho)}{d\rho} = \frac{1}{2}(1 - e^{-\lambda_2 x^*}) \geq 0$$

Next recall that the expected number of searches is just $1/Pr(\tilde{\mu} \geq x^*)$.

This can be expressed as $\left(\frac{2\lambda_1 c}{\rho\lambda_2}\right)^{-1/2}$.

Back