

The Inner Beauty of Firms

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Abstract

This paper studies how the internal organization of firms interacts with labor and product markets. Using data from a software company, I am able to observe millions of task assignments within hundreds of hair salons, many of which are competitors. I develop a measure of organization complexity, which is the amount of information required to implement a given task assignment. Using this measure, I provide evidence of firm-specific organization costs, which grant complex salons a comparative advantage in producing high-quality products. Based on these facts, I develop a model where oligopolistic firms with different organization costs choose their internal structure. Complexity is costly, but it allows firms to improve product quality by better matching workers with multidimensional skills to tasks. I characterize the profit-maximizing organization as solving a rational inattention problem, and use the characterization to identify and estimate the model for Manhattan hair salons. Counterfactuals reveal that allowing internal organization to be heterogeneous and endogenous changes the equilibrium effects of policy. A sales tax cut increases specialization and therefore the productivity of all workers, while a minimum wage increase generates new types of wage spillovers.

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"Of all the things I've done, the most vital is coordinating those who work with me and aiming their efforts at a certain goal." - **Walt Disney**

1 Introduction

Greater specialization allows markets to better use the unique talents of individuals. As early as Adam Smith's pin factory, economists have recognized that much of this division of labor occurs within the firm. In practice, firms differ in their ability to organize people, and are observed using a wide variety of organization structures. How do firms choose their internal organization, and how does this choice interact with product markets, labor markets and government policy?

I propose a framework to study firms' equilibrium choice of internal organization. Based on a set of stylized facts from management software data, I model firms as deciding which workers to hire and how to assign these workers to tasks. More complex assignments are costly, but they improve product quality by better matching skills to tasks. Because firms differ in their organizational capabilities, they choose different internal structures. Because firms share a product and a labor market, the internal organization of competing firms are intertwined in equilibrium. I estimate the model for Manhattan hair salons, and show that allowing internal organization to be heterogeneous and endogenous qualitatively changes the effect of counterfactual policies. For example, a minimum wage raises equilibrium specialization for minimum wage workers, reduces specialization for non-minimum wage workers and causes wage spillovers which are not monotone in initial wage.

In the first part of this paper, I use novel data to establish empirical patterns in firm internal organization. The data come from a management software company, and they allow me to observe the assignment of millions of tasks to individual workers across hundreds of hair salons. I conceptualize firms as choosing organization structures, which are matrices where rows represent workers and columns represent tasks and each element is the fraction of total time assigned to each worker-task pair. I create a measure called organization complexity, which quantifies the amount of information that must be communicated within a firm in order to implement a given organization structure.

I document three facts about salon internal organization. First, complexity varies significantly across salons but very little across time, with almost no salons engaging in complete specialization. This is evidence of firm-specific and time invariant organization costs which prevent full specialization. Second, complex firms have higher revenue and employment. This is evidence firms with lower organizational costs have a com-

petitive advantage in the product market. Third, complex firms have higher prices and more repeat customers. This is evidence the organizational competitive advantage operates through quality rather than quantity. It grants organizationally efficient salons a comparative advantage at producing higher quality products.

In the second part of the paper, I build a model consistent with these facts. In this model, firms with product market power choose product prices, the composition of their workforce, and worker task assignments. Workers differ in their skill at each task. Assigning tasks to the most skilled worker raises product quality, but also increases organization complexity. Firms differ in the cost of complexity and their task-based production function, which causes them to choose different internal structures. Firms compete in a common product and labor market, so that their choices of internal structures both shape and are shaped by wages, prices and qualities.

The main theoretical result in this paper is a characterization of the firm's optimal organization structure enabling analysis, identification, and estimation. My model differs from past task assignment models along three dimensions: firms face heterogeneous organization costs which prevent full specialization; firms have market power; and workers have horizontal skill heterogeneity.¹ Because of these differences, I cannot use existing approaches to make the firm's problem tractable. Instead, I show that the profit maximizing organization also solves an equivalent rational inattention problem with mutual information attention costs. This equivalence allows me to weave together existing results to prove the other propositions in the paper.

Using this model I analyze the theoretical forces that shape a firm's choice of internal structure. I show that firms navigate a complexity-wage-quality three-way trade-off, where they attempt to produce the highest quality product using the simplest task assignment and the lowest wage bill. Even though firms are choosing the task assignment of each individual worker, I prove that at a high level the firm is choosing a point along a two-dimensional, convex frontier. One dimension is organization complexity and the other is wages adjusted for product quality. The firm chooses the point along the frontier that is tangent to its isoprofit curves—which I show are straight lines with a slope given by the firm's organization cost. In this way, the complexity a firm chooses reveals its unobserved organization cost parameter.

In the third part of the paper, I identify and estimate a structural version of the model for hair salons in New York City. The distribution of organization costs is identified by

1. Horizontal skill heterogeneity arise when one worker is good at coloring but bad at cutting while another is good at cutting but bad at coloring. They cause a technical problem because workers cannot be ordered along a single dimension.

the complexity of a firm's task assignments. The identification proof shows that they are known function of the data and the other parameters. Thus, if the other parameters are known, so is the distribution of organization costs.² Variation in the interaction of task intensity and organization complexity across firms in the same market identifies the other parameters. Intuitively, firms intense in task k and organizationally complex hire a large share of task k specialists and assign a large amount of task k to these specialists. The quality of these firms identifies the skill of task k specialists, while the cost of these firms identifies the wage of task k specialists. I provide a computationally light, nested fixed point estimation procedure which implements this identification strategy.

The estimated model reveals that even within a single industry (hair salons) and occupation (cosmetologists), variation in task-specialization is large and depends on unobserved worker skills and unobserved firm organizational differences. Salons in the bottom quartile of organization costs (efficient salons) on average assign 90% of tasks to the associated specialist while firms in the top quartile (inefficient salons) assign 67%. Haircut specialists spend the vast majority of their time cutting, but blow dry specialists spend less than half of their time blow drying. I also show that internal organization is a large source of productivity differences across firms, accounting for 40% of variation in marginal costs.

In the fourth part of the paper, I study two counterfactual policy changes, one in the product market and one in the labor market. In both cases, the fact that internal organization is heterogeneous and endogenous introduces new economic forces and qualitatively changes the total economic impact. The structure of the model allows any policy to be cleanly decomposed into a reallocation effect, where labor shifts across firms but internal organization remains fixed, and a reorganization effect, where task assignments within the firm are allowed to adjust. The reallocation effect is driven by the heterogeneity of internal organization, while the reorganization effect is driven by the endogeneity of internal organization.

In the first counterfactual, I eliminate the 4.5% New York City sales tax on services. The reallocation effect improves the competitive position of complex salons who were initially providing high-quality services. The reorganization effect induces almost all salons to reorganize in order to increase quality. Both effects increase equilibrium task-specialization across all workers and increase equilibrium labor productivity. Workers capture most of the productivity gains through higher wages.

In the second counterfactual, I increase the minimum wage from \$15 to \$20. The reallocation effect reduces the competitive position of firms with internal structures that rely

2. This is because complexity reveals organization costs, by the logic in the last paragraph.

on minimum wage workers. This effect improves the relative position of some salons enough that they increase employment in response to the minimum wage. The reorganization effect causes firms to layoff more minimum wage workers and shift their tasks onto other workers. This increases task-specialization for minimum wage workers but reduces task-specialization for other workers. Despite the fact that the labor market is competitive, organizational heterogeneity and endogeneity allow the model to generate aggregate labor-labor substitution patterns that are not possible with standard models. For Manhattan hair salons, wage spillovers are non-monotonic in initial wage, with workers close and far from the minimum wage seeing wage increases and worker in the middle seeing wage decreases.

This paper draws from insights in organizational economics and the task-based literature in labor economics in order to understand how internal organization decisions shape economic outcomes. The primary contribution is to build and estimate a model where organizationally unique firms make task assignment decisions which have labor and product market consequences.

The literature in organizational economics provides many ways in which firms can allocate talent better than markets. These include monitoring (Alchian and Demsetz 1972), relational contracts (Baker, Gibbons, and Murphy 2002), knowledge (Garicano and Wu 2012), coordination (Dessein et al. 2003), trust (Meier, Stephenson, and Perkowsky 2019) and culture (Martinez et al. 2015). Just as Holmstrom and Milgrom (1994) view the firm as an incentive system, this paper views the firm as a system of organizational practices. Once one adopts this view, firms should have heterogeneous organizational capabilities depending on their particular mix of practices (Argyres et al. 2012). My contribution is to capture this heterogeneity using firm-specific costs, in order to study its impact on market outcomes. I find that organizational heterogeneity is important, both for determining the division of labor across the economy and for understanding the distributional impact of policy changes.

I model labor as being divisible into tasks which can be assigned to workers with different skills, a tradition that dates back to at least Sattinger (1975) but has seen growing use since Autor, Levy, and Murnane (2003). I incorporate features present in different parts of the literature, including multidimensional worker types (Lindenlaub 2017), firms with multiple worker types (Haanwinckel 2020), organization costs (Adenbaum 2021, Garicano 2000), and firm-specific task demands (Lazear 2009). I also incorporate product market power. This combination of features allows for flexible labor-labor substitution patterns that are determined by the distribution of skills, organization costs and task demands in the economy. This flexibility is why I find that a minimum wage generates

non-monotonic wage spillovers even in a competitive labor market.³ Additionally, my model generates jobs which are bundles of tasks and vary from firm to firm even for the same type of worker. This is more realistic than past models, which typically generate fully-specialized jobs that are homogeneous within industry.

I also contribute to the empirical methodology in the task literature. To my knowledge, this is the first paper to make use of data on actual tasks completed across many competing firms. In other papers, task information is usually limited to a single firm, inferred using occupation or industry level data like O*NET or job-titles, or obtained via surveys. These papers often have access to rich worker data, which allows them to identify aspects of the task-based production function indirectly from wage distributions. I show that when a researcher has rich data on tasks but less data on workers, worker types can be inferred by equilibrium differences in qualities and costs across firms intense in different tasks. My approach is useful for studying industry equilibria, where within-industry firm differences, within-occupation worker differences and product market power are of first-order importance.

The remainder of the paper is organized as follows. Section 2 describes the management software data. Section 3 describes three stylized facts about hair salons that are used to build the model. Section 4 specifies the theoretical model. Section 5 theoretically analyzes the model. Section 6 discusses identification and estimation of a structural version of the model. Section 6 presents parameter estimates and assesses model fit. Section 7 performs two counterfactual policy experiments. Sections 8 and 9 discuss implications and conclusions

2 Data

This section describes the salon management software data.

2.1 Context and Institutional Details

The data set results from a data sharing agreement I negotiated with a salon management software company.⁴ The software helps with many aspects of running a beauty business,

3. This is similar to how Teulings (2000) showed that imperfect substitution along a single dimension changes how we should analyze minimum wages.

4. I designed the agreement as part of a consulting arrangement with the software company. During the first part of the consulting arrangement, the data was used to conduct internal analyses for the company. After, the agreement stipulated that I own the de-identified data, and the firm only had the right to review publications for privacy concerns.

including scheduling, pricing, payments, inventory, staffing, business reporting, client profiling, and marketing. As of July 19, 2022 a monthly subscription has a base price of \$175. Although the company markets its products to spas, tanning salons and massage parlors, hair salons and barbers make up the majority of clients. For this reason, I analyze only hair salons and barbershops.

The software is sold to beauty businesses throughout the United States, but the data indicate uptake is largest in Los Angeles (where the company was founded) and New York City. An important aspect of the data is that it allows me to observe the internal organization of salons that are geographically close and therefore likely to be direct competitors in labor and product markets. For example, I observe 10 salons in the lower Manhattan zip code 10013, which is a 0.55 square mile area.

The data document the assignment of individual stylists to particular services with particular clients. It describes the duration of the appointment, the price paid, and a custom text description of the services performed. If more than one employee is assigned to a single client, this is recorded as multiple entries describing what each employee contributed. Although the data is de-identified, a unique client ID allows a researcher to track employees and clients across time within salon.⁵

A sample from the data is provided in Table 3, with IDs replaced with pseudonyms. Looking at this sample, we can see the different way two salons coordinate employees to meet customer demand. Blake requested a cut, highlights and a treatment at salon 1A. The salon had a single employee, Rosy, perform all three services. Grace requested a cut and a single process (color) at salon 2A. Unlike salon 1A, salon 2A chose to assign each of these tasks to two separate employees, Tyler and Ben. Both of these salons are in the same zip code.

While the data is rich in terms of task content and worker assignments, information about employee compensation is not available. The software is capable of tracking some compensation information (tips, commissions and employment relationship, etc.) but discussions with the company and the data reveal that these additional functions are not consistently used by client salons.

2.2 Mapping Descriptions to Tasks

The data contain 20,560 unique text descriptions of services. This section describes the process used to categorize these descriptions into five tasks.

5. IDs are salon-specific, so I cannot track employees or clients if they move across salons.

A licensed cosmetologist was hired to group the services into tasks. Appendix Section B.1 describes the instructions sent to the cosmetologist and displays part of the final spreadsheet. I use the 6-category grouping provided by the cosmetologist with one modification: I combine the extension task with the blow dry task to create five final task categories. I do this because the extension task is very sparse—for Manhattan in 2021 Q2, less than 10 hours were devoted to this task. This sparsity leads to estimation problems, as parameters tied to this task have a negligible effect on observable outcomes.

In some cases, a service is marked as multiple task categories. In these cases, I divide the service into unique tasks in the following way. First, I compute the average amount of time spent on each task among services that are marked only as one task. I then compute the fraction of time to assign to each task as the corresponding task average divided by the sum of the averages of all other tasks marked for that service. I then distribute the total time spent on the service across the tasks using this imputed fraction. This process generates task categories that are mutually exclusive.

2.3 Descriptive Statistics

The data used in this section and the stylized facts include all observed firm-quarters where revenue per customer is positive. I exclude 2021 Q3 because I only observe part of the quarter. I also exclude an establishment in Kentucky with revenue that is implausibly high. The data contain information on 445 hair salon establishments, which represent 316 unique businesses, 9,179 hair stylists, 1,654,233 customers and 10.8 million services performed. Establishments first appear in the data when they adopt the management software. The last complete month of data is August of 2021. Although the software is used by salons across the country, users are concentrated in New York and California.

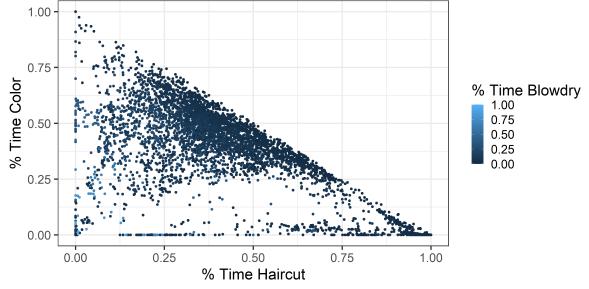
I aggregate the data to the firm-quarter level for analysis. Descriptive statistics at this level are provided in Table 4. The salons in the sample are higher end, bringing on average \$200 in revenue per customer in a given quarter. Throughout the paper, I refer to the price as the average revenue per customer per quarter. Even though there is significant variation in the relative importance of services offered, most salons offer at least 4 of the 5 task categories. Organizational complexity using the natural log scale with five tasks is bounded above by approximately 1.61 and below by 0. Some firms have an organizational complexity of 0, while others have a complexity as high as 1.02. I visually represent firm-quarter heterogeneity in the task-mix and complexity in Figure 1. Firms vary in their intensity in each task, and there is a long right tail of high-complexity firms.⁶

6. To see variation across jobs, see Table B3 and Figure B6.

Figure 1: Variation in Salon-Quarter Task-Mix

Share of Labor	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Haircut/Shave	4,558	0.41	0.23	0.00	0.26	0.52	1.00
Color/Highlight/Wash	4,558	0.38	0.20	0.00	0.29	0.52	1.00
Blowdry/Etc	4,558	0.09	0.12	0.00	0.03	0.11	1.00
Administrative	4,558	0.05	0.11	0.00	0.002	0.04	1.00
Nail/Etc	4,558	0.06	0.16	0.00	0.05	1.00	

(a) Summary Statistics



(b) Scatter Plot of Three Main Tasks

Note: Panel A provides summary statistics about the share of time spent on each task across all firm-quarters. Panel B visualizes this variation for the three most common tasks. Each point is a firm-quarter.

3 Stylized Facts

The model I use to study the effect of internal organization on product and labor markets is inspired by three stylized facts. In order to analyze internal organization, I define two concepts which will be used throughout the paper to think about internal task assignments. To begin, denote workers by the index i , firms by the index j , and tasks by the index k .

Definition 1 *A firm's organization structure, denoted B_j , is a matrix where element $B_j(i, k)$ is the fraction of labor assigned to worker i and task k .*

An example of two different organizational structures is given in Figure 2. The column totals represent the firm's task-mix, that is, the amount of each task needed to produce one unit of output. This mix is assumed to be fixed either by technology constraints or demand. The row margins represent the composition of a firm's workforce, how much work is assigned to each worker. The Figure 2 represents two different organization structures which have the same task-mix (column totals) but which assign work very differently. The left structure is staffed by specialists; it is called an employee salon because each person has a distinct role and all workers coordinate to produce each service. The right structure is staffed by generalists; it is called a chair renter salon because it is what we would expect if each worker rented a space and essentially operate as a salon within a salon. Workers are exchangeable and roles are not distinct.

Figure 2: Two Organization Structures

		"Employee" Salon			"Chair Renter" Salon					
		Tasks			Tasks					
Employee		1	2	3		1	2	3		
	A	1/2	0	0	1/2	A	1/6	1/12	1/12	1/3
	B	0	1/4	0	1/4	B	1/6	1/12	1/12	1/3
	C	0	0	1/4	1/4	C	1/6	1/12	1/12	1/3
Tot.		1/2	1/4	1/4	1	Tot.	1/2	1/4	1/4	1

Note: Two organizational structures a firm with a task-mix of $1/2, 1/4, 1/4$ could choose. Column sums represent the task-mix and row sums represent the fraction of work performed by each employee.

The second concept—complexity—establishes the minimum amount of information that must flow through the firm in order to implement the structure⁷ and builds on a lengthy literature in information theory starting with Shannon (1948).

Definition 2 *The complexity of an organization structure B_j is:⁸*

$$I(B_j) = \sum_{i,k} B_j(i, k) \log \left(\frac{B_j(i, k)}{\underbrace{\sum_{k'} B_j(i, k')}_{\text{Worker } i \text{ Labor Share}} \underbrace{\sum_{i'} B_j(i', k)}_{\text{Task } k \text{ Labor Share}}} \right)$$

Consider the two structures in Figure 2. The firm can implement the chair renter structure (right) by randomly assigning workers to tasks. This does not require information about tasks or worker identities, so complexity is 0. To implement the employee structure (left), the firm must tell each worker exactly which task to perform. The firm can write an employee manual, saying: “assign the task to employee A if you observe ‘0’, assign to B if you observe ‘01’, and assign to C if you observe ‘10’.” The expected number of bits (or amount of information) is $1 \times 1/2 + 2 \times 1/2 = 1.5$. This is the minimum information required to communicate this assignment, so the complexity is 1.5.

I now present three stylized facts about internal organization.

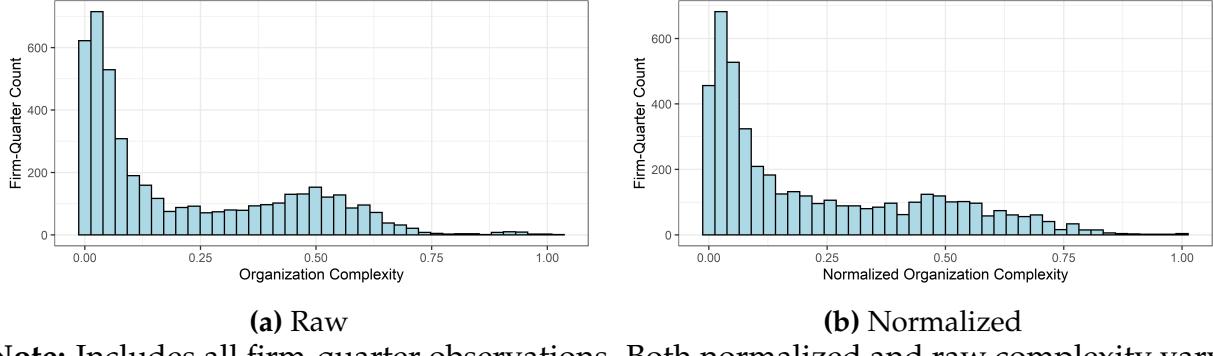
Fact 1 *Complexity varies significantly across firms and varies little across time.*

To establish this fact, I first compute I_j^{\max} which is the maximum value of complexity given a firm’s task-mix in a given quarter. I construct normalized complexity \bar{I}_j as

7. The measure is the mutual information of the joint distribution over workers and tasks. Mutual information is a well-understood way to measure information costs. It has many desirable properties, and gives the model many micro-foundations.

8. When computing this measure, I assume that $0 \log(0) = 0$.

Figure 3: Histogram of Normalized Complexity



Note: Includes all firm-quarter observations. Both normalized and raw complexity vary significantly. Normalized complexity is observed achieving both its lower bound (0) and upper bound (1).

raw complexity divided by I_j^{max} . Normalized complexity \bar{I}_j has a minimum of 0 (like raw complexity) and a maximum of 1 (unlike raw complexity). I plot a histogram of normalized complexity in 3 and observe that complexity varies significantly across firm-quarters and has a long right tail. In particular, I observe that while some firms have very complex organizations (close to the upper bound) others have very simple organizations (complexity of 0). To understand whether the variation is across time or across salons, I decompose complexity into a salon-specific component, a time-specific component and a residual component:

$$\bar{I}_{j,t} = \bar{I}_j + \bar{I}_t + e_{j,t}$$

I estimated the firm and year components by regressing normalized complexity on time and salon fixed effects. This allows me to decompose the total variance of complexity into the three components, as shown:

$$Var(\bar{I}_{j,t}) = Var(\bar{I}_j) + Var(\bar{I}_t) + 2Cov(\bar{I}_j, \bar{I}_t) + Var(e_{j,t})$$

.0516	.0464	.0002	-.0009	0.0059
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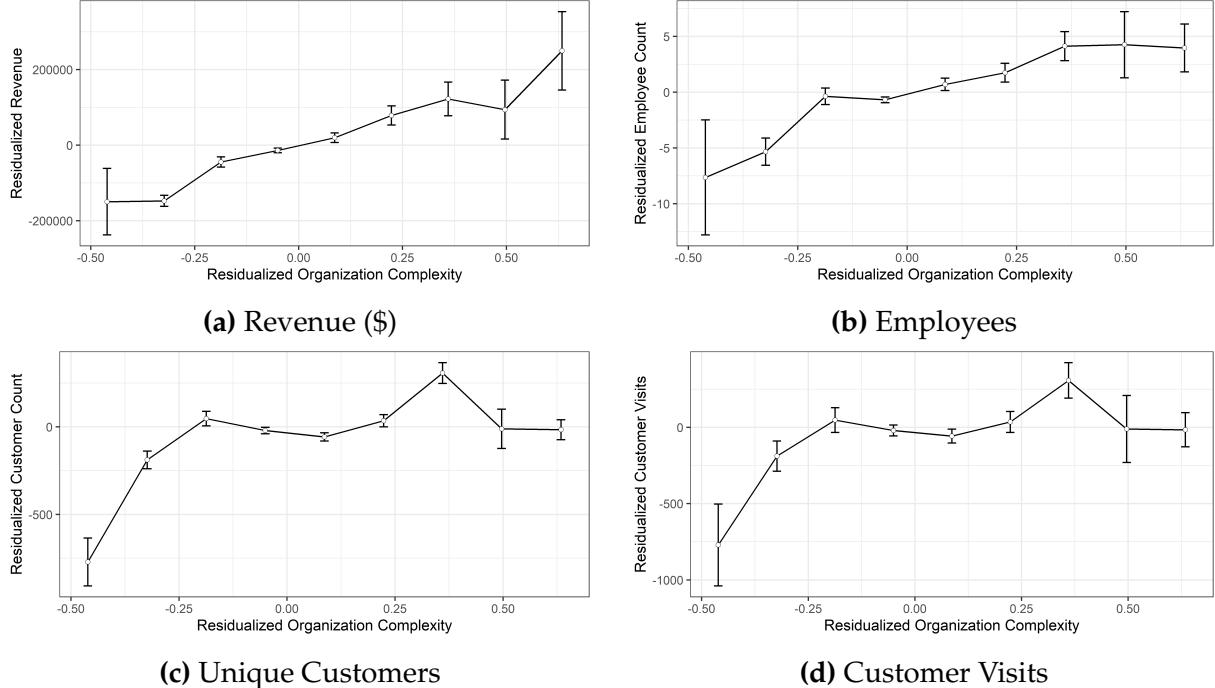
These results demonstrate that 90 percent of the variance in normalized complexity is attributable to the firm component, while only 0.4 percent is attributable to the time component. Therefore, complexity varies significantly across firms but little across time. This is evidence the choice of complexity is driven by a deep, time invariant characteristic which varies significantly across firms.

Fact 2 *Complex firms have higher revenue and employment.*

Complexity is positively correlated with revenue and employment as well as several other measures of firm size. After controlling for task-mix, county and year-quarter fixed

effects, I depict the residualized relationship between firm in Figure 4. The regression results, shown in Table 2, demonstrate that the correlation is positive for all variables and statistically significant at the 5 percent level for revenue and employment. The positive relationship between revenue and complexity is robust. It remains when only Manhattan hair salons are analyzed and when employee count is interacted with complexity.⁹

Figure 4: Organization Complexity and Firm Size



Note: Each panel illustrates the positive relationship between organization complexity and a different measure of firm size. All variables residualized for quarter, county and task-mix. Firm-quarters are grouped into equally spaced bins based on complexity.

