The Inner Beauty of Firms

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Pin Factories in 18th Century France

Adam Smith in *The Wealth of Nations*

"One man draws out the wire, another straights it, a third cuts it, a fourth points it...Those ten persons, therefore, could make among them upwards of forty-eight thousand pins in a day...But if they had all wrought separately and independently...they certainly could not each of them have made twenty, perhaps not one pin in a day"

- There was a spectrum of specialization across competing workshops.
- There were both specialists and generalists in the "pin" labor market.
- Pin makers tried to subdivide the longest task, but ultimately found a single worker more efficient.

Hair Salons in 21st Century Los Angeles

Westwood Barber Shop

[redacted]

1 2

12/10/2014 · Updated review

A lovely stylist named Minoo did an incredible job. She colored my hair, freshened up my bob and gave me a great blow dry. The prices are unbelievable, 25 for color, 20 for haircut and 20 for blow dry.

[redacted]

0 18 12

3/10/2019

Throughly enjoyable quality cut from the delightful owners of the salon. At 81 she cut while he cleaned.

John Frieda Salon

[redacted]

33 65 14

6/9/2011

In addition to seeing a different person for your cut and color all the stylists have assistants and they are usually the ones that take you back for washing and drying if your stylist is busy. I've had days where I swear 4-5 people worked on me like I'm a celebrity or something, which speaking of there are often quite a few getting their hair done as well.

[redacted]

24 54 14

1/23/2013

A cut and color here costs more than a monthly payment for some cars.

Source: Yelp.com. Review text truncated for brevity.

Summary of Paper

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3. How does heterogeneous and endogenous internal organization shape the economy?
   - **Method:** An estimated industry equilibrium model with endogenous and heterogeneous internal organization.
     Answer: (Partial Equilibrium) Workforce diversity is non-monotonic in org. productivity. 2 workers can be complements at 1 firm and substitutes at another in the same market. Firms increase specialization and productivity when price sensitivity falls. (Industry Equilibrium) These imply a sales tax cut raises productivity and a min. wage hike generates new wage spillovers.

Contribution

Endogenous and Firm-Specific Task Specialization

Lazear 2009 (task-mix); Haanwinckel 2020 (multi-worker firms); Garicano 2000 (vertical workers); Adenbaum 2021 (org. costs); Lindenlaub 2017 (multi-skill workers); Baker, Gibbons, and Murphy 2002 (relational contracts); Garicano and Wu 2012 (knowledge); Meier, Stephenson, and Perkowski 2019 (trust); Martinez et al. 2015 (culture); Alchian and Demsetz 1972, Baker and Hubbard 2003 (monitoring)

Task Assignment as a Determinant of Productivity Dispersion

Bassi et al. 2023 (across firms); Minni 2023 (across managers); Bloom and Van Reenen 2007 (management); Syverson 2011 (survey across fields); De Loecker and Syverson 2021 (IO survey)

Estimation of Task-Based Production Functions

Key features: no wage data, multi-dim. workers, not Hicks neutral

Caliendo et al. 2012 (vertical wage-based approach); Berry, Levinsohn, and Pakes 1995 (demand + firm conduct); Caplin and Nalebuff (1991) (uniqueness); Matêjka and McKay 2015 (key tool); Rubens 2023 (non-Hicks neutral example)

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Data

- Salon management software company founded in 2016
- Nationwide, but clients are concentrated in NYC and LA
- Observe 13 million assignments of tasks to hair stylists across hundreds of salons from 2016 to Q3 2021

A Data Snapshot

Firm	Salon	App.	Cust.	Task	Staff	Time Stamp	Price	Duration
1	1A	123	Blake	Advanced Cut	Rosy	3/26/2021 16:15	100	72
1	1A	123	Blake	Full Head - Highlights	Rosy	3/26/2021 16:15	243	127
1	1A	123	Blake	Treatment Add On (Olaplex)	Rosy	3/26/2021 16:15	39	72
2	2A	9982	Grace	Women's Cut	Tyler	3/17/2021 11:00	225	43
2	2A	9982	Grace	Single Process	Ben	3/17/2021 11:00	200	77

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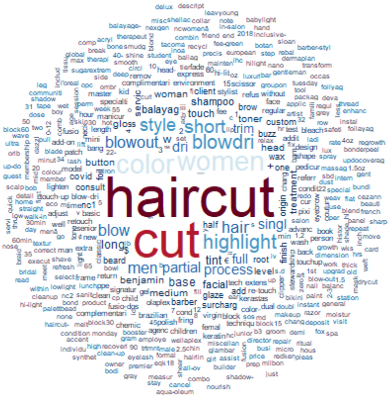
- Tasks are aggregated to form one representative product per firm-quarter.
- A firm's **price** is the sum of service prices divided by total customers.
- A firm's **required labor** is the sum of durations divided by total customers.
- A firm's **task-mix** is the fraction of labor classified as each task.

Creating Task Categories

- 20,560 unique task descriptions.
- A certified cosmetologist was paid to group into 6 categories.
- Two categories merged due to sparsity to yield 5 task categories.



Task Categories

Share of Labor	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Haircut/Shave	4,558	0.41	0.23	0.00	0.26	0.52	1.00
Color/Highlight/Wash	4,558	0.38	0.20	0.00	0.29	0.52	1.00
Blowdry/Etc	4,558	0.09	0.12	0.00	0.03	0.11	1.00
Administrative	4,558	0.05	0.11	0.00	0.002	0.04	1.00
Nail/Etc	4,558	0.06	0.16	0.00	0.00	0.05	1.00

Firm-Quarter Stats.

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What is an Organization?

Definition

A firm's organization (B_j) is a matrix where element (i, k) is the fraction of labor assigned to worker i and task k.

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Tasks

	Cut	Color	Dry	Worker Share (E)
A	.1	.2	.1	.4
B	.1	.1	.1	.3
C	.2	.05	.05	.3
Tot.	.4	.35	.25	

Task-Mix (α)

Measuring Internal Task-Specialization

Suppose we observe this organization:

	Cut	Color	Dry	
A	.1	.2	.1	.4
B	.1	.1	.1	.3
C	.2	.05	.05	.3
Tot.	.4	.35	.25	

Tasks

Task-Mix (α)

Worker Share (E)

Measuring Internal Task-Specialization

Construct a generalist benchmark ($B^G(i, k)$):

	Tasks			
	Cut	Color	Dry	
A	.1	.2	.1	.4
B	.1	.1	.1	.3
C	.2	.05	.05	.3
Tot.	.4	.35	.25	

	Tasks			
	Cut	Color	Dry	
A				
B				
C				
Tot.				

Worker Share (E)

Task-Mix (α)

Measuring Internal Task-Specialization

Hold fix what needs to be done (task-mix):

	Tasks			
	Cut	Color	Dry	
A	.1	.2	.1	.4
B	.1	.1	.1	.3
C	.2	.05	.05	.3
Tot.	.4	.35	.25	

	Tasks			
	Cut	Color	Dry	
A				
B				
C				
Tot.	.4	.35	.25	

Worker Share (E)

Task-Mix (α)

Measuring Internal Task-Specialization

Hold fix who is employed (worker share):

	Tasks			
	Cut	Color	Dry	
A	.1	.2	.1	.4
B	.1	.1	.1	.3
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	Tasks				
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A				.4	Worker Share (E)
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Task-Mix (α)

Measuring Internal Task-Specialization

Randomly assign workers to tasks ($B^G(i,k) = E_i \cdot \alpha_k$)

	Tasks			
	Cut	Color	Dry	
A	.1	.2	.1	.4
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Task-Mix (α)

Measuring Internal Task-Specialization

A firm is task-specialized if it is "far" from the counterfactual generalist firm.

