

# The Inner Beauty of Firms

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# Motivation

- ▶ Organizational differences are well-documented across firms.
- ▶ **Motivating Question:** Do these differences matter?
- ▶ They determine firm ability to make use of individual talents.

# Motivation

- ▶ **Research Question:** How do differences in firm ability to assign workers to tasks interact with product and labor markets?
- ▶ **Empirical Challenge:** Rarely observe task assignments within competing firms.
- ▶ **Theoretical Challenge:** Do not have a model where firms with different abilities choose their structure based on product/labor market conditions.

# Summary of Results

1. **Facts:** Internal complexity is heterogeneous and firm-specific. Complex firms have  $\uparrow$  revenue, employees, prices, repeat customers.
2. **Theory:** The firm's optimal organization structure also solves a more tractable, well-understood problem.
3. **Trilemma:** Firms trade-off complexity, quality and wages when choosing an organization structure.
4. **Policy Experiments for Manhattan Hair Salons**
  - ▶ **Minimum wage to \$20:**  $\uparrow$  ( $\downarrow$ ) specialization of (non-)binding workers, + and – wage spillovers based on whether worker is employed alongside binding workers
  - ▶ **Service Sales Tax Elimination:** +10% service quality,  $\uparrow$  task-specialization, workers capture most welfare gains through higher wages

# Contributions

- ▶ **Task-Based Perspective:** Imperfectly competitive product market, horizontal worker skill heterogeneity, firm-specific organizational heterogeneity, activity data
  - ▶ Lazear 2009, Haanwinckel 2020, Adenbaum 2021
- ▶ **Organizational Heterogeneity:** Aggregate sources of differences and understand market impact
  - ▶ Baker, Gibbons, and Murphy 2002 (relational contracts), Garicano and Wu 2012 (knowledge), Dessein et al. 2003 (coordination), Meier, Stephenson, and Perkowski 2019 (trust), Martinez et al. 2015 (culture)

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# Data

- ▶ Salon management software company founded in 2016
- ▶ Clients concentrated in New York City and Los Angeles, but scattered salons throughout US
- ▶ Observe  $> 10$  million assignments of services to hair stylists across hundreds of salons from 2016 to Q3 2021

## A Data Snapshot

Firm	Salon	App.	Cust.	Service	Staff	Time Stamp	Price	Duration
1	1A	123	Blake	Advanced Cut	Rosy	3/26/2021 16:15	100	72
1	1A	123	Blake	Full Head - Highlights	Rosy	3/26/2021 16:15	243	127
1	1A	123	Blake	Treatment Add On (Olaplex)	Rosy	3/26/2021 16:15	39	72
2	2A	9982	Grace	Women's Cut	Tyler	3/17/2021 11:00	225	43
2	2A	9982	Grace	Single Process	Ben	3/17/2021 11:00	200	77



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- ▶ Services are aggregated to form one representative product per firm-quarter.
- ▶ A firm's **price** is the sum of service prices divided by total customers.
- ▶ A firm's **required labor** is the sum of durations divided by total customers.
- ▶ A firm's **task-mix** is the fraction of labor classified as each task.

# Creating Tasks

- ▶ 20,560 unique text descriptions of services.
- ▶ Hired a certified cosmetologist via UpWork to classify into 6 categories.
- ▶ For multi-category services: time is split across the tasks.
- ▶ Patterns are robust to method of classification (i.e. NLP classification)

# Firm-Quarter Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Revenue	4,558	213,201.30	248,359.90	5	58,912.5	271,236.5	2,559,703
Price	4,558	199.73	135.16	0.20	111.71	261.88	3,180.44
Employees	4,558	13.38	10.79	1	6	17	92
Customers	4,558	1,159.23	1,098.45	1	397	1,619	16,768
Task Categories	4,558	4.45	0.86	1	4	5	5
Labor per. Customer	4,558	2.15	1.63	0.10	1.52	2.57	61.33
Organization Complexity	4,558	0.22	0.22	0.00	0.03	0.42	1.02
Share Haircut/Shave	4,558	0.41	0.23	0.00	0.26	0.52	1.00
Share Color/Highlight/Wash	4,558	0.38	0.20	0.00	0.29	0.52	1.00
Share Blowdry/Style/Treatment/Extensions	4,558	0.09	0.12	0.00	0.03	0.11	1.00
Administrative	4,558	0.05	0.11	0.00	0.002	0.04	1.00
Nail/Spa/Eye/Misc.	4,558	0.06	0.16	0.00	0.00	0.05	1.00

Task-Mix Variation

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# What is an Organization Structure?

## Definition 1

A firm's **organization structure** ( $B_j$ ), is a matrix where element  $(i, k)$  is the fraction of labor assigned to worker  $i$  and task  $k$ .

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"Employee" Salon					"Chair Renter" Salon					
Employee	Tasks					Tasks				
		Cut	Color	Dry			Cut	Color	Dry	
	A	1/2	0	0	1/2	A	1/6	1/12	1/12	1/3
	B	0	1/4	0	1/4	B	1/6	1/12	1/12	1/3
	C	0	0	1/4	1/4	C	1/6	1/12	1/12	1/3
Tot.	1/2	1/4	1/4		Tot.	1/2	1/4	1/4		

Task-Mix

# What is Organizational Complexity?

## Definition 2

The complexity of an organization structure  $B_j$  is:

$$I(B_j) = \sum_{i,k} B_j(i, k) \log \left( \frac{B_j(i, k)}{\sum_{k'} B_j(i, k') \sum_{i'} B_j(i', k)} \right)$$

- ▶ **Intuition:** the amount of instructions (measured in bits) that must be communicated within the firm to implement  $B_j$ 
  - ▶ Data plan required to communicate assignments to employees
- ▶ **Several Micro-Foundations:** Manager Attention Task-Specialization Highly Correlated with Specialization

# Complexity of the Two Structures

"Employee" Salon				
Tasks				
Employee		Cut	Color	Dry
	A	1/2	0	0
	B	0	1/4	0
	C	0	0	1/4
	Tot.	1/2	1/4	1/4

Exactly match tasks and workers

If cut send "0" assign to A

If color send "01" assign to B

If dry send "10" assign to C

$$\frac{1}{2}(1\text{bit}) + \frac{1}{4}(2\text{bit}) + \frac{1}{4}(2\text{bit}) = 1.5$$

"Chair Renter" Salon				
Tasks				
		Cut	Color	Dry
A	1/6	1/12	1/12	1/3
B	1/6	1/12	1/12	1/3
C	1/6	1/12	1/12	1/3
Tot.	1/2	1/4	1/4	

Randomly match tasks and workers

If cut send nothing roll dice

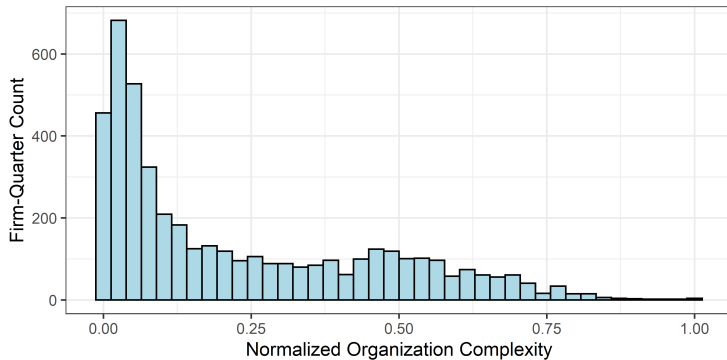
If color send nothing roll dice

If dry send nothing roll dice

$$\frac{1}{2}(0\text{bit}) + \frac{1}{4}(0\text{bit}) + \frac{1}{4}(0\text{bit}) = 0$$