This positive correlation suggests some salons have an organizational competitive advantage, that is, they find it easier than competitors to adopt productive organizational practices. This allows them to implement more complex task assignments at a lower cost.

Fact 3 *Complex firms have higher prices and more repeat customers.*

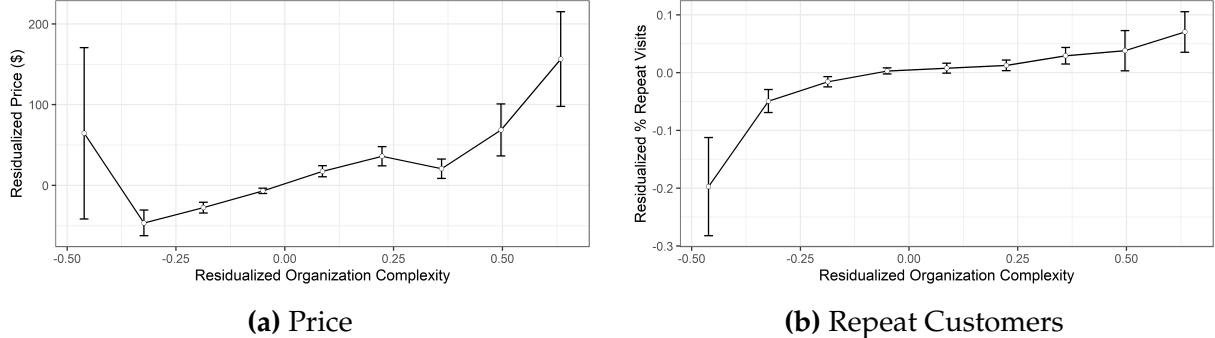
Complexity is positively correlated with price. I show this visually in Panel A of Figure 5. Section A.12.5 proves that this pattern in the data is inconsistent with a model where organizational competitive advantages operate only through marginal cost reductions. In such a model, prices should be decreasing in complexity. The fact that the oppo-

9. See Appendix Section B.3 for additional regression tables.

site is true suggests salons with higher internal complexity are producing services with higher unobserved quality and thus higher costs.

To test this quality channel, I used the share of repeat visits as a proxy for quality in Panel B of Figure 5. It is reasonable to assume that a customer who returns was satisfied with the quality of the original service. The fraction of visits by return customers rises with complexity, evidence of a link between quality and organization. This suggests that the organizational advantage described in Fact 2 operates through unobserved quality rather than quantity. In the next section, I build a model inspired by this and the other two facts.

Figure 5: Organization Complexity, Prices and Repeat Customers



(a) Price

(b) Repeat Customers

Note: Panel A illustrates the positive relationship between organization complexity and price, while panel B illustrates the relationship between organization complexity and the fraction customers that return. All variables residualized for quarter, county and task-mix. Firm-quarters are grouped into equally spaced bins based on complexity.

4 Model

This section specifies a model where firms choose prices and organizational structures simultaneously in order to compete for consumers. Consistent with the stylized facts, firms choose their organization structure subject to heterogeneous organization costs. The main benefit of a complex organization is the ability to produce a higher quality product. There are three important groups of objects in the economy. Firms, indexed by $j = 1, \dots, J$. Worker types, indexed by $i = 1, \dots, N$. Tasks, indexed by $k = 1, \dots, K$.

Firms and Tasks. The J firms differ in their organization cost $\gamma_j \in \mathbb{R}_+$ discussed below. Each firm produces a single good¹⁰ using a Leontief task-based production function

10. A single good allows us to focus on internal organization. Nocke and Schutz (2018) shows we can represent a pricing game with multi-product firms as one with single product firms by adjusting qualities

described by $\alpha \in \mathbb{R}_+^K$, which I refer to throughout the paper as the task-mix. To produce one unit of the good, it must allocate α_k labor to task k , where I normalize $\sum_k \alpha_k = 1$. The firm can choose how these tasks are assigned to workers in a process that is described shortly.

Workers and Labor Markets. Each of the N workers types are characterized by inelastic total labor supply L_i and skill set vector θ_i . Element $\theta_i(k)$ is the quality with which worker i performs task k . The labor market is competitive with type-specific wages w_i , which I collect into a wage vector w .

Firm Strategies. Firms choose the price of their product $p_j \in \mathbb{R}_+$ and their organizational structure $B_j \in \Delta^{N \times K}$, where $\Delta^{N \times K}$ is a $N \times K$ -dimensional unit simplex. Element $B_j(i, k)$ of an organization structure specifies the fraction of total labor allocated to worker-type i and task k .¹¹ An organizational structure B_j is feasible if it is consistent with the task-mix vector: $\sum_i B_j(i, k) = \alpha_k \forall k$. The workforce composition, $E_j(i) = \{E_j(1), \dots, E_j(N)\}$ is the fraction of total labor demanded that is from each worker type. By definition, we have that:

$$E_j(i) = \sum_k B_j(i, k) \quad (1)$$

The cost of a firm's organization structure is the firm-specific parameter γ_j multiplied by the complexity of the organization structure $I(B_j)$. Recall complexity is defined as:¹²

$$I(B_j) = \sum_{i,k} B_j(i, k) \log \left(\frac{B_j(i, k)}{\underbrace{\sum_{k'} B_j(i, k')}_{\text{Type } i \text{ Labor Share, } E_i} \underbrace{\sum_{i'} B_j(i', k)}_{\text{Task-mix, } \alpha_k}} \right)$$

A firm's organizational structure determines the match between worker skills and tasks. As a result, it determines product quality ($\xi(B_j)$). I specify that product quality is a

and costs when demand takes the multinomial logit form.

11. This definition of task assignment treats all workers with a given set of skills symmetrically. In the model brought to the data, workers with the same skills may have different labor supplies. I show in Appendix Section A.5 that, due to an invariance property of the organization cost function, even if firms could treat the different workers with the same skills differently, it would not choose to do so in the optimal task assignment. Thus this abstraction is without loss under mutual information organization costs.

12. This mutual information-based functional form is used because it is the only cost function in a certain class where complexity over types will be equal to complexity over worker identities under a general matching process (Bloedel and Zhong 2021).

weighted-average of task quality:

$$\xi(B_j) = \sum_{i,k} B_j(i, k) \theta_i(k)$$

Since quality is valued by consumers, this is the main benefit of carefully assigning workers to tasks. A firm's organization structure also determines its per-unit wage bill ($W(B_j)$):

$$W(B_j) = \sum_{i,k} w_i B_j(i, k)$$

Demand. Total market demand for good j is given by a function D_j which maps the prices and qualities (ξ_j) of all firms into a quantity demanded for firm j . I assume that demand for good j depends on own-price and own-quality only through the quality-price index $\xi_j - \rho p_j$, and demand for good j is strictly increasing in the quality-price index. This implies the demand for good j can be written as $D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j})$.¹³ I do not place parametric assumptions on consumer utility (or functional form assumptions on market demand) until I estimate the model.

The Firm's problem. Per-unit organization costs and competitive labor markets imply marginal costs are constant. I denote the feasible set of organization structures $\mathbb{B} = \{B \in \Delta^{N \times K} \mid \sum_i B(i, k) = \alpha_k \forall k\}$. The firm's problem can now be defined:

$$\max_{p_j \in \mathbb{R}_+, B_j \in \mathbb{B}} D_j(\underbrace{\xi(B_j)}_{\text{quality}} - \rho p_j, p_{-j}, \xi_{-j}) \left[p_j - \underbrace{\left(\gamma_j I(B_j) + \sum_{i,k} w_i B_j(i, k) \right)}_{\text{constant marginal cost, } MC_j} \right] \quad (2)$$

Equilibrium. An equilibrium consists of firm strategies $\{p_j, B_j\}_{j=1}^J$ and wages w such that:

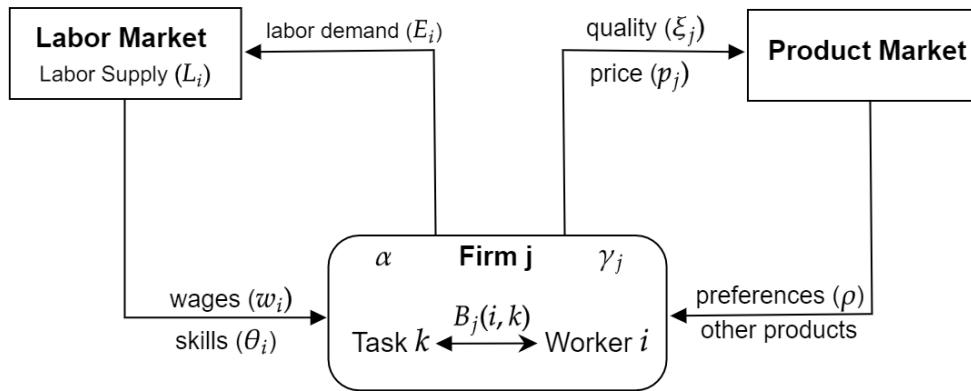
1. Firms choose prices p_j and organizational structures B_j to maximize (2).
2. Labor markets for each worker type clear:

$$\sum_j D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j}) \sum_k B_j(i, k) = L_i \forall i = 1, \dots, N$$

13. There are several random utility models and representative consumer models consistent with this assumption, including multinomial, nested logit and CES. The assumption rules out demand systems with consumer heterogeneity in price sensitivity, like the pure vertical model or the main specification in Berry, Levinsohn, and Pakes (1995).

Model Summary. Figure 6 illustrates the model from the perspective of a single firm. The firm chooses B_j (determining who they hire and how hired workers are assigned to tasks) and prices taking into account internal factors (like the task-mix (α) and organization costs (γ_j)), labor market factors (like wages and skills), and product market factors (like price sensitivities (ρ) and the prices and qualities of other products). This choice impacts the product market directly by determining product quality and indirectly through prices and the labor market by determining labor demand across worker types.

Figure 6: Illustration of the Model



Note: Each firm chooses B_j . In equilibrium, the firm's choice feeds back into both the product and labor market because labor markets must clear and prices must form best-responses to other firm strategies.

4.1 Discussion of Organization Costs

This section describes different ways of interpreting γ_j , the cost of increasing the complexity of firm j 's organizational structure by 1 unit. It can be interpreted as the collection of relationships, management practices, contract knowledge, human resource technology and other factors which are inherent to firm j .

Coordination Costs. Under this interpretation, γ represents the fact that firms are “second-best solutions to transactions plagued by various forms of contractual incompleteness” (Gibbons 2020) and that “firms can never succeed in conquering the nonrational dimensions of organizational behavior” (Williamson and Williamson 1984). As γ approaches 0, coordination costs disappear, and a firm can design any organizational structure it chooses at 0 cost. When γ becomes sufficiently large, firms will resort to assigning every worker the same job. In the latter case, workers are essentially firms since they perform all of the tasks the firm performs and do not rely on coworkers. In this way

the distribution of γ traces out the value of firms. A firm with low γ is greater than the sum of its people, producing a product superior to that which any could produce alone.

Rational Inattention. The mutual information form of organization costs gives it a rational inattention micro-foundation. We can interpret γ as the level of “managerial talent” (Lucas 1978) which determines the attention cost needed to allocate tasks to workers. Also, in some sense we can think of organization costs as capturing contractual inattention, as described by Tirole (2009). That is, different firms may find it more or less costly to write down complex contracts in order to support complex organizational structures.

Incentives for Teams. Under this interpretation, organization costs reflect losses due to free-riding. Dai and Toikka (2022) show that the residual or profit of a firm managing a team of multiple workers increases with the productivity of the known technology. Thus, heterogeneity in γ_j reflects the fact that some firms know a larger number of technologies, or ways to combine tasks and workers.

Costly Specialization. Less abstractly we can assume that specialization is costly, and this cost differs across firms according to γ . This interpretation reflects the fact that optimal $I(b, \alpha)$ is a measure of task-specialization.¹⁴

Thus, the multifaceted nature of γ_j can account for several dimensions of organizational heterogeneity all at once. Unfortunately, this also means the mechanisms driving the value of γ_j at any particular firm are not identified.

An example from the hair salon industry makes the model concrete. There are two main ways to organize a salon. In the chair renter arrangement, stylists pay a fixed fee to the salon owner and keep all revenue. Chair renters set their own hours and develop their own client lists. In the employee arrangement, stylists are paid by the salon owner and do not run their own business. The chair renter arrangement is characterized by a simple contracts and little need for coordination. In the language of the model, there is little organizational complexity. In contrast, the employee arrangement exhibits complex contracts (including non-competes, commission-based compensation, etc) and requires coordination. In the language of the model, there is significant organizational complexity. Although the employee arrangement is complex, it allows the firm to leverage the individual talents of workers to produce a higher quality product than any individual could produce alone.

14. The proof is in Section A.8.

5 Theoretical Results

This section analyzes the theoretical model. I first show the profit maximizing organization structure is also the solution to a simpler problem that is well-studied in information theory and behavioral economics. I use this equivalence to understand the economic forces which determine each firm's internal structure.

5.1 Main Characterization

The firm's problem as written in (2) appears complicated at first glance; there are $1+N \times K$ choice variables and the objective is highly non-linear. The following theorem reveals it can be greatly simplified.

Theorem 1 *An organizational structure (B_j^*) is profit-maximizing if and only if it solves:*

$$\min_{B_j \in \mathbb{B}} \gamma_j I(B_j) + W(B_j) - \rho^{-1} \xi(B_j) \quad (3)$$

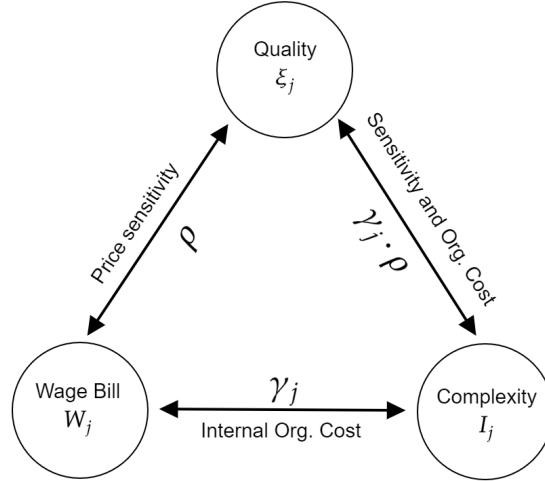
which is a rate-distortion problem and a rational inattention problem.

The proof of the result is provided in Appendix Section A.2. The main idea of the proof is that if an organization structure does not solve (3) the firm can switch to a structure that does and adjust prices to strictly improve profit. In this way, even though price and organization structure appear entangled in the firm's problem they can be separated during analysis. The result relies on the fact that the quality-price index $\xi(B_j) - \rho p_j$ is sufficient for price and organization structure in demand, and demand is strictly increasing in the quality-price index. The result does not rely on the functional form of organization costs.

Why does it matter that (3) is a rate distortion and rational inattention problem? The reason is that rate-distortion problems are well-studied in information theory and rational inattention problems are well-studied in behavioral economics. This allows me to weave together results across these two literatures to identify firm-specific organization costs, prove a form of equilibrium existence and uniqueness, construct an estimation algorithm, and solve for counterfactual equilibria.

Theorem 1 also reveals the forces that shape a firm's internal organization. Examining (3), firms face a triple trade-off which I depict in Figure 7. Each firm wishes to achieve the lowest complexity and wages while achieving the highest quality. How it navigates this trade-off depends on its internal organization cost γ_j , consumer price sensitivity ρ and their interaction $\gamma_j \cdot \rho$.

Figure 7: The Complexity-Wage-Quality Trade-Off



Note: Theorem 1 reveals that firms face a three-way trade-off when designing their internal structure. This trade-off does not depend on other firms actions except through wages in equilibrium. The firm must choose between higher quality, higher organization costs, or a higher wage bill. The exact trade-off depends on consumer price sensitivity and the firm-specific organization cost parameter γ_j .

If a firm wishes to increase quality, it has two options. It can hire better workers and incur a wage cost. Or it can rearrange its current workforce in order to better leverage existing working skills and incur an organization cost. Intuitively, when ρ is high, consumers are price sensitive and the firm cannot pass on costs to consumers via prices. Thus, firms prioritize minimizing costs over maximizing quality, and choose less complex organizations.

To analyze how the firm navigates the complexity-wage-quality trade-off, I define the *organization frontier* as the set of all organization structures which minimize complexity for some quality-adjusted wages (Q). The frontier consists of the simplest organizations that achieves some quality-adjusted wages. I wish to study the relationship between quality-adjusted wages and complexity along the frontier:

$$I^*(Q) = \min_{B_j \in \mathbb{B}} I(B_j) \text{ s.t. } W(B_j) - \rho^{-1} \xi(B_j) \leq Q$$

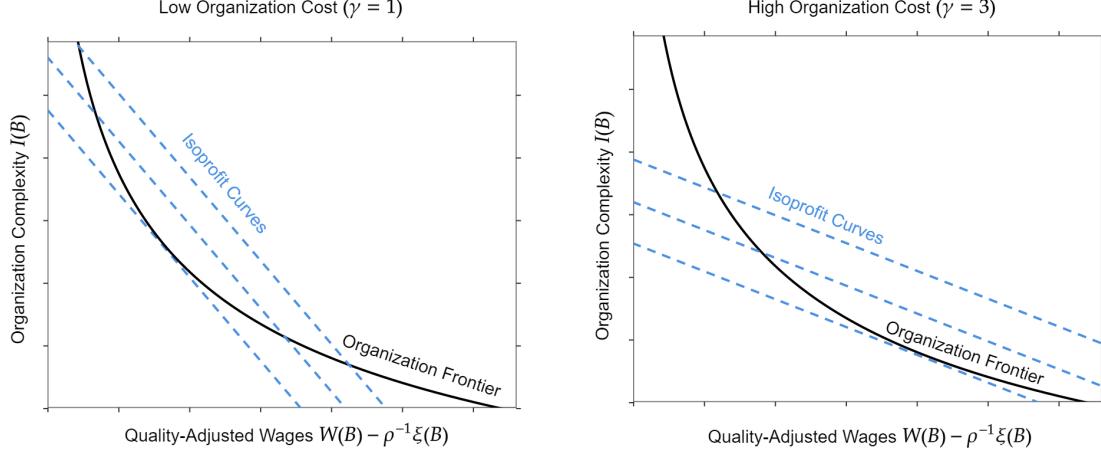
The characterization provided in Theorem 1 allows me to apply existing results from information theory to understand the general shape of this relationship.

Proposition 1 *Organization complexity along the organization frontier ($I^*(Q)$) is continuous, convex and decreasing in quality-adjusted wages.*

The proof is provided in Appendix Section A.3. The proposition means the choice of

a high-dimensional organization structure can be thought of on a high-level as a two-dimensional choice similar to a classic expenditure minimization problem from consumer theory. This is illustrated in Figure 8.

Figure 8: Choosing an Organizational Structure



Note: Although the B_j is a high-dimensional object, the firm essentially solves a two-dimensional trade-off between complexity and quality-adjusted wages. The firm's optimal structure will be the point of tangency between the organization frontier and the best possible isoprofit curve. As organization costs rise, the complexity of the chosen structure falls. This foreshadows identification of the firm-specific organization cost parameters.

Although B_j is a high-dimensional object, the firm essentially solves a two-dimensional trade-off between complexity and quality-adjusted wages. The firm's optimal structure will be the point of tangency between the organization frontier and the best possible(i.e., leftmost) isoprofit curve. The firm's isoprofit curves have a slope equal to $-\gamma_j^{-1}$. So, as γ_j rises, the curves become flatter, causing the tangent point to shift right and reducing organizational complexity while increasing quality-adjusted wages. A more complex organization allows a firm to produce a higher quality good at lower wage but it requires a greater organization cost. An immediate consequence is that a lower organization cost parameter grants the firm an organizational competitive advantage in the product market.

Proposition 2 *In equilibrium, firms with lower organization cost (γ_j) have higher organization complexity, market share, and profits.*

The proof is provided in Appendix Section A.3. Recall that γ_j represents the management technology, relationships, knowledge, and practices specific to the firm which

make it easier or harder to implement a given organizational structure. Proposition 2 implies more organizationally efficient firms are larger and more profitable. They can produce better quality goods at a lower cost. Importantly, this proposition confirms that the model is consistent with Fact 2: complexity should be positively correlated with measures of firm-size.

One implication of the above results is that while firms interact strategically in their price decisions, it is as if firms have a dominant organizational strategy. That is, firms choose B_j to minimize quality-adjusted cost, which depends on other firm decisions only indirectly through wages. This greatly improves tractability because we can solve the organization problem first and then use the solution to solve for optimal prices. This is critical to the estimation procedure outlined later in the paper.

In many contexts this result may be reasonable. That is, for many firms internal structures are connected to competitors solely through wages. However, there are situations where the product market choices of other firms may impact internal organization in ways other than wages. For example, in a model where consumers are more likely to substitute to products with similar qualities, the quality choice of the other firm may directly impact the returns to quality, and thus the returns to more complex organizations. Section A.12.5 describes such a model in more detail and outlines some challenges involved.

5.2 Optimal Jobs Within the Firm

The last result shows the originally high-dimensional problem of the firm can be reduced to a tractable two-dimensional trade-off. However, one of the goals of the model is to understand how firms assign workers to tasks. This section describes the properties of task assignments within the firm and shows that the firm customizes the bundles of tasks it assigns individual worker types. For this, I define the job of worker type i at firm j as a vector $(b_j(i, \cdot))$ where element k denotes the amount of i 's time spent on each task. The jobs a at a firm are simply the rows of the organization structure divided by the total labor of worker type i :

$$b_j(i, k) = \frac{B_j(i, k)}{\sum_{k'} B_j(i, k')}$$

Proposition 3 *The profit-maximizing organizational structures satisfies the following properties.*

1. **Law of Demand:** *The share of workers of type i ($E_j(i)$) decreases as their wage increases.*
2. **Incomplete Specialization:** *All hired worker types spend a positive amount of time on each task whenever $\gamma_j > 0$.*

3. *Optimal Jobs:* Jobs take the following logit-like form:

$$b_j(i, k) = \alpha_k \frac{\exp(-\gamma^{-1}w_i + (\rho\gamma)^{-1}\theta_{i,k}))}{\sum_{i'} E_j(i')\exp(-\gamma^{-1}w_{i'} + (\rho\gamma)^{-1}\theta_{i',k}))}$$

I prove this result by appealing to results from the rational inattention literature. I derive the expression for optimal jobs by manipulating the first-order conditions and the constraints. The proof is provided in Appendix Section A.4. Even though at a high level the firms are trading off complexity and quality-adjusted wages, under the surface, they customize jobs for individual workers and tasks. The proposition illustrates that task-assignments depend on skills through $\theta_{i,k}$, wages through w_i , consumer price sensitivity through ρ , the task-mix through α_k , and organization costs through γ_j . This proposition highlights two important feature of the model. First, whenever there are some organizational frictions within a firm, complete specialization will not occur. Every "job" will be a bundle of multiple tasks. Second, because jobs depend on organization costs, where someone works matters for what they do. That is, two identical workers will not perform the same tasks even in the same product and labor market. The tasks included in any job will depend on the type of firm where a worker works.

6 A Structural Model of Internal Organization

Understanding the quantitative relationship between internal organization and the labor and product market requires a structural model that can be taken to the data. This section describes a model which preserves the spirit of the theoretical model while allowing for additional firm and worker heterogeneity. I prove identification of the distribution of organization costs and provide a computationally light nested-fixed point generalized method of moments estimation procedure.

6.1 Econometric Model

I define labor markets and product markets as counties and time periods as quarters. I estimate the model for New York County (Manhattan) 2021 Quarter 2, the last full quarter of data in my sample. I add several types of heterogeneity to the theoretical model introduced in Section 4 to better fit the data. The theoretical results in Section 5 continue to apply to the econometric model.

Consumers. I assume a parametric form for demand. There are a mass M of consumers interested in purchasing at most one of the J final products, where M is set to be the population of Manhattan. Consumer z 's utility for good j is represented by the logit utility function:

$$u_{z,j} = \xi(B_j) - \rho p_j + \epsilon_{z,j}$$

where $\epsilon_{z,j}$ is distributed i.i.d. Type-1 extreme value across consumers and products. The outside option for consumers is assigned index $j = 0$ and its utility is normalized to $u_{z,0} = \epsilon_{z,0}$. As is in McFadden (1973), market demand for good j can be written as:

$$D_j(\xi(B_j) - \rho p_j, p_{-j}, \xi_{-j}) = \frac{\exp(\xi(B_j) - \rho p_j)}{\sum_{j'} \exp(\xi(B_{j'}) - \rho p_{j'})} \quad (4)$$

From this expression, it is clear that this demand system satisfies the quality-price index restriction assumed during theoretical analysis.

Production Function Heterogeneity. I allow the task-mix (α_j) to be firm-specific. Since α will be equal to the distribution of time across tasks, it is fully observable in the data. This allows firms in the same product and labor market to have different organizational frontiers.

Quality Heterogeneity. I specify that in addition to endogenously chosen quality ξ , each firm also has unobserved quality ν which it cannot change, where $\mathbb{E}[\nu_j] = 0$. This represents reputation and other attributes that are not related to labor. This assumption assures that only quality differences correlated with observed organization complexity (I_j) will be attributed to internal organization. Quality is now: $\xi(B_j) + \nu_j$.