Definition 1

The task-specialization index **(s-index)** of a firm with org. structure B is given by:

$$\underbrace{I(B, B^G)}_{\text{Kullback-Leibler divergence}} := \sum_{i,k} B(i,k) log\left(\frac{B(i,k)}{.B^G(i,k)}\right)$$

Measuring Internal Task-Specialization

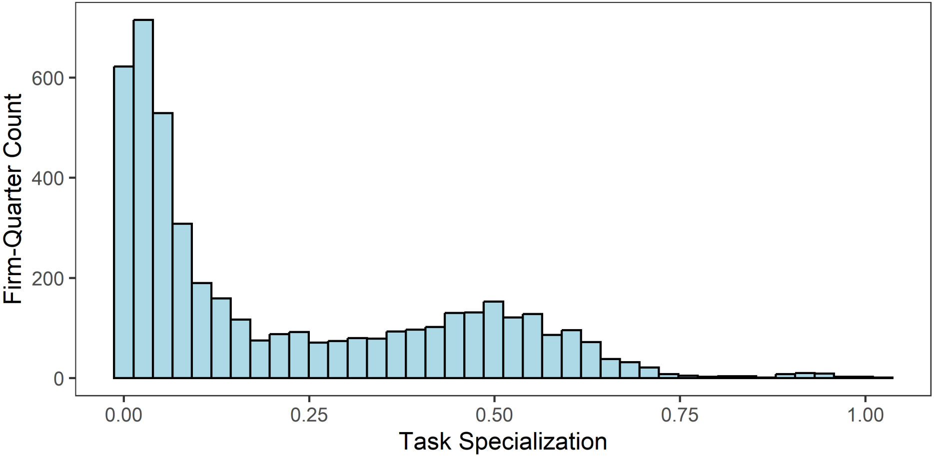
A firm is task-specialized if it is “far” from the counterfactual generalist firm.

Definition 2

The task specialization index **(s-index)** of a firm with org. structure B is given by:

$$\underbrace{I(B, B^G)}_{\text{Kullback-Leibler divergence}} := \sum_{i,k} B(i,k) log \left(\frac{B(i,k)}{\underbrace{\alpha_k}_{\text{task-mix}} \cdot \underbrace{E_i}_{\text{labor demand}}} \right)$$

Fact 1: Task Specialization Follows a Power Law



Takeaway: Specialization is heterogeneous, and full specialization rarely occurs.

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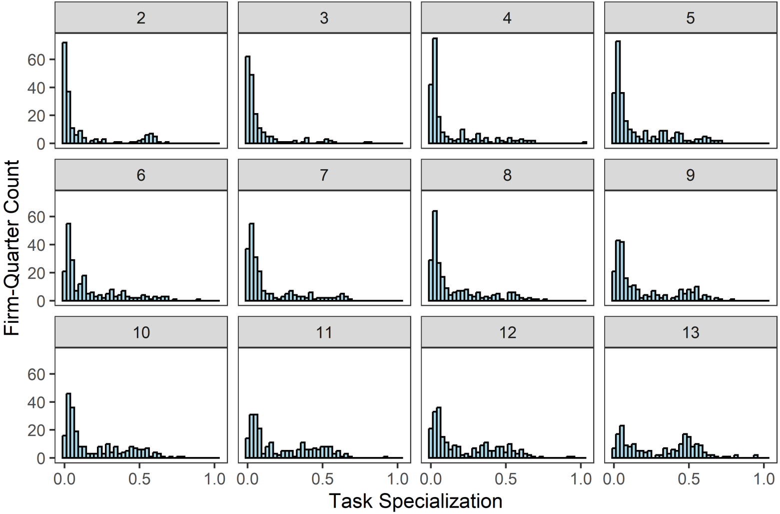
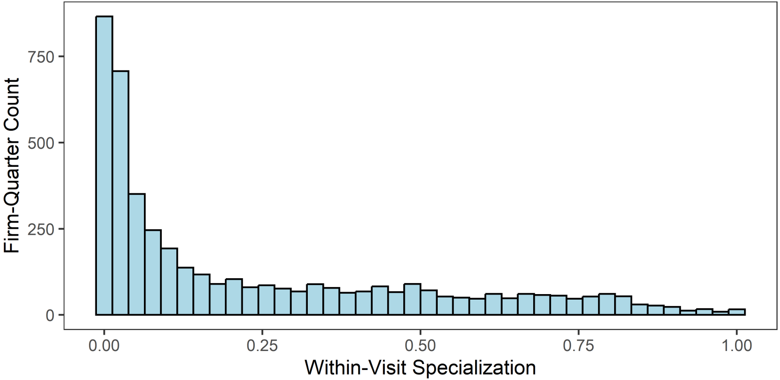


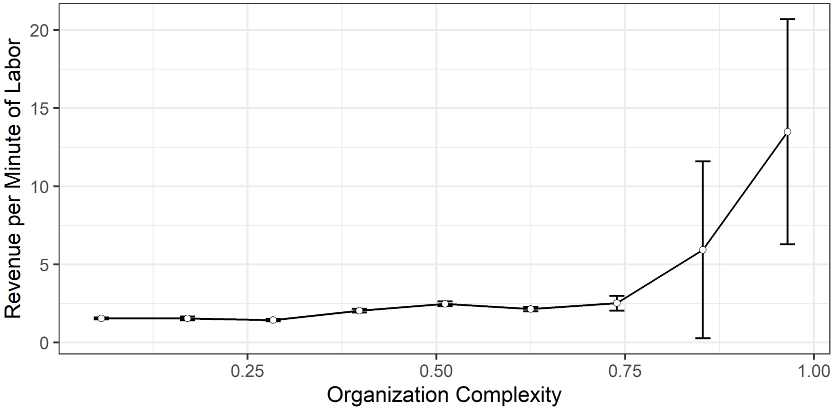
Takeaway: The power-law persists even within firm size.

Fact 1: Task-Specialization Follows a Power Law



Takeaway: It persists even if we consider a simpler, more restrictive definition of specialization.

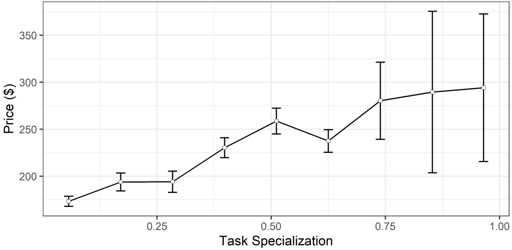
Fact 2: Task Specialized Salons are More Productive



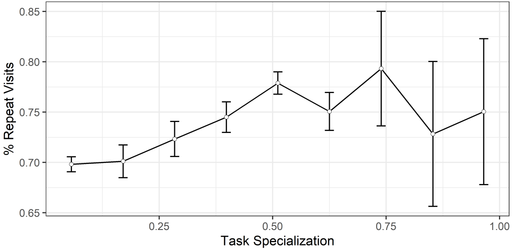
Regression Version | Was Staff Requested?

Takeaway: Specialized salons appear more productive even among a selected subset of peers.

Fact 3: Task-specialized salons produce higher quality services



(a) Prices



(b) Repeat Customers

Manhattan Only Within Firm Size Within-Visit Specialization

Takeaway: Specialization-productivity relationship is mediated by quality upgrading rather than marginal cost reductions. Theory

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Model

Firms: $j = 1, ..., J$

- Firm j communicates 1 bit of info. at cost γ_j (not Hicks neutral)
- Firm j requires $\bar{a}_j$ labor and must assign a fraction $\alpha_j(k)$ to task k
- Firm j incurs a per-unit cost: $\alpha_j \cdot c + \omega_j$ (material cost + Hicks neutral)

Workers: $m = 1, ..., M$

- Skill level $\bar{\theta}_m \in \mathbb{R}$, skill set $\theta_m \in \mathbb{R}^K$ and labor supply $l_m \in \mathbb{R}_+$
- Worker m performs task k with quality $\bar{\theta}_m + \theta_m(k)$
- Worker-specific wages $w \in \mathbb{R}_+^M$

Model

Firm Actions

(simultaneously chosen)

- Price $p_j \in \mathbb{R}_+$ (Bertrand-style)
- Relative Labor demand $E_j \in \mathbb{R}_+^M$ (fraction of work done by each worker)
- Task assignment $A_j \in \mathbb{R}_+^M \times \mathbb{R}_+^K$ (how each worker spends their time)

Organization Costs

- Workers know the task-mix of firms (α_j) but their task assignment must be communicated (knowledge hierarchy-style)
- Org. cost of task assignment A is γ_j times minimum info. required to communicate A to workers

Model

Product Market

- Consumers observe task assignments and prices and purchase based on utility $u_{z,j} = \xi_j + \nu_j - \rho p_j + \epsilon_{z,j}$ with $\epsilon_{z,j}$ i.i.d. Type-1 EV (no purchase normalized to $\epsilon_{z,0}$)
- ξ_j is average quality across all workers and tasks given assignment

Equilibrium

- Firm strategies $\{p_j, E_j, A_j\}_{j=1}^{J}$ are a Nash Equilibrium under wage w
- Call this a fixed w-subgame
- Wages w are such that the labor market clears in the fixed w-subgame

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Firm Strategies Vastly Simplify in Equilibrium

- The firm strategy space is large: $\{p_j, E_j, A_j\} \in \mathbb{R}_+ \times \mathbb{R}_+^M \times (\mathbb{R}_+^M \times \mathbb{R}_+^K)$

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- In EQ firms must be indifferent between all workers w/ same "horizontal" skill set but different "vertical" skill level. Prop.