## Fact 1: Complexity is heterogeneous and firm-specific.

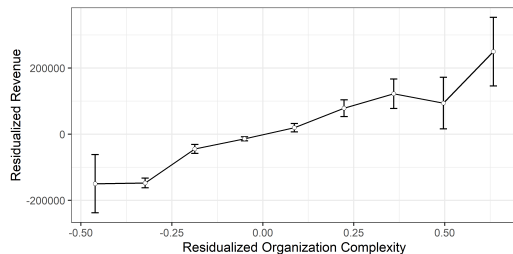


$$l_{j,t} = \bar{l}_j + \bar{l}_t + e_{j,t}$$

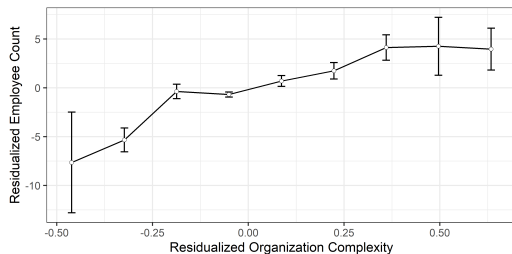
$$\begin{array}{cccccc} \text{Var}(l_{j,t}) = & \text{Var}(\bar{l}_j) & + & \text{Var}(\bar{l}_t) & + & 2\text{Cov}(\bar{l}_j, \bar{l}_t) & + & \text{Var}(e_{j,t}) \\ .0516 & & .0464 & & .0002 & & -.0009 & & 0.0059 \end{array}$$

**Takeaway:** Internal complexity is driven by a deep characteristic of the firm.

## Fact 2: Complex salons have higher revenue and employment



(a) Revenue



(b) Employees

Regressions

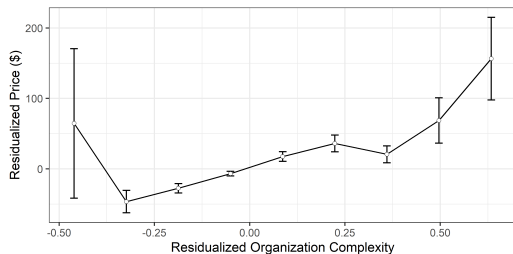
Manhattan

Manhattan Regs.

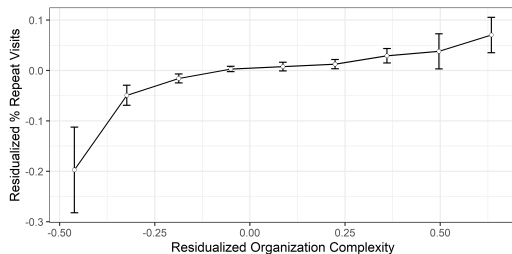
Customers and Visits

**Takeaway:** There is an organizational competitive advantage.

## Fact 3: Complex salons have higher prices and repeat customers



(a) Prices



(b) Repeat Customers

Manhattan Only

**Takeaway:** This advantage operates through quality NOT quantity. Theory

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# Model: Salons and Workers

## $J$ Salons

- ▶ Salon-specific internal organization cost  $\gamma_j \geq 0$
- ▶ Leontief task-based production function with parameter  $\alpha \in \mathbb{R}_+^K$ 
  - ▶ Producing 1 unit requires assigning  $\alpha_k$  labor to task  $k$ . Normalize  $\sum_k \alpha_k = 1$

## $N$ Worker Types

- ▶ Skill set  $\theta_i = \{\theta_{i,1}, \dots, \theta_{i,k}, \dots, \theta_{i,K}\}$
- ▶ Inelastic total labor supply  $L_i$  and wage  $w_i$  determined in equilibrium

# Model: Salon Choices and Consumers

## Salon Choices

- ▶ Org. structure  $B_j \in \Delta^{N \times K}$  s.t.  $\sum_i B_j(i, k) = \alpha_k$ 
  - ▶ Product Quality:  $\xi(B_j) = \sum_{i,k} \theta_{i,k} B_j(i, k)$
  - ▶ Per-Unit Wage Bill:  $W(B_j) = \sum_{i,k} w_i B_j(i, k)$
  - ▶ Per-Unit Internal Organization Cost:  $\gamma_j I(B_j)$  where  $I(B_j)$  is complexity
- ▶ Price  $p_j \in \mathbb{R}_+$

## Mass $M$ Consumers

- ▶ Utility for good  $j$ :  $u_{z,j} = \xi(B_j) - \rho p_j + \epsilon_{z,j}$ ,  $\epsilon \sim$  i.i.d. Type-1 E.V.

# The Firm's Problem

Denote feasible organization structures  $\mathbb{B}_j = \{B_j \in \Delta^{N \times K} \mid \sum_i B_j(i, k) = k\}$

$$\max_{p_j, B_j \in \mathbb{B}_j} \underbrace{\frac{\exp(\overbrace{\xi(B_j)}^{\text{quality}}) - \rho p_j}{\sum_{j'} \exp(\xi(B_{j'}) - \rho p_{j'})}}_{\text{market share, } s_j} \left[ p_j - \underbrace{\left( \overbrace{\gamma_j I(B_j)}^{\text{org.}} + \overbrace{W(B_j)}^{\text{avg. wage}} \right)}_{\text{constant marginal cost, } MC_j} \right]$$

# Equilibrium

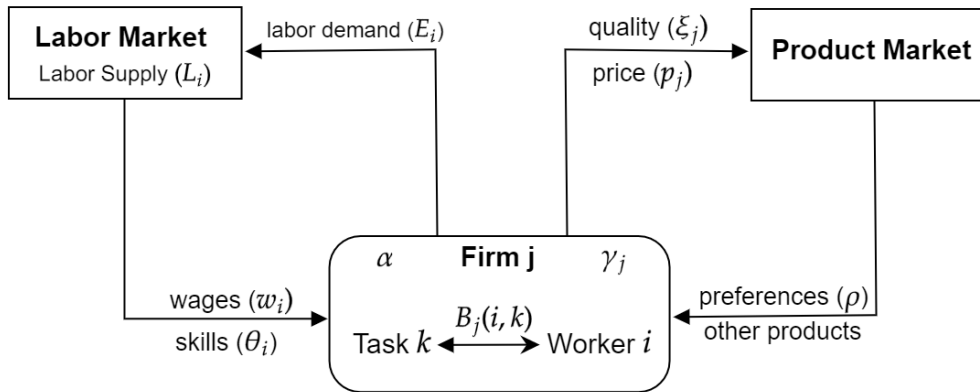
An equilibrium consists of firm strategies  $\{p_j, B_j\}_{j=1}^J$  and wages  $w$  such that:

1. Firm strategies maximize profits.
2. Labor markets for each worker type clear:

$$\sum_j \frac{\exp(\xi(B_j) - \rho p_j)}{\sum_{j'} \exp(\xi(B_{j'}) - \rho p_{j'})} \sum_k B_j(i, k) = L_i \forall i = 1, \dots, N$$



# Summary of the Model



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# Characterizing the Firm's Problem

## Theorem

*The profit-maximizing organizational structure  $B_j^*$  also solves:*

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} W(B_j) - (\gamma_j \rho)^{-1} \xi(B_j)$$

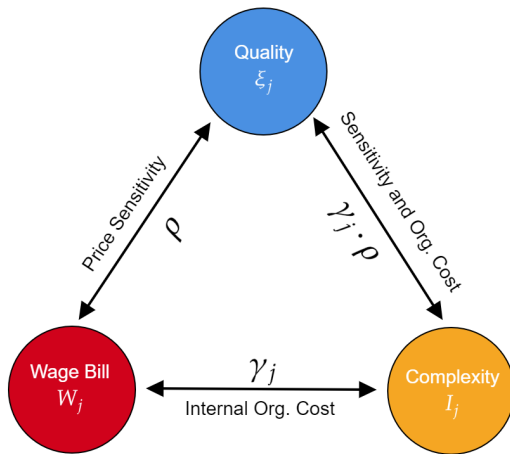
*which is a rate-distortion problem and a linear rational inattention problem.*

- ▶ Rate-distortion problems are well-studied in information theory.
- ▶ Rational inattention problems are well-studied in behavioral economics.
- ▶ Org. structure depends on other salon actions only indirectly via wages.