Marginal Cost Heterogeneity. Marginal cost may depend on the firm-specific task mix (α_j) to capture the costs of materials relating to specific tasks (like dyes) as well as an idiosyncratic marginal cost shifter (ϕ_j where $\mathbb{E}[\phi_j] = 0$). I measure \bar{a}_j as the average number of hours salon j spends on a customer in a quarter. I specify that organization costs and wages are per hour of labor. This allows each firm to have a different required labor per unit (\bar{a}_j) to capture traditional productivity differences across firms.¹⁵ Marginal cost can

15. This is similar to specifying a production function of the form $\bar{a}_j \min \left\{ \frac{a_1}{\alpha_1}, \dots, \frac{a_k}{\alpha_k}, \dots, \frac{a_K}{\alpha_K} \right\}$.

then be expressed as:

$$MC_j = \bar{a}_j \left[\gamma_j I(B_j) + W(B_j) \right] + \sum_k m_k \alpha_j(k) + \phi_j$$

Worker Labor Supply Heterogeneity. I allow workers with the same skill set to differ in their labor supply. This clarifies the relationship between worker identities (observed in the data) and worker types in the model. Specifically, in addition to their skill set, individual workers also have an inelastic labor supply l_i . I augment the game by specifying that firms first demand an amount of labor of each skill set. Then an unspecified process matches workers to firms. The only assumption I place on this process is that firm's labor demand from the first stage is exactly met. Thus if a firm demands 10 hours of a skill set, it may be met by any combination and number of workers, but no more or less than 10 hours is supplied in total. Following the matching process, firms then select an organization structure \tilde{B} , which is over worker identities (rather than just worker skills).¹⁶

Worker Skill Sets. I assume there is one specialist worker type for each of the five tasks. Tasks are performed with a base skill level β_k when assigned to a non-specialist, and are performed with an additive skill gap θ_k when assigned to a task k specialist. The matrix of skill sets where each row denotes a worker type and each column a task can be written as:

$$\Theta = \begin{bmatrix} \theta_1 + \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 \\ \beta_1 & \theta_2 + \beta_2 & \beta_3 & \beta_4 & \beta_5 \\ \beta_1 & \beta_2 & \theta_3 + \beta_3 & \beta_4 & \beta_5 \\ \beta_1 & \beta_2 & \beta_3 & \theta_4 + \beta_4 & \beta_5 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 & \theta_5 + \beta_5 \end{bmatrix}$$

Sales Tax. The state of New York does not tax hair services. However, New York City (which includes New York County/Manhattan) levies a 4.5 percent tax on beauty services (New York City Department of Finance 2022). Therefore I denote the sales tax τ and assume it is 4.5% initially.

Outside Option. I define the consumer's outside option to be not buying services from a salon. I use Consumer Expenditure Expenditure survey micro-data to compute the share of individuals from a county in a quarter who spend \$0 at salons and take this to be the

16. This means that in principle a firm may employ multiple workers of the same skill set and assign them different distributions of tasks. This allows me to bring the model to the data, where I observe worker identities but not worker types.

share of people who choose the outside option. I do not use weights because counties are smaller than states. For New York County (Manhattan) Q2 this methodology generates an outside share of 40%.

Profit. With these additions, firm profit can be written as:

$$\frac{\exp(\xi(B_j) - \rho(1 + \tau)p_j + \beta\alpha_j + \nu_j)}{\sum_{j'} \exp(\xi(B_{j'}) + -\rho(1 + \tau)p_{j'} + \beta\alpha_{j'} + \nu_{j'})} \left[p_j - \bar{a}_j \left(\gamma_j I(B_j) + W(B_j) + m\alpha \right) - \phi_j \right]$$

where the features added to the theoretical model are written in blue. Fixing an equilibrium, we can divide the parameters of the model into two groups. The first are the firm-specific organization cost coefficients $\{\gamma_j\}_{j=1}^J$. The second are the parameters that define worker skills (10 parameters), wages (5 parameters), material costs (5 parameters), and price sensitivity (1 parameter). I call these market parameters and denote them Ω .

6.2 Equilibrium Existence and Uniqueness with Fixed Wages

In the empirical application of the model, I treat wages as fixed parameters to be estimated. Prior to identification and estimation, I establish that for fixed wages, there almost always exists a unique equilibrium.

Proposition 4 *Suppose wages are fixed parameters. A pure strategy equilibrium always exists, and it is unique except over a set of parameters with measure 0.*

The proof of this result is provided in Appendix Section A.6, and it relies on the equivalence to a rational inattention problem established in Theorem 1. This result means that multiplicity arises only in knife-edge cases. Proposition 4 does not establish equilibrium uniqueness or existence in the full model with wages determined endogenously by labor market clearing. Nevertheless, this result is crucial for the main identification result. It implies that if we have a uniform prior over the the parameters, the probability the true parameters induce multiple equilibria is 0.

Several aspects of the model make Proposition 4 surprising. First, firms each have 26 choice variables and quality and marginal cost are endogenous. Many models where product positioning is endogenous (including the canonical two-stage Hotelling model) suffer from equilibrium existence and uniqueness problems.¹⁷

17. In a 2 firm Hotelling model with product positioning, it is known that a pure strategy equilibrium does not exist for linear transportation costs. When transportation costs are quadratic, there are two equilibria.d'Aspremont, Gabszewicz, and Thisse (1979)

The result is also useful for counterfactual analysis, because it means the model almost always delivers one and only one internal organization structure for each firm. In this sense, the model will almost never suffer from the inverse identification problem and counterfactual analysis is therefore straightforward.

6.3 Identification of Firm-Specific Organization Costs

The organization costs $\{\gamma_j\}_{j=1}^J$ are important parameters, determining product quality, organization complexity and marginal costs for each firm. However, the fact that there is one parameter per firm and that I place no restrictions on their distribution raises concerns for identification and estimation. I alleviate this concern with the following result.

Proposition 5 *Organization costs (γ_j) and organization structures (B_j) are a known function of firm task-mixtures (α_j), complexities (I_j) and market parameters (Ω) for all firms with positive complexity, except for a set of market parameters with measure 0.*

The proof is fully described in Appendix Section A.5. A key hurdle is the fact that I do not observe worker types. Rather, I observe worker identities within firms. Because I allow for a flexible matching process, a given firm may in principle employ multiple workers of the same skill set, and assign these workers different tasks. However, a property of the mutual-information based organization cost assures that if firms do employ multiple workers with the same skill set, they assign these workers the same tasks. This implies that the observed organization complexity based on worker identities is equal to the true organization complexity based on worker skill sets.

The intuition for the identification of γ_j was demonstrated in Figure 8. Suppose I see two firms with the same task-mix (α_j) operating in the same product and labor market. This means they have the same organization frontier. If one firm has a higher complexity, it must be that the slope of their isoprofit line is steeper, which can only be because they have a lower organization cost. Therefore I am able to order the firms by organization cost without knowing the market parameters. Once we know the market parameters, we can find the cardinal values of each firm's organization cost.

This proposition implies that organization costs do not need to be estimated in the statistical sense. For any market parameters, there are unique organization cost parameters which rationalize the observed organization complexities and task-mixtures. This is similar to the way unobserved product quality is a known function of market shares in Berry, Levinsohn, and Pakes (1995). Another similarity is the lack of a closed-form for the known function. Instead I provide a fixed point algorithm as part of my suggested estimation routine.

Beyond the main implication for estimation, Proposition 5 has two additional implications. Statistically, it means that observing the task-mix (a vector of length K) and organizational complexity (a scalar) is enough to estimate the model. It is not necessary to observe the individual assignments of workers to tasks, only an average measure of the complexity of the assignment. This means the model can be estimated in settings where rich assignment data is not available.¹⁸ It also means that if a researcher has assignment data, they can estimate the model using only summary measures from the data and use the full data to conduct a validation exercise. This is precisely what I do in Section 7.2.

Economically, the result means that the task-mix (a vector of length K) and organizational complexity (a scalar) are sufficient statistics for a firm's internal organization structure (B_j , an $N \times K$ matrix). When all firms share the same task-mix, a firm's internal structure can be fully inferred by competitors through complexity alone. On the one hand, this illustrates that logit demand and mutual information-based organization costs are imposing a large amount of structure. On the other hand, to the extent that logit demand and mutual information-based organization costs are reasonable, this framework has the potential to maintain tractability in complicated settings.

6.4 Estimation of Market Parameters

Organization costs are a known function of the data and market parameters. This section derives a set of moments and assumptions under which the market parameters can be estimated via the generalized method of moments. I note that material costs are not separately identified from $\mathbb{E}[\phi_j]$ and the skill base is not separately identified from $\mathbb{E}[\nu_j]$. I therefore estimate $\mathbb{E}[\nu_j] + \beta_{cut}$ and $\mathbb{E}[\phi_j] + m_{cut}$ and call them the demand and cost intercepts. This means all skills parameters and material costs parameters are relative to the haircut/shave task.

To construct moment conditions, I follow a common approach in the literature and use one demand side and one supply side equation. Starting with the supply side, we can write the equilibrium pricing equation as:

$$p_j = \frac{1}{\rho(1+\tau)(1-s_j)} + \bar{a}_j \left[\gamma(\Omega, I_j, \alpha_j) I_j + W(\Omega, I_j, \alpha_j) \right] + m\alpha_j + \phi_j \quad (5)$$

Because the demand system takes a multinomial logit form, market shares can be expressed as:

$$\log(s_j) - \log(s_0) = \xi(\Omega, I_j, \alpha_j) - \rho(1+\tau)p_j + \beta\alpha_j + \nu_j \quad (6)$$

18. For example, privacy concerns may often prevent the disclosure of employee-client assignments.

I interact firm-level covariates with Equations 5 and 6 using covariates that are relevant to the determination of prices and market shares but also independent of ν, ϕ . Firm organizational complexity (I) and task-mix vector (α) fit these requirements, since under my model they change organization costs but are not impacted by ν, ϕ . Additionally, I include their interaction, $\alpha \cdot I$. A discussion of how this variation identifies specific parameters is provided in Section 6.6.

I add one additional wage moment. For each county and quarter, I compute the average wage of hair stylists using the Quarterly Census of Employment and Wages. For each county and quarter, I take the total quarterly wages of establishments with NAICS code 812112 and divide by the number of establishments. This generates the average total wage bill, which corresponds to $W(\Omega, I_j, \alpha_j)$ multiplied by the number of customers. Assuming some measurement error in wages denoted e_j , we have that:

$$W_j = Ms_j a_j W(\Omega, I_j, \alpha_j) + e_j$$

The moment conditions can then be written as:

$$\mathbb{E} \left[\begin{pmatrix} \phi_j(\Omega, I_j, \alpha_j, p_j, s_j) \\ \nu_j(\Omega, I_j, \alpha_j, p_j, s_j) \end{pmatrix} \begin{pmatrix} \alpha_j & \alpha_j I_j \end{pmatrix} \right] = 0 \quad \mathbb{E}[e_j(\Omega, I_j, \alpha_j)] = 0$$

For a single market and quarter, this yields a total of 21 moments to estimate 21 market parameters. The model is globally identified if I assume that Ω is the unique vector of parameter values which satisfies the moment conditions. With this assumption, I estimate the model using the generalized method of moments (GMM). Denote the sample analogue of the moments as $G(\Omega)$. Then to estimate Ω , I solve:

$$\arg \min_{\hat{\Omega}} G(\hat{\Omega})' W G(\hat{\Omega}) \tag{7}$$

where W is a weighting matrix. Note that to evaluate this GMM objective, I must recover the vector of organization costs implied by the data and each guess of the market parameters. This requires solving each firm's internal organization problem many times per evaluation of the GMM objective. The next section provides a result which greatly reduces the computational resources needed to do this.

For my hair salon application the weighting matrix W is a diagonal matrix, where each diagonal element is the sample variance of the independent variable involved in the moment. This effectively standardizes the moments in the objective function, preventing random variables with large nominal values (like average hours per unit) from dominat-

ing during estimation. I constrain wages to be between \$15 (the minimum wage) and \$200 per hour. I require that the algorithm only search over parameters values yielding a positive demand share for each type of labor.

6.5 A Computationally Light Estimation Procedure

Because there is no closed-form for the mapping from organization complexities and market parameters to organization costs, it is necessary to numerically solve each firm's internal organization problem many times, until the model-produced complexities match the complexities in the data. This is computationally intensive when there are many tasks and many firms, because each firm's problem is a high-dimensional non-linear minimization problem. This section provides a nested-fixed point algorithm which efficiently solves the firm's problem and is proven to globally converge.

To derive the algorithm, I utilize the equivalence to a rate-distortion problem proved in Theorem 1.

Lemma 1 *Given market parameter values, the Blahut-Arimoto algorithm with Lagrangian multiplier $(\bar{a}_j \gamma_j)^{-1}$ delivers an organizational structure B_j which maximizes firm profit.*

The lemma follows directly from Theorem 1 and well-known results in information theory.¹⁹ The Lagrangian multiplier involves \bar{a}_j because marginal costs are $\bar{a}_j(\sum_i w_i E_i + \gamma_j I(B_j)) - \rho^{-1}\xi(B_j)$. The Blahut-Arimoto algorithm is a fixed-point algorithm which iterates on two optimality conditions (Blahut 1972) and can be described as follows (suppressing firm subscripts):

0. Guess some labor demand E^0 . Create matrix V :

$$V_{i,k} = \exp[(\bar{a}\gamma)^{-1}(\rho^{-1}\theta_{i,k} - w_i)]$$

1. Compute B^t as:

$$B(i, k)^t = \alpha_k \frac{V_{i,k} E^t(k)}{\sum_i E^t(i) V_{i,k}}$$

2. Compute E^{t+1} as:

$$E^{t+1}(i) = \sum_k B(i, k)^t$$

3. If converged exit, else return to Step 1 and advance t .

19. See Tishby, Pereira, and Bialek 2000 or Blahut 1972.

The Blahut-Arimoto algorithm is proven to converge to a global optimum from any feasible starting point (Tishby, Pereira, and Bialek 2000). It avoids the need to repeatedly use nonlinear optimization routines, and because it is a fixed point method, acceleration routines like Du and Varadhan (2020) can be used to increase the speed of convergence. The algorithm also delivers the entire internal organization of the firm, B_j . Practically, a researcher can use the algorithm to search for the γ_j which makes the model generated complexities match the complexities observed in the data. Because complexity is monotone in γ_j , a researcher can use the bisection method for this task. Thus, the full estimation procedure is as follows:

1. Given a guess of the market parameters $\hat{\Omega}$, use the Blahut-Arimoto algorithm to find the organization costs $\gamma_j(\hat{\Omega})$ which rationalize each firm's observed organizational complexity I_j .
2. Using $B_j(\hat{\Omega}), \gamma_j(\hat{\Omega})$ compute firm-specific wage bills and endogenous quality.
3. Evaluate the GMM objective given by Equation 7.²⁰ If the objective is minimized stop, otherwise return to step 1 with a new market parameter guess, $\hat{\Omega}$.

This estimation algorithm is similar in spirit to the demand estimation procedures that have become popular in industrial organization since Berry, Levinsohn, and Pakes (1995). Just as those procedures invert market shares using a contraction mapping to derive unobserved product qualities, my procedure inverts organization complexities using a contraction mapping to obtain unobserved organization costs.

6.6 Identifying Variation

Proposition 5 establishes that given fixed values for the market parameters, organization costs (γ_j) are identified by variation in complexity and the task-mix across firms. The purpose of this section is to discuss the sources of identifying variation for the market parameters.

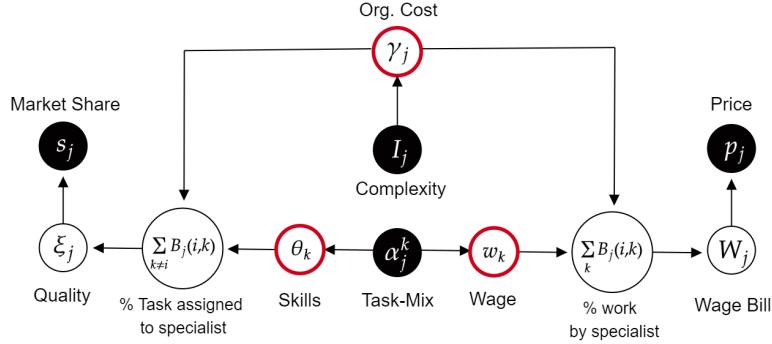
Consumer price sensitivity is identified by the pass-through of average wages to consumers. If most of wages are passed through to consumers via higher prices, this indicates consumers are not very price sensitive and ρ is low. Once price sensitivity (ρ) is known, the marginal cost of each firm can be obtained by subtracting the markup from prices. Similarly, unobserved quality can be obtained from observed prices and market shares.

²⁰ I use the R package "gmm" described in Chausse (2021) to perform estimation.

For this reason, I discuss identification of the other parameters as if quality and cost are observed.

The other market parameters are identified using variation across firms in the task-mix and complexity. The relationship between market parameters and the observed data is demonstrated in Figure 9. Black objects are observed, white objects are not.

Figure 9: Identifying Variation for Skills and Wages



Note: The diagram visualizes the way market parameters (skill sets and wages) are identified from observable data. Black objects are observed, while white objects are parameters to be estimated or unobserved variables like organization costs. Organization costs mediate the relationship between particular skill and wage parameters and costs and qualities. Since organization costs are known functions of complexity, we can use variation in the interaction of complexity and the task-mix to identify the different parameters.

Recall that a highly complex firm generally has a highly specialized workforce. When I observe a firm that is complex and is intense in a task k , this implies two things. First, the firm likely uses a large share of task k specialists. Therefore the observed price (and thus cost) of that firm largely reflects the wage of task k specialists (w_k). Second, the firm assigns a large amount of task k to those specialists. Therefore the observed market share (and thus quality) largely reflect the skill gap of task k specialists (θ_k). In this way, variation in the interaction of complexity and the task-mixtures identifies wages and skill gaps.

The base skill parameters (β) and the material costs (m) are identified by variation solely in the task-mixtures across firms. When I observe a firm that is intense in task k , I know that its cost is largely determined by m_k and its quality is largely determined by β_k . This is why I interact firm price and market share moments with α_j and $\alpha_j \cdot I_j$ in the GMM procedure.

7 Empirical Results

This section summarizes parameter estimates and uses the model to analyze the sources of variation in task content for hair stylists in Manhattan.

7.1 Parameter Estimates

Estimated wages, skill parameters and material costs are organized by task in Table 5. Price sensitivity and the intercepts are provided in Table B4. Standard errors are computed as the sample standard deviation of the parameter estimates from 500 bootstrap replications, with the procedure described in Appendix Section B.5.

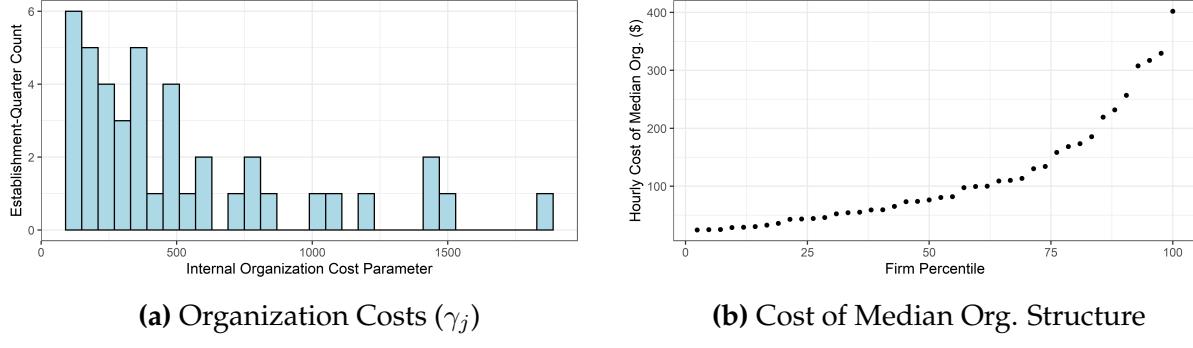
The coefficients associated with the color and haircut tasks are the most precisely estimated. This is not surprising, as these tasks are the most common and as a consequence their associated parameters will have the most statistical power. Across all tasks, the skill gap parameters are positive, indicating that assigning the task to the associated specialist increases. The skill gap parameters can be interpreted as the dollar value to a consumer of increasing task-specialization in that task by 4 percentage points. Wages are in 2021 dollars per hour. Material costs are dollars per service.

Wages for color specialists are more than double that for haircut specialists, and the skill gap for color specialists is nearly double the skill gap for haircuts. This is in line with folk wisdom in the industry that it is hard to master coloring. The material costs are largest for the color task, in line with our intuition that color uses the most expensive non-labor supplies, like dye.

The organization costs (γ_j) for each firm are provided in Figure 10a. To provide a magnitude for these estimates, I also plot the cost of implementing the median complexity organization structure across all firms in Figure 10b. There are large differences in organization costs. Firms in the bottom quartile of organization costs can implement the structure for less than \$50 an hour. It would cost firms in the top quartile over \$150 an hour. The estimates imply that variation in organization costs explains 40% of total variation in prices across firms.²¹

21. This is obtained as the r-squared of regressing price on $\bar{a}_j \gamma_j I(B_j)$.

Figure 10: Estimated Organization Costs



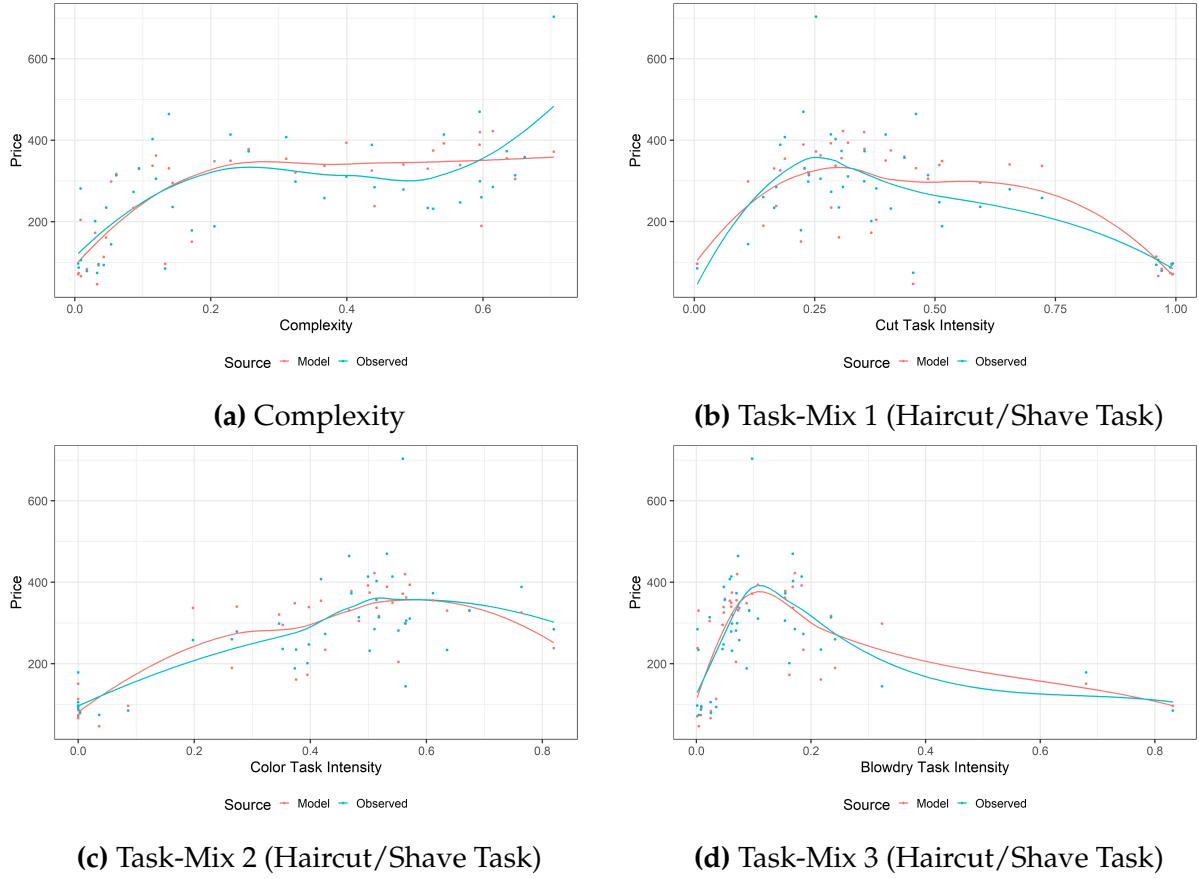
Note: Panel A displays the estimated γ_j parameters for Manhattan. These can be interpreted as a measure of organization frictions at each firm, with lower values indicating less frictions. Panel B displays the magnitude of these differences, by plotting the cost to each firm of implementing the median complexity organization structure in dollars.

For each firm, I also can recover the unobserved, equilibrium organization structure B_j . This is the amount of time allocated to each task and each worker type. I present two examples in Table 6, one for a very complex, task-specialized firm and another for a generalist, low-complexity firm. The complex firm hires 4 worker types and assigns a large share of tasks to the associated specialist. The low complexity firm hires only 2 worker types and spreads all tasks across these two types.

7.2 Model Fit and Validation

I assess model fit by comparing the predicted and actual relationship between prices and various organizational variables in Figure 11. The model captures the shape of the relationships.

Figure 11: Model Fit



Note: Each panel plots the model and observed relationship between price and different firm variables. Dots represent individual firms, while lines are Loess smoothed fitted curves.