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- Same-skill workers w/ diff labor supply assigned same task distribution. Prop.
- In EQ firms must be indifferent between all workers w/ same "horizontal" skill set but different "vertical" skill level. Prop.
- Thus firms assign tasks to N representative workers of each skill set
- Call this the firm's organization $B_j^* \in \mathbb{R}_+^N \times \mathbb{R}_+^K$ $N << M$

Communication is Task-Specialization

Proposition

The communication required to implement the profit-maximizing B^* is equal to the observed s-index. Both are strictly decreasing in γ_j for all values of firm-level heterogeneity $(\alpha_j, \nu_j, \omega_j)$ until they reach 0.

- Microfoundation: specialization is costly because it requires communication.
- Can also view directly as a catch-all specialization cost.
- Observed s-index is monotone in unobserved org. cost parameter γ_j

Equilibrium Worker Jobs

Definition

A worker's job is their distribution of time across tasks.

Theorem

The job and labor demand of a worker w/ skill set i at firm j:

1. ***Characterization:***

$$b_j(i,k) = \alpha_j(k)\frac{exp[\gamma_j^{-1}(\rho^{-1}\theta_i(k) - w(i))]}{\sum_{i'} E_j(i')exp[\gamma^{-1}(\rho^{-1}\theta_{i'}(k) - w(i'))]}$$

2. ***Law of Demand:*** *As $w(i)$ rises, $E_j(i)$ falls*
3. ***Incomplete Specialization:*** *All workers spend some time on all tasks* (unless $\alpha_j(k) = 0$)
4. ***Maximum Coworker Diversity:*** *Either # skill sets at firm $\leq$ # tasks, or there exists another profit max. strategy where this is true.*

Essential Uniqueness

Assumption

There are $N = K$ horizontal skill sets, which can be collected into a matrix Θ which is positive definite.

Proposition

There exists a unique Nash equilibrium in prices (p_j) and organizations (B_j) for every fixed-w subgame.

Proof Sketch

- Every wage vector implies a unique organization and price.
- Multiple task assignments may map to one org., but all yield the same profit.
- Not full uniqueness because multiple wages may clear the market.

Simple Example

- 3 tasks with uniform task-mix $\alpha = (1/3, 1/3, 1/3)$, price sensitivity $\rho = 1$
- 3 worker types with wages $w = (21, 20, 15)$ and skill set:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 15 & 19 & 26 \\ 23 & 19 & 15 \\ 15 & 15 & 15 \end{bmatrix}$$

- Wage-adjusted quality:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} - \rho w = \begin{bmatrix} -6 & -2 & 5 \\ 3 & -1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

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Data and Identifying Assumptions

- The econometrician observes jobs ($\{b_m(i,k)\}_{m=1}^{M}$) of workers.
- The econometrician observes the required labor, task-mix, price and market share ($\{a_j, \alpha_j, p_j, s_j\}_{j=1}^{J}$) of firms.
- The wage-adjusted skill matrix $\Theta - \rho(we')$ is full rank. (e is a vector of ones)
  - Θ is full-rank already, so this rules out a measure 0 set of wages
- Idiosyncratic quality (ν_j) and cost (ω_j) are mean zero and independent of firm heterogeneity.
- Standard linear GMM rank assumptions. Rank Conditions

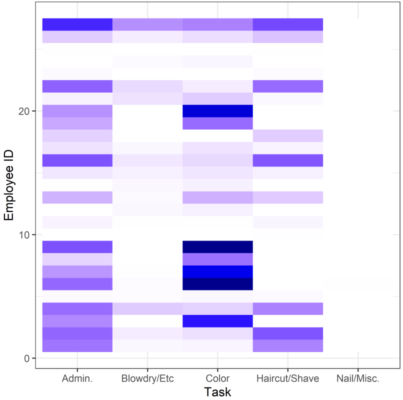
Identification

Theorem

Wages (w), price sensitivity (ρ), material costs (m) and the skill sets of all workers ($\{\theta_m\}_{m=1}^{M}$) are identified. The organization cost parameters (γ_j) of firms with a strictly positive s-index ($I_j > 0$) are identified. A lower bound on the organization cost parameters of firms with an s-index of 0 is identified.

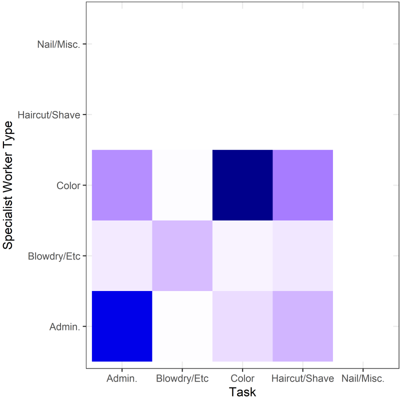
The proof sketch develops an estimation procedure.

Worker Skills are Unobserved, So B_j^* is Unobserved



What We Have (jobs $b_j(m)$)

$\Rightarrow$



What We Want (B_j^*)

Grouping Workers By Skill Set Within Firm

- Firms may employ two workers with the same skill type.
- All workers with the same skill set have the same job if they are employed at the same firm.
- We can group workers into skill sets within a firm.
- We set aside all firms with an s-index of 0 or that do not perform one or more tasks ($\alpha_j(k) = 0$).
- Implementation: hierarchical clustering based on time-use within firms.

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Recall the job of worker 1 at firm j with unknown skill set t_l is:

$$b_j(t_1, k) = \alpha_j(k) \frac{exp(-\gamma^{-1} w(t_1) + (\rho\gamma_j)^{-1}\theta_{t_1}(k)))}{\sum_{i'} E_j(i') exp(-\gamma_j^{-1} w(i') + (\rho\gamma)^{-1}\theta_{i'}(k))}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Take another worker at firm j but with a different skill set. Call them worker 2:

$$b_j(t_2, k) = \alpha_j(k) \frac{exp(-\gamma^{-1} w(t_2) + (\rho\gamma_j)^{-1}\theta_{t_2}(k)))}{\sum_{i'} E_j(i') exp(-\gamma_j^{-1} w(i') + (\rho\gamma)^{-1}\theta_{i'}(k))}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- Divide the job of worker 1 by that of worker 2 across all tasks:

$$\frac{b_j(t_1,k)}{b_j(t_2,k)} = \frac{\alpha_j(k)\frac{exp(-\gamma^{-1}w(t_1)+(\rho\gamma_j)^{-1}\theta_{t_1}(k)))}{\sum_{i'} E_j(i')exp(-\gamma_j^{-1}w(i')+(\rho\gamma)^{-1}\theta_{i'}(k))}}{\alpha_j(k)\frac{exp(-\gamma^{-1}w(t_2)+(\rho\gamma_j)^{-1}\theta_2(k)))}{\sum_{i'} E_j(i')exp(-\gamma_j^{-1}w(i')+(\rho\gamma)^{-1}\theta_{i'}(k))}}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- This removes most of the firm-level confounding:

$$\frac{b_j(t_1, k)}{b_j(t_2, k)} = \frac{exp(-\gamma^{-1} w(t_1) + (\rho\gamma_j)^{-1}\theta_{t_1}(k)))}{exp(-\gamma^{-1} w(t_2) + (\rho\gamma_j)^{-1}\theta_{t_2}(k)))}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- Take logs:

$$log\left(\frac{b_j(t_1,k)}{b_j(t_2,k)}\right) = (\rho\gamma_j)^{-1}\left([\theta_{t_1}(k) - \rho w(t_1)] - [\theta_{t_2}(k) - \rho w(t_2)]\right)$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Divide the vector by its Euclidean norm:

$$\frac{log\left(\frac{b_j(t_1,k)}{b_j(t_2,k)}\right)}{\left|\{log\left(\frac{b_j(t_1,k')}{b_j(t_2,k')}\right)\}_{k'=1}^{K}\right|} = \frac{(\rho\gamma_j)^{-1}\left(\theta_{t_1}(k) - \rho w(t_1) - [\theta_{t_2}(k) - \rho w(t_2)]\right)}{\left(\sum_{k'}\left[(\rho\gamma_j)^{-1}(\theta_{t_1}(k') - \rho w(t_1) - [\theta_{t_2}(k') - \rho w(t_2)])\right]^2\right)^{1/2}}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- This removes the org. cost parameter:

$$\frac{log\left(\frac{b_j(t_1,k)}{b_j(t_2,k)}\right)}{\left|\{log\left(\frac{b_j(t_1,k')}{b_j(t_2,k')}\right)\}_{k'=1}^{K}\right|} = \frac{\left(\theta_{t_1}(k) - \rho w(t_1) - [\theta_{t_2}(k) - \rho w(t_2)]\right)}{\left(\sum_{k'}\left[(\theta_{t_1}(k') - \rho w(t_1) - [\theta_{t_2}(k') - \rho w(t_2)])\right]^2\right)^{1/2}}$$

Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Call it the coworker log-ratio vectors.

