Two Dimensional Delegated Search

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Motivation

- In many situations, a principal delegates search to an intermediary.
  - Example: A firm delegates employee search to a recruiter.
- The value of an object is uncertain until it is consumed. (even to the principal).
  - Example: You don't truly know if you like a person until after the first date.
- Thus, searched objects can be modeled as **random variables.**
- Ex-ante, searched objects differ both in terms of expected value and variance.
  - Example: The expected return and the variance of a stock.
- In many practical settings, contract is contingent on some binary event.
  - Example: If employee stays for 90 days, if actor gets the role, if athlete signs with a team.
  - Usually the agent refunds some of the upfront payment if the realization is bad.

Research Questions

We will consider two cases:

1. First-best: search is undertaken directly by the principal.
2. Second-best: Principal delegates search to an agent using binary bonus contracts.

And answer three questions:

1. How do the objects the principal finds acceptable differ from the objects the agent finds acceptable?
   - *Preview: The agent selects too many low variance objects (safe bets) at the expense of high mean higher variance objects (diamonds in the rough).*
2. How does the distribution of ex-ante characteristics impact the efficiency of delegation?
   - *Preview: Efficiency depends crucially on the "variance of the variance."*
3. In which settings will firms/principals outsource search?

Applications: Matching platforms (Booking.com, Tinder), talent agents/managers, recruiters.

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Literature

- **Delegated Search:** Ulbricht (2016), Foucart (2020), Lewis (2012)
- **Delegated Choice:** Armstrong and Vickers (2009), Frankel (2014), Frankel (2016)
- **Labor search and matching models with heterogeneity:**
  1. One dimension: Postel-Vinay and Robin (2002), Moscarini (2003), **Lazear (1998)**
  2. Multidimensional: Lindenlaub and Postel Vinay (2017)

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Environment

To fix ideas, we use the firm/recruiter example.

The Players

1. One risk-neutral principal (firm) wishes to hire a worker (object).
2. One risk-neutral recruiter (agent) operates the search technology.
3. Worker productivity a is ex-ante a random variable.

Timing

1. Firm proposes a contract.
2. The recruiter accepts or rejects the contract.
3. The recruiter sequentially searches for a worker and proposes one worker to the firm.
4. Productivity realizes as does the contract.

Maintained Assumption: Search is profitable: $E[a] > c$

Search Process

- Workers are ex-ante heterogeneous with two dimensions: μ, σ
- μ, σ have joint distribution G in the labor market with finite moments.
- Conditional productivity: $a \big| (\mu, \sigma) \sim N(\mu, \sigma^2)$
- Search is sequential in the style of McCall 1970.
- Searches are i.i.d. draws from G at cost c.

Definition

A **binary refund contract** is an upfront payment α and a payment β conditional on the event $a \leq 0$.

Payoffs Under This Contract

- Firm ex-post profit: $a - \beta \mathbb{I}\{a \leq 0\} - \alpha$
- Recruiter ex-post utility: $\alpha + \beta \mathbb{I}\{a \leq 0\}$ less search costs.

Our Motivating Example

1. Interviewed 3 recruiters who mentioned external recruiters are usually paid a fixed percent of salary if their candidate is hired and stays for a sufficient period.

2. Can think of a as net of a market wage, where after the first period if $a \geq 0$ wage adjusts so $a - \Delta w = 0$. If $a < 0$ then employee is terminated (wages are downwards sticky).

3. No limited liability: Allows us to focus on the inefficiency generated by misaligned preferences rather than through IR.

Acceptance Regions

Definition

An **acceptance region**, denoted $\mathcal{D}_i$, is the set of applicant types (μ, σ) which are accepted.

Definition

Standardized productivity, denoted $\tilde{\mu}$, of a candidate is the ratio of her expected productivity over her productivity uncertainty

$$\tilde{\mu} = \frac{\mu}{\sigma}$$

First-Best Benchmark

Suppose the firm can search **directly.** Then σ is irrelevant due to risk neutrality:

Lemma

In the first-best benchmark, where the firm operates the search technology directly, the acceptance region is given by.

$$\mathcal{D}_{FB} = \{\mu, \sigma | \mu \geq \mu^*\}$$

where μ^* *solves:*

$$c = \int_{\mu \geq \mu^*} (1 - G_\mu(\mu)) d\mu$$

Or equivalently

$$(\mathbb{E}[\mu | \mu \geq \mu^*] - \mu^*) \cdot \Pr(\mu \geq \mu^*) = c$$

Delegated Search (Second Best)