Although the model delivers an entire predicted organization structure (B_j) for each firm, the estimation procedure uses only some of this information. I use the additional predicted information to validate the model. In particular, I compare the model generated distribution of task content to the observed distribution of task-content. Recall that the jobs within firm j are denoted b_j , and it is a matrix where element i, k denotes the time worker type i spends on task k . Using the model, we can compute b_j for each firm among worker types that it hires. In the data, we can compute \tilde{b}_j , which are jobs within firm j where element i, k denotes the time worker i spends on task k . The main difference between \tilde{b}_j and b_j is that the first is with respect to worker identity and the second is with respect to worker types. To make them comparable, we can weight each job by the total amount of labor it represents. Once we combine all J firms, this yields an unobserved and model based distribution of job task content for each of the 5 tasks, where jobs are weighted by their effective labor.

Tables 7a, 7b, and 7c compare the model and observed mean, median, variance, 25th percentile and 75th percentile of job task-content. The estimated results exactly match the mean and between-firm variance of job task-content because the model imposes that organization structures must be consistent with the task-mix α_j , which is exactly the average amount of time spent on each task at each firm. The estimates are also reasonable approximations of the total variances of task content and the 75th percentile of the job task-content distribution. But the model does a poor job of fitting the median and the 25th percentile.

The statistics related to the color/highlight/wash task are the hardest for the model to replicate. The reason for this is that the empirical distribution for this task is triple peaked. The empirical and model generated task-content distributions are presented in full in Appendix Table B5.²²

7.3 The Determinants of Task-Specialization

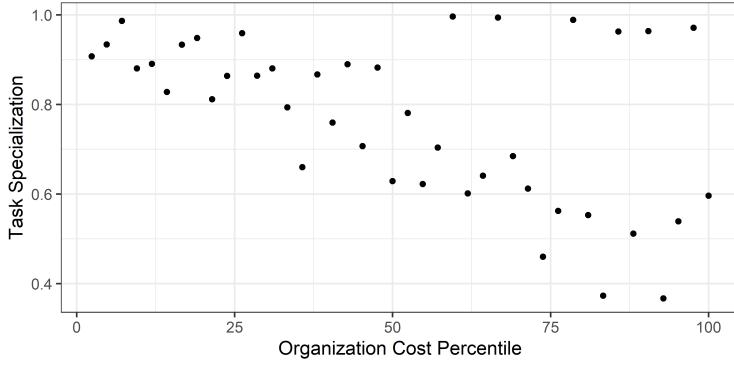
The estimated model allows me to understand how worker skills and firm internal organization determine the task-specialization of jobs. Measuring task-specialization as the amount of time a worker spends on their specialty task, I find that 45% of the variation in task-specialization is attributable to firms, while 55% is attributable to worker skills.²³

I calculated the variation in task specialization due to the firm component by computing the fraction of total firm labor spent on any worker's specialty. This measure is plotted for each firm against its organization cost percentile in Figure 12.

22. Comparing the entire distribution of content is another, much more rigorous validation test.

23. In the decomposition, I separate task-specialization variance into a within worker type and across worker type component. Since the only difference across worker types is firms, I can call the across component the firm component of variance.

Figure 12: Task-Specialization and Firm Organization Costs

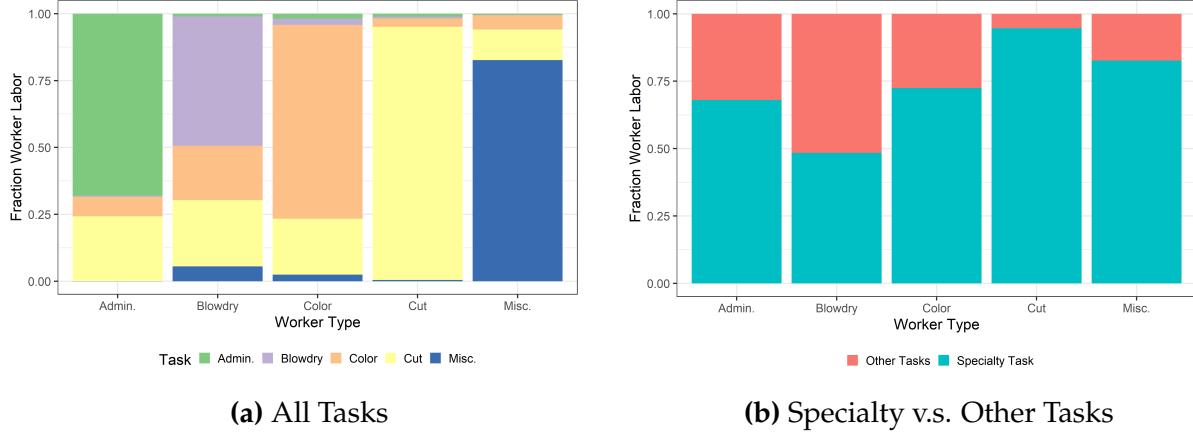


Note: The figure displays the model predicted relationship between task-specialization (fraction of time spent by workers on their focal task) and firm organization cost.

The negative relationship shown in Figure 12 is consistent with the theoretical analysis earlier in the paper. Firms with higher organization costs exhibit less task-specialization. The magnitude of this effect is large: firms in the bottom quartile of organization costs (efficient firms) assign on average 90% of tasks to the associated specialist, while firms in the top quartile (inefficient firms) assign on average 67%.

Finally, variation due to worker skills is shown in Figure 13. Haircut/shave specialists work the most specialized jobs, spending 95% of their time on their specialty task. Blow dry/extension/style specialists work the most generalized jobs, spending only 48% of their time on their specialty task.

Figure 13: Equilibrium Division of Tasks Across Workers



Note: Panel A provides the breakdown of time spent on each task by worker type. Panel B provides the breakdown of time spent on specialty task vs. all other tasks. Haircut/shave specialists are the most task-specialized, while blow dry specialists are the least.

8 Counterfactuals

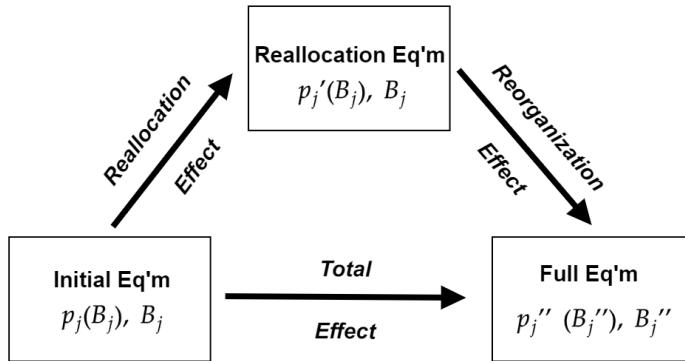
This section uses the estimated model to study two counterfactual policy changes, one impacting the product market and one impacting the labor market. Internal organization qualitatively alters responses to these well-studied policies. The procedure used to solve for equilibria and conduct the analyses is described in Appendix Section B.7. The model allows me to distinguish between two effects of any policy: a reallocation effect and a reorganization effect.

To do this, I first define the reallocation equilibrium. The reallocation equilibrium is the outcome when firms are allowed to adjust prices (p_j) but organization structures (B_j) are fixed at the initial equilibrium choices. Because prices control quantities, this equilibrium allows firms to adjust the total labor they hire, but not the division of labor within the firm. The reallocation effect of any policy change is the change in outcomes between the reallocation equilibrium and the initial equilibrium. It captures changes due to reallocation of labor across firms. Because firms differ in their organization costs and task mixtures, reallocation will change the task content of jobs, relative wages, and other outcomes.

The reorganization effect of any policy change is the change in outcomes between the full equilibrium and the reallocation equilibrium. It captures changes due to reorganization of labor within firms. I define the total effect of any policy change as the change in outcomes from the initial to the full equilibrium. These relationships are summarized in

Figure 14.

Figure 14: Reallocation, Reorganization and Total Effect



Note: The diagram shows how the total effect of a counterfactual policy can be decomposed into two parts. The first part fixes the organization structure but allows firms to update prices. This is the reallocation effect, because labor flows across firms but does not change within firms. The second part allows organization structures to adjust. This is the reorganization effect, because firms are now responding to wages and prices by shifting their internal structure.

In the reallocation equilibrium firms are acting as if they employ a composite worker. The worker's skills and wage are determined exogenously by the initial internal organization of the firm, B_j . The firm has the option of adjusting the total amount of labor they demand from this composite worker, but cannot adjust the skills and wages of this composite worker. In the full equilibrium, the firm is free to fully adjust its internal structure.

8.1 Minimum Wage Increase

I study an increase in the minimum wage in New York City from \$15 (the minimum in 2021) to \$20. An increase to \$20 is similar in magnitude to the increases that would occur if the minimum wage were pegged to inflation, as proposed in several pending pieces of legislation.²⁴

To implement the counterfactual, I require that all equilibrium wages be at least \$20, and that markets clear for all worker types for which the wage is not binding. I allow there to be excess labor supply (unemployment) for those worker types facing a binding minimum wage. I believe my model is well-suited for studying large increases in the minimum wage, because I allow salons to reorganize as well as raise prices. There are

24. Senate Bill S3062C and Assembly Bill A7503B. Analysis by EPI: *Proposed New York state minimum wage increases would lift wages for more than 2 million workers through 2026: Minimum wages would range by region from \$16.35 to \$21.25 per hour by 2026 — Economic Policy Institute*

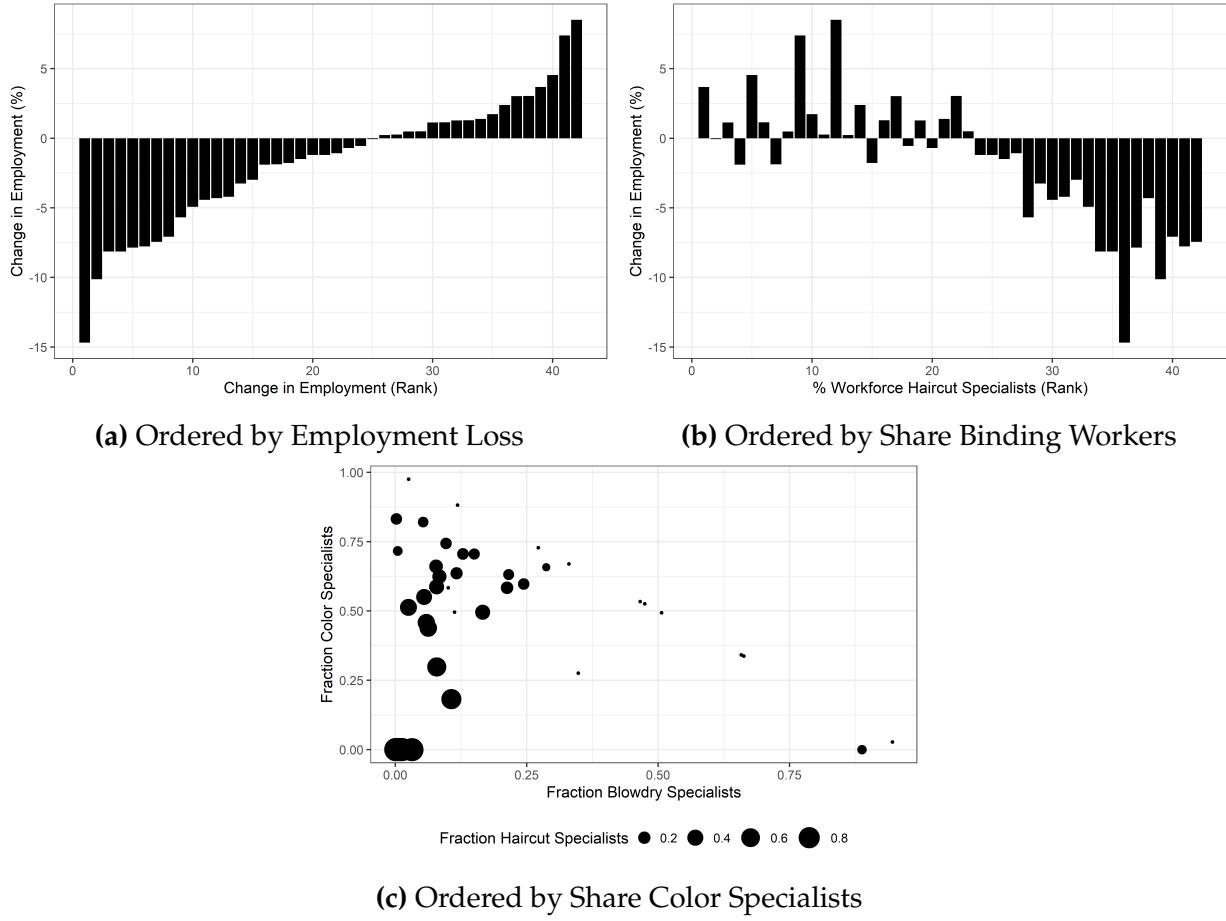
technical details that must be addressed when implementing the minimum wage counterfactuals, including the possibility of multiple equilibria and numeraire goods. I discuss these in Appendix Section B.7.2.

The minimum wage binds for the haircut/shave specialist only. The wages and employment levels across worker types are given in Table B6 (including values for the reallocation equilibrium). I first discuss the reallocation and reorganization effects of this policy change. I then analyze the overall impact of the new policy, using the reallocation and reorganization effects to understand the underlying forces.

8.1.1 The Reallocation Effect

The impact of the minimum wage on individual salons depends partly on their initial internal structure. As a result, the minimum wage changes the competitive positions of salons and reallocates labor. By comparing the initial and reallocation equilibrium, I can hold each firm's internal structure fixed but allow firms to adjust prices. This captures the extensive margin adjustment of salons but prevents internal reorganization. Figure 15 presents the reallocation effect of the minimum wage in a series of three panels.

Figure 15: The Minimum Wage Reallocation Effect



Note: In panels A and B, each bar is a firm, and employment changes are comparing the reallocation equilibrium to the initial equilibrium, holding fixed internal organization. Panel A orders firms by employment losses. Panel B reorders firms by the fraction of the workforce that are haircut specialists in the initial equilibrium. Panel C plots firms by their initial share of color specialists vs. blow dry, admin. and misc. specialists. The minimum wage disadvantages firms with internal structures that rely on minimum wage workers, reducing labor demand for coworkers and generating negative wage spillovers.

The minimum wage has a disproportionately negative impact on salons whose internal organization relies heavily on minimum wage workers. These salons see the largest increases in marginal costs and thus the largest decreases in output and thus employment. Because the minimum wage increases some salon's costs more than others, it changes the competitive position of firms in the product market. As can be seen in the figure, this effect is large enough that some salons even see employment increases.

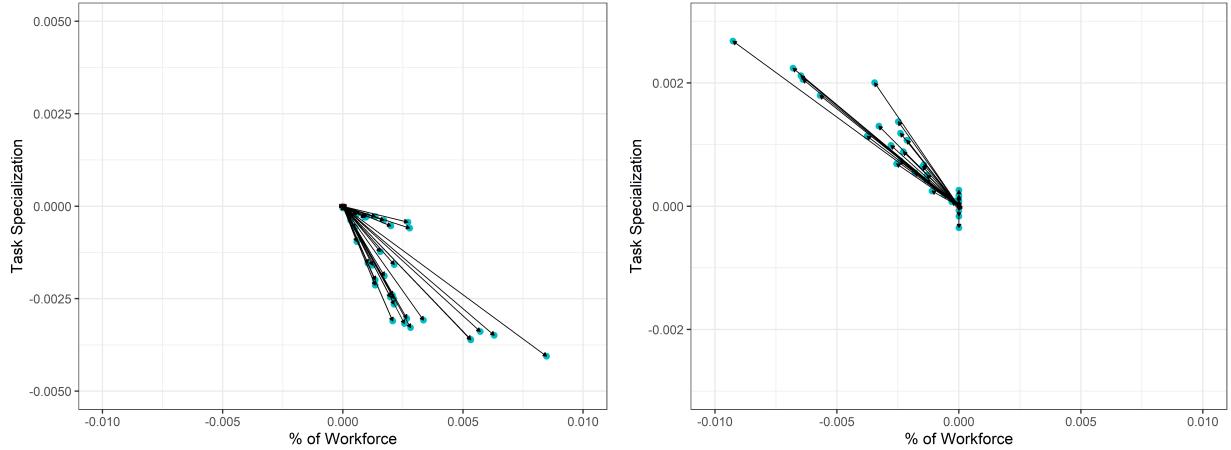
Workers that are often employed alongside minimum wage workers initially see negative wage spillovers because the minimum wage erodes the competitive position of these firms. In the opposite way, workers employed at salons with few minimum wage workers

initially see positive wage spillovers, because the minimum wage improves the competitive position of these firms. The effects of the minimum wage is contagious, and are spread across workers based on firm internal organization. In equilibrium, the minimum wage reallocates labor towards high-complexity, task-specialized salons and away from low-complexity, task-generalized salons, raising industry task-specialization and average worker productivity.

8.1.2 The Reorganization Effect

By comparing the full equilibrium and the equilibrium where firms can only adjust prices, I can study the effect of internal reorganization. In Figure 16, I plot vectors each representing a firm, where the length and direction of the vector represents the change in the firm's relative labor demand for that worker type and the change in task-specialization of that worker type at the firm.

Figure 16: Reorganization Effect Under a Minimum Wage Increase



(a) Non-Binding (Color/Highlight Specialists) **(b) Binding (Haircut/Shave Specialists)**

Note: Each arrow in both panels is a firm, with the blue dot at the end of the arrow representing the firm after the reorganization effect (the final position). Panel A displays the change in task-specialization and relative employment for color/highlight specialists, a type for which the minimum wage is not binding. Relative employment increases while task-specialization decreases. Panel B displays the change in task-specialization and relative employment for haircut/shave specialists, a type for which the minimum wage is binding. Relative employment decreases while task-specialization increases. This illustrates how firms are asking surviving workers to pick up the slack.

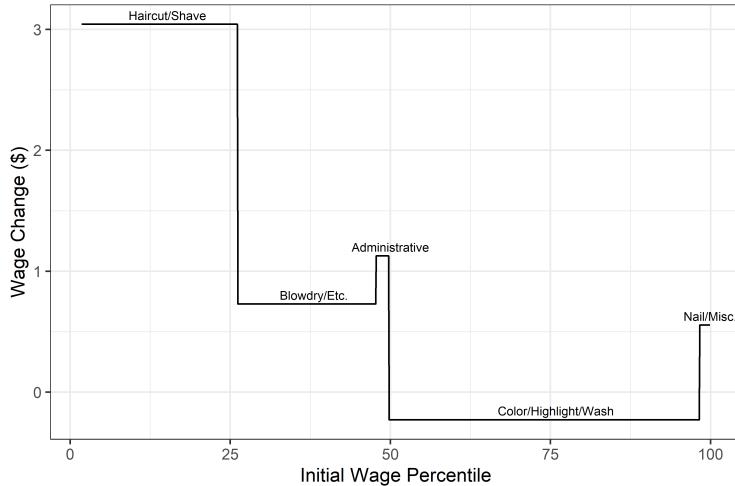
The figure illustrates a general pattern. Salons reduce relative employment and increase task specialization of minimum wage workers. Salons reduce task specialization and increase relative employment for workers above the minimum wage. I call this a

“pick-up-the-slack” effect. Intuitively, the minimum wage reduces the comparative advantage of workers for which the minimum wage binds in all tasks relative to other (non-binding) workers. Firms respond by laying off minimum wage workers and shifting tasks performed by them onto relatively less expensive workers, non-binding workers. Only minimum wage workers that are sufficiently productive survive, which are those who are task-specialized. This implies that the minimum wage increases the productivity of binding workers.

8.1.3 Total Impact

Although the minimum wage is binding for only one worker type, all workers see wage changes. Table 8a shows that there are both positive and negative wage spillovers. The largest positive spillover is for administrative specialists, who see a wage increase of 4.2% (+\$1.13). Color/highlight/wash specialists see a small wage decrease of 0.7% (-\$0.23). What is notable about these spillovers is that they are non-monotonic in initial wage. To see this, I plot the wage change experienced by different workers ordered by initial wage in Figure 17. Non-monotone spillovers occur because substitution patterns in the model are determined endogenously based on the distribution of firm organization costs and task-mixtures. Appendix Section B.9 discusses this at greater length.

Figure 17: Minimum Wage Spillovers Across the Initial Wage Distribution



Note: This figure plots the wage change experienced by different workers ordered by the initial wage of the worker. Haircut/shave specialists are the only binding type, so their wage increase is due to the direct effect of the minimum wage. All other wage changes are spillovers. While the majority of workers see wage increases some see decreases. Spillovers are not monotone in initial wage.

In Table 9 I decompose wage spillovers into those arising from the reallocation and

the reorganization effect. As discussed in the prior two subsections, spillovers for each worker type are a combination of forces, with the reorganization and reallocation effects sometimes moving in opposite directions. For example, color specialists see negative wage spillovers because they are employed alongside minimum wage workers and the minimum wage increase disadvantages the salon where they work. But they also see positive wage spillovers because firms shift tasks from minimum workers to them during reorganization. The total wage spillover for color specialists is negative, as the reallocation effect is about double the reorganization effect. For binding workers (haircut/shave specialists) the two effects work in the same direction, increasing unemployment. In this sense, internal reorganization amplifies unemployment losses.

Table 9b shows that reorganization wage spillovers follow a pattern. Workers that see an increase in task-specialization see a wage decrease (or unemployment increase) while workers that see a decrease in task-specialization see a wage increase. Because task-specialization determines worker productivity, this implies that internal reorganization causes absolute productivity and wages to move in opposite directions.

Table 10 summarizes the welfare impacts of the policy. Wage gains for employed workers amount to \$1.6 million, and they are greater than salon losses (-\$714,413) and unemployed wage losses (\$600,240) combined. A little less than a quarter of the wage gains (\$389,847) come from positive spillovers. Welfare losses are concentrated among consumers, who see welfare reductions due to an average 1.7% price increase and an average 0.5% quality decrease. The minimum wage has heterogeneous impacts across firms, with most firms seeing employment losses but some firms seeing employment gains. This is discussed in depth in the reallocation effect subsection.

8.2 Sales Taxes

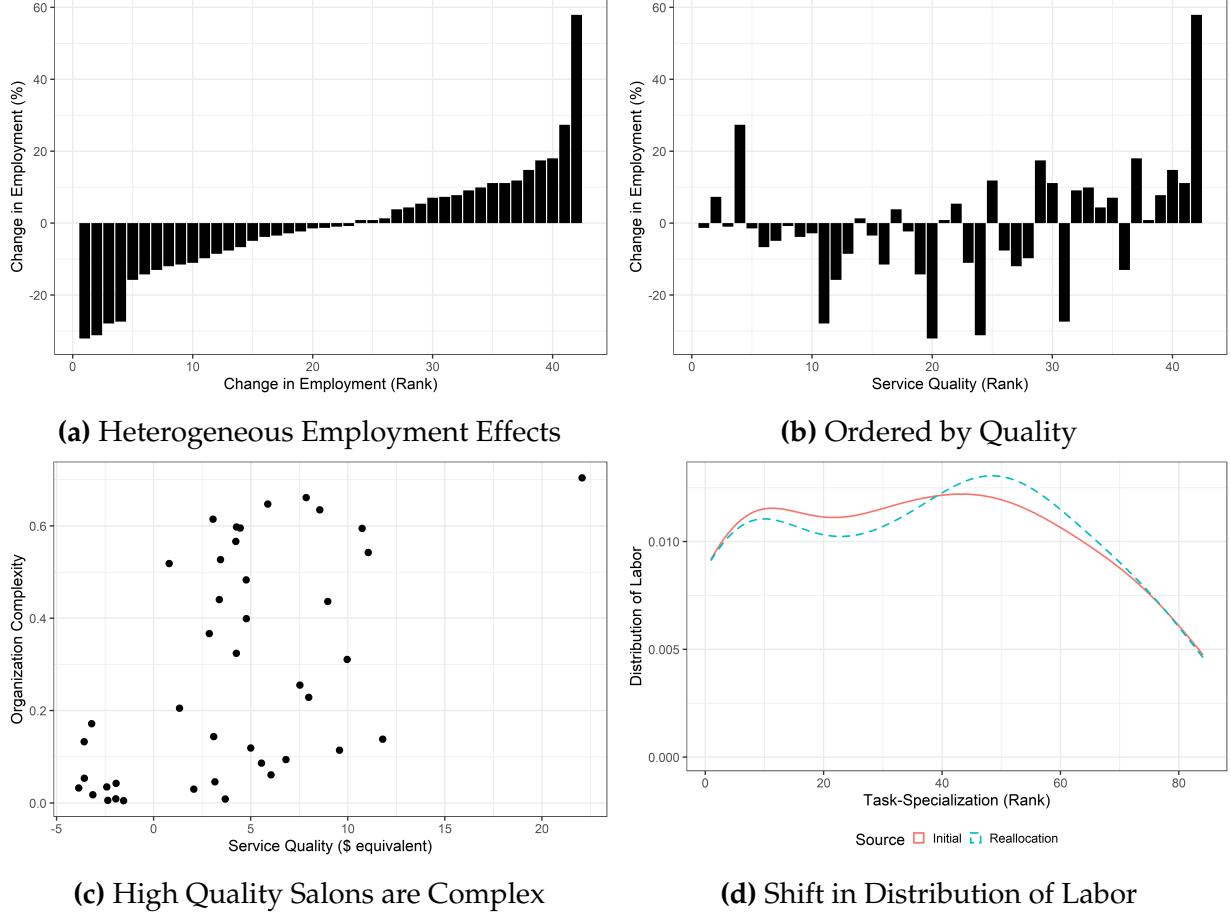
New York City is unique in that it levies a 4.5% sales tax on certain services, including those performed at hair salons. This section studies the effect of eliminating this sales tax. Formally, I estimate a new equilibrium with $\tau^{NEW} = 0$. The wages in this new equilibrium are provided in Table B7. I first discuss the reallocation and reorganization effects of the policy. I then analyze the overall impact of the policy, using the reallocation and reorganization effects to understand mechanisms.