Proof

# The Complexity-Wage-Quality Trilemma

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} W(B_j) - (\gamma_j \rho)^{-1} \xi(B_j)$$



# Organization Frontier

$$\min_{B_j \in \mathbb{B}} \underbrace{I(B_j)}_{\text{complexity}} + \gamma_j^{-1} \left[ \underbrace{W(B_j) - \rho^{-1} \xi(B_j)}_{\text{quality-adjusted wages}} \right]$$

## Definition

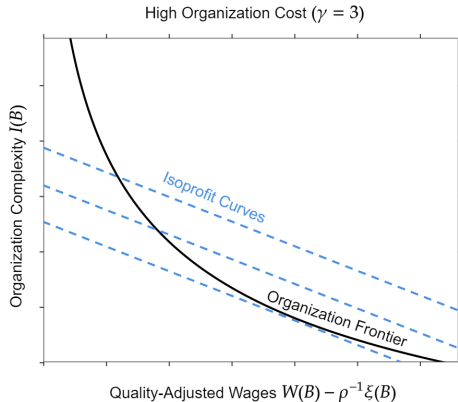
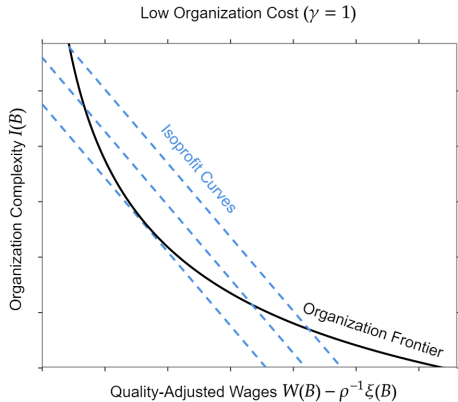
*The organization frontier is the set of organization structures which minimize complexity for some quality-adjusted wages.*

Because this is a rate-distortion problem:

## Proposition

*Complexity along the organization frontier is continuous, convex and decreasing in quality-adjusted wages.* [Proof](#)

# Choosing an Organizational Structure



[Back](#)

# Fitting the Facts

1. Fact 1: Complexity is heterogeneous and firm-specific
  - ▶ Firms in the same product and labor market choose different internal structures based on individual org. cost ( $\gamma_j$ ).
2. Fact 2: Complex salons have more employees and higher revenue
  - ▶ In equilibrium complexity and market share are positively correlated.
3. Fact 3: Complex salons have higher prices and higher quality
  - ▶ Price-complexity relationship depends on parameters.
  - ▶ Quality is the main benefit of complexity.

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# The Econometric Model

- ▶ **Market:** Analyze Manhattan 2021 Q2 with sales tax  $\tau = 4.5\%$ 
  - ▶ Outside option is no purchase. Compute via Consumer Expenditure Survey.
- ▶ **Workers:** Base skill  $\beta_k$ , skill gap  $\theta_k$ 
  - ▶ Color Specialist:  $\theta = \{\beta_{cut}, \beta_{color} + \theta_{color}, \beta_{dry}, \beta_{admin}, \beta_{misc}\}$
- ▶ **Labor Supply:** Individual workers also differ in their labor supply
  - ▶ For example: two color specialists may supply different number of hours.
- ▶ **Task Heterogeneity:** Different material costs ( $m$ )
- ▶ **Firm Heterogeneity:** Firm-specific task-mix ( $\alpha_j$ ), effective labor per unit ( $\bar{a}_j$ ), exogenous quality ( $\nu_j$ ), marginal cost shifter ( $\phi_j$ )

What does firm profit look like?

# The Econometric Model

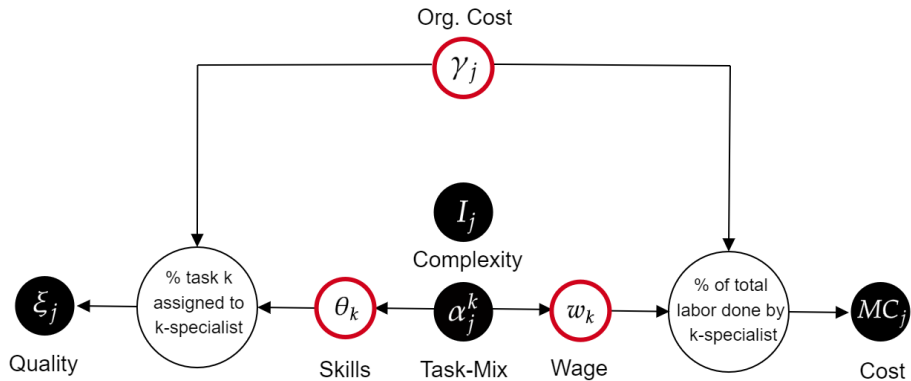
- ▶ A natural notion of task-specialization:

## Definition 3

Task-specialization is the fraction of total labor spent on specialty tasks.

- ▶ 6 utility + 5 cost + 5 wages + 5 skills = 21 parameters
- ▶ Call these market parameters and denote  $\Omega$
- ▶ 42 salons  $\implies$  42 org. cost parameters

# Identification Problem



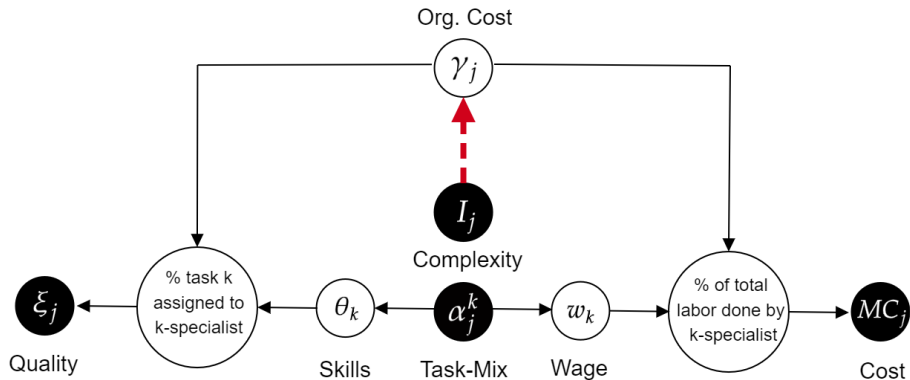
# Identification: Firm-Specific Organization Costs $\gamma_j$

## Proposition 1

*Organization costs ( $\gamma_j$ ) and organization structures ( $B_j$ ) are a known function of firm task-mixtures ( $\alpha_j$ ), complexities ( $I_j$ ) and market parameters ( $\Omega$ ) for all firms with positive complexity, except for a set of market parameters with measure 0.* Visual Intuition

- ▶  $\{\gamma_j\}_{j=1}^J$  do not need to be estimated
- ▶  $\alpha_j, I_j$  are sufficient for the internal organization of the firm
- ▶ There is a one-to-one mapping from observed complexity to unobserved  $\gamma_j$
- ▶ 2-part Proof: Complexity = Mutual Information Essential Equilibrium Uniqueness