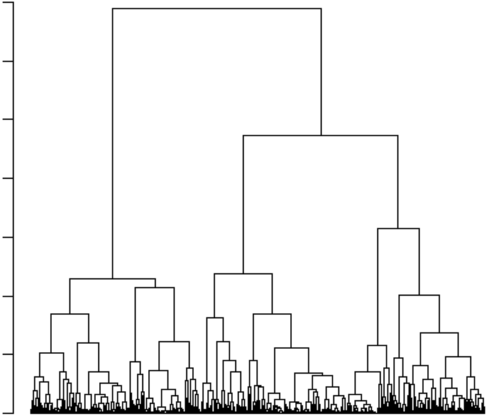
$$\frac{log\left(\frac{b_j(t_1,k)}{b_j(t_2,k)}\right)}{\left|\{log\left(\frac{b_j(t_1,k')}{b_j(t_2,k')}\right)\}_{k'=1}^{K}\right|} = \frac{\left(\theta_{t_1}(k) - \rho w(t_1) - [\theta_{t_2}(k) - \rho w(t_2)]\right)}{\left(\sum_{k'}\left[(\theta_{t_1}(k') - \rho w(t_1) - [\theta_{t_2}(k') - \rho w(t_2)])\right]^2\right)^{1/2}}$$

- We can compute this for every coworker at the firm who is a different skill set.
- If 4 skill sets employed at my firm, I have 3 coworker log-ratios vectors

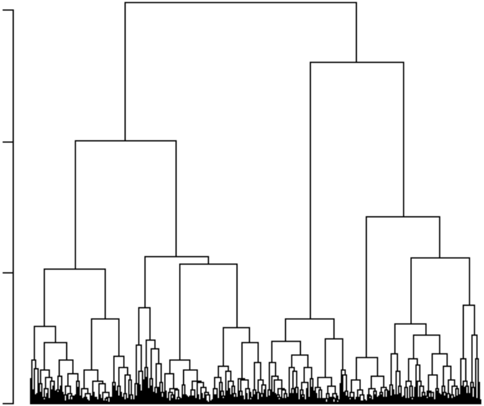
Groupings Workers by Skill Sets Across Firms

- Take two pairs of pairs of workers at different firms.
- Compare the coworker log ratios. They will match if and only if numerator workers AND denominator workers have the same skill set. (add button)
- Disconnectedness will generally not happen in large samples
  (sufficient for 1 firm to hire all skill sets)
- Implementation: define distance between workers as the smallest Euclidean distance of any pair of coworker log ratios.
- Implementation: constrained hierarchical clustering until we get to N groups, where constraints prevent contradictions with prior step.

Grouping Workers By Skill Sets

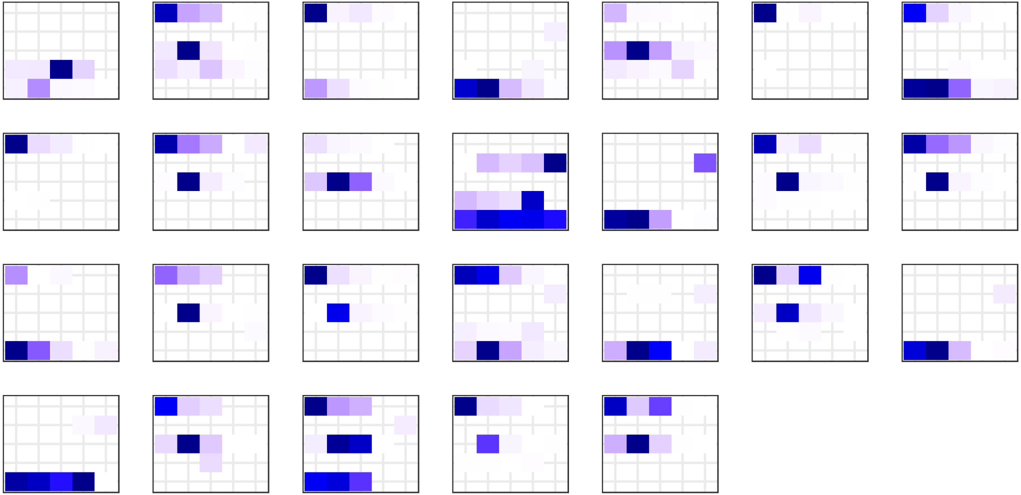


Manhattan (2018-2021)



Los Angeles (2018-2021)

Organizations are Now Data



Firm-Specific Organization Costs (γ_j)

- Recall our coworker log ratios before we divided by the norm:

$$log\left(\frac{b_j(t_1, k)}{b_j(t_2, k)}\right) = (\rho\gamma_j)^{-1}\left([\theta_{t_1}(k) - \rho w(t_1)] - [\theta_{t_2}(k) - \rho w(t_2)]\right)$$

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- We can use this to obtain org. costs $\tilde{\gamma}_j := \gamma_j/\gamma_1$ relative to a reference firm 1.

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- Two firms are connected if they employ two pairs of workers with the same skill set.

Firm-Specific Organization Costs (γ_j)

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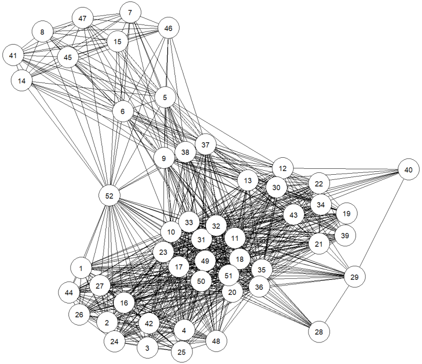
- We can use this to obtain org. costs $\tilde{\gamma}_j := \gamma_j/\gamma_1$ relative to a reference firm 1.

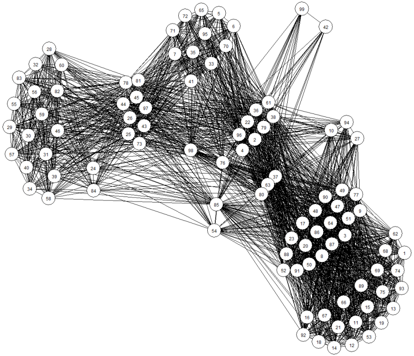
- Two firms are connected if they employ two pairs of workers with the same skill set.

- We can identify relative org. costs $\tilde{\gamma}_j$ within any connected set (a set where there exists a path between any two firms).

Firm-Quarter Networks Linked by Labor Input Combinations



Manhattan (2019 Q1-Q4)



Los Angeles (2019 Q1-Q4)

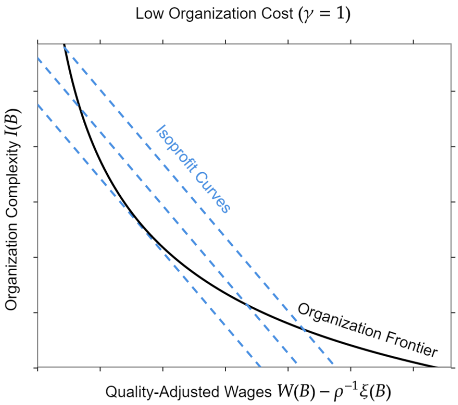
Wages, Skills, Reference Firm Org. Cost, Etc.