8.2.1 Reallocation Effect

Eliminating the sales tax confers a competitive advantage on firms producing high quality services in the initial equilibrium. Since salons with low organization costs tend to pro-

duce high quality services, this reallocates labor towards organizationally efficient firms, as seen in Figure 18.

Figure 18: Sales Tax Reallocation Effect



Note: Each bar is a salon. The sales tax elimination decreases employment at some salons and increases it at others (Panel A). Salons with high quality service see an improvement in their competitive position (Panel B). These salons have a complex, task-specialized internal structure (Panel C). As a result, labor is reallocated to task-specialized firms and workers become more specialized (Panel D).

Organizationally efficient salons produce high quality services using a more task-specialized internal structure. Thus the reallocation effect increases market-wide task specialization because more workers are working at task-specialized firms.

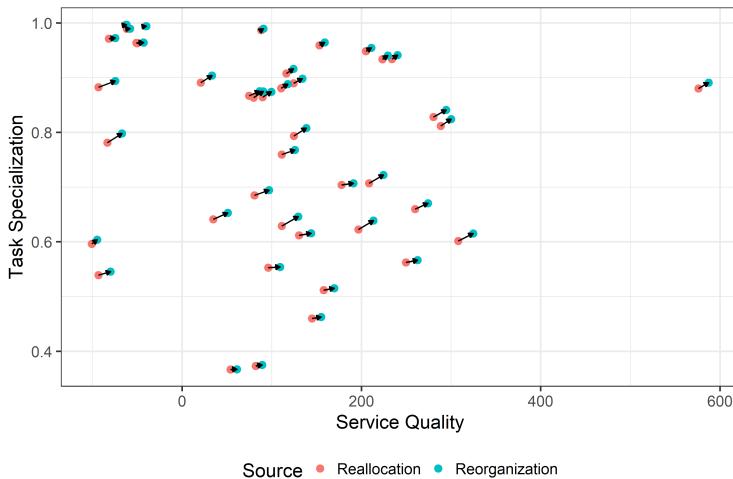
8.2.2 Reorganization Effect

Eliminating the sales tax makes producing higher quality product more attractive. In order to produce higher quality products, firms choose internal organizations which are on average 5.5% more complex, increasing average labor market task specialization by

0.9%. In terms of the three-way trade-off introduced in Figure 7, it has the same effect as reducing consumer price sensitivity (ρ). Average firm service quality rises by 10%. This is consistent with the quality-complexity-wage three-way trade-off discussion in the theoretical section.

Figure 19 illustrates that these market-wide patterns also happen at the firm-level. However, the extent to which salons increase quality and increase task specialization depends on the firm's internal organization costs and its particular task-mix. Thus, the slopes and lengths of the arrows in Figure 19 differ. Changing sales tax, a product market policy, influences what workers do and what workers are paid in the labor market.

Figure 19: Reorganization Effect Under a Sales Tax



Note: Each pair of dots connected by an arrow represents a firm, with red representing the firm before the sales tax and blue representing the firm after the sales tax. The direction of the arrows indicates that most salons increase quality by raising task specialization internally. The magnitude of this change (given by the length and angle of the arrow) depends on the firm's particular organization costs and task-mixture.

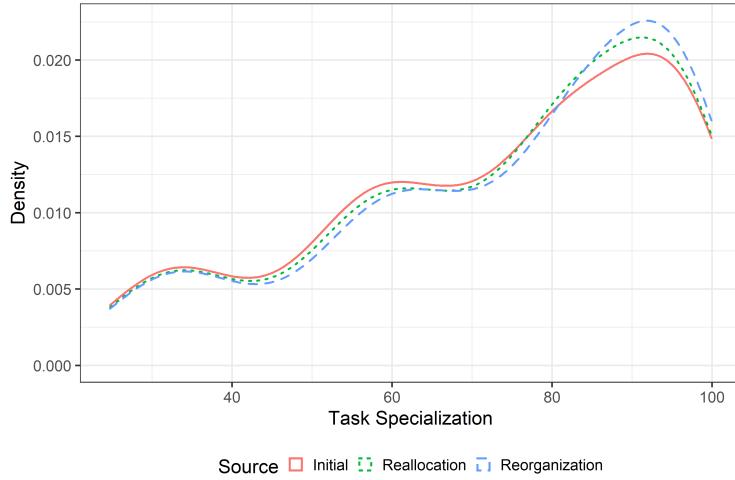
8.2.3 Total Impact

Table 12a summarizes the effect of the policy on wages and task-specialization. All worker types see wage increases and task-specialization increases. Wage increases are not proportional to task-specialization increases: even though blow dry specialists see the largest increase in specialization, they see the lowest increase in wages. This is because the size of wage increases are partly driven by how the policy impacts the competitive position of firms.

Figure 20 shows that eliminating the sales tax generates reallocation and reorganization effects, each of which increases task-specialization by about the same amount. Labor

shifts towards organizationally efficient firms who were already producing high quality products (the reallocation effect). Most salons also increase quality and task specialization (the reorganization effect). Both effects make workers more productive because they are spending more of their time on their specialty task.

Figure 20: The Task Specialization of Jobs Before and After



Note: The figure plots the distribution of labor across jobs of different task-specialization (amount of time spent on specialty task). Eliminating the sales tax increases the amount of time workers spend on their specialty task, with both the reorganization and reallocation effects contributing equally to this total change.

The welfare effects of the policy are summarized in Table 12b. Overall, eliminating the sales tax leads to a small welfare increase of 0.19%. However, the effects are quite different for different actors in the model. Firms respond to the sales tax elimination by increasing quality by 10%. Firms capture the surplus from improved quality and reduced taxes from consumers by raising prices by 8.7%. Firm profit increases by a modest 0.58% because workers capture most of the surplus from firms through higher wages, which rise by a dollar amount that is comparable to the total lost tax revenue. This is consistent with workers capturing almost all of the productivity improvements from increased task-specialization.

Eliminating the sales tax reduces consumer welfare. Why does this occur? In the model, salons can only control two aspects of their products: prices and vertical quality. In the reallocation equilibrium I hold fixed quality and only allow price adjustment. I see that prices rise by 4.7%, as salons both pass on higher wage costs to consumers and increase markups. This reduces consumer welfare by 0.18%. The remaining 0.57% of lost consumer welfare is due to quality over-provision. When the sales tax is eliminated,

salons reorganize to increase quality. Reorganization increases organization costs and increases wages, which salons pass on to consumers via higher prices. Consumers would prefer a cheaper, lower quality product. Quality over-provision in imperfectly competitive markets is not common but can occur.²⁵

9 Discussion

This paper provides a model which, when estimated, allows the researcher to study internal organization in equilibrium. The theoretical section highlights that common forces govern internal organization across firms. It also provides tractable ways to think about the complex choice of organization. The counterfactual exercises emphasize that while common forces govern internal organization, and we can think about these forces tractably within the model, the equilibrium effects of policies are still quite rich. Specifically, internal organization opens new mechanisms for policies to change market outcomes like wages, prices and task-specialization.

In the following subsections I discuss the implications of my results for workers, and ways in which the model can be applied to other contexts.

9.1 Productivity and the Minimum Wage

The minimum wage literature has put forward the idea that minimum wages may increase worker productivity. A main rational for this is efficiency wage theory, which holds that in response to better pay, workers may exert more effort. My model provides an alternative, firm-driven reason for the same phenomena.

The model in this paper does not feature efficiency wages and labor markets are competitive. Nonetheless, the minimum wage increases the productivity of low-wage workers. This improvement in productivity comes from the reorganization effect. Increasing the minimum wage induces firms to reduce the labor they demand from minimum wage workers. They do this by first shifting the tasks minimum wage workers are least good at to other workers. This increases specialization among minimum wage workers that are employed, and thus raises their productivity.

25. Crawford, Shcherbakov, and Shum (1997) present a case where it occurs in cable television markets.

9.2 Implications for Workers

An area for future work is the welfare impact of task-specialization on workers. This paper shows that product and labor market policies change the task composition of jobs in an industry. Minimum wage increases raise specialization for some workers and lower it for others, while sales tax decreases lower specialization for all workers. What are the welfare implications of these changes for workers?

This is ultimately an empirical question. On the one hand, greater task specialization could deepen worker experience in certain tasks, improving the production possibilities when workers bring their skills together. Greater task specialization also means workers are less exchangeable with other workers at the same firm. On the other hand, greater task specialization may limit task exploration. It may also make jobs more skill-specific, making it more difficult for a worker to find a new job and limiting a worker's outside options. Workers may also intrinsically value generalized jobs more than specialized jobs.

The relative importance of these forces depend crucially on the size of job search frictions, the nature of task-based human capital accumulation, and the amenity value of specialization. Thus, understanding the effect on worker welfare requires extending the model and better data on wages and employee outcomes.

9.3 Model Generality

Although this paper applies my model of internal organization to hair salons, the model can be used to study internal organization in other settings. In many cases, like restaurants and hotels, the model can be applied as is. In other settings, like manufacturing, the model can be adjusted while preserving the core features. The flexibility of the model make it ideal for studying questions like the adoption of information and communication technology, immigration and robotics.

Appendix Section A.12.5 shows how to adjust the model to accommodate quantity-based (rather than quality-based) productivity, continuous task spaces, labor market power and more sophisticated demand systems. I discuss the complications with some of these extensions, and some potential ways to get around these complications. In addition to the usefulness of the model, the extensions highlight that the central insights of the model are robust.

When estimating the model for Manhattan hair salons, I restrict the worker type space to 5 specialists (horizontal types). This restriction is not theoretical: the results in the first part of the paper hold under any worker type space, including a space where one worker has higher skills at all tasks than another (vertical types). Rather it is empirical: I restrict

the type space because I do not have data on wages or worker demographics. If I had such data, the model could be estimated with a richer type space.

10 Conclusion

This paper studies how internal organization decisions within firms interact with markets outside firms. I develop a structural model, grounded in a set of stylized facts, which allows firms to differ in their internal organization and to change their internal organization in response to market conditions. Workers have multidimensional skills and different wages. Internal organization matters because the match between workers and tasks determines wage costs and product qualities. Firms in the same market choose different internal organizations because they vary both in their ability to internally organize and in their task-based production functions.

The model allows me to look inside the black box of the firm, and understand how organizational decisions are made in equilibrium. Firms face a trade-off where they want to design the simplest organization that achieves the lowest wage cost and the highest product quality. In equilibrium, the aggregate assignment of workers to tasks is determined both by worker skills and the internal organization decisions of many competing firms. Despite the richness of the model, I am able to identify and estimate it using data on hair salon task assignments. This allows me to analyze policy counterfactuals.

The counterfactual exercises illustrate that allowing internal organization to be endogenous and heterogeneous qualitatively changes the impact of policy. Minimum wage increases generate new types of wage spillovers that cannot occur in many other models of the labor market. Sales tax cuts induce firms to reorganize their workforce, changing the task-composition of jobs. Although these effects are specific to the salon industry, they indicate that internal organization is an important force that deserves careful study in a variety of contexts. The framework in this paper is meant to provide researchers with a way to do exactly this.

References

- Adenbaum, Jacob. 2021. "Endogenous Firm Structure and Worker Specialization."
- Alchian, AA, and H Demsetz. 1972. "Production, information costs, and economic organization." *American Economic Review* 62 (5): 777–795.

- Aravindakshan, Ashwin, and Brian Ratchford. 2011. "Solving share equations in logit models using the LambertW function." *Review of Marketing Science* 9.
- Argyres, Nicholas S., Teppo Felin, Nicolai Foss, and Todd Zenger. 2012. "Organizational Economics of Capability and Heterogeneity." <https://doi.org/10.1287/orsc.1120.0746> 23, no. 5 (October): 1213–1226.
- Autor, David H., Frank Levy, and Richard J. Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *The Quarterly Journal of Economics* 118, no. 4 (November): 1279–1333.
- Baker, George, Robert Gibbons, and Kevin J. Murphy. 2002. "Relational Contracts and the Theory of the Firm." *The Quarterly Journal of Economics* 117, no. 1 (February): 39–84.
- Berry, Steven, James Levinsohn, and Ariel Pakes. 1995. "Automobile Prices in Market Equilibrium." *Econometrica* 63 (4): 841–890.
- Blahut, Richard E. 1972. "Computation of Channel Capacity and Rate-Distortion Functions." *IEEE TRANSACTIONS ON INFORMATION THEORY*, no. 4.
- Bloedel, Alexander, and Weijie Zhong. 2021. "The Cost of Optimally Acquired Information*." June.
- Caplin, Andrew, and Barry Nalebuff. 1991. "Aggregation and Imperfect Competition: On the Existence of Equilibrium." *Econometrica* 59, no. 1 (January): 25.
- Card, David, Ana Rute Cardoso, Joerg Heining, and Patrick Kline. 2018. "Firms and labor market inequality: Evidence and some theory." *Journal of Labor Economics* 36, no. S1 (January): S13–S70.
- Chausse, Pierre. 2021. *Package 'gmm'*.
- Chen, Po-Ning. n.d. "Chapter 6 Lossy Data Compression and Transmission."
- Crawford, Gregory S, Oleksandr Shcherbakov, and Matthew Shum. 1997. "Quality Over-provision in Cable Television Markets †." *American Economic Review* 2019 (3).
- d'Aspremont, C., J. Jaskold Gabszewicz, and J.-F. Thisse. 1979. "On Hotelling's "Stability in Competition"." *Econometrica* 47, no. 5 (September): 1145.
- Dai, Tianjiao, and Juuso Toikka. 2022. "Robust Incentives for Teams." *Econometrica* 90, no. 4 (July): 1583–1613.

- Dessein, Wouter, Tano Santos, Gary Becker, Mathias Dewatripont, Jonathan Levin, Kevin Murphy, Canice Prendergast, and John Roberts. 2003. "Adaptive Organizations *."
- Du, Yu, and Ravi Varadhan. 2020. "SQUAREM: An R package for off-the-shelf acceleration of EM, MM and other EM-like monotone algorithms." *Journal of Statistical Software* 92:1–41.
- Garicano, L. 2000. "Hierarchies and the organization of knowledge in production." *Journal of Political Economy* 108 (5): 874–904.
- Garicano, Luis, and Yanhui Wu. 2012. "Knowledge, communication, and organizational capabilities." *Organization Science* 23, no. 5 (February): 1382–1397.
- Gibbons, Robert. 2020. "March-ing toward organizational economics." *Industrial and Corporate Change* 29, no. 1 (February): 89–94.
- Gregory, Terry, and Ulrich Zierahn. 2022. "When the minimum wage really bites hard: The negative spillover effect on high-skilled workers." *Journal of Public Economics* 206 (February): 104582.
- Haanwinckel, Daniel. 2020. "Supply, Demand, Institutions, and Firms: A Theory of Labor Market Sorting and the Wage Distribution."
- Holmstrom, Bengt, and Paul Milgrom. 1994. "The Firm as an Incentive System." *American Economic Review* 84 (4): 972–991.
- Jung, Junehyuk, Jeong Ho (John) Kim, Filip Matějka, and Christopher A. Sims. 2019. "Discrete Actions in Information-Constrained Decision Problems." *The Review of Economic Studies* 86, no. 6 (November): 2643–2667.
- Kremer, Michael. 1993. "The o-ring theory of economic development." *Quarterly Journal of Economics* 108 (3): 551–575.
- Lazear, Edward P. 2009. "Firm-specific human capital: A skill-weights approach." *Journal of Political Economy* 117, no. 5 (October): 914–940.
- Lindenlaub, Ilse. 2017. "Sorting Multidimensional Types: Theory and Application." *The Review of Economic Studies* 84, no. 2 (April): 718–789.
- Lipnowski, Elliot, Doron Ravid, Ben Brooks, Tommaso Denti, Mira Frick, Daniel Gottlieb, Ryota Iijima, et al. 2022. "Predicting Choice from Information Costs" (May).
- Lucas, Robert E. 1978. "On the Size Distribution of Business Firms." *The Bell Journal of Economics* 9 (2): 508.

- Martinez, Elizabeth A., Nancy Beaulieu, Robert Gibbons, Peter Pronovost, and Thomas Wang. 2015. "Organizational Culture and Performance." *American Economic Review* 105, no. 5 (May): 331–35.
- Matějka, Filip, and Alisdair McKay. 2015. "Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model." *American Economic Review* 105, no. 1 (January): 272–98.
- Matějka, Filip, Alisdair Mckay, Michal Bauer, Levent Celik, Satyajit Chatterjee, Faruk Gul, Christian Hellwig, et al. 2015. "Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model †." *American Economic Review* 105 (1): 272–298.
- McFadden, D. 1973. "Conditional logit analysis of qualitative choice behavior."
- Meier, Stephan, Matthew Stephenson, and Patryk Perkowski. 2019. "Culture of trust and division of labor in nonhierarchical teams." *Strategic Management Journal* 40, no. 8 (August): 1171–1193.
- New York City Department of Finance. 2022. *New York State Sales and Use Tax*, May.
- Nocke, Volker, and Nicolas Schutz. 2018. "Multiproduct-Firm Oligopoly: An Aggregative Games Approach." *Econometrica* 86, no. 2 (March): 523–557.
- Proposed New York state minimum wage increases would lift wages for more than 2 million workers through 2026: Minimum wages would range by region from \$16.35 to \$21.25 per hour by 2026 — Economic Policy Institute.*
- Sattinger, Michael. 1975. "Comparative Advantage and the Distributions of Earnings and Abilities." *Econometrica* 43, no. 3 (May): 455.
- Shannon, C. E. 1948. "A Mathematical Theory of Communication." *Bell System Technical Journal* 27 (3): 379–423.
- Shkel, Yanina, and Sergio Verdú. 2018. "A Coding Theorem for f-Separable Distortion Measures." *Entropy* 20, no. 2 (February).
- Song, Jae, David J. Price, Fatih Guvenen, Nicholas Bloom, and Till Von Wachter. 2019. "Firming Up Inequality." *The Quarterly Journal of Economics* 134, no. 1 (February): 1–50.
- Teulings, Coen N. 2000. "Aggregation Bias in Elasticities of Substitution and the Minimum Wage Paradox." *International Economic Review* 41, no. 2 (May): 359–398.

- Tian, Jianrong. 2019. "Attention Costs without Distraction and Entropy."
- Tirole, Jean. 2009. "Cognition and Incomplete Contracts." *American Economic Review* 99, no. 1 (March): 265–94.
- Tishby, Naftali, Fernando C Pereira, and William Bialek. 2000. "The information bottleneck method."
- Williamson, Oliver E, and Oliver F Williamson. 1984. "The Incentive Limits of Firms: A Comparative Institutional Assessment of Bureaucracy," 736–763.

Tables

Table 1: Regressions of Worker Specialization on Organization Complexity

Dependent Variable:	Worker Task Specialization		
Model:	(1)	(2)	(3)
<i>Variables</i>			
Organization Complexity	0.2853*** (0.0313)	0.2862*** (0.0310)	0.2922*** (0.0392)
<i>Fixed-effects</i>			
Quarter-Year		Yes	Yes
County			Yes
Observations	62,452	62,452	62,452
R ²	0.10184	0.10901	0.21483

Standard errors clustered at the salon level.

Signif. Codes: ***: 0.001, **: 0.01, *: 0.05

Note: Task-specialization is measured as the maximum fraction of time spent on a single task by a worker. Complexity is measured at the salon level. Across all specifications, complexity (a salon-level measure) can account for 10% of the variation in worker specialization.

Table 2: Regressions of Salon Size on Organization Complexity

Dependent Variables:	Revenue	Employees	Utilized Labor	Customers	Visits
Model:	(1)	(2)	(3)	(4)	(5)
<i>Variables</i>					
Org. Complexity	347549.2*** (79546.2)	9.75** (3.016)	26481 (35653.2)	334.6 (259.6)	731.7 (450.1)
<i>Fixed-effects</i>					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
County	Yes	Yes	Yes	Yes	Yes
<i>Fit statistics</i>					
Observations	4,558	4,558	4,558	4,558	4,558
R ²	0.32465	0.34319	0.28918	0.34901	0.35004

Standard-errors clustered at the salon level.

Signif. Codes: ***: 0.001, **: 0.01, *: 0.05

Note: Observations are salon-quarters. Regressions illustrate a positive correlation between complexity and several measures of salon size after controlling for county and quarter fixed effects and the composition of tasks performed at the salon in the quarter. This relationship is statistically significant for revenue and employees.

Table 3: Salon Activity Data Sample

Firm	Salon	App.	Cust.	Service	Staff	Time Stamp	Price	Duration
1	1A	123	Blake	Advanced Cut	Rosy	3/26/2021 16:15	100	72
1	1A	123	Blake	Full Head - Highlights	Rosy	3/26/2021 16:15	243	127
1	1A	123	Blake	Treatment Add On (Olaplex)	Rosy	3/26/2021 16:15	39	72
2	2A	9982	Grace	Women's Cut	Tyler	3/17/2021 11:00	225	43
2	2A	9982	Grace	Single Process	Ben	3/17/2021 11:00	200	77

Note: This table is a snapshot displaying two actual appointments at salons in the same zip code from the data used for the estimation. Customer IDs are replaced by pseudonyms. Both appointments include multiple services, but one salon assigns all services to a single worker while the other assigns different types of services to different workers.

Table 4: Summary Statistics for all Salon-Quarters

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Revenue	4,558	213,201.30	248,359.90	5	58,912.5	271,236.5	2,559,703
Price	4,558	199.73	135.16	0.20	111.71	261.88	3,180.44
Employees	4,558	13.38	10.79	1	6	17	92
Customers	4,558	1,159.23	1,098.45	1	397	1,619	16,768
Task Categories	4,558	4.45	0.86	1	4	5	5
Labor per. Customer	4,558	2.15	1.63	0.10	1.52	2.57	61.33

Note: The table displays summary statistics for the main variables of interest with data aggregated at the salon-quarter level. There is significant variation across salons in complexity, number of employees, revenue and many other dimensions.

Table 5: Parameters Estimates, Tasks

Task	Associated Specialist			
	Skill Gap	Wage	Skill Base	Material Cost
Administrative	43.29*	26.99	-16.16	-147.60*
	(21.66)	(63.75)	(14.58)	(13.47)
Blowdry/Etc.	141.69*	20.91	-70.56*	12.39
	(36.67)	(40.22)	(13.57)	(16.65)
Color/Highlight/Wash	60.03*	37.75*	-9.69	56.49*
	(21.24)	(7.00)	(11.97)	(15.79)
Haircut/Shave	32.45*	16.96*	.	.
	(13.07)	(8.32)	.	.
Nail/Spa/Eye/Misc.	66.48	81.16	-252.58*	-1061.12*
	(37.72)	(53.52)	(11.47)	(10.73)

Note: Standard errors from 500 bootstrap replications in parentheses. * indicates significance at the 0.05 level. For each task, the table lists the skill gap and wage of the associated specialist in 2021 dollars. The skill gap is the change in quality when a task is assigned to a specialist. Also listed are the skill base, the quality when the task is performed by a non-specialists, the material cost, and the non-wage costs associated with the task (i.e. dye for coloring). Material costs and skill base are relative to the haircut task. Wages are per hour, while material costs and skills are per unit.

Table 6: Two Estimated Organization Structures

Specialist	Task					Total	Task					Total
	Cut	Color	Blow Dry	Admin.	Nail/Misc.		Cut	Color	Blow Dry	Admin.	Nail/Misc.	
Cut	0.15	0.01	0.001	0.06	0	0.22	0.180	0.003	0	0.006	0.003	0.193
Color	0	0	0	0	0	0	0.057	0.553	0	0.016	0.009	0.116
Blow Dry	0	0	0	0	0	0	0.012	0.002	0.097	0.003	0.002	0.636
Admin.	0.31	0.03	0.003	0.45	0	0.784	0	0	0	0	0	0
Nail/Misc.	0	0	0	0	0	0	0.004	0.001	0	0.001	0.050	0.055
Tot.	0.455	0.036	0.004	0.505	0	1	0.253	0.559	0.097	0.026	0.064	1

(a) Salon 1, $I_j = 0.03$

(b) Salon 2, $I_j = 0.70$

Note: These are estimated organization structures (B_j) for a high and a low complexity salon in New York Quarter 2, 2021.