# Identification: Firm-Specific Organization Costs $\gamma_j$



## Identification: Market Parameters

- Use firm price FOC (supply side moments):

$$p_j = \frac{1}{\rho(1+\tau)(1-s_j)} + \bar{a}_j \left[ \gamma(\Omega, l_j, \alpha_j) l_j + wE(\Omega, l_j, \alpha_j) \right] + m\alpha_j + \phi_j$$

- Use market-share equation (demand side moments):

$$\log(s_j) - \log(s_0) = \xi(\Omega, l_j, \alpha_j) - \rho(1+\tau)p_j + \beta\alpha_j + \nu_j$$

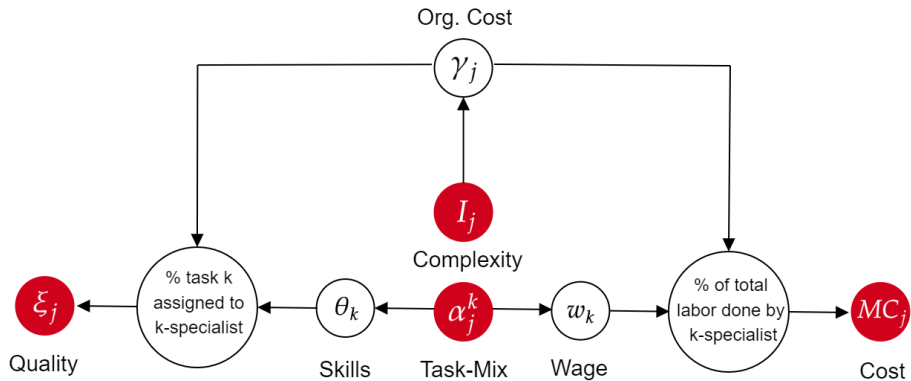
- Match avg. beauty salon QCEW wage bill with measurement error:

$$W_j = Ms_j a_j wE(\Omega, l_j, \alpha_j) + e_j$$

- The model is globally identified if  $\Omega$  uniquely satisfies:

$$\mathbb{E} \left[ \begin{pmatrix} \phi_j(\Omega, l_j, \alpha_j) \\ \nu_j(\Omega, l_j, \alpha_j) \end{pmatrix} \begin{pmatrix} \alpha_j & \alpha_j l_j \end{pmatrix} \right] = 0 \quad \mathbb{E}[e_j(\Omega, l_j, \alpha_j)] = 0$$

# Identification: Identification: Market Parameters ( $\Omega$ )



# From Identification to Estimation

## ► Issue

- $B_j$  is a  $5 \times 5$  matrix which solves a non-linear minimization problem
- Must solve for  $B_j$  repeatedly to get  $\gamma_j$
- Must solve for  $\gamma_j$  for each set of market parameters  $\Omega$

## ► Solution: Blahut-Arimoto algorithm

- Fixed point algorithm which is globally convergent
- Can use because of equivalence to a rate-distortion problem
- Algorithm



## Estimation Routine Summarized

Construct the sample analogue of the moment conditions, call it  $G(\cdot)$ . Then solve:

$$\arg \min_{\hat{\Omega}} G(\hat{\Omega})' W G(\hat{\Omega})$$

This amounts to:

1. Guess  $\hat{\Omega}$ .
2. Recover implied quality, marginal costs, and organization cost parameters using the Blahut-Arimoto algorithm.
3. Evaluate GMM objective. If minimum achieved, stop. Otherwise return to 1.

Nested fixed-point GMM, similar to Berry, Levinsohn, and Pakes (1995)

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# Task Parameter Estimates

Task	Associated Specialist			
	Skill Gap	Wage	Skill Base	Material Cost
Administrative	43.29*	26.99	-16.16	-147.60*
	( 21.66)	(63.75)	( 14.58)	( 13.47)
Blowdry/Etc.	141.69*	20.91	-70.56*	12.39
	( 36.67)	(40.22)	( 13.57)	( 16.65)
Color/Highlight/Wash	60.03*	37.75*	-9.69	56.49*
	( 21.24)	( 7.00)	( 11.97)	( 15.79)
Haircut/Shave	32.45*	16.96*	.	.
	( 13.07)	( 8.32)	.	.
Nail/Spa/Eye/Misc.	66.48	81.16	-252.58*	-1061.12*
	( 37.72)	(53.52)	( 11.47)	( 10.73)

Standard errors from 500 bootstrap replications in parentheses.

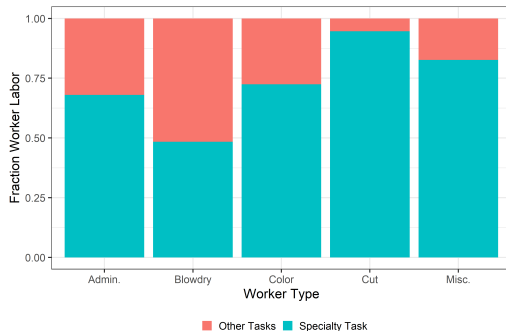
\* indicates significance at the 0.05 level.