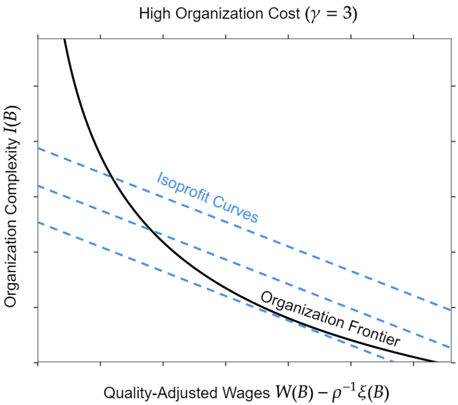
- Demand-side: $log(s_j/s_0) = \sum_{i,k} \theta_i(k) a_j B_j(i,k) - \rho p_j + \nu_j$
- Supply-side: $p_j = \frac{1}{\rho(1-s_j)} + \gamma_1 \tilde{\gamma}_j a_j l_j + w \cdot a_j \cdot E_j + c \cdot \alpha_j + \omega_j$
- Use relative org. costs $\tilde{\gamma}_j a_j l_j$ as instrument for price in demand-side.
- Linear GMM with $K^2 + 1$ equations and $K^2 + 1$ unknowns.
- Adjust prices by markup: $p_j - \frac{1}{\rho(1-s_j)} = \gamma_1 \tilde{\gamma}_j a_j l_j + w \cdot a_j \cdot E_j + c \cdot \alpha_j + \omega_j$
- Linear GMM with $2K + 1$ equations and $2K + 1$ unknowns (no instruments).

The Set Aside Firms

- We set aside firms with s-index of 0 or that did not perform one task at all.
- Therefore we have not identified their org. costs (γ_j).
- But now we have all market parameters.
- And we proved monotonicity of the s-index in γ_j.
- For firms with s-index > 0: invert the s-index to recover γ_j.
- For firms with s-index $= 0$: invert the s-index to recover a lower bound $\bar{\gamma}_j$.

Monotonicity of S-Index in γ_j





A Globally Convergent Contraction Mapping

The Blahut–Arimoto algorithm (Blahut 1972):

0. Guess some relative labor demands E^0. Create matrix V:
   $V_{i,k} = exp[\gamma_j^{-1}(\rho^{-1}\Theta(i,k) - w(i))]$.

1. Compute interim organization structure $B_j(i,k)^t = \alpha_j(k)\frac{V_{i,k}E^t(i)}{\sum_{i'} E_j^t(i')V_{i,k}}$.

2. Compute interim relative labor demands $E_j^{t+1}(i) = \sum_k B(i,k)^t$.

3. If converged, exit; else return to Step 1 and advance t.

It can also be used for solving for counterfactual equilibrium.

Summary of Estimation Procedure

- Cluster workers within firm based on their job's task content.
- Cluster workers across firms using their job's task content relative to coworkers.
- Obtain relative org. costs of a connected set of firms.
- Estimate Θ, ρ via 2SLS of relative market shares on prices and orgs.
- Estimate wages and material costs using OLS of relative market shares on prices and orgs.
- Invert s-index via contraction mapping to get γ_j for set-aside firms.

Panel Estimation Details

- Counties: Cook, Los Angeles, New York (counties=markets)
- Time Periods: 2018Q1 - 2021Q2 (Exclude 2020Q1, Q2 - COVID)
- Two substantive panel assumptions (for power)
  - Parallel Wage Trends (can cluster across time)
  - Time-invariant Firm-Specific Org. Cost Parameters ($\hat{\gamma}_j$ is avg.)
- Restrict wages to be at least minimum wage in time period-county.
- Jointly estimate demand and supply equations. (for power)

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Manhattan Wage and Price Sensitivity Estimates

Base Wage	Estimate
1	48.31
2	37.66
3	30.17
4	12.00
5	39.30
ρ	-0.06

Manhattan Skill Estimates

Skill Set	Haircut/Shave	Color/Highlight/Wash	Blowdry/Etc.	Admin	Nail/Etc.
1	-19.08	12.94	8.66	5.91	-28.73
2	1.78	8.76	-7.41	1.55	-2.01
3	-1.79	-0.57	11.87	-8.24	-11.39
4	2.61	-19.88	-68.75	-2.96	31.44
5	-4.91	3.77	14.12	21.91	8.28

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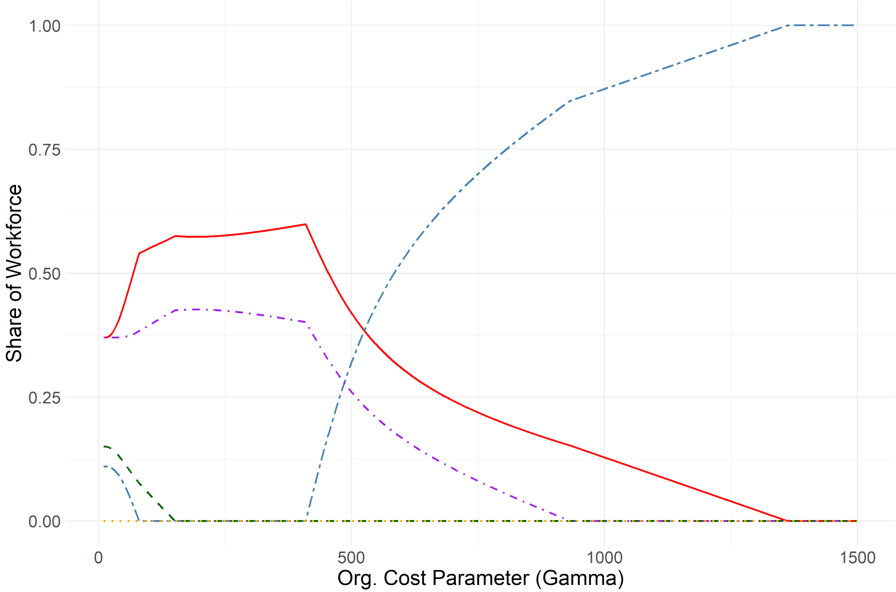
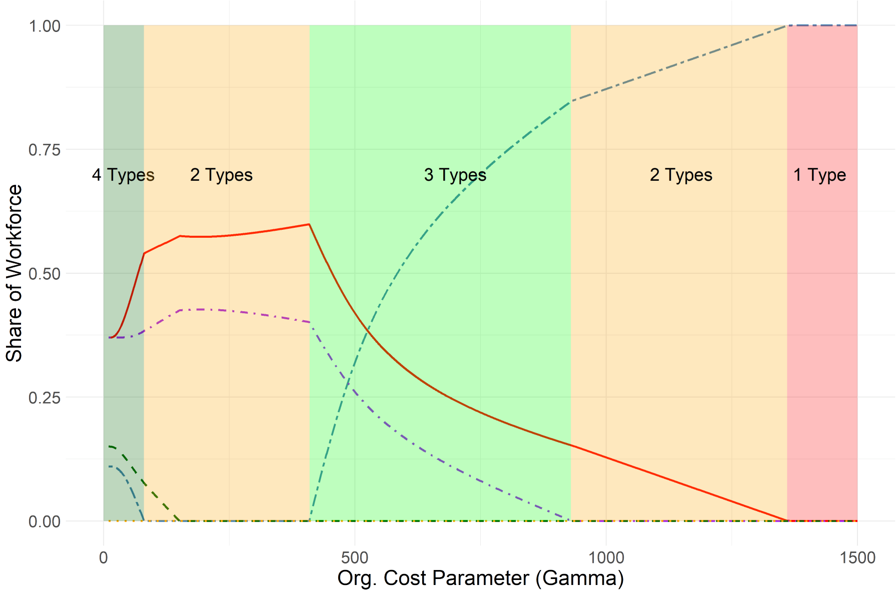
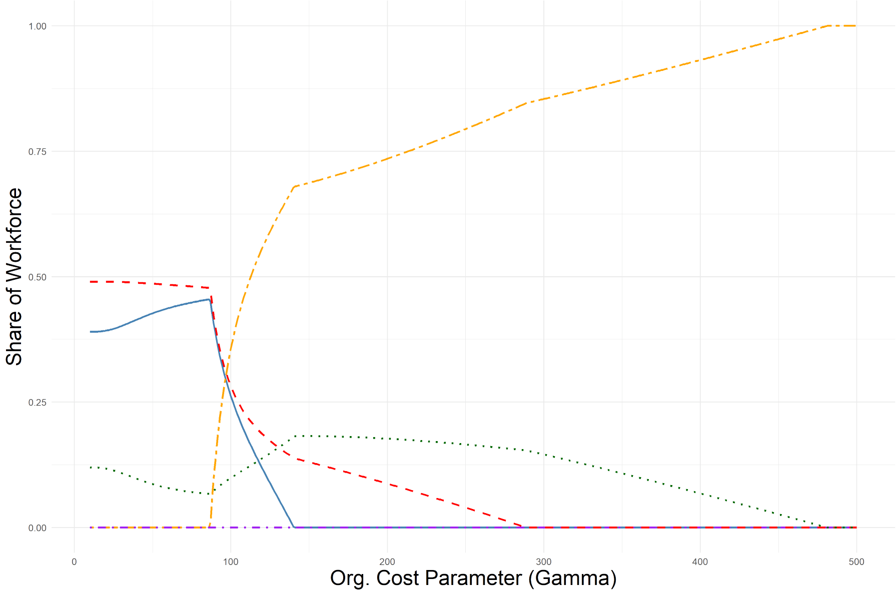
The Path of Workforce Composition: Cook County