Table 7: Model Validation: Estimated vs. Observed Job Task Content

(a) Mean and Median

Task	Mean		Median	
	Model	Observed	Model	Observed
Haircut/Shave	0.4094	0.4094	0.2816	0.3357
Color/Highlight/Wash	0.4058	0.4058	0.3067	0.4042
Blowdry/Style/Treatment/Extension	0.1179	0.1179	0.0162	0.0704
Administrative	0.0278	0.0278	0.0050	0.0040
Nail/Spa/Eye/Misc.	0.0391	0.0391	0.0049	0.0000

(b) Variance

Task	Total Variance		Between Firm Variance	
	Model	Observed	Model	Observed
Haircut/Shave	0.1110	0.1268	0.0597	0.0597
Color/Highlight/Wash	0.1127	0.1105	0.0365	0.0365
Blowdry/Style/Treatment/Extension	0.0472	0.0194	0.0111	0.0111
Administrative	0.0098	0.0080	0.0063	0.0063
Nail/Spa/Eye/Misc.	0.0120	0.0171	0.0050	0.0050

(c) Interquartile Range

Task	p25		p75	
	Model	Observed	Model	Observed
Haircut/Shave	0.1583	0.0469	0.8013	0.7577
Color/Highlight/Wash	0.0417	0.0388	0.7020	0.6383
Blowdry/Style/Treatment/Extension	0.0004	0.0110	0.0726	0.1892
Administrative	0.0027	0.0000	0.0166	0.0108
Nail/Spa/Eye/Misc.	0.0000	0.0000	0.0329	0.0106

Note: The table compares model generated and observed job task content along several dimensions. The model is designed to exactly match the average market-wide amount of time spent on each task and the between firm variance. The other moments were not targeted, and assessing their match serves as a validation exercise.

Table 8: Total Effects of Increasing the Minimum Wage

(a) Wage Changes by Worker Type

Type	Wage Change	Total Wages Gained/Lost
Haircut/Shave - UNEMPLOYED	-100.00%	-\$600,240
Haircut/Shave - EMPLOYED	17.95%	\$1,528,205
Color/Highlight/Wash	-0.61%	-\$228,453
Blowdry/Style/Treatment/Extension	3.48%	\$323,374
Administrative	4.17%	\$47,154
Nail/Spa/Eye/Misc.	0.68%	\$19,319

(b) Welfare Breakdown

Source	Change	Percent Change
Salon Profit	-\$714,413	-0.472%
Consumer Welfare	-\$2,528,784	-1.671%
Employed Wages	\$1,689,600	1.116%
Unemployed Wages	-\$600,240	-0.397%
Total Welfare	-\$2,153,838	-1.423%

Note: Increasing the minimum wage generates both positive and negative wage spillovers for workers on whom it is not binding. Positive spillovers are larger and occur for most worker types. Overall, wage increases for employed workers are more than salon profit losses and wage losses of unemployed workers combined. Total welfare declines, as consumers see higher prices and slightly lower quality.

Table 9: Spillovers from an Increase in the Minimum Wage

(a) Reallocation Effect

Type	Reallocation Change		
	Employment	Task-Spec.	Wage
Haircut/Shave	-5.85%	-0.04%	17.95%
Color/Highlight/Wash	0%	-0.17%	-1.13%
Blowdry/Style/Treatment/Extension	0%	-0.40%	4.63%
Administrative	0%	0.09%	5.22%
Nail/Spa/Eye/Misc.	0%	-0.03%	0.58%

(b) Reorganization Effect

Type	Reorganization Change		
	Employment	Task-Spec.	Wage
Haircut/Shave	-0.73%	0.12%	0%
Color/Highlight/Wash	0%	-0.33%	0.52%
Blowdry/Style/Treatment/Extension	0%	0.03%	-1.15%
Administrative	0%	0.03%	-1.05%
Nail/Spa/Eye/Misc.	0%	-0.00%	0.10%

Note: The minimum wage increase has positive spillovers for some workers and negative spillovers for others. These spillovers can be further decomposed as those resulting from reorganization and those resulting for reallocation. Most spillovers come from the fact that the policy favors salons that have internal organizations intense in binding workers initially (reallocation). Some spillovers occur because the policy induces firms to shift tasks from binding to non-binding workers (reorganization).

Table 10: Summary of All Minimum Wage Effects

Statistic	Reallocation	Reorganization	Total
Avg. Price	1.96%	-0.29%	1.67%
Avg. Complexity	0.00%	-0.46%	-0.46%
Avg. Quality	0.00%	-0.54%	-0.54%
Avg. Hourly Wage	3.40%	0.20%	3.60%
Std. Dev. Wage	-8.91%	1.03%	-7.88%
Task Specialization	-0.61%	-0.18%	-0.79%
Employment	-1.53%	-0.19%	-1.72%
Market Served	-2.69%	-0.12%	-2.81%
Total Profit	-2.69%	-0.12%	-2.81%
Consumer Welfare	-2.64%	-1.19%	-3.83%
Total Wages	1.81%	0.00%	1.82%
Total Welfare	-0.88%	-0.54%	-1.42%

Note: This table summarizes the impact of increasing the minimum wage from \$15 to \$20 on different actions and market outcomes in the Manhattan hair salon market.

Table 11: Summary of All Sales Tax Elimination Effects

Statistic	Reallocation	Reorganization	Total
Avg. Price	4.70%	3.99%	8.68%
Avg. Complexity	0.00%	5.53%	5.53%
Avg. Quality	0.00%	10.03%	10.03%
Avg. Hourly Wage	18.32%	1.02%	19.34%
Std. Dev. Wage	22.67%	-6.32%	16.35%
Task Specialization	0.90%	0.93%	1.83%
Total Profit	4.32%	-0.60%	3.71%
Consumer Welfare	-0.18%	-0.57%	-0.75%
Total Wages	18.32%	1.02%	19.34%
Total Welfare	0.14%	0.05%	0.19%

Note: This table summarizes the impact of eliminating the service sales tax on different actions and market outcomes in the Manhattan hair salon market.

Table 12: Total Effects of a Sales Tax Elimination

(a) Wage Changes by Worker Type			(b) Welfare Breakdown		
Type	Wage Change	Task-Spec. Change	Source	Change	Percent Change
Haircut/Shave	31.99%	0.29%	Salon Profit	\$942,740	0.58%
Color/Highlight/Wash	20.09%	2.57%	Consumer Welfare	-\$494,199	-0.30%
Blowdry/Style/Treatment/Extension	6.06%	3.01%	Wages	\$11,603,777	7.12%
Administrative	17.99%	1.03%	Tax Revenue	-\$11,739,300	-7.20%
Nail/Spa/Eye/Misc.	12.74%	2.39%	Total Welfare	\$313,017	0.19%

Note: Eliminating the sales tax raises wages most in percentage terms for haircut specialists. Workers gain the most from eliminating the sales tax: wage increases are almost equal to the lost revenue to the government.

A Theoretical Appendix

A.1 Rate-Distortion and Rational Inattention Equivalence

(3) from Theorem 1 can be re-written as:

$$\gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \left[W(B_j) - \rho^{-1} \xi(B_j) \right] \right\} \quad (8)$$

I can re-write (8) as a maximization problem:

$$\max_{B_j \in \mathbb{B}} \left\{ \sum_{i,k} B_j(i, k) (\rho^{-1} \theta_{i,k} - W_i) \right\} - \gamma_j I(B_j) \quad (9)$$

Comparing (9) to formulations in papers like Jung et al. (2019) illustrates that this is a rational inattention problem with mutual information attention costs. I re-write 8 one last time:

$$\gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i, k) (W_i - \rho^{-1} \theta_{i,k}) \right\} \quad (10)$$

Comparing (10) to formulations like Equation 6 in Tishby, Pereira, and Bialek (2000) demonstrates this is a well-understood minimization problem from information theory called a rate-distortion problem.

A.2 Proof of Theorem 1

Note that for any given organization structure, the firm will only choose prices weakly above marginal cost, otherwise it receives negative profit. Without loss I therefore restrict the set of price-structure pairs considered to be those where price exceeds marginal cost.

First, I prove that if an organization structure B_j^* solves the simpler problem (3) then it is profit-maximizing (only if direction). I need to show that for any price-organization structure pair (p'_j, B'_j) there exists p such that profit under (p_j, B_j^*) is weakly higher than profit under (p'_j, B'_j) . I do this by construction. Denote B_j^* as a structure which solves (3). Such a structure always exists because (3) is a rate-distortion/rational inattention problem as shown in Appendix Section A.1.

For any price p'_j and any structure B'_j we can construct $p_j = p'_j + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B'_j) - W(B'_j)$. Notice that the price p_j is positive and therefore feasible. Recall that

profit evaluated at (p_j, B_j^*) is:

$$D_j(\xi(B_j^*) - \rho p_j, p_{-j}, \xi_{-j}) \left[p_j - \gamma_j I(B_j^*) - W(B_j^*) \right]$$

From this, we see that the second multiplicative term of profit is equal under (p_j, B_j^*) and (p'_j, B'_j) . Turn now to the first term (demand). We have that demand is strictly increasing in the quality-price index $\xi(B_j) - \rho p_j$ therefore it is sufficient to show that this index is weakly higher for (p_j, B_j^*) . I show this by re-writing $\xi(B_j^*) - \rho p_j$:

$$= \xi(B_j^*) - \rho[p'_j + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B'_j) - W(B'_j)] \quad (11)$$

$$= \xi(B_j^*) - \rho[p'_j + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B'_j) - W(B'_j)] + \xi(B'_j) - \xi(B_j^*) \quad (12)$$

$$= \xi(B'_j) - \rho[p'_j + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B'_j) - W(B'_j) - \rho^{-1}\xi(B_j^*) + \rho^{-1}\xi(B'_j)] \quad (13)$$

$$= \xi(B'_j) - \rho p'_j - \rho \underbrace{[\gamma_j I(B_j^*) + W(B_j^*) - \rho^{-1}\xi(B_j^*) - \{\gamma_j I(B'_j) + W(B'_j) - \rho^{-1}\xi(B'_j)\}]}_{\leq 0 \text{ because } B_j^* \text{ minimizes}} \quad (14)$$

$$\geq \xi(B'_j) - \rho p'_j \quad (15)$$

This proves the only if direction. I now prove that if a structure B_j^* is profit-maximizing it solves (3) (the if direction). Suppose for sake of contradiction there exists B'_j which is profit-maximizing but does not solve (3). Then as in the first part of the proof there exists B_j^* which does solve (3). Then I can construct p_j as before for any p'_j that is weakly higher than marginal cost under B'_j . However, because B_j does not minimize (3), we have that $\xi(B_j^*) - \rho p_j > \xi(B'_j) - \rho p'_j$ and thus profit is strictly higher under B_j^*, p_j . This contradicts optimality of B'_j and concludes the proof.

A.3 Proof of Proposition 1 and 2

We have already shown in Theorem 1 that optimal B solves a rate-distortion problem. In this problem, distortion is adjusted wages less quality. Considering the optimal organization complexity (I) for each quality-adjusted wage (denoted as D), we know from information theory that this function is continuous, convex and strictly decreasing in D above some upper bound (Chen, n.d.). Thus there exists some \bar{D} , such that $I(\bar{D}) = 0$. Suppose we are below this threshold. Then we have that we can write the firm's problem as solving:

$$\min_D \gamma I(D) + D$$

The first-order condition of this problem is:

$$\gamma I'(D) + 1 = 0 \implies I'(D) = -\gamma^{-1}$$

We notice that because $I(D)$ is a rate-distortion function, it is decreasing and convex. Thus the D^* which solves the FOC will be increasing in γ , as depicted in Figure 8. But D^* is increasing in γ and I , as a rate-distortion function, is strictly decreasing in D . Therefore, I will be decreasing in γ .

For profit and output, apply the envelope theorem to $\gamma I(D) + D$:

$$\frac{\partial}{\partial \gamma} \gamma I(D^*) + D^* = I(D^*)$$

This is positive whenever D^* is below some threshold or equivalently whenever complexity is positive. Profit and market share are both decreasing functions of this quantity. Thus profit and output are decreasing in γ .

A.4 Proof of Proposition 3

For the purposes of this proof only, we define $h_{i,k}$ as the fraction of task k performed by worker i . Then the optimal job of worker i is given by:

$$h_{i,k} = \frac{E_i}{Z(k, \lambda)} \exp\left(-\lambda(\rho w_i - \theta_{i,k})\right)$$

Summing over i , we have:

$$\sum_i h_{i,k} = \frac{1}{Z(k, \lambda)} \sum_i E_i \exp\left(-\lambda(\rho w_i - \theta_{i,k})\right) = 1$$

Therefore:

$$Z(k, \lambda) = \sum_i E_i \exp(-\lambda(\rho w_i - \theta_i \delta^{\{k_i \neq k\}}))$$

And:

$$h_{i,k} = \frac{e_i \exp(\lambda(-\rho w_i + \theta_{i,k}))}{\sum_{i'} e_{i'} \exp(\lambda(-\rho w_{i'} + \theta_{i',k}))}$$

Substituting for λ :

$$h_{i,k} = \frac{E_i \exp(-\gamma^{-1} w_i + (\rho \gamma)^{-1} \theta_{i,k})}{\sum_{i'} E_{i'} \exp(-\gamma^{-1} w_{i'} + (\rho \gamma)^{-1} \theta_{i',k})}$$

By the definition of $h_{i,k}$, we have:

$$B_{i,k} = \alpha_k h_{i,k}$$

To get to jobs, we divide by E_i :

$$b_{i,k} = B_{i,k}/E_i = \alpha_k/E_i h_{i,k} = \frac{\alpha_k \exp(-\gamma^{-1}w_i + (\rho\gamma)^{-1}\theta_{i,k})}{\sum_{i'} E_{i'} \exp(-\gamma^{-1}w_{i'} + (\rho\gamma)^{-1}\theta_{i',k})}$$

This illustrates that optimal jobs take a multinomial logit form. We can also derive this result by applying Theorem 1 from Mat  jka and McKay (2015).

The fact that all hired worker types spend a positive amount of time on each task is a direct application of Lemma 1 from Jung et al. (2019). An increase in the wage corresponds to a decrease in the “pay-off” to the firm of using workers of type i in all tasks (i.e., states of the world in the rational inattention literature). This means I can apply Proposition 3 from Mat  jka and McKay (2015) to say that an increase in w_i leads to a decrease in E_i all else constant. We can even say that e_i is strictly decreasing in w_i whenever the initial share of worker i is strictly interior, i.e. $0 < E_i < 1$.

A.5 Proof of Proposition 5

As stated earlier in the main text, the proof of this proposition will consist of three parts. For simplicity, firm index j is suppressed throughout this section.

First, we prove that observed organization complexity based on worker identities is equal to unobserved true complexity based on worker skill sets. Consider the augmented model proposed in Section 6.1. In particular, recall that workers with different labor supplies match to firms by some unspecified matching process. I then prove the following:

Lemma 2 *All workers with the same skill-set are assigned the same distribution of tasks regardless of their labor supply.*

Proof. A well-known property of mutual information attention costs is that they satisfy compression monotonicity or are “distraction-free” (Tian 2019). I will use this in the proof.

Suppose for the sake of contradiction the firm assigned two workers of the same skill set different distributions of tasks. Consider a different assignment of work such that the same amount of each task is accomplished, and both workers still are assigned the same total amount of work. Such a assignment always exists: we can just take the total time spent in each task by both workers and split it based on effective units of labor. By the strict distraction free property of mutual information, this new assignment reduces organization costs. Notice that this does not impact the wage bill, since both workers have

the same wage. Also, it does not impact quality because the total amount of each task accomplished remains the same, and both workers have the same skill set. Thus quality-adjusted cost strictly decreased, so profit strictly decreased, contradicting the optimality of the original assignment. Therefore all workers with the same skill set are assigned the same distribution of tasks regardless of their effective units of labor.

This lemma means that the firm treats workers with different labor supplies but the same skill sets as if they are a single, aggregate worker. Denote worker identities as indexed by n and worker skill sets by i . Denote the organizational structure over worker identities as \tilde{B} . Then we have:

$$\frac{\tilde{B}_{n,k}}{\sum_{k'} \tilde{B}_{n,k'}} = \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} \quad \forall i, k \text{ s.t. } \theta_n = \theta_i$$

Because total amount of each task is fixed at α_k :

$$\sum_{n'} \tilde{B}_{n',k} = \alpha_k = \sum_{i'} B_{i',k}$$

Plugging these results into organization complexity:

$$I(\tilde{B}) = \sum_{n,k} \tilde{B}_{n,k} \log \left(\frac{\tilde{B}_{n,k}}{\sum_{k'} \tilde{B}_{n,k'} \sum_{n'} \tilde{B}_{n',k}} \right) = \sum_{n,k} \sum_i \frac{B_{i,k} \sum_{k'} \tilde{B}_{n,k'}}{\sum_{k'} B_{i,k'}} \mathbb{I}\{\theta_n = \theta_i\} \log \left(\frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right)$$

And rearranging terms:

$$= \sum_{i,k} \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} \log \left(\frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) \sum_{n,k'} \tilde{B}_{n,k'} \mathbb{I}\{\theta_n = \theta_i\}$$

Sum of all $\tilde{B}_{n,k}$ of workers with the same skill set but different labor supply is exactly E_i which is exactly equal to $\sum_{k'} B_{i,k'}$. Therefore we can write:

$$I(\tilde{B}) = \sum_{i,k} \frac{B_{i,k}}{\sum_{k'} B_{i,k'}} \log \left(\frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) \sum_{k'} B_{i,k'} = \sum_{i,k} B_{i,k} \log \left(\frac{B_{i,k}}{\sum_{k'} B_{i,k'} \sum_{i'} B_{i',k}} \right) = I(B)$$

Therefore organization complexity based on worker identities is equal to organization complexity based on worker skill sets. Since we observe identities, this implies that we can compute organization complexity as the mutual information between worker identities and tasks.

For the second part of the proof, we appeal to Proposition 2 which shows that organization complexity is strictly decreasing in γ . This implies a one-to-one mapping when-

ever Ω is known and $I > 0$. For the third part of the proof, we appeal to the equivalence with a rate-distortion problem to say that we can find the firm's optimal organizational structure, B_j using the Blahut-Arimoto algorithm.

A.6 Proof of Proposition 4

To recover the best-responses of the firm's problem, we utilize the fact that the joint maximization of any function is equivalent to the sequential maximization. Thus we can write the firm's problem as:

$$\max_{B_j \in \mathbb{B}} \max_{p_j \in \mathbb{R}_+} \underbrace{\frac{\exp(\xi(B_j) - \rho p_j)}{\sum_{j'} \exp(\xi(B_{j'}) - \rho p_{j'})}}_{\text{market share, } s_j} \left[p_j - \underbrace{\left(\gamma_j I(B_j) + \sum_{i,k} w_i B_j(i, k) \right)}_{\text{constant marginal cost, } MC_j} \right]$$

I first study the inner pricing problem. Fixing an organization structure, the model reduces to a logit Bertrand game with heterogeneous costs and qualities. Proposition 7 of Caplin and Nalebuff (1991) proves that such a game has a unique pure strategy Nash equilibrium in prices. Therefore for any chosen organizational structure, there is a single best-response price. In the course of the proof of Theorem 1, I substituted the equation characterizing the optimal price into profit, and showed that the best-response B_j also solves:

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i, k) (W_i - \rho^{-1} \theta_{i,k})$$

Note that the best-response structure will therefore depend on other firm actions only through wages. The theorem also establishes that this is equivalent to a rational inattention problem with a mutual information cost function. With the equivalence to a rational inattention problem, we can establish existence. With this in hand, we can appeal to Matějka et al. (2015) to say that there exists an organization structure which maximizes profit for each firm. This establishes equilibrium existence. For uniqueness, the online Appendix of Matějka et al. (2015) contains a result which implies that all firms satisfying the following condition have a unique organization structure which maximizes profits:

Assumption 1 Define the wage-quality vector of a worker of type i at firm j as $v_{i,j} = \{\exp(\gamma_j^{-1}(\rho^{-1}\theta_{i,k} - w_i))\}_{k=1}^K$. The set of wage-quality vectors $\{v_{i,j}\}_{i \in \mathcal{I}}$ is affinely independent.

Whenever this holds for all firms there is a unique cost and quality for each firm which, according to Caplin and Nalebuff (1991), implies there are unique equilibrium prices.

Thus this condition is sufficient (but not necessary) to guarantee uniqueness. This condition is testable, but it requires many parameters, including wages, to be known. Since we wish to estimate the parameters it is not satisfactory. To get a more general result, we now appeal to Lipnowski et al. (2022).

Note that a rational inattention problem with mutual information costs is a special case of the problems considered by Lipnowski et al. (2022). A stochastic choice rule, in their language, is an organization structure in mine. Proposition 1 of their paper (translated to the language of my model) states that if γ_j is known, the set of quality-adjusted wages which generate multiple organization structures is “meager and shy.” Since I consider the case of finite tasks (finite Ω in their language), meager and shy implies a null set. This is only for one firm with a specific γ_j . The set of quality-adjusted wages which generate multiplicity for at least one firm will be the union of all sets which generate multiplicity for each individual firm. The union of countable null sets is also null, therefore the set of quality-adjusted wages that generate multiplicity is null.

Denote the set of quality-adjusted wages which generate multiplicity as \mathbb{M} . The mapping from market parameters Ω to quality-adjusted wages is defined by a multivariate, vector-valued function $F : \mathbb{R}_+^{N \times K + N + 1} \rightarrow \mathbb{R}_+^{N \times K}$. It can be shown that if F is smooth and the rank of the Jacobian of F is at least $N \times K$, then the measure of the pre-image any measure 0 set is 0.

I now prove that F satisfies the rank condition. Recall that the quality-adjusted wage of worker i and task k has the form $w_i - \rho^{-1}\theta_{i,k}$. Collapse i, k into a single index, $y = 1, \dots, N \times K$, where $\mathcal{I}(\cdot)$ and $\mathcal{K}(\cdot)$ return the task and worker type associated with the index y . Then we have that element y of F is:

$$F(\Omega) = w_{\mathcal{I}(y)} - \rho^{-1}\theta_{\mathcal{I}(y), \mathcal{K}(y)}$$

The Jacobian of this function is rank of at least $N \times K$ because each skill parameter $\theta_{i,k}$ impacts only one quality-adjusted wage. Formally, there exist at least $N \times K$ columns of the Jacobian which are linearly independent of each other. Thus we have that the pre-image of the null set \mathbb{M} on F will be measure 0. Since the pre-image is the set of parameters which generate multiplicity, we have that the set of parameters which generate multiplicity is measure 0.

An implication of this result is that a pure strategy Nash equilibrium exists for any fixed wages. I conjecture that this proof could be extended to show equilibrium existence with endogenous wages determined by market clearing. One approach would be to prove that excess labor demands satisfy Kakutani’s fixed point theorem. Extending the

uniqueness result to say that the equilibrium is unique for almost any total labor supplies may not be possible. This is because in general, worker types may be complements or substitutes depending on their skill sets. If firms are homogeneous with respect to task mixtures and organization costs, the wages that clear the market may very well be the wages which induce indifference across multiple organization structures and multiple equilibria.

A.7 Welfare

Preferences take a random utility form with Type 1 extreme value distribution for the horizontal taste heterogeneity $\epsilon_{i,j}$ in the population. I assumed throughout that this heterogeneity is distributed i.i.d. across consumers and alternatives. Therefore expected utility of consumer i has the well-known closed form:

$$V_i = \mathbb{E}[\max_j \{\xi_j - \rho p_j + \epsilon_{i,j}\}] = \ln \left[\sum_{j=1}^J \exp(\xi_j - \rho p_j) \right] + C$$

where C is Euler's Constant. There are a mass M of consumers, therefore total consumer expected utility is $M \cdot V_i$.²⁶ We then can denominate this in dollar terms by dividing by the coefficient on price, ρ . Our measure of total consumer welfare in dollar terms is:

$$CS = \frac{M}{\rho} \left\{ \ln \left[\sum_{j=1}^J \exp(\xi_j - \rho p_j) \right] + C \right\}$$

With a sales tax τ , it is:

$$CS = \frac{M}{\rho} \left\{ \ln \left[\sum_{j=1}^J \exp(\xi_j - \rho(1+\tau)p_j) \right] + C \right\}$$

Total welfare is measured as the sum of consumer surplus, firm profits and worker wages. This assumes an additive welfare function which weights all consumers, firms, and workers equally.

A.8 Organization Complexity As Task-Specialization

This section illustrates that complexity is a measure of average task-specialization. To see this, first define a job as a vector, where component k is the fraction of a worker's total

26. This assumes an additive welfare function which gives equal weight to all consumers.

labor spent performing task k :

$$b_i(k) = \frac{B(i, k)}{E_i}$$

We can measure the specialization of any job by comparing it to a benchmark “generalist job.” I define the generalist job as the job where all workers are assigned exactly the task-mix:

$$b_j^G(k) = \alpha_k$$

Notice that under this assumption each worker is working as a miniature version of the firm itself. There is no sense in which a worker needs a coworker in order to produce output. With these two concepts in hand we have the following result.

Proposition 6 *Complexity ($I(B_j)$) is the weighted-average Kullback-Leibler divergence between the jobs at a firm and the firm’s generalist job $b_j^G(k)$, where the weights are the share of each worker type.*

Proof. Using the definition of mutual information, we can write out complexity as:

$$\begin{aligned} I(B_j) &= \sum_{i,k} B(i, k) \log \left(\frac{B(i, k)}{\sum_k B(i, k') \sum_{i'} B(i', k)} \right) \\ &= \sum_{i,k} E_i \frac{B(i, k)}{E_i} \log \left(\frac{B(i, k)}{E_i \alpha_k} \right) \\ &= \sum_i E_i \sum_k b_i(k) \log \left(\frac{b_i(k)}{\alpha_k} \right) \\ &= \sum_i E_i \sum_k b_i(k) \log \left(\frac{b_i(k)}{b_j^G(k)} \right) \\ &= \sum_i E_i D_{KL}(b_i || b_j^G) \end{aligned}$$

This can also be proved more quickly using the well-known fact that mutual information is the expected K-L divergence of the conditionals with respect to a marginal distribution. An implication of this result is that assuming mutual information organization costs is then isomorphic to assuming the cost of an organization structure is proportional to its distance from the generalist structure.