# Equilibrium Task Specialization Across Workers

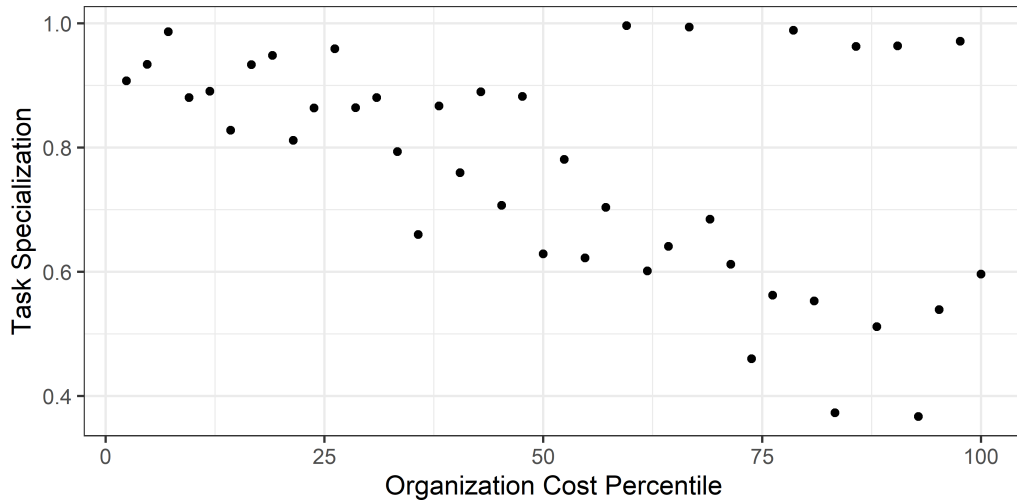
## All Tasks



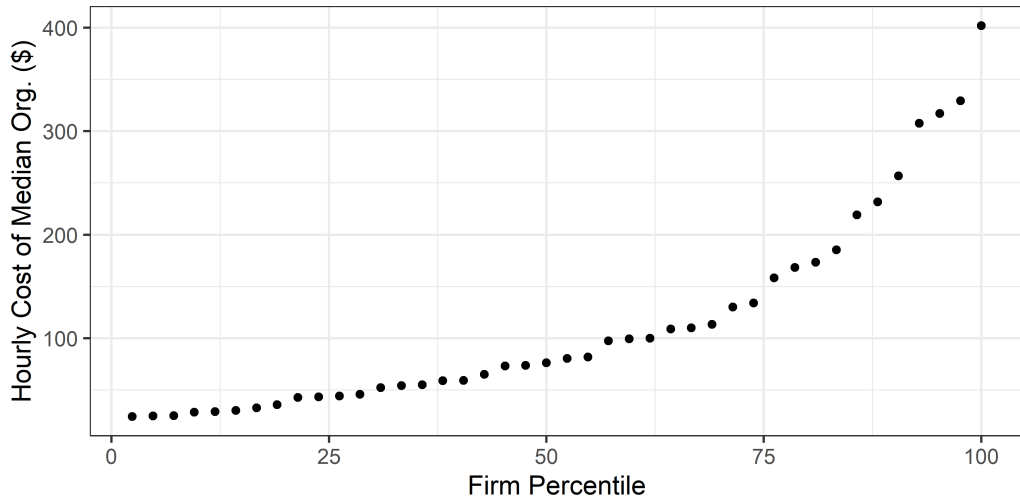
## Specialty v.s. Other Tasks



# Equilibrium Task Specialization Across Firms



# Cost of Median Complexity Organization Across Firms



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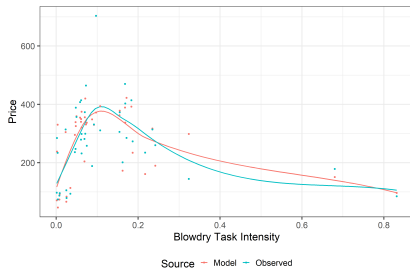
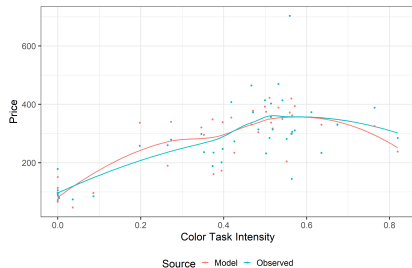
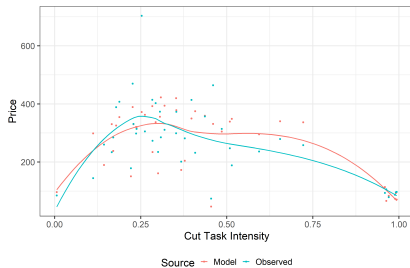
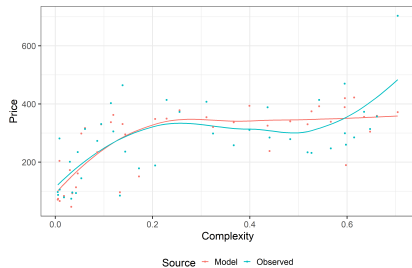
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# Fit: Supply Side Relationships





# Validation: The Task Content of Jobs

Model generated jobs:

$$b_j(i, k) = \alpha_k \frac{\exp(-\gamma^{-1}w_i + (\rho\gamma)^{-1}\theta_{i,k})}{\sum_{i'} E_j(i') \exp(-\gamma^{-1}w_{i'} + (\rho\gamma)^{-1}\theta_{i',k})}$$

Task	Total Variance		Between Firm Variance	
	Model	Observed	Model	Observed
Haircut/Shave	0.1110	0.1268	0.0597	0.0597
Color/Highlight/Wash	0.1127	0.1105	0.0365	0.0365
Blowdry/Style/Treatment/Extension	0.0472	0.0194	0.0111	0.0111
Administrative	0.0098	0.0080	0.0063	0.0063
Nail/Spa/Eye/Misc.	0.0120	0.0171	0.0050	0.0050

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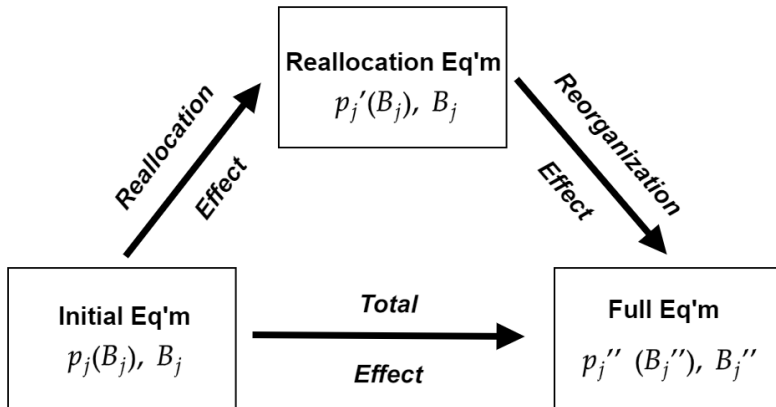
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# Counterfactual Implementation

- ▶ Preparation
  - ▶ Weight salons so that observed demand equals CEX market demand.
  - ▶ Derive type total labor using estimated wages and firm labor demands.
  - ▶ Change deep parameter.
- ▶ Solving for a New Equilibrium
  1. Guess wages, solve for organization structures.
  2. Use org. structures to solve for prices.
  3. Check labor market clearing, if not return to 1 and repeat.
  4. Compute consumer welfare using closed form.

# Decomposing Mechanisms



# Minimum Wage Increase from \$15 to \$20

## Wages Changes

Type	Wage Change	Total Wages Gained/Lost
Haircut/Shave - UNEMPLOYED	-100.00%	-\$600,240
Haircut/Shave - EMPLOYED	17.95%	\$1,528,205
Color/Highlight/Wash	-0.61%	-\$228,453
Blowdry/Style/Treatment/Extension	3.48%	\$323,374
Administrative	4.17%	\$47,154
Nail/Spa/Eye/Misc.	0.68%	\$19,319

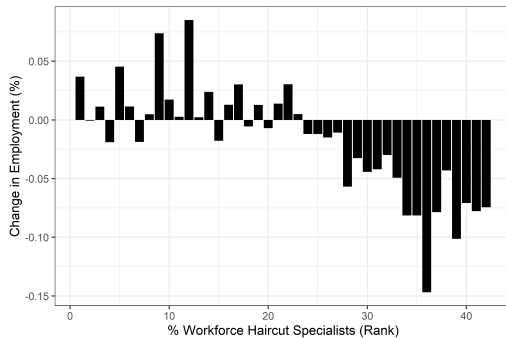
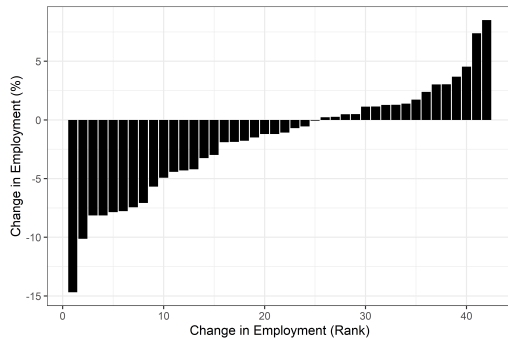
## Welfare

Source	Change	Percent Change
Salon Profit	-\$714,413	-0.472%
Consumer Welfare	-\$2,528,784	-1.671%
Employed Wages	\$1,689,600	1.116%
Unemployed Wages	-\$600,240	-0.397%
Total Welfare	-\$2,153,838	-1.423%

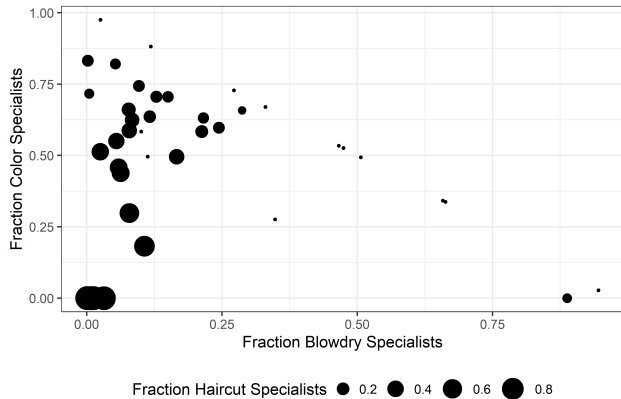
Employment and Wages

Technical Details

# The Reallocation Effect



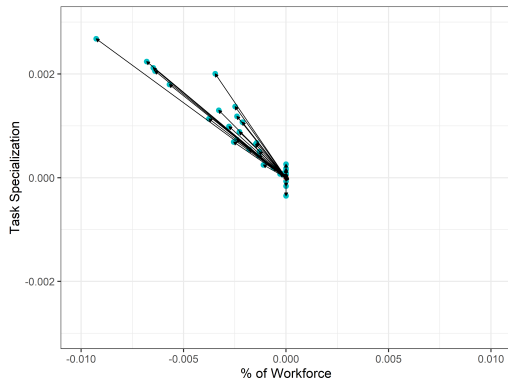
# The Reallocation Effect: Wage Spillovers