Suppose the firm must delegate search to the recruiter using a binary bonus contract.

$$\max_{\alpha,\beta,\mathcal{D}_R} E[a|(\mu,\sigma)\in\mathcal{D}_R]-\alpha-\beta E[\mathbb{I}\{a\leq 0\}|(\mu,\sigma)\in\mathcal{D}_R] \tag{OBJ}$$

s.t.

$$\mathcal{D}_R=\{\mu,\sigma|\beta E_a[\mathbb{I}\{a\leq 0\}|(\mu,\sigma)]-U\geq 0\} \tag{IC}$$

$$\alpha+E[U|(\mu,\sigma)\in\mathcal{D}_R]\geq 0 \tag{IR}$$

where U is the value function of the recruiter (less α) during the sequential search problem, defined as:

$$U=-c+\int\max\{\beta E_a[\mathbb{I}\{a\leq 0\}|(\mu,\sigma)],U\}dG(\mu,\sigma) \tag{VAL}$$

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Reducing Dimensions

The prior problem technically involves sequential search over two dimensional objects. We can reduce the problem using this Lemma:

Lemma

Given β, define M as the CDF of $u := \beta\Phi(-\tilde{\mu})$. In any incentive compatible contract, the recruiter's acceptance region is given by:

$$\mathcal{D}_R = \left\{ \mu, \sigma | \tilde{\mu} \geq \Phi^{-1}\left(\frac{u^*}{\beta}\right) \right\}$$

where u^* solves:

$$c = \int_{u \geq u^*} (1 - M(u)) du$$

Alignment of Preferences

Assumption

$\mathbb{E}[\mu|\tilde{\mu} = x]$ *is weakly increasing in x.*

Comment: This assumption means that $\tilde{\mu}$ is a noisy proxy for μ. Thus the contract induces in the agent the same preferences over $\tilde{\mu}$ as the principal's preference.

The Reduced Problem

Proposition

The firm-optimal contract reduces to solving the unconstrained problem:

$$\max_{x} \mathbb{E}[\mu|\tilde{\mu} \geq x] - \frac{c}{Pr(\tilde{\mu} \geq x)}$$

which has the below F.O.C., uniquely defining x^* (which can then be used to derive α, β):

$$(\mathbb{E}[\mu|\tilde{\mu} \geq x^*] - \mathbb{E}[\mu|\tilde{\mu} = x^*]) \cdot \Pr(\tilde{\mu} \geq x^*) = c$$

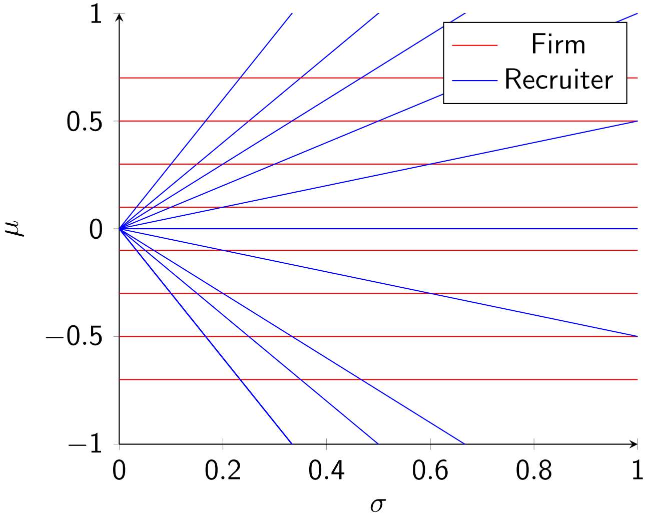
Comment:

1. Suppose the distribution of σ is a point mass at some value d.
2. Then define a new variable $\tilde{x} = d \cdot x^*$. FOC is:

$$(\mathbb{E}[\mu|\mu \geq \tilde{x}] - \tilde{x}) \cdot \Pr(\mu \geq \tilde{x}) = c$$

3. This is the same as first-best FOC, so we can achieve first-best (Ulbricht (2016)).

Misalignment: Indifference Curves



The "Refund" Form of the Optimal Contract

Corollary

If x^* is on the interior of supp$(\tilde{\mu})$, the firm's profit is positive and equal to $\mathbb{E}[\mu|\tilde{\mu} = x^*]$. Then $x^* \geq 0$, $\beta \leq 0$, $\alpha \geq 0$.

Induced Risk Aversion

Proposition

If μ and σ are independent, the distribution of σ in the first-best acceptance region $\mathcal{D}_{FB}$ first-order stochastically dominates the distribution of σ in the recruiter's acceptance region $\mathcal{D}_R$. Proof

Intuition: The contract form induces risk aversion in the recruiter (agent) causing them to under select diamonds in the rough and overselect safe bets.