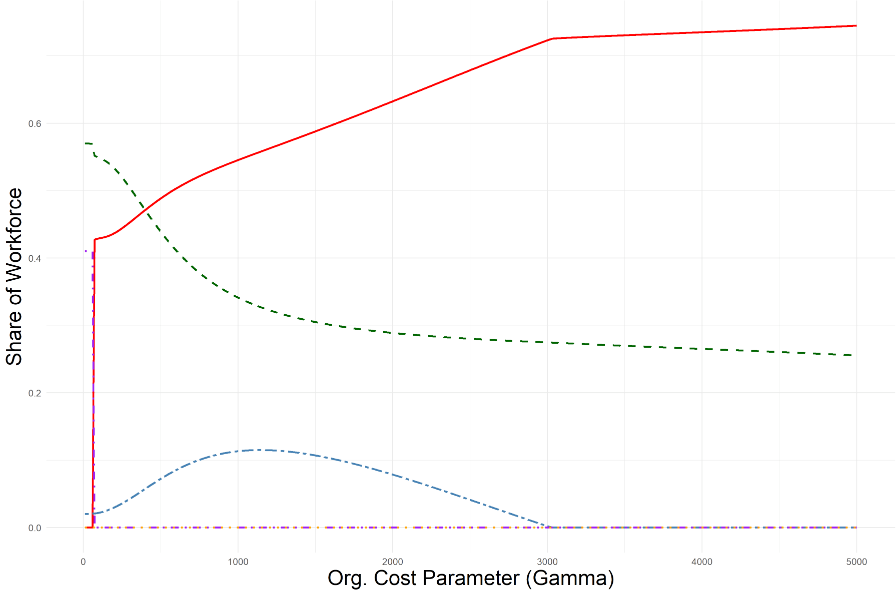
The Path of Workforce Composition: Cook County



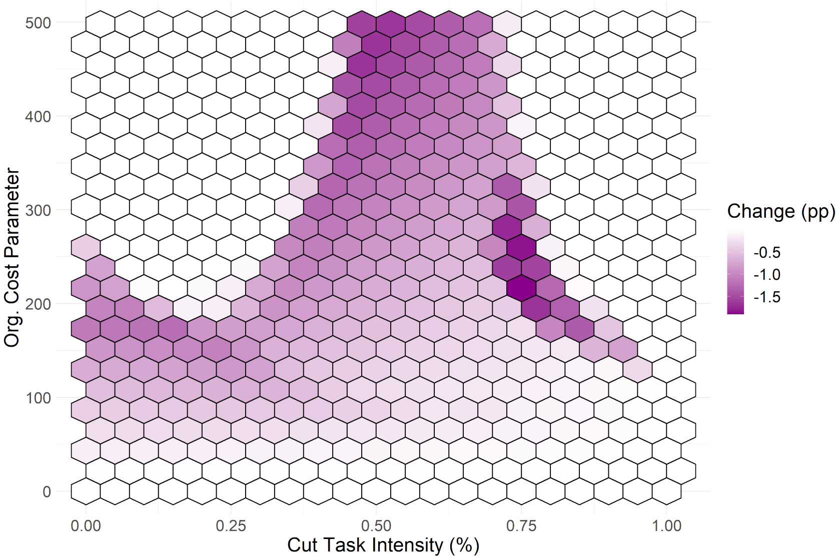
The Path of Workforce Composition: Manhattan



The Path of Workforce Composition: Los Angeles County

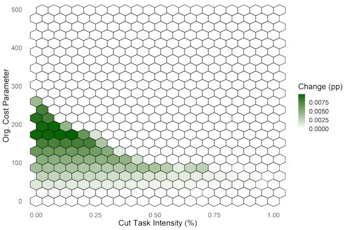


Own Wage Elasticity of Labor Demand

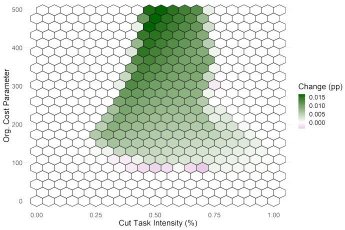


From a $1 increase in Skill Set 5's wage.

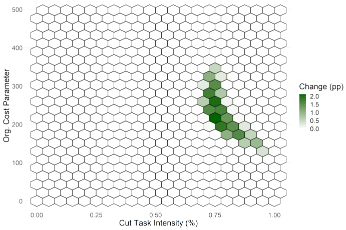
Cross Wage Elasticity of Labor Demand



Skill Set 1



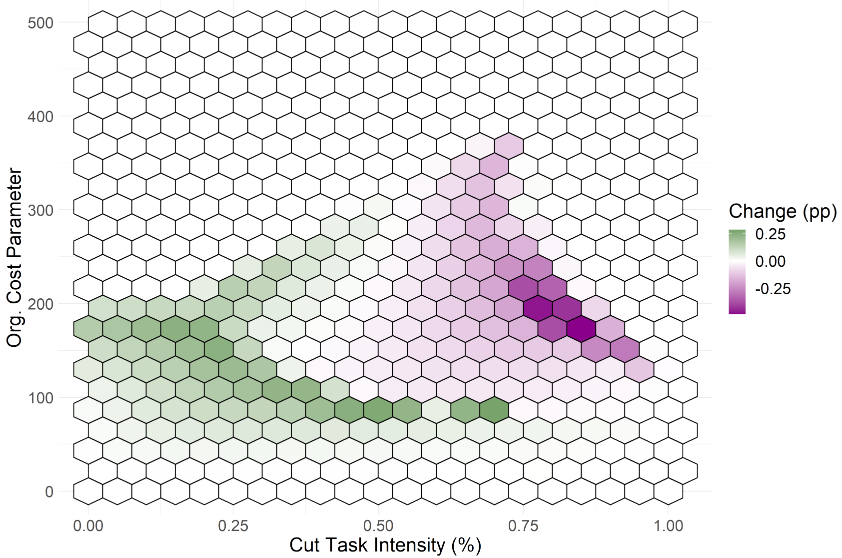
Skill Set 2



Skill Set 3

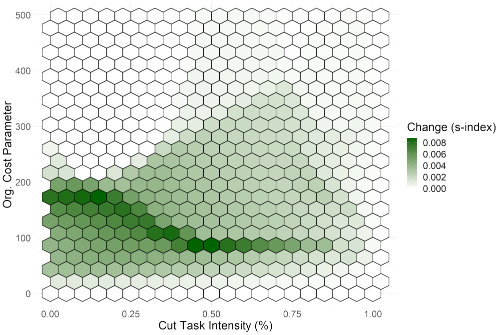
From a $1 increase in Skill Set 5’s wage.

Complements at Some, Substitutes at Others

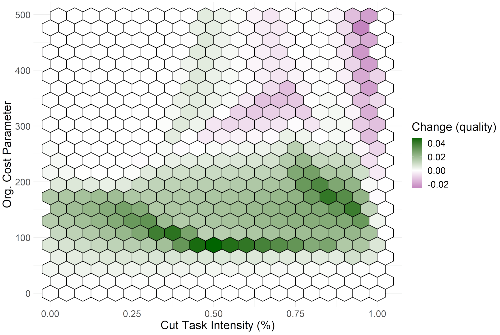


From a $1 increase in Skill Set 5's wage.