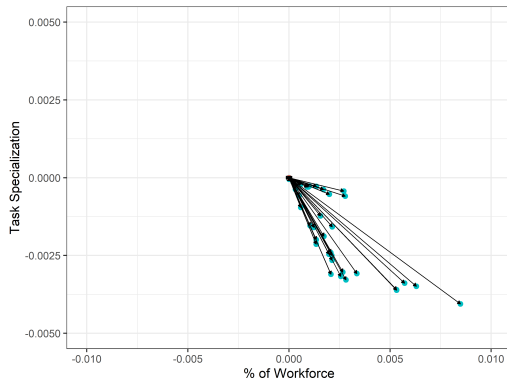
Type	Reallocation Change	
	Employment	Wage
Haircut/Shave	-5.85%	17.95%
Color/Highlight/Wash	0%	-1.13%
Blowdry/Style/Treatment/Extension	0%	4.63%

# The Reorganization Effect

Haircut Specialists (Binding)



Color Specialists (Non-Binding)





## The Reorganization Effect: Wage Spillovers

Type	Reorganization Change		
	Employment	Task-Spec.	Wage
Haircut/Shave	-0.73%	0.12%	0%
Color/Highlight/Wash	0%	-0.33%	0.52%
Blowdry/Style/Treatment/Extension	0%	0.03%	-1.15%
Administrative	0%	0.03%	-1.05%
Nail/Spa/Eye/Misc.	0%	-0.00%	0.10%

## Service Sales Tax Elimination (4.5% to 0%)

Firm Choices		Welfare		
Statistic	Total	Source	Change	Percent Change
Avg. Price	8.68%	Salon Profit	\$942,740	0.58%
Avg. Complexity	5.53%	Consumer Welfare	-\$494,199	-0.30%
Avg. Quality	10.03%	Wages	\$11,603,777	7.12%
Task Specialization	1.83%	Tax Revenue	-\$11,739,300	-7.20%
		Total Welfare	\$313,017	0.19%

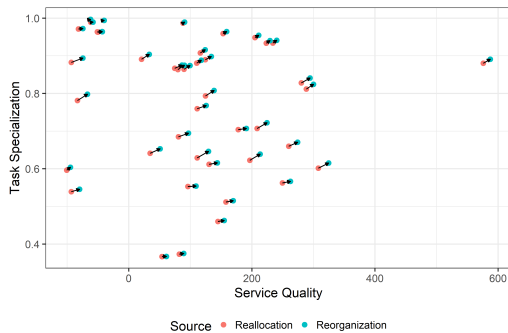
Effects by Worker Type

# Service Sales Tax Elimination (4.5% to 0%)

## Reallocation



## Reorganization



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Identification and Estimation

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Counterfactuals

**Conclusions**

# Extensions

- ▶ In the paper:
  - ▶ Multiplicative quality (i.e. Kremer's O-Ring)
  - ▶ Quantity-based productivity (i.e. manufacturing)
  - ▶ Large firms (continuous tasks, worker types)
- ▶ Future work:
  - ▶ Imperfectly competitive labor markets with task-content as amenity
  - ▶ Accumulation of task-specific human capital

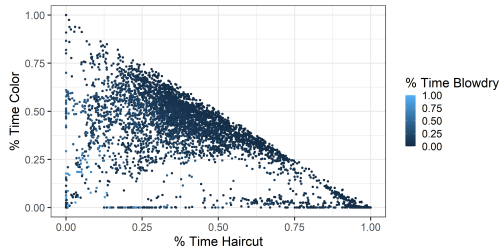
# Summary of Results

1. **Facts:** Salon complexity explains over 10% of variation of individual worker specialization. Internally complex firms have  $\uparrow$  revenue, employees, prices, repeat customers.
2. **Theory:** The firm's optimal organization structure also solves a more tractable, well-understood problem.
3. **Trilemma:** Firms trade-off complexity, quality and wages when choosing an organization structure.
3. **Policy Experiments for Manhattan Hair Salons**
  - ▶ **Minimum wage to \$20:**  $\uparrow$  ( $\downarrow$ ) specialization of (non-)binding workers, + and – wage spillovers based on whether worker is employed alongside binding workers
  - ▶ **Service Sales Tax Elimination:** +10% service quality,  $\uparrow$  task-specialization, workers capture most welfare gains through higher wages

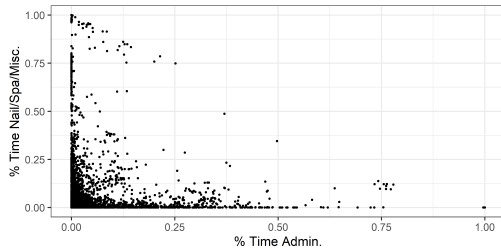
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Appendix

# Task-Mix Variation



(a) Cut, Color, Blowdry



(b) Admin., Misc.

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# Organization Costs As Average Task-Specialization

Define the generalist job as the job as:  $b_j^G(k) = \alpha_k$

## Proposition 2

*Complexity ( $I(B_j)$ ) is the weighted-average Kullback-Leibler divergence between the jobs at a firm and the firm's generalist job  $b_j^G(k)$ , where the weights are the share of each worker type.*

**Proof.** Using the definition of mutual information, we can write out complexity as:

$$\begin{aligned} I(B_j) &= \sum_{i,k} B(i,k) \log \left( \frac{B(i,k)}{\sum_{k'} B(i,k') \sum_{i'} B(i',k)} \right) = \sum_{i,k} E_i \frac{B(i,k)}{E_i} \log \left( \frac{B(i,k)}{E_i \alpha_k} \right) \\ &= \sum_i E_i \sum_k b_i(k) \log \left( \frac{b_i(k)}{\alpha_k} \right) = \sum_i E_i \sum_k b_i(k) \log \left( \frac{b_i(k)}{b_j^G(k)} \right) \\ &= \sum_i E_i D_{KL}(b_i || b_j^G) \end{aligned}$$

# Managerial Attention

- ▶  $X$  is the task type, with prior  $\alpha$ .  $Y$  is assigned worker type. Manager's payoff from the assignment of workers to tasks is  $m(X, Y)$ .
- ▶ Manager chooses any signal  $Z$  with info about the task-type and an assignment function  $\delta(Z)$  mapping signal to an assignment.
- ▶ Cost of signal is  $\gamma I(Z, X)$  where  $I$  is the mutual information:

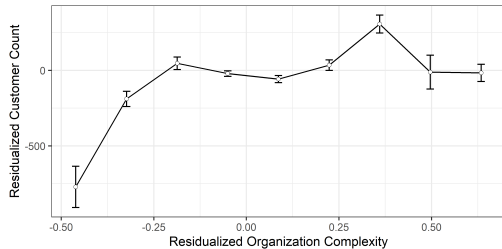
$$\max_{\delta, Z} \mathbb{E}[m(X, \delta(Z))] - \gamma I(X, Z)$$

Jung et al. (2019) (and others) show this is equivalent to:

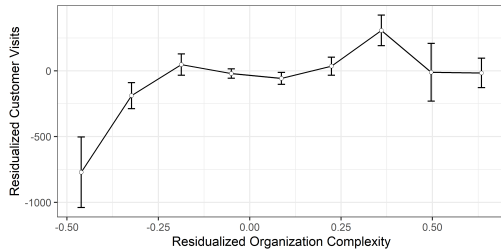
$$\max_{B_j} \mathbb{E}[m(X, Y)] - \gamma I(X, Y)$$

- ▶ Thus complexity measures the attention paid to assignments.