Application to Statistical Discrimination

1. Consider the case where whoever is searching sees only a binary variable X and continuous Y which satisfy:

   $$a|X, Y = N(Y, X\sigma_H^2 + (1 - X)\sigma_L^2) \qquad X \perp\!\!\!\perp Y$$

   Thus X only impacts the variance.
2. This induces a joint distribution μ, σ to search over.
3. In the first-best, the firm ignores X and accepts all applicants with $Y \geq Y^*$.
4. Not so in the second best: to get the recruiter to select on Y, the firm will have to make the recruiter prefer $X = 0$ over $X = 1$.

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Parametric Assumptions

Assumption

μ, σ are distributed independently with marginal Type-1 Pareto distributions. That is, their joint pdf is given by:

$$f(x,y) = \frac{\alpha_1 c_1^{\alpha_1}}{x^{\alpha_1+1}} \frac{\alpha_2 c_2^{\alpha_2}}{y^{\alpha_2+1}} \mathbb{I}\{x \geq c_1\}\mathbb{I}\{y \geq c_2\}$$

Assumption

(Non-degenerate Search Condition:) Search cost c and the parameters of the joint Pareto distribution satisfy:

$$\frac{c_1 \alpha_2}{(\alpha_1 + \alpha_2 - 1)(\alpha_1 - 1)} \geq c$$

Remark: Without the second assumption, $\mathcal{D}_R$ may be equal to the whole support of (μ, σ). Thus the firm is paying the recruiter to randomly draw an applicant. Assuming the firm can do this itself for free makes this assumption without loss

Closed Forms

Under joint Pareto with non-degenerate search, the thresholds have a closed form.

Proposition

The first-best acceptance region is given by:

$$\mathcal{D}_F = \{\mu, \sigma | \mu \geq \mu^*\} \qquad \mu^* := \left(\frac{c_1^{\alpha_1}}{c(\alpha_1 - 1)} \right)^{\frac{1}{\alpha_1 - 1}}$$

Proposition

The recruiter acceptance region is given by:

$$\mathcal{D}_R = \{\mu, \sigma | \mu/\sigma \geq x^*\} \qquad x^* = \frac{1}{c_2} \left(\frac{c_1^{\alpha_1} \alpha_2}{c(\alpha_1 + \alpha_2 - 1)(\alpha_1 - 1)} \right)^{\dfrac{1}{\alpha_1 - 1}}$$

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Relative Agency Loss

Definition

Relative agency loss (RAL) is the fraction of social surplus lost due to delegation.

In our model, social surplus is profit:

$$RAL = 1 - \frac{\Pi_{SB}}{\Pi_{FB}} = 1 - (\alpha_1 + \alpha_2)\left(\frac{\alpha_2}{(\alpha_1 + \alpha_2 - 1)^{\alpha_1}}\right)^{\dfrac{1}{\alpha_1 - 1}}$$

Relative Agency Loss

Theorem

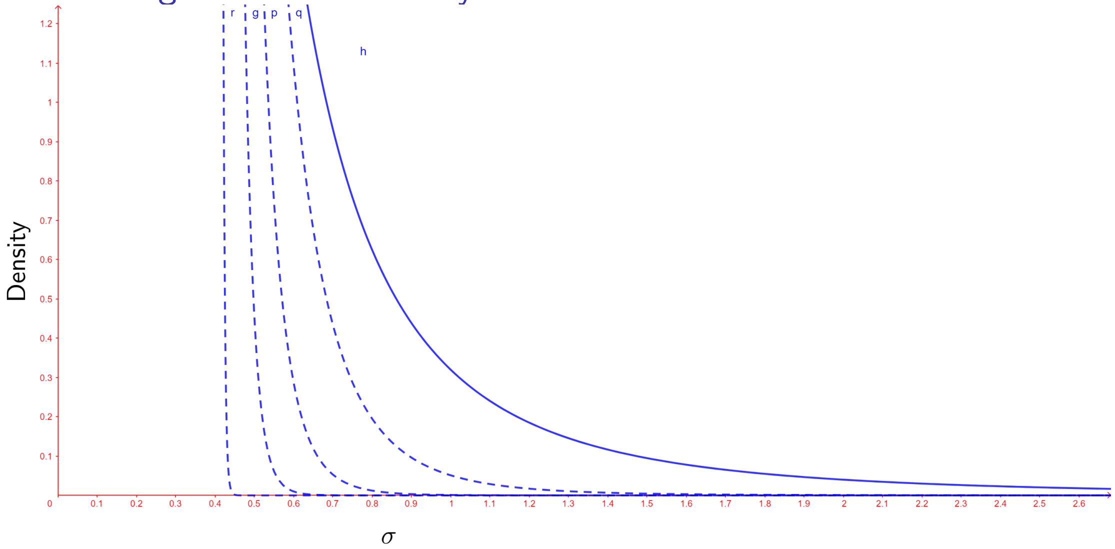
Relative agency loss has the following characteristics:

1. *invariant to* c, c_1, c_2
2. *increasing in* α_2
3. $\lim_{\alpha_1 \to \infty} RAL(\alpha_1) = 0$
4. $\lim_{\alpha_2 \to \infty} RAL(\alpha_2) = 0$

Comments:

1. The general level of variance (c_2) is not important for efficiency.
2. We can have arbitrarily high variance in productivity a and still achieve first-best.
3. What matters is the "variance of the variance" (α_2).
4. If most objects have a similar σ, the agent does not waste search effort chasing safe-bets.

Understanding the Limit: Density of σ



The Choice to Delegate

Suppose the principal can choose between delegating recruitment or doing it on their own.

- The benefit: less agency loss
- The cost: higher opportunity cost

Model this as the principal having search cost c_P and the agent having search cost $c_A < c_P$. The principal delegates if:

$$(\alpha_1 + \alpha_2)^{\alpha_1 - 1} \frac{\alpha_2}{(\alpha_1 + \alpha_2 - 1)^{\alpha_1}} \geq \frac{c_A}{c_P}$$

Prediction: The left side is increasing in α_2, meaning that as the variance distribution becomes more concentrated, delegation becomes more viable.

Other Applications

1. **Recruiter utilization across industries/occupations:** Each labor market can have different G distribution.
2. **Search delegation across settings:** Delegation is more likely when agents do not possess the ability to predict σ.
3. **Information Angle:** Differences in perceived ability among different groups (college degree vs. no college degree, racial bias, etc.).
   - One issue is that we do not formally introduce an information structure.
   - Any suggestions about how to apply result without doing this?

Thank you!

Solutions for Optimal Contract Payments

Then β can be obtained from the recruiter's IC constraint:

$$\beta = \frac{c}{(E[\Phi(\tilde{\mu})|\tilde{\mu} \geq x^*] - E[\Phi(\tilde{\mu})|\tilde{\mu} = x^*]) \cdot \Pr(\tilde{\mu} \geq x^*)}$$

and α from the recruiter's IR constraint:

$$\alpha = -\left(\beta \cdot E[\Phi(\tilde{\mu})|\tilde{\mu} \geq x^*]) - \frac{c}{\Pr(\tilde{\mu} \geq x^*}\right) = -\beta \cdot E[\Phi(\tilde{\mu})|\tilde{\mu} = x^*]$$

Proof of Proposition

Proof. Note that under independence, $\sigma|\mathcal{D}_F$ is the same as the unconditional distribution of σ. Then:

$$\begin{aligned} Pr(\sigma \leq y|(\mu,\sigma) \in \mathcal{D}_R) &= Pr((\mu,\sigma) \in \mathcal{D}_R)^{-1} \int \mathbb{I}\{x^* y \leq \mu\} + \mathbb{I}\{x^* y \geq \mu\} G_\sigma(y) dG_\mu(\mu) \\ &= Pr((\mu,\sigma) \in \mathcal{D}_R)^{-1} \Big(G_\mu(y/x^*) + (1 - G_\mu(y/x^*)) G_\sigma(y) \Big) \\ &\geq \Big(G_\mu(y/x^*) + (1 - G_\mu(y/x^*)) G_\sigma(y) \Big) \\ &\geq G_\sigma(y) \end{aligned}$$

Notice that the first quantity is the conditional CDF in the recruiter acceptance region. The second to last line shows that the this CDF is essentially a weighted average of 1 and $G_\sigma(y)$ which is always weakly greater than $G_\sigma(y)$. This proves first-order stochastic dominance of σ by $\sigma|\mathcal{D}_F$. Back

Expected Number of Searches

Corollary

The expected number of searches, given by $Pr(\tilde{\mu} \geq x^*)^{-1}$:

- *is unchanged by c_2 and decreasing in c_1.*
- *is decreasing in search cost, c.*
- *is decreasing in α_2.*

Intuition: A uniform increase in the variance of all candidates does not impact search activity.