Endogenous Specialization



Specialization (S-index)



Service Quality (Productivity)

From a 5%↑ in price sensitivity (ρ) in Manhattan.

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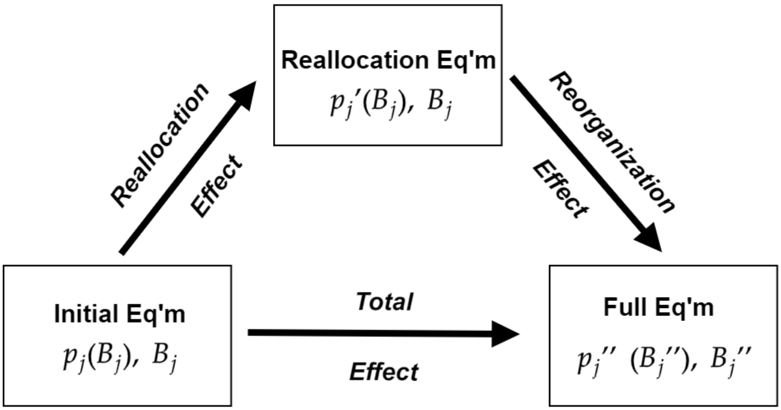
Decomposing Mechanisms

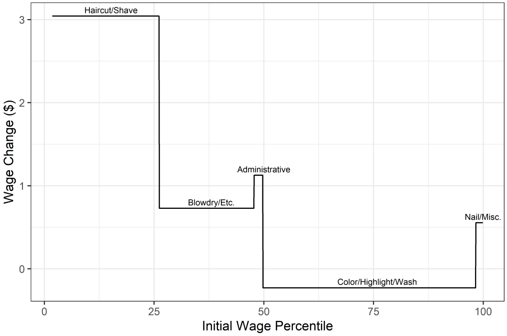


Minimum Wage Increase from $15 to $20

Wage Changes

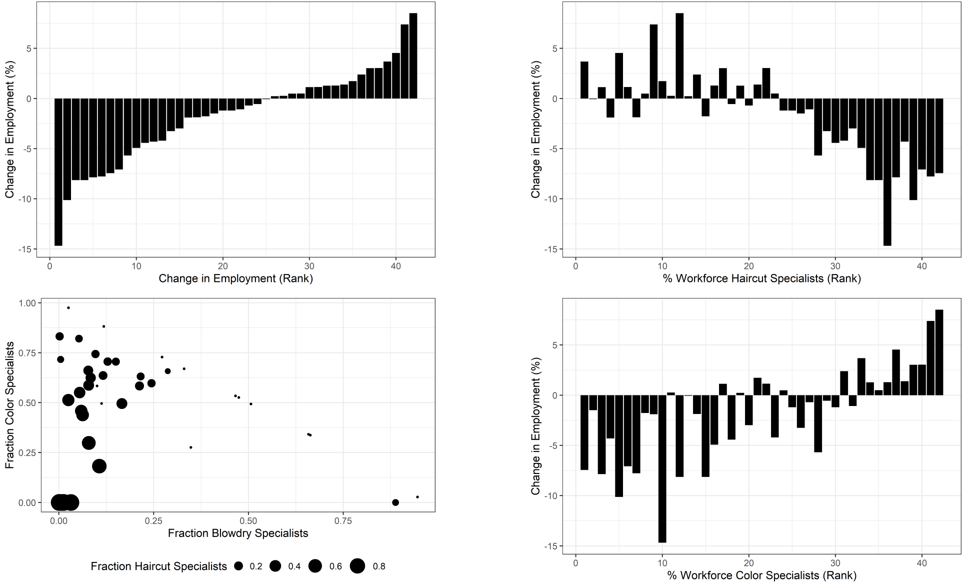
Type	Wage Change	Total Wages Gained/Lost
Haircut/Shave - UNEMPLOYED	-100.00%	-$600,240
Haircut/Shave - EMPLOYED	17.95%	$1,528,205
Color/Highlight/Wash	-0.61%	-$228,453
Blowdry/Style/Treatment/Extension	3.48%	$323,374
Administrative	4.17%	$47,154
Nail/Spa/Eye/Misc.	0.68%	$19,319

Wage Changes by Initial Wage Percentile



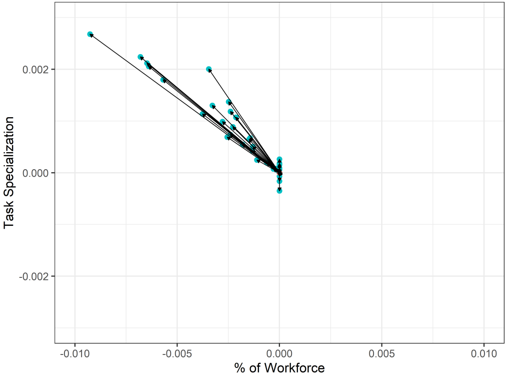
Employment and Wages
Technical Details

The Reallocation Effect

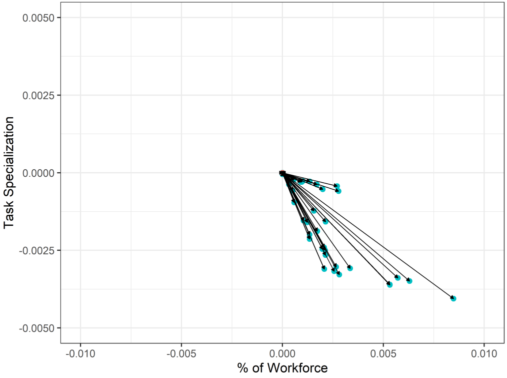


The Reorganization Effect

Haircut Specialists (Binding)



Color Specialists (Non-Binding)



Decomposing Minimum Wage Spillovers

Type	Reallocation Change		
	Employment	Task-Spec.	Wage
Haircut/Shave	-5.85%	-0.04%	17.95%
Color/Highlight/Wash	0%	-0.17%	-1.13%
Blowdry/Style/Treatment/Extension	0%	-0.40%	4.63%
Administrative	0%	0.09%	5.22%
Nail/Spa/Eye/Misc.	0%	-0.03%	0.58%

Type	Reorganization Change		
	Employment	Task-Spec.	Wage
Haircut/Shave	-0.73%	0.12%	0%
Color/Highlight/Wash	0%	-0.33%	0.52%
Blowdry/Style/Treatment/Extension	0%	0.03%	-1.15%
Administrative	0%	0.03%	-1.05%
Nail/Spa/Eye/Misc.	0%	-0.00%	0.10%

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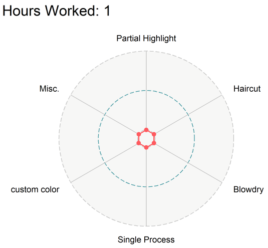
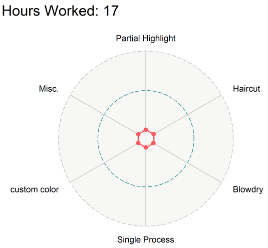
Future Work: Firms Shaping Human Capital Accumulation



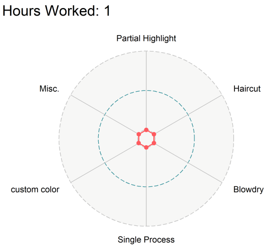
(a) Under Specialization



(b) Under Generalization

Week 1

Future Work: Firms Shaping Human Capital Accumulation



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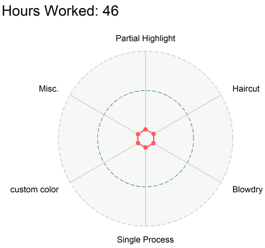
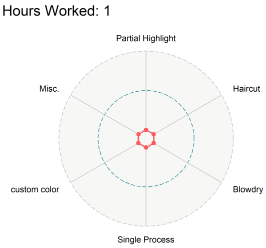


(b) Under Generalization

Week 6

Future Work: Firms Shaping Human Capital Accumulation



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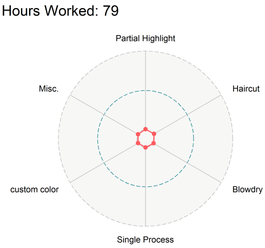
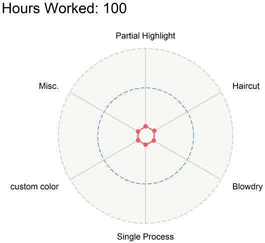