# Relationship Between Complexity and Customers/Visits



(a) Customers



(b) Visits

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# Firm Size and Complexity Regressions

Dependent Variables:	Revenue	Employees	Utilized Labor	Customers	Visits
Model:	(1)	(2)	(3)	(4)	(5)
<i>Variables</i>					
Org. Complexity	347549.2*** (79546.2)	9.75** (3.016)	26481 (35653.2)	334.6 (259.6)	731.7 (450.1)
<i>Fixed-effects</i>					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
County	Yes	Yes	Yes	Yes	Yes
<i>Fit statistics</i>					
Observations	4,558	4,558	4,558	4,558	4,558
R <sup>2</sup>	0.32465	0.34319	0.28918	0.34901	0.35004

*Standard-errors clustered at the salon level.*

*Signif. Codes: \*\*\*: 0.001, \*\*: 0.01, \*: 0.05*

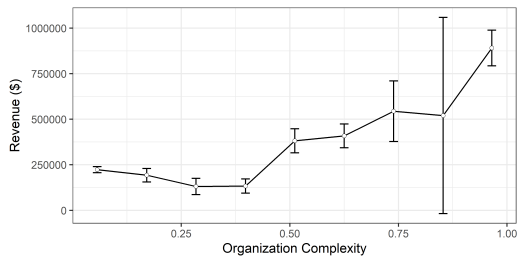
# Manhattan Firm Size and Complexity Regressions

Dependent Variables:	Revenue	Employees	Utilized Labor	Customers	Visits
Model:	(1)	(2)	(3)	(4)	(5)
<i>Variables</i>					
Org. Complexity	430406.6*	12.55	-17733.9	277.2	876.9
	(179977.4)	(6.531)	(70765.2)	(600)	(907.1)
<i>Fixed-effects</i>					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
<i>Fit statistics</i>					
Observations	595	595	595	595	595
R <sup>2</sup>	0.33485	0.21039	0.20359	0.44164	0.48831

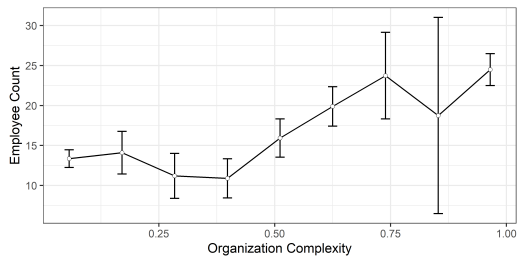
*Clustered standard-errors in parentheses*

*Signif. Codes: \*\*\*: 0.001, \*\*: 0.01, \*: 0.05*

## Fact 2: Complex salons have higher revenue and employment



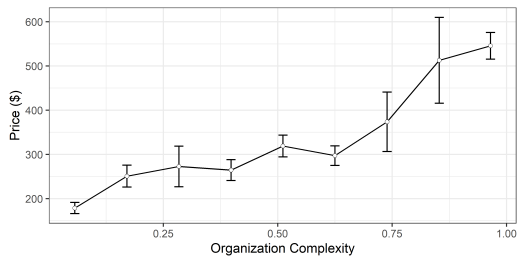
(a) Revenue



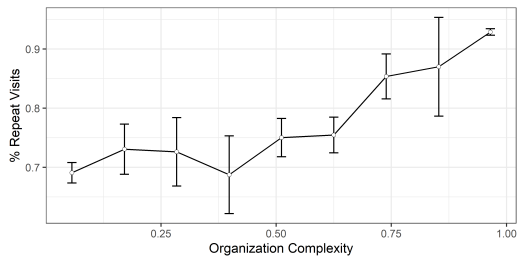
(b) Employees

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## Fact 3: Complex salons have higher prices and repeat customers



(a) Prices



(b) Repeat Customers

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## Quantity Model Implies Price $\downarrow$ Complexity

$$F_{\alpha,B}(a_j) = \min \left\{ \frac{a_1}{\alpha_1 \sum_i \theta_{i,1} B_j(i, 1)}, \dots, \frac{a_k}{\alpha_k \sum_i \theta_{i,k} B_j(i, k)}, \dots, \frac{a_K}{\alpha_K \sum_i \theta_{i,K} B_j(i, K)} \right\}$$

Given any fixed organizational structure, the efficient way to produce a single unit of output is to set  $a_k = \alpha_k \sum_i \theta_{i,k} B_j(i, k)$ . Thus marginal costs are constant and consist of the per-unit wage bill and organization costs:

$$MC_j = \sum_i w_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i, k) + \gamma_j I(B_j)$$

### Proposition 3

*Under these assumptions, prices are decreasing with organizational complexity.*



## Proof of Main Result: Part 1/3

Take the FOC of profit to get the best-response price:

$$p_j^* = \frac{1}{\rho(1 - s_j)} + MC_j$$

Once this is plugged into profit, I prove:

### Lemma 4

*Maximized profit depends on organization structure only through quality-adjusted cost. Additionally, maximized profit is strictly decreasing in quality-adjusted cost.*

Firms choose an organization structure which achieves the lowest quality-adjusted marginal cost:

$$\min_{B_j \in \mathbb{B}_j} MC_j(B_j) - \rho^{-1} \xi_j(B_j) \tag{1}$$

## Proof of Main Result: Part 2/3

$$\min_{B_j \in \mathbb{B}_j} MC_j(B_j) - \rho^{-1} \xi(B_j) = \min_{B_j \in \mathbb{B}} W(B_j) + \gamma_j I(B_j) - \rho^{-1} \xi_j(B_j) \quad (2)$$

$$= \gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \left[ W(B_j) - \rho^{-1} \xi(B_j) \right] \right\} \quad (3)$$

I can re-write 3 as a maximization problem:

$$\max_{B_j \in \mathbb{B}} \left\{ \sum_{i,k} B_j(i, k) (\rho^{-1} \theta_{i,k} - W_i) \right] - \gamma_j I(B_j) \right\} \quad (4)$$

Comparing 4 to formulations in Jung et al. (2019) illustrates that this is a linear rational inattention problem.

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## Proof of Main Result: Part 3/3

I re-write 3 one last time:

$$\gamma_j \min_{B_j \in \mathbb{B}} \left\{ I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i, k) (W_i - \rho^{-1} \theta_{i,k}) \right\} \quad (5)$$

Comparing 5 to formulations in like Equation 6 in Tishby, Pereira, and Bialek (2000) demonstrates this is well-understood minimization problem from information theory called a rate-distortion problem.