In this sense, we are assuming “task-specialization” is costly. On the other hand, the mapping from complexity to task-specialization is not exact. The firm in principle could have a worker spend all of their time on two tasks. Whenever the task-mix is uniform, this is complex, in the sense that it is far from the generalist job. However, this is not a specialized job, since the worker is doing two tasks that may or may not be tasks they are

particularly skilled at performing.

A.9 Quality and Price Competition Logit Demand

Elements of the proof of Theorem 1 imply a more general result for oligopoly games with quality and price competition and per-unit quality costs. Suppose there are J firms which compete by choosing prices (p_j) and qualities (ξ_j) simultaneously. Denote the total cost of firm j as: $C_j(q, \xi_j)$. Suppose total cost is multiplicatively separable in the following way:

Assumption 2 *Total costs take the form:*

$$C_j(q, \xi_j) = q \cdot c_j(\xi_j)$$

where $c_j(\xi_j)$ is a strictly increasing, convex function.

There are a mass 1 of consumers interested in purchasing at most one of the J final products. Consumer z 's utility for good j is represented by the following utility function:

$$u_{z,j} = a_j + \xi_j - \rho p_j + \epsilon_{z,j}$$

where $\epsilon_{z,j}$ is distributed i.i.d. Type-1 extreme value across consumers and products and $\rho > 0$. Exogenous quality is represented by a_j . Then optimal quality solves:

$$\max_{\xi_j} \xi_j - \rho c_j(\xi_j)$$

Giving us the following result:

Proposition 7 *In the unique equilibrium, qualities for all firms are given by:*

$$c'_j(\xi_j^*) = \frac{1}{\rho}$$

Thus, while prices depend on the actions of other firms, equilibrium qualities depend only on consumer price sensitivity. This result also implies that the equilibrium is robust to timing. Suppose the game is sequential: firms first choose quality and then price. Regardless of the pricing strategies of the other firms, every firm will choose ξ_j^* . This result appears to come from the fact that quality and price are perfect substitutes for consumers.

A.10 Closed-Form Logit Price Expression

Demand for a product j is given by:

$$s_j(p_j) = \frac{\exp(-\rho p_j + \xi_j)}{\sum_{j'=0}^J \exp(-\rho p_{j'} + \xi_{j'})}$$

Optimal pricing in a Bertrand Nash equilibrium with single product firms is then given by:

$$p_j = MC_j + \frac{1}{\rho(1 - s_j(p_j))}$$

I now follow the arguments laid out in Aravindakshan and Ratchford (2011). We can re-write this expression as:

$$p_j = c_j + \frac{1}{\rho(1 - \frac{\exp(-\rho p_j + \xi_j)}{\exp(-\rho p_j + \xi_j) + \sum_{j' \neq j} \exp(-\rho p_{j'} + \xi_{j'})})}$$

Re-write again as:

$$p_j = c_j + \frac{1}{\rho} + \frac{\exp(-\rho p_j + \xi_j)}{\rho \sum_{j' \neq j} \exp(-\rho p_{j'} + \xi_{j'})}$$

Multiplying by ρ and subtracting ξ_j yields:

$$\rho p_j - \xi_j = \rho c_j + 1 + \frac{\exp(-\rho p_j + \xi_j)}{\sum_{j' \neq j} \exp(-\rho p_{j'} + \xi_{j'})} - \xi_j$$

Now denote:

$$\begin{aligned} E_j &= \sum_{j' \neq j} \exp(-\rho p_{j'} + \xi_{j'}) \\ \frac{\exp(-\rho p_j + \xi_j)}{E_j} + \xi_j - \rho p_j &= -1 - \rho c_j + \xi_j \\ \exp\left(\frac{\exp(\xi_j - \rho p_j)}{E_j}\right) \exp\left(\xi_j - \rho p_j\right) E_j^{-1} &= \exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1} \end{aligned}$$

And:

$$\tilde{W} = \exp\left(\xi_j - \rho p_j\right) E_j^{-1}$$

Then the expression becomes:

$$\tilde{W} e^{\tilde{W}} = \exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}$$

The left expression is the form required by Lambert's W, so we have that the \tilde{W} which solves is given by Lambert's W function of the right hand side by definition. Thus optimal price solves:

$$W\left(\exp\left(-1 + \xi_j - \rho c_j\right)E_j^{-1}\right) = \exp\left(\xi_j - \rho p_j\right)E_j^{-1}$$

A property of this function is that $\log(W(x)) = \log(x) - W(x)$. Using this fact, we have:

$$-1 + \xi_j - \rho c_j - \log(E_j) - W\left(\exp\left(-1 + \xi_j - \rho c_j\right)E_j^{-1}\right) = \xi_j - \rho p_j - \log(E_j)$$

Which can be solved for the optimal price:

$$\frac{1}{\rho} + c_j + \rho^{-1}W\left(\exp\left(-1 + \xi_j - \rho c_j\right)E_j^{-1}\right) = p_j^* \quad (16)$$

A.11 A Microfoundation for the Organization Complexity Measure

This section provides a full microfoundation for thinking of mutual information as complexity when there are many tasks to be assigned.

Suppose the process of producing each unit of the final product consists of T steps. Each step is a type of task $k = 1, \dots, K$ that is randomly drawn i.i.d. from the task-mix α . Denote the random vector of task-types X^T . A firm must choose a business plan, which consists of task categorization system f_T and an employee handbook g_T , which assigns task categories to workers. f_T is a vector-valued function which maps every possible combination of T tasks to an index, where without loss we say the index is of the form $\{1, 2, \dots, 2^{T \cdot I}\}$. g_T is a function which maps this index back into a vector of length T where element t specifies which worker type ($i = 1, \dots, N$) does task t . The firm pays the wage cost as before. Average quality is similar to before, but with more notation:

$$\sum_{t=1}^T Pr(X^T = x^T) T^{-1} \sum_i \theta_{x_i^T, g_T(f_t(x^T))}$$

The complexity of the business plan is the number of contingencies that need to be written in the employee handbook per step:

$$\log_2(2^{T \cdot I})/T = I$$

Organization cost is then complexity times the firm-specific organization cost coefficient γ_j . Equivalence to a rate-distortion problem allows us to say that as $T \rightarrow \infty$, opti-

mal B_j from the original problem approximates the organization complexity and quality-adjusted wages which maximize profit in this more general problem. Put another way, each optimal B_j approximates the optimal business plan. This is quite interesting because the business plan is an even more complicated object than B_j , as it provides an assignment of tasks to workers for every possible realized set of tasks. Additionally, we can think of $I(B_j)$ as measuring the length of the business plan needed to implement a certain organization structure. This is why we call $I(B_j)$ complexity.

A.12 Extensions

Many of the modeling assumptions are made solely for tractability or to match the hair salon application. The core idea behind the model is general, and this section outlines several extensions which accommodate other contexts and additional economic forces. I also provide ways to use recent work in labor economics, rational inattention, and rate-distortion theory to implement these extensions in future work.

A.12.1 Labor Market Power

The model presented in this paper focuses on situations where firms have product market power but not labor market power. These assumptions are realistic when the product market is small relative to the labor market, either geographically or due to worker's ability to work in multiple industries. In many situations such assumptions are not realistic and we might expect firms to hold labor market power as well.

Introducing labor market power raises an interesting theoretical question which could make it the most important area for future work. Firms with labor market power have an incentive to reduce the number of workers they hire in order to markdown wages. How does this incentive interact with internal organization, and how does it change competition? Unlike firms in a competitive labor market, firms with labor market power will realize that demanding more of a certain type of worker increases wages.

Such an extension has the potential to help us understand two features of modern labor markets. First, we can measure the amenity value of task specialization to workers. In some industries, workers may find a specialized job unfulfilling or limiting, restricting their long term career goals by pigeonholing them. However in others, workers may find specialized jobs valuable because they deepen expertise. Second, in highly concentrated industries, we can study how internal organization choices are driven by a desire to make workers scarce for competitors. Anecdotal accounts in the technology sector suggest such talent wars occur. My model provides a way to study the trade-off between over-hiring a

certain type of worker while still trying to operate the firm.

A model with labor market power could be made tractable by assuming monopolistic competition in the product market and monopsonistic competition in the labor market. The labor market could be modeled using the framework introduced in Card et al. (2018). The novel internal organization cost introduced in this paper extends to such a model. However, because output would impact marginal costs (through wage markdowns) the characterization in Theorem 1 will no longer hold. We will need new tools to solve and estimate the model. Because the new problem will be a non-linear rational inattention problem, results from Jung et al. (2019) may be helpful in this regard.

A.12.2 Large Firms

The model can be extended to the case where firms are “large,” with a continuum of tasks and worker types.

Consider a firm which must complete a continuum of tasks to produce the final good. The task-mix is now a distribution, which I assume to be normal: $k \sim N(0, \sigma^2)$. Suppose workers have a single specialty task, and that they are indexed by i in the order of their specialty task. An organization structure B is now a continuous bivariate joint distribution.

Suppose the quality of a performed task is given by the squared distance between the specialty of the worker and the task assigned, that is $\xi = -\int (i - k)^2 dB(i, k)$ and denote $D = -\xi$. Finally, assume all workers have the same wages (skills are not priced by the market).

It can be shown that the organizational frontier in this special case has a closed form, and an organization structure B which maximizes profit is:

$$\binom{i}{k} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 - \ln(2)\gamma\rho & \sigma^2 - \ln(2)\gamma\rho \\ \sigma^2 - \ln(2)\gamma\rho & \sigma^2 \end{bmatrix}\right)$$

To interpret this result, note that as γ approaches 0, the correlation between tasks and workers approaches 1 and the marginal distribution of hired worker types widens and approaches the distribution of tasks. In other words, the firm assigns each task entirely to the appropriate specialist. Whenever there are positive organization costs, the task-content of a worker of type i is a normal distribution centered on their specialty with variance $\sigma^2 - \ln(2)\gamma\rho$. Greater organization costs or higher price sensitivity reduce task-specialization. This illustrates two things. First, it is easy to extend the model to accommodate the large firm case, where the task space is uncountable (as is the worker type

space). Second, the key role of organizational friction is a deep property of the model that does not go away when we make organizations large and less “lumpy.”

A.12.3 A Quantity-Based Model

In some contexts, like manufacturing, we may wish to model organizational efficiency as allowing firms to produce greater quantity rather than greater quality. Indeed, this is the default definition of productivity in economics. The model can also be extended to accommodate this: we simply interpret the skill sets as denoting the amount of time required by the worker to complete task k (therefore smaller $\theta_{i,k}$ are better). Then the production function becomes a function of organization structure:

$$F_{\alpha,B}(a_j) = \min \left\{ \frac{a_1}{\alpha_1 \sum_i \theta_{i,1} B_j(i,1)}, \dots, \frac{a_k}{\alpha_k \sum_i \theta_{i,k} B_j(i,k)}, \dots, \frac{a_K}{\alpha_K \sum_i \theta_{i,K} B_j(i,K)} \right\}$$

Given any fixed organizational structure, the efficient way to produce a single unit of output is to set $a_k = \alpha_k \sum_i \theta_{i,k} B_j(i,k)$. Thus the per-unit wage bill is given by:

$$\sum_i W_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k)$$

Marginal costs are constant and consist of the per-unit wage bill and organization costs:

$$MC_j = \sum_i w_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k) + \gamma_j I(B_j)$$

All of the benefits of a more complex organization then come through a reduction in the per-unit wage bill. The logic of the original model extends directly to the quantity case: firms with greater organizational efficiency (lower γ_j) can produce more of the good with the same workforce. Some readers may wonder why I did not use this as the main model. My reason was based on the following property of the quantity productivity model.

Proposition 8 *Under the quantity model, prices are decreasing with organizational complexity.*

The proof of this proposition is given in the next paragraph. Intuitively, under the quantity model with logit demand, all the benefits of a complex organization come from greater output rather than from greater revenue per unit. The reduction in marginal cost outpaces the increase in the markup, resulting in lower prices. This property may be desirable in some settings, like manufacturing, but Stylized Fact 2 shows it is not true for hair salons.

Proof. Equation 16 from an earlier proof provides a closed-form expression for price in any Nash Equilibrium under logit demand:

$$\frac{1}{\rho} + c_j + \rho^{-1} W\left(\exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right) = p_j^*$$

Taking the derivative w.r.t. c_j :

$$\frac{\partial p_j^*}{\partial c_j} = 1 - \exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1} W'\left(\exp\left(-1 + \xi_j - \rho c_j\right) E_j^{-1}\right)$$

A property of the LambertW function is that:

$$W'(x) = \frac{W(x)}{(1 + W(x))x}$$

Thus we can simplify the expression to:

$$\frac{\partial p_j^*}{\partial c_j} = 1 - \frac{W(\exp[-1 + \xi_j - \rho c_j] E_j^{-1})}{1 + W(\exp[-1 + \xi_j - \rho c_j] E_j^{-1})}$$

The Lambert W function is weakly positive for values which are weakly positive, therefore the derivative is positive, and price is decreasing in cost. The firm minimizes cost:

$$\min_{B \in \mathbb{B}} \gamma I(B_j) + W(B_j)$$

This is again a rate-distortion problem. Denoting the optimal wage-bill as $D = W(B_j^*)$, we can reformulate the problem as before, with the firm choosing D given some optimal organization cost and wage bill:

$$\min_D \gamma I(D) + W(D)$$

where I and W are expressed as functions of D instead of B_j . Then, as before, we have a negative cross-partial derivative:

$$\frac{\partial \gamma I(D) + W(D)}{\partial D \partial \gamma} = I'(D) < 0$$

with strict inequality whenever $I(D)$ is strictly positive. This establishes strict decreasing differences of D in γ , thus D is strictly decreasing in γ , and since $I(D)$ is a strictly decreasing function, it is also strictly decreasing in γ . Therefore we have that prices should be decreasing as γ decreases, while complexity should be increasing.

A.12.4 Non-Additive Quality

The model developed in this paper required the effect of the quality of each individual task to have an additive impact on overall quality. This assumption is natural in some settings, but unnatural in others. An excellent example is the launching of space shuttles. A single task performed poorly can be catastrophic, as illustrated by the Challenger explosion. In these contexts, nonlinear quality aggregation is necessary to model the production process. I can accommodate this within the model using multiplicative quality, similar in spirit to Kremer (1993):

$$\xi_j = \prod_{i,k} \theta_{i,k}^{B_{j,i,k}}$$

Re-write using logarithms:

$$\xi_j = \exp\left(\sum_{i,k} B_{j,i,k} \log(\theta_{i,k})\right)$$

This is now an f-separable distortion measure, meaning we can apply recent work in information theory (Shkel and Verdú 2018) to adapt the Blahut-Arimoto algorithm and other tools to work with this extended model.

A.12.5 Quality Positioning and Richer Demand Systems

One surprising result from the theoretical section is that the choice of organization structure depends only on other firm choices via wages. This derives from the independence of irrelevant alternatives property of logit demand. In my setting, this property manifests as consumers substituting uniformly to other products regardless of quality. In some contexts this may be unrealistic, and we may believe that there is a “positioning effect,” where the return to higher quality depends in part on how many other firms are also producing high quality. This section illustrates that this effect can be incorporated using mixed logit demand systems if a researcher is willing to sacrifice analytical tractability and incur greatly increased computational requirements.

Suppose consumers differ in their taste for quality. The utility of consumer z for product j is now given by:

$$u_{z,j} = q_z \xi_j - \rho p_j + \epsilon_{z,j}$$

where q_z is distributed i.i.d. across consumers according to some distribution G . This utility specification now nests both pure vertical and pure horizontal differentiation models.

When we specify that $\epsilon_{z,j}$ is type-1 extreme value and q_z is normally distributed the result is a random coefficients logit model. Market share for product i among consumer segment z is given by:

$$s_{j,z} = \frac{\exp(q_z \xi_j - \rho p_j)}{\sum_{j'} \exp(q_z \xi_{j'} - p_{j'})}$$

To understand the effect of quality on market share, we can compute the derivative:

$$\frac{\partial s_j}{\partial \xi_j} = \int q_z s_{j,z} (1 - s_{j,z}) dG(z)$$

Two facts are apparent from this expression. First, the marginal revenue from increasing quality now depends on the quality position of other firms. Firms will find it more beneficial to raise quality when high quality segments are relatively untapped by other firms. Second, the optimal organizational structure B_j will depend on the equilibrium quality choices of other firms.

The cost of this more flexible demand system is tractability. Because of the dependence on the quality choices of other firms, the characterization in Theorem 1 no longer holds. In particular, the firm's problem is not a rate-distortion problem. Estimation requires solving the model for each firm using nonlinear convex optimization. Additionally, demand no longer takes a multinomial logit form, so there does not exist a closed-form solution relating market shares, prices and unobserved qualities. Estimation now also requires numerical integration and and a BLP-style contraction mapping to invert market shares.

B Empirical Appendix

B.1 Task Classification Process: Further Details

A licensed cosmetologist was paid to categorize 20,560 salon services performed according to their descriptions. As part of the agreement, the person provided a picture of their cosmetology license. The cosmetologist was provided with a blank spread sheet with pre-defined subcategories and was instructed to mark all subcategories where the description matched with a one. They were instructed that some subcategories may not be mutually exclusive, so they should mark all that applied. The initial job description was as follows:

I have a list of approx. 20,560 short descriptions of salon services (mainly hair salons, but also some nail/spas). I would like someone with knowledge of the industry to mark whether each descriptions fits into one of several categories (male/female service, coloring, cutting, highlighting, washing, etc). This amounts to putting a 1 in each column that fits the description.

In a follow-up message I further clarified:

Here are the descriptions. I did the first few to give you a sense of the task. Basically read the description and then put a 1 in all categories that fit. Sometimes a description may match many, sometimes 1, rarely none. If you start reading them and see that it may be worth adding a separate category let me know. The idea though is to capture the core "tasks" or services performed at hair salons, like cut, color, highlight, style, etc and also to get some info on gender and typos.

After the first draft was submitted, I checked the coding, looking for any mistakes or missed descriptions and sent the document back to the cosmetologist several times for revision. A sample from the final spreadsheet is displayed in Figure B1.

	B Service Description	C Typo	D Cut	E Shave	F Style	G Extensions	H Color/Bleach	I Highlight/Blayage	J Other Treatment	K Blowdry	L Wash/Shampoo	M Eyebrow/Eye Lash Service	N Admin/Consult	O Spa Service	P Nail service	Q Male	R Female	S Child	T No Info
1319	'Add-on Camo															1			
1320	'Add-on Clipper cut (for booking with other Fi	1								1							1		
1321	'Add-on Color (10Z)							1									1		
1322	'Add-on Color Melt							1									1		
1323	'Add-on Curling Iron									1							1		
1324	'Add-on Extra 20oz Lightener										1						1		
1325	'Add-on Haircut		1															1	
1326	'Add-on Haircut - (Thick hair)							1									1		
1327	'Add-on Haircut - with Color								1								1		
1328	'Add-on Haircut - (Fine Hair)									1							1		
1329	'Add-on Iron Service									1								1	
1330	'Add-on Kids Cut (when added to service of a	1																	1
1331	'Add-on Lightener										1							1	
1332	'Add-on Olaplex										1							1	
1333	'Add-on Pixie cut										1							1	
1334	'Add-on Salon Metal Detox																1		
1335	'Add-on Scalp Treatment											1					1		
1336	'Add-on Service Only																1		
1337	'Add-on Toner										1							1	
1338	'Add-on Treatment											1						1	
1339	'Added Base										1							1	
1340	'Added Biologic Color										1							1	
1341	'Added Color For Eyebrows																1		
1342	'Added Deep Conditioner										1							1	
1343	'Added Elumen Glaze											1						1	
1344	'Added Glaze										1							1	
1345	'Added Highlight											1						1	
1346	'Added K18											1						1	
1347	'Added Nectaya Single																	1	

Figure B1: Final Task Sub-Categorization Spreadsheet from Cosmetologist

Since the subcategories were very detailed, I hired the same cosmetologist, at a rate of \$100, to classify the subcategories into six task categories. The specific instructions given to the cosmetologist were as follows:

Please categorize the 13 tasks from before into "groups." For the 6 group column, put the 13 tasks into 6 groups that are most similar in terms of who would do them/tasks they would require. for example, if color and highlight are similar, mark both as number 1. Number the groups 1 through 6. For the four group column, make 4 groups, etc. Underneath, please write a small note describing why you put the tasks together the way you did.

The final categories in their original spreadsheet are provided in Figure B2. The cosmetologist's reasons for grouping subcategories together are provided underneath the appropriate column.

	A Task Category	B Task Group (6 Groups)	C Task Group (5 Groups)	D Task Group (4 Groups)
1	Cut	4	1	1
2	Shave	4	1	1
3	Style	3	1	1
4	Extensions	2	2	1
5	Color/Bleach	1	1	1
6	Highlight/Balayage	1	1	1
7	Other Treatment	3	3	2
8	Blowdry	3	1	1
9	Wash/Shampoo	1	1	1
10	Eyebrow/Eye Lash Service	6	5	3
11	Admin/Consult	5	4	4
12	Spa Service	6	5	3
13	Nail service	6	5	3
14				
15				
16				
17				
18				
19	Notes:	1 - Hair colouring services to the hair performed by a hairstylist. 2- Hair service that could be completed by a full cosmetologist or someone that is trained only in this service. 3. Hair service that could be performed by licenced cosmetologist but also an assistant / trainer or a service specific individual such as an aesthetics practitioner. 4. Hair cutting service typically performed by a hairstylist or barber. 5.A sevice that could be performed by either a hairstylist OR a Non licenced manager/ front of house staff member. 6. A beauty service to the body. Typically performed by an esthetician.	1 - Hair service only done by a licenced hairstylist, assistant hairstylist or barber. 2- Hair service that could be completed by a full cosmetologist or someone that is trained only in this service 3. Service that could be performed by a licenced professional or an assistant / trainee. 4. A sevice that could be performed by either a hairstylist OR a Non licenced manager/ front of house staff member. 5.A beauty service to the body. Typically performed by an esthetician.	1- Under most circumstances this service would be performed by a Hairstylist, as they are services to the hair/ scalp. 2. This treatment/ service could be performed by varied amount of staff members from esthetics practitioner to hairstylists. 3. These services would typically be performed by an esthetician as they are services to the face / body 4. This service could be performed by licenced professionals or non licenced team members such as front of house staff.

Figure B2: Final Task Sub-Categorization Spreadsheet from Cosmetologist

B.2 An Alternative Task Category Generation Process

The analysis in this paper relies on professionally categorized descriptions. This section describes an alternative natural language processing approach used to generate task categories. This process has the benefit of being less labor intensive and not reliant on human judgment. It has the disadvantage of making certain types of classification errors. An earlier version of this paper used the NLP method. Many of the stylized facts continue to hold using this alternative method of classification.

In order to minimize issues related to spelling and grammar, I cleaned the descriptions using standard natural language pre-processing techniques. This reduces issues related to spelling errors. Some descriptions contain only punctuation, symbols or “stop words” and as a result are removed from the analysis.

To group descriptions into tasks, I used a hierarchical clustering method where the only choices are the number of categories and the measure of distance. I set the number of categories to be four and the distance measure to be the euclidean distance.²⁷ Figure B3 presents word clouds which capture the most common words appearing in each category. The size of a word in the cloud is roughly proportional to the number of times the word appeared in a unique description.

27. The quality-specialization relationship which inspired this paper is robust to both the number of categories and the distance metric.