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## Proof of Frontier Shape and Profit Increasing

- Denote  $Q$  as quality-adjusted wages. Because the problem is an RD problem:

$$\min_Q R(Q) + \gamma^{-1} Q$$

where  $R$  is continuous, convex and decreasing. Taking the cross-partial derivative:

$$\frac{\partial R(Q) + \gamma^{-1} Q}{\partial Q \partial \gamma} = -\gamma^{-2} \leq 0$$

- This implies decreasing differences, which implies  $Q$  is increasing in  $\gamma_j$ .
- $R(Q)$  is decreasing, therefore complexity is increasing in  $\gamma_j$
- Envelope condition implies quality-adjusted marginal cost is decreasing in  $\gamma_j$ .
- Profit and quantity are decreasing in quality-adjusted marginal cost, therefore they are decreasing in  $\gamma_j$ . [Back](#)

# Profit Under the Econometric Model

$$\frac{\exp(\xi(B_j) - \rho(1 + \tau)p_j + \beta\alpha_j + \nu_j)}{\sum_{j'} \exp(\xi(B_{j'}) - \rho(1 + \tau)p_{j'} + \beta\alpha_{j'} + \nu_{j'})} \left[ p_j - \bar{a}_j \left( \gamma_j I(B_j) + W(B_j) + m\alpha \right) - \phi_j \right]$$

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# A Full Solution Method for Optimal Organization Structure

A globally convergent fixed point algorithm to fully solve based on Blahut (1972):

0. Guess some labor demand  $E^0$ . Create matrix  $V$ :

$$V_{i,k} = \exp[\gamma^{-1}(\rho^{-1}\theta_{i,k} - W_i)]$$

1. Compute  $B^t$  as:

$$B_{i,k}^t = \alpha_k \frac{V_{i,k} E_k^t}{\sum_i E_i^t V_{i,k}}$$

2. Compute  $E^{t+1}$  as:

$$E_i^{t+1} = \sum_k B_{i,k}^t$$

3. If converged exit, else return to Step 1 and advance  $t$ .

## Minimum Wage Counterfactual Details

- ▶ Counterfactuals assume the utility of not getting a service remains fixed.
- ▶ Ruling out Multiple Equilibria
  - ▶ Assume beforehand which wages bind (i.e. which wages are \$20)
  - ▶ 0 excess labor supply for all types except binding types.
  - ▶ Check that assumed binding types have excess labor supply.
  - ▶ If yes, count as an equilibria. If not excluded.
- ▶ I do this for all possible binding combinations ( $2^5$ ).
- ▶ There is only one equilibrium.

## Consumer Welfare

Therefore expected utility of consumer  $i$  has the well-known closed form:

$$V_i = \mathbb{E}[\max_j \{\xi_j - \rho p_j + \epsilon_{i,j}\}] = \ln \left[ \sum_{j=1}^J \exp(\xi_j - \rho p_j) \right] + C$$

where  $C$  is Euler's Constant. There are a mass  $M$  of consumers, therefore total consumer expected utility is  $M \cdot V_i$ . We then can denominate this in dollar terms by dividing by the coefficient on price,  $\rho$ . Our measure of total consumer welfare in dollar terms is:

$$CS = \frac{M}{\rho} \left\{ \ln \left[ \sum_{j=1}^J \exp(\xi_j - \rho p_j) \right] + C \right\}$$

With a sales tax  $\tau$ , it is:

$$CS = \frac{M}{\rho} \left\{ \ln \left[ \sum_{j=1}^J \exp(\xi_j - \rho(1 + \tau)p_j) \right] + C \right\}$$



# Equilibrium Uniqueness

## Proposition 4

*Suppose wages are fixed parameters. A pure strategy equilibrium exists, and is unique for almost any parameter values.*

### Proof Sketch:

- ▶ Bertrand oligopoly with logit demand has unique NE Caplin and Nalebuff (1991)
- ▶ Profit is strictly incr. in quality-adjusted cost Algebra + Aravindakshan and Ratchford (2011)
- ▶ Quality-adjusted cost matrices with mult. equilibria is null Lipnowski et al. (2022)
- ▶ QA matrices are function  $F$  of params. Jacobian of  $F$  is rank  $N \times K$
- ▶ Parameters which generate mult. equilibria are measure 0

## Identification Proof Sketch

- ▶ All workers with the same skill set are assigned the same distribution of tasks.
- ▶ To see why, suppose not. Then we can take any pair of tasks where the two have different distributions, and re-distributed so that they are more similar.
- ▶ Reduces org. cost, does not change wage or quality. This contradicts optimality.
- ▶ This with algebra implies complexity wrt identities is equal to complexity wrt just skill set.
- ▶ When  $l_j > 0$ , we have that  $l_j$  is strictly decreasing in  $\gamma_j$ .

# Condition for Uniqueness

For price uniqueness use Caplin and Nalebuff (1991). For uniqueness of  $B_j$  we require this condition derived from Matêjka and McKay (2015):

## Assumption

*Define the wage-quality vector of a worker of type  $i$  as  $v_i = \{w_i - \rho^{-1}\theta_{i,k}\}_{k=1}^K$ . The set of wage-quality vectors  $\{v_i\}_{i \in \mathcal{I}}$  is affinely independent.*

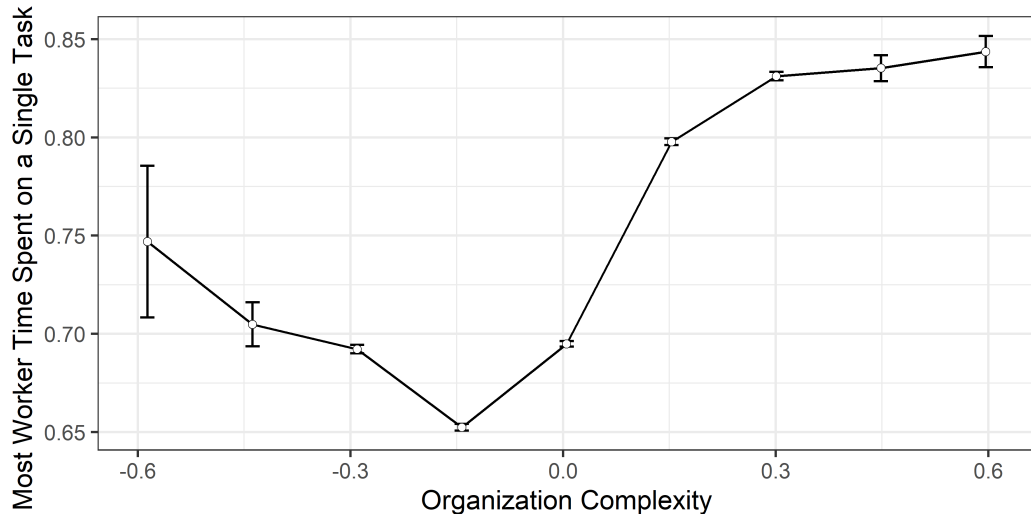
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# Minimum Wage Counterfactual Employment and Wages

Worker Type	Initial		Counterfactual	
	Hours	Wage	Hours	Wage
Haircut/Shave	537550	\$16.96	502152	\$20.00
Color/Highlight/Wash	997053	\$37.75	997053	\$37.52
Blowdry/Style/Treatment/Extension	444040	\$20.91	444040	\$21.64
Administrative	41860	\$26.99	41860	\$28.12
Nail/Spa/Eye/Misc.	34844	\$81.16	34844	\$81.71

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# Regressions of Worker Specialization on Organization Complexity



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## Why Aggregation?

- ▶ A single product allows me to focus on the effects of organization on overall salon quality.
- ▶ Consumers buy a bundle of services at salons.
- ▶ It buys significant numerical/theoretical tractability.
- ▶ Nocke and Schutz (2018): any pricing game with multi-product firms and MNL demand can be represented as a single product firm game with transformed qualities and costs:

$$\tilde{q}_j = \rho \log \left( \sum_k \exp((q_k - c_k)/\rho) \right) + 1 \quad \tilde{c}_j = 1$$

## Sales Tax Elimination Effects by Worker Type

Type	Wage Change	Task-Spec. Change
Haircut/Shave	31.99%	0.29%
Color/Highlight/Wash	20.09%	2.57%
Blowdry/Style/Treatment/Extension	6.06%	3.01%
Administrative	17.99%	1.03%
Nail/Spa/Eye/Misc.	12.74%	2.39%

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