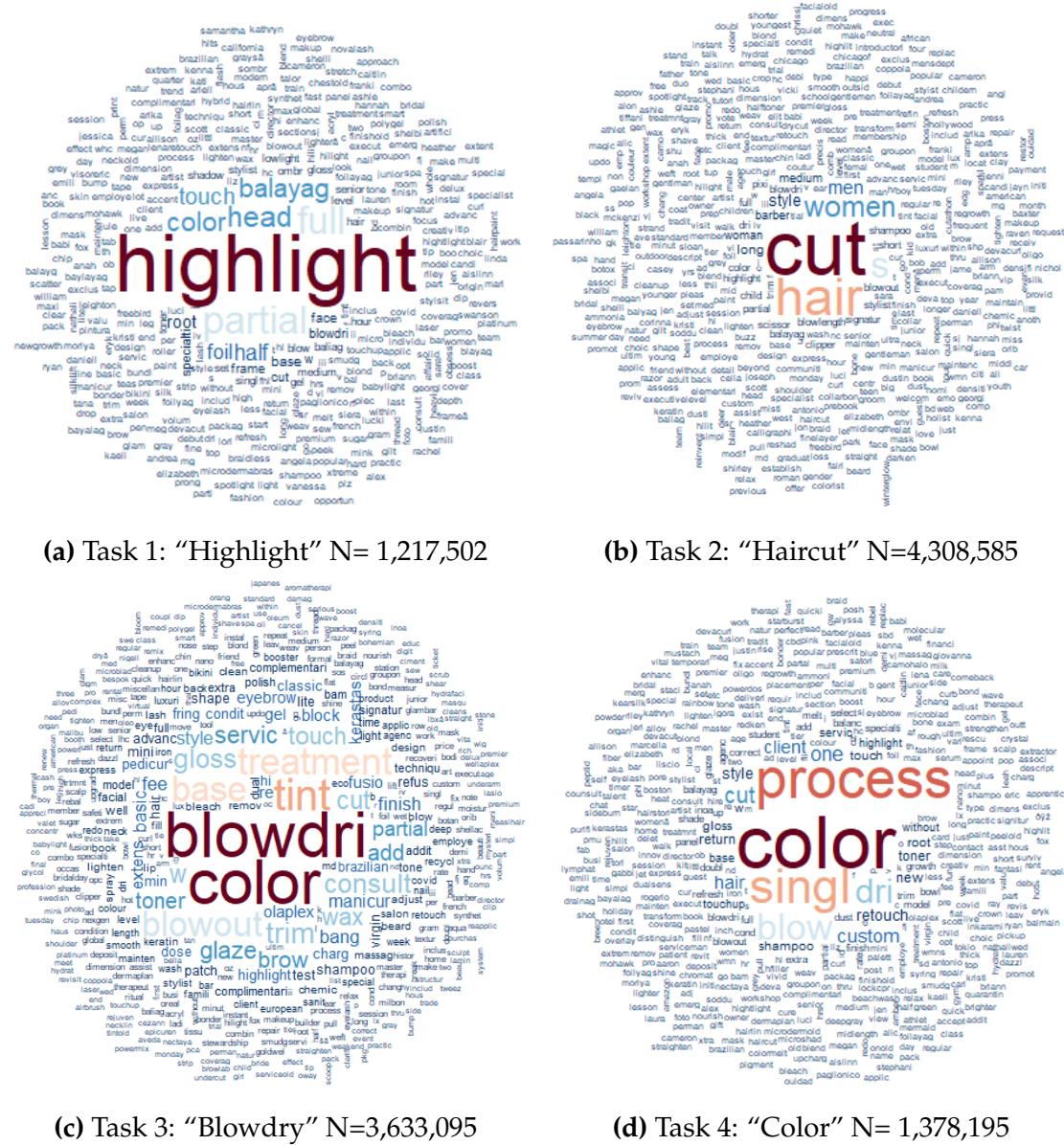


Figure B3: Tasks Generated from Hierarchical Clustering

The four tasks roughly correspond to three services: coloring ('single process'), hair cutting, blow drying and highlighting.²⁸

28. This is born out in the word clouds, but also if one looks at the modal description in each category.

B.3 Firm Size and Complexity Associations: Raw and Robustness

Figure B4: Organization Complexity and Firm Size

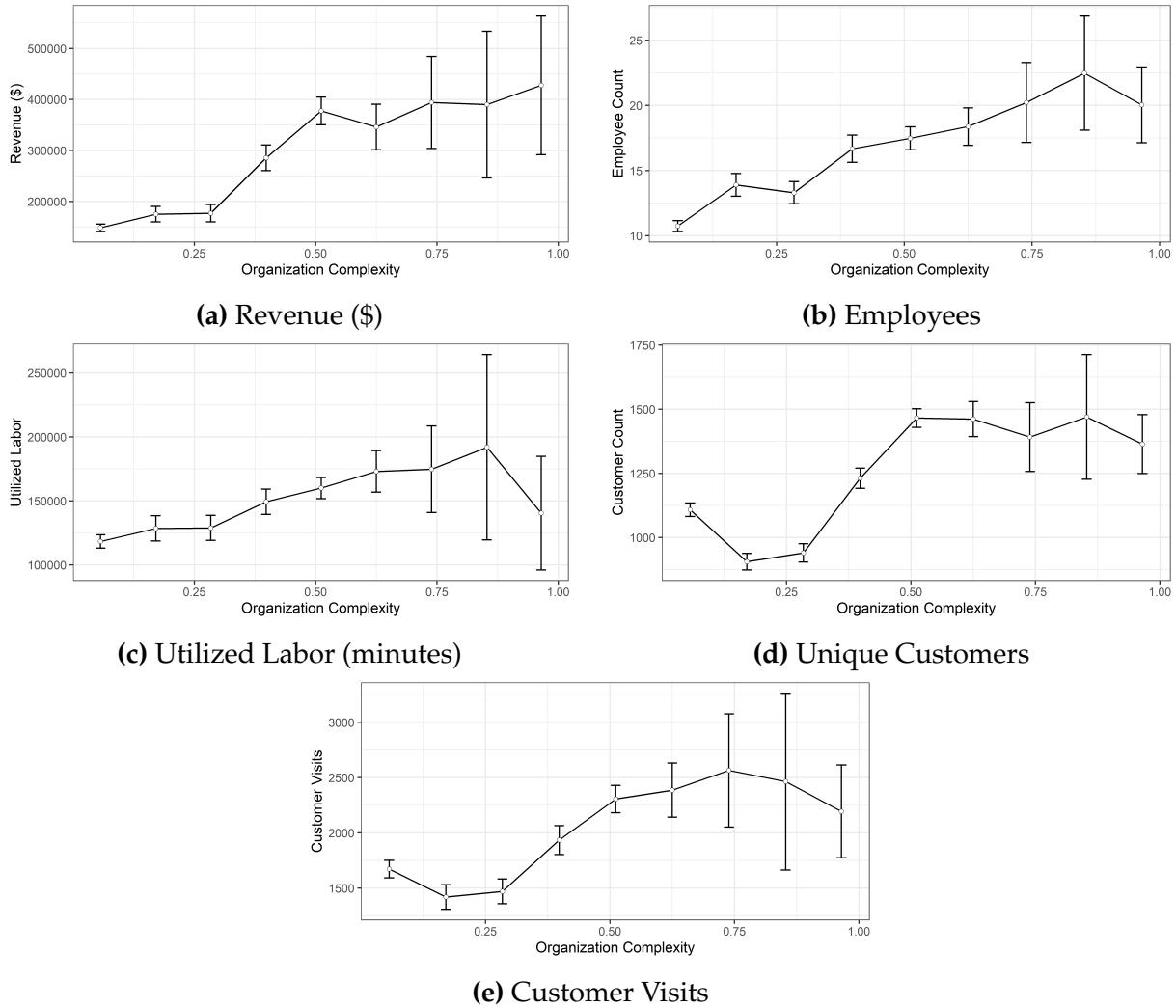


Table B1: Regressions of Firm Size on Complexity, Manhattan Only

Dependent Variables:	Revenue (1)	Employees (2)	Utilized Labor (3)	Customers (4)	Visits (5)
<i>Model:</i>					
Org. Complexity	430406.6* (179977.4)	12.55 (6.531)	-17733.9 (70765.2)	277.2 (600)	876.9 (907.1)
<i>Variables</i>					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
<i>Fixed-effects</i>					
Observations	595	595	595	595	595
R ²	0.33485	0.21039	0.20359	0.44164	0.48831
<i>Fit statistics</i>					
<i>Clustered standard-errors in parentheses</i>					

Signif. Codes: ***: 0.001, **: 0.01, *: 0.05

Note: This table repeats the regressions of revenue and other measures of firm size on complexity, but only for New York County (Manhattan). The positive relationship between revenue and complexity remains statistically significant.

Table B2: Regressions of Revenue on Complexity and Employee Count Interacted

Dependent Variable:	Revenue		
Model:	(1)	(2)	(3)
<i>Variables</i>			
(Intercept)	79487.9*** (19103.3)		
Complexity	-226181.8* (111684)	-242961.5* (110939.4)	-320973.6** (117545.2)
Employee Count	5652.8* (2315.3)	4871.6* (2257)	3878.9 (2192.2)
Complexity × Employee Count	29487.9*** (8587.8)	30187.8*** (8507.4)	35052.8*** (8528.5)
<i>Fixed-effects</i>			
Quarter-Year		Yes	Yes
County			Yes
<i>Fit statistics</i>			
Observations	4,558	4,558	4,558
R ²	0.4913	0.52042	0.61654

Clustered standard-errors in parentheses

Signif. Codes: ***: 0.001, **: 0.01, *: 0.05

Note: This table presents regressions of revenue on complexity interacted with employee count. The mean number of employees is 13.38, so the marginal effects in all specifications evaluated at the mean are positive.

B.4 Task Content Variance Decomposition

Using the estimated model we can study the determinants of the task content of hair salon jobs in Manhattan. As a first step, we can decompose the variation in task content into a worker and firm component. Using the distribution of model generated jobs, we can write:

$$b_j(i, k) = \bar{b}(i, k) + (b_j(i, k) - \bar{b}(i, k))$$

I then adapt the method used by Song et al. (2019) to my setting. Fixing k and taking the distribution to be weighted by effective units of labor, we can then decompose the

variance into a worker type component and a within-worker type component, where recall ω_i is the share of total labor represented by workers of skill set i :

$$Var_{i,j}(b_j(i,k)) = Var_i(\bar{b}(i,k)) + \sum_i \omega_i Var_j(b_j(i,k)|i)$$

However, because $b_j(i,k)$ is generated by the structural model, and we are considering a single labor and product market, the within-type component comes entirely from variation in firm attributes. Therefore we have decomposed the total variance in task content into a worker and firm component. Dividing through by $Var_{i,j}(b_j(i,k))$, Table B8 shows the share of variance due to each component. For the main tasks (cut, color, blow dry) between 8 and 22 percent of variation in job task content is attributable to firms.

B.5 Bootstrap Procedure

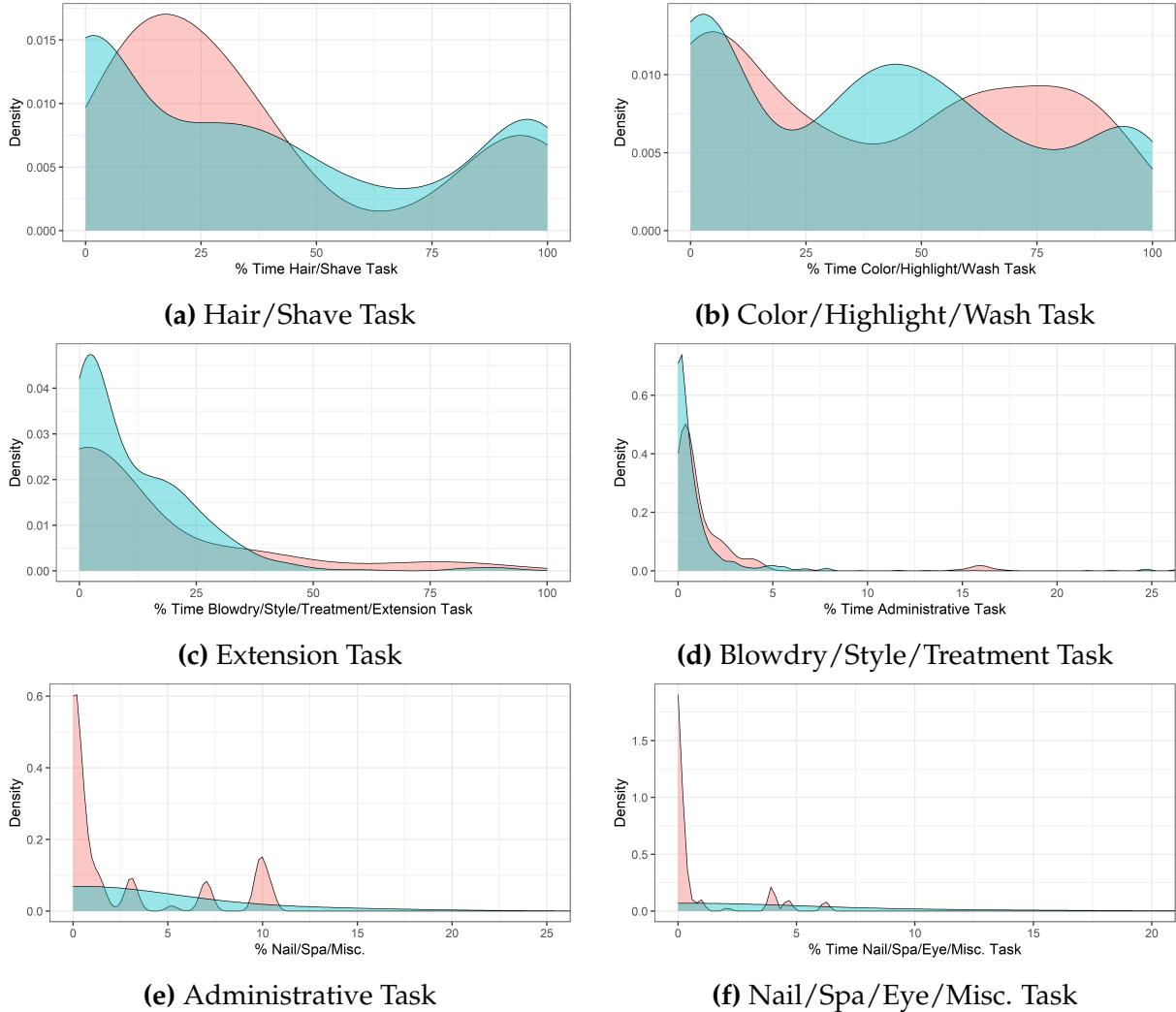
During each bootstrap replication, the model is fully re-estimated. The estimation procedure has three loops, which are run with slightly looser tolerances than the primary estimation algorithm. The inner most loop, which is the Blahut-Arimoto algorithm, is run with a convergence tolerance of 10^{-10} . The middle loop, which choose γ_j to match each firm's complexity, is run with a tolerance of 10^{-8} . The outer loop, which finds the market parameters, uses the Nelder-Mead method. Relative and absolute tolerance are set to 10^{-8} with a maximum number of iterations set to 4000.

Standard errors are computed as the sample standard deviation of the bootstrap distribution of each parameter. To check the stability of standard errors, I ran an additional 23 replications. The standard errors with these additional replications are within 4% of the reported standard errors.

B.6 The Full Distribution of Task Content

We can also go beyond the variance and compare the entire distribution of model and observed job task content. This is a strong test of the model, because we observe 509 stylists in Manhattan during 2021Q2 (the estimation period), and we are asking the model to match their jobs using only firm-based moments. Figure B5 plots the two distributions for each of the six tasks. Although the match is not perfect, the model is able to replicate important features of the data. As an example, in panel b, we can see that the fraction of time stylists spend on the coloring task is tri-modal in the data, with peaks at 0%, 40% and 90%. The model is able to approximate this pattern with a bimodal distribution, with a wide first peak that merges the 0% and 40% observed in the data.

Figure B5: Model (Red) vs. Observed (Blue) Job Task Content in Manhattan



B.7 Counterfactual Procedures

B.7.1 General Procedure

The general procedure used in all counterfactuals is as follows.

1. Weight each firm such that the observed total market share matches the share of people purchasing some amount of hair salon services in the Consumer Expenditure Survey. This means that each Manhattan salon in the data is assumed to represent 23 salons.²⁹

29. It is necessary to weight firms for counterfactual analysis but not estimation. This is because during estimation we fix an equilibrium, but in counterfactuals we must find a new equilibrium, and firm pricing strategies depend on the number of other firms in the market.

2. Compute the implied total labor supply of each worker skill set by summing all labor demands at initial wages over all firms.
3. Make the relevant parameter changes that correspond to the counterfactual.
4. Guess wages.
5. Solve for organization structure: If I allow internal reorganization, use the Blahut-Arimoto algorithm described in the estimation procedure to solve for each firm's organization structure. Otherwise maintain the same organization structure for each firm.
6. Compute optimal pricing: Given that the organization structures, qualities and costs of all firms are now known, optimal pricing is computed by iterating on each firm's best response pricing function until convergence.
7. Check labor market clearing by comparing the new labor demands to the total labor supply computed in step 3. If supply and demand for each type match, exit. Otherwise, return to step 4 and guess a new wage vector.

I assume that the exogenous quality (ν_j) and exogenous marginal cost (ϕ_j) remain the same in the counterfactual analyses. To solve for market clearing wages, I minimized the sum of squared excess labor demand. I used the L-BFGS-B routine and stop only when the objective is less than 0.1. This corresponds to a very close match between labor supply and demand. I found it more efficient to use a minimization routine because the labor demands of each worker type depend in a complex manner on the entire vector of wages. That is, while each labor demand is monotone decreasing in own-wage, firms can use other worker types as substitutes, so we cannot simply find each of the six market clearing wages sequentially.

Total welfare is defined as the sum of total wages, consumer welfare and total profit. Task specialization is defined as the total amount of labor spent on worker's specialty tasks. Reported average prices, qualities and complexities are at the firm-level, and are not weighted by market share. Wage statistics are weighted by the labor supply of each worker type.

B.7.2 Minimum Wage Technical Issues

For the minimum wage counterfactuals, it is necessary to discuss two additional technical issues: the numeraire good and multiple equilibria.

The numeraire good in my model is the outside option, which is not getting a haircut. I normalized its utility to be 0 in the random utility framework developed earlier. Wages are estimated using observed prices and hours, so they can be interpreted directly as nominal wages, or the wages we discuss in everyday language. When considering a counterfactual minimum wage increase to \$30, I require equilibrium wages to be at or above \$30, without any transformations. This is valid under the partial equilibrium assumption that the value of not getting a haircut remains the same before and after the minimum wage increase. In general equilibrium models, counterfactual minimum wage changes must be implemented more carefully (for example, Haanwinckel (2020)).

In general, a minimum wage can result in multiple equilibria. To ensure that there are not multiple equilibria, I solved the model under every possible permutation of binding minimum wages. That is, I assumed the minimum wage binds for worker types 1 and 2 only, 1,2 and 3 only, etc. With 6 worker types, this amounts to solving the model $2^6 = 64$ times. Each time I solved the model, I fixed the wages of the binding types at \$30, and then solved for the wages of the other types which clear the labor market for only those other types. Afterwards, I checked three things:

1. Worker types with non-binding wages have wages greater than \$30.
2. Worker types with binding minimum wages have excess labor supply.

Any solution which passed this check was considered a valid equilibrium. For example, for the case when the minimum wage is binding only for type 1, I set type 1's wage to \$30 up front, then solved for the other 5 wages which clear the market for the other five types. I then checked that types 1 through 6 have wages above \$30 and type 1's excess labor supply is positive.

This process indicates there is a unique equilibrium in both counterfactuals. For both the full adjustment and no adjustment counterfactuals, only one of the 64 cases satisfied the checks as a valid equilibrium. One additional case in the full adjustment counterfactual never converged, meaning I could not find wages that cleared the labor market for the non-binding worker types. The wages, employment and task specialization in the initial, reallocation and full equilibrium are provided in Table B6.

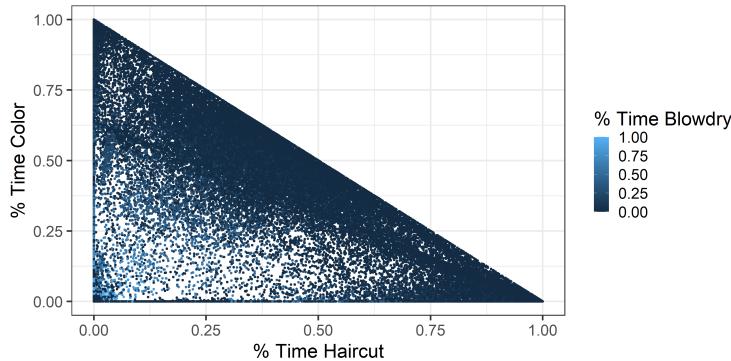
B.8 Job-Level Heterogeneity

Table B3: Job Task-Mix

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Share Haircut/Shave	62,671	0.387	0.344	0.000	0.008	0.669	1.000
Share Color/Highlight/Wash/Extensions	62,586	0.371	0.322	0.000	0.025	0.599	1.000
Share Blowdry/Style/Treatment	62,564	0.102	0.162	0.000	0.008	0.124	1.000
Share Administrative	62,702	0.061	0.168	0.000	0.000	0.027	1.000
Share Nail/Spa/Eye/Misc.	63,012	0.076	0.227	0.000	0.000	0.010	1.000

Note: This table displays summary statistics about the time spent on each task at the worker level. While worker averages correspond roughly to firm averages, there is greater heterogeneity across workers, supporting the idea that within firms there are distinct roles.

Figure B6: The Job Task-Mix Distribution



B.9 Flexible Labor-Labor Substitution

A key difference between the model in this paper and others in the task literature is that workers differ horizontally and firms differ in their organization costs and task-based production functions. These features allow for richer forms of labor-labor substitution, and as a result richer responses to policy change. Just as allowing for richer consumer substitution patterns is important for understanding the impact of policies on consumers, allowing for richer labor-labor substitution is important for understanding the impact of policies on workers.

In most models of task assignment, tasks can be ordered in a single dimension. For expositional purposes, let us call this dimension difficulty. Workers can also be ordered by their skill at completing difficult tasks. As articulated by Teulings (2000), this leads to distance-dependent complementarity, a strong assumption on substitution patterns.

Across all firms, workers closer in education are substitutes while those farther away in education are complements.

Distance-dependent complementarity restricts the effects of policies changes. As an example, consider a minimum wage increase in a model with competitive labor markets and only distance-dependent complementarity.³⁰ Beyond the direct effect for low skill workers, the policy has two effects. First, it causes labor-labor substitution towards the closest available substitutes (medium skill workers). Second, it increases costs, reducing prices and overall labor demand. Wage spillovers are decreasing in initial wage, potentially becoming negative near the top of the distribution. This is theoretically unambiguous and occurs regardless of model parameters.

In my model, distance-dependent complementarity does not hold. Instead, whether a worker is a substitute or a complement of the binding worker type varies from firm to firm and depends on the firm-specific organization cost and firm-specific task-based production function. Aggregate employment/wage effects then depend on how market share is distributed across firms. If there are two firms, and worker type A is a substitute for minimum wage workers at firm 1 and a complement at firm 2, whether the minimum wage increases or decreases the worker's wage depends on the market share of firm 1 vs. firm 2. This is why, when I apply the model to Manhattan hair salons, I find a minimum wage increase generates non-monotonic wage spillovers. I visualize these in Figure 17.

B.10 Supplementary Tables

Table B4: Parameter Estimates, Other

Parameter	Estimate
Price Sensitivity	0.04* (0.01)
Cost Intercept	27.95 (15.21)
Utility Intercept	-24.77* (8.36)

Note: Standard errors from 500 bootstrap replications in parentheses. * indicates significance at the 0.05 level. Consumer price sensitivity (ρ) is the main determinant of demand elasticities.

30. For an empirical example of such a model, see Gregory and Zierahn (2022).

Table B5: Variance Decomposition: Without a Model

Across Firms				Across Quarters			
Task	Share of Labor	Share of Variance		Task	Share of Labor	Share of Variance	
		Firm	Within-Firm			Quarter	Within-Quarter
Haircut/Shave	0.4049	0.3744	0.6256	Haircut/Shave	0.4049	0.0057	0.9943
Color/Highlight/Wash	0.3902	0.2899	0.7101	Color/Highlight/Wash	0.3902	0.0062	0.9938
Blowdry/Style/Treatment/Extension	0.0850	0.5056	0.4944	Blowdry/Style/Treatment/Extension	0.0850	0.0111	0.9889
Administrative	0.0590	0.4900	0.5100	Administrative	0.0590	0.0193	0.9807
Nail/Spa/Eye/Misc.	0.0610	0.4124	0.5876	Nail/Spa/Eye/Misc.	0.0610	0.0118	0.9882

Table B6: Minimum Wage Counterfactual Type-Specific Wages, Employment and Specialization

Worker Type	Initial			Reallocation			Reorganization		
	Hours	Wage	Task-Spec.	Hours	Wage	Task-Spec.	Hours	Wage	Task-Spec.
Haircut/Shave	537550	\$16.96	0.9463	506090	\$20.00	0.9459	502152	\$20.00	0.947
Color/Highlight/Wash	997053	\$37.75	0.7245	997053	\$37.33	0.7233	997053	\$37.52	0.7209
Blowdry/Style/Treatment/Extension	444040	\$20.91	0.4837	444040	\$21.88	0.4817	444040	\$21.64	0.4819
Administrative	41860	\$26.99	0.6801	41860	\$28.40	0.6807	41860	\$28.12	0.6809
Nail/Spa/Eye/Misc.	34844	\$81.16	0.8262	34844	\$81.63	0.826	34844	\$81.71	0.826

Note: This table displays employment and wage levels across the initial, reallocation and full equilibrium under a \$20 minimum wage. It provides context for the main counterfactual results, which are reported in percentages. In both counterfactual equilibria, the minimum wage is binding only for haircut specialists.

Table B7: Sales Tax Counterfactual Type-Specific Wages, Employment and Specialization

Worker Type	Initial			Reallocation			Reorganization		
	Hours	Wage	Task-Spec.	Hours	Wage	Task-Spec.	Hours	Wage	Task-Spec.
Haircut/Shave	537550	\$16.96	0.9463	537550	\$21.18	0.9471	537550	\$22.38	0.9491
Color/Highlight/Wash	997053	\$37.75	0.7245	997053	\$45.99	0.7326	997053	\$45.34	0.7432
Blowdry/Style/Treatment/Extension	444040	\$20.91	0.4837	444040	\$21.01	0.4946	444040	\$22.18	0.4982
Administrative	41860	\$26.99	0.6801	41860	\$30.15	0.6786	41860	\$31.85	0.6872
Nail/Spa/Eye/Misc.	34844	\$81.16	0.8262	34844	\$90.75	0.8351	34844	\$91.49	0.846

Note: This table displays employment and wage levels across the initial, reallocation and full equilibrium under the elimination of the service sales tax. It provides context for the main counterfactual results, which are reported in percentages.

Table B8: Model-Based Decomposition of Job Task-Content Variance

Task	Share of Task-Content Variance	
	Firm	Worker
Haircut/Shave	0.0761	0.9239
Color/Highlight/Wash	0.1194	0.8806
Blowdry/Style/Treatment/Extension	0.2180	0.7820
Administrative	0.0965	0.9035
Nail/Spa/Eye/Misc.	0.0865	0.9135

Note: The table displays a variance decomposition which uses the model to separate the variance of job task content into a worker and firm component.