(b) Under Generalization

Week 11

Future Work: Firms Shaping Human Capital Accumulation

(a) Under Specialization

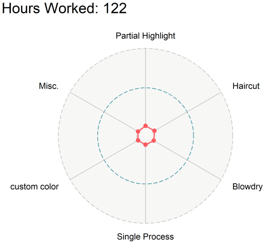


(b) Under Generalization

Week 16

Future Work: Firms Shaping Human Capital Accumulation

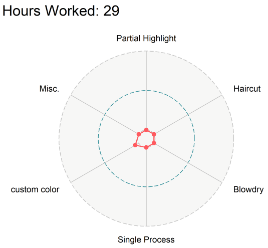
(a) Under Specialization



(b) Under Generalization

Week 21

Future Work: Firms Shaping Human Capital Accumulation



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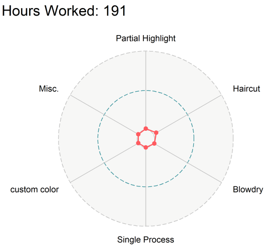

(b) Under Generalization

Week 26

Future Work: Firms Shaping Human Capital Accumulation

(a) Under Specialization

(b) Under Generalization

Week 31

Future Work: Firms Shaping Human Capital Accumulation

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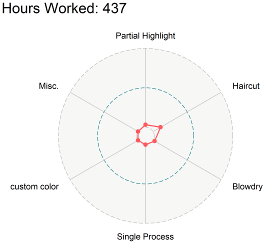

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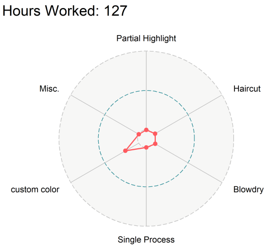
Week 36

Future Work: Firms Shaping Human Capital Accumulation



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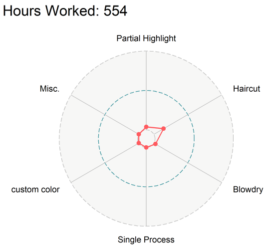
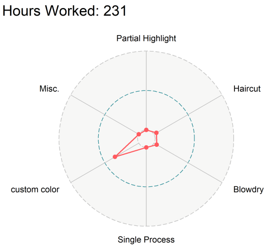


(b) Under Generalization

Week 41

Future Work: Firms Shaping Human Capital Accumulation

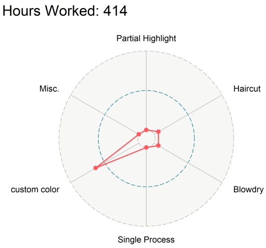


(a) Under Specialization

(b) Under Generalization

Week 46

Future Work: Firms Shaping Human Capital Accumulation



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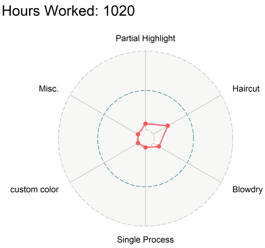

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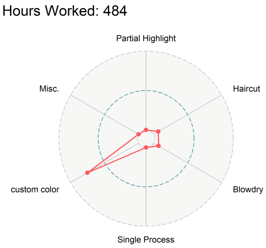
Week 51

Future Work: Firms Shaping Human Capital Accumulation



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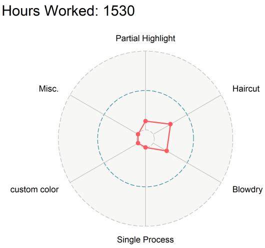
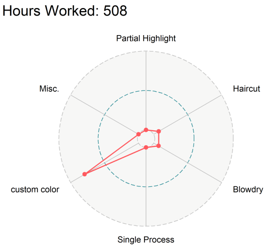


(b) Under Generalization

Week 56

Future Work: Firms Shaping Human Capital Accumulation



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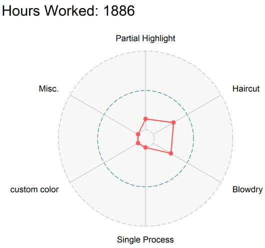
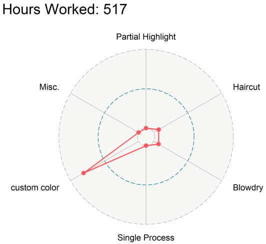


(b) Under Generalization

Week 61

Future Work: Firms Shaping Human Capital Accumulation



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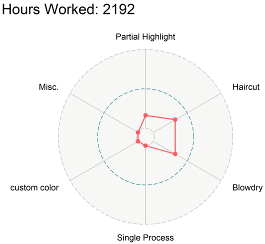
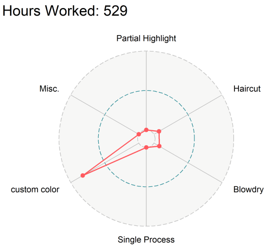


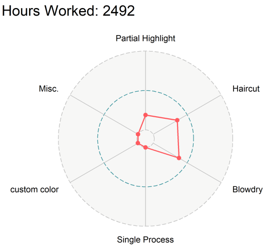
(b) Under Generalization

Week 66

Future Work: Firms Shaping Human Capital Accumulation



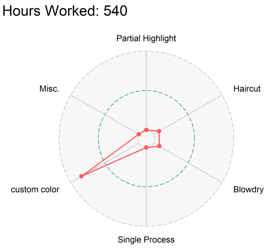
(a) Under Specialization



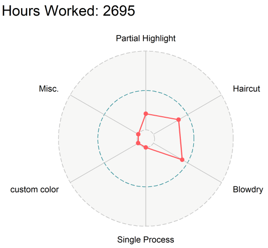
(b) Under Generalization

Week 71

Future Work: Firms Shaping Human Capital Accumulation



(a) Under Specialization



(b) Under Generalization

Week 76

Part of a Broader Agenda: Motivating Question

How do human resource decisions within firms shape markets?
(intersection of IO and Labor)

Part of a Broader Agenda: Working Papers

- “The Inner Beauty of Firms” (R&R at JPE): How organizationally unique firms assign tasks shapes labor productivity and the impact of policy.
- “Delegated Recruitment and Hiring Distortions” (R&R at JET, w/ Stepan Aleksenko): When firms hire via recruiters they statistically discriminate against diamonds in the rough.
- “Workplace Injury and Labor Supply within an Organization” (Reject&Resubmit at JoLE, w/ Robert McDonough): LA traffic officers choose to work when they are less likely to be injured. Shift auctions can leverage this selection to reduce injury.
- “Consumer Reviews and Dynamic Price Signaling” (w/ Stepan Aleksenko): A privately informed monopolist underprices in order to improve consumer reviews if vertical quality differentiation dominates horizontal taste differentiation.

Table of Contents

GMM Rank Conditions

$$rank\{E[\left(\{B_j(i,k)\}_{i,k}^{N,K} \quad \frac{\gamma_j}{\gamma_1} I_j\right)\left(\{B_j(i,k)\}_{i,k}^{N,K} \quad \frac{\gamma_j}{\gamma_1} I_j\right)']\} = N \times K + 1$$

$$rank\left\{E\left[\left(\{\frac{E_j(i)}{\sum_{i'} E_j(i')}\}_i^N \quad \frac{\gamma_j}{\gamma_1} I_j\right)\left(\{\frac{E_j(i)}{\sum_{i'} E_j(i')}\}_i^N \quad \frac{\gamma_j}{\gamma_1} I_j\right)\right]\right\} = N \times K + 1$$

Back

Same Skills, Same Job

Proposition

In any profit maximizing task assignment, all workers with the same skill set and skill level are assigned the same distribution of time across tasks.

Proof. Notice that the only difference between two workers with the same skill set is their labor supply. Because consumers care only about the average quality of tasks performed, it follows immediately that if a firm employs multiple workers with the same skill set, the only reason to assign them different amounts of each task is to economize on organization costs. However, it turns out that any differences in the distribution of time across tasks only increases the necessary communication. Therefore firms alway assign workers with the same skills the same distribution of time across tasks. Back

Different Skill Level, Indifference, Same Job

Proposition

In any equilibrium, wages are such that all firms are indifferent between all workers with the same skill set but different skill levels. Further, firms assign all workers with the same skill set but different skill levels the same distribution of time across tasks.

Proof. Consider two workers with the same skill set but different skill level $\bar{\theta}_m > \bar{\theta}_{m'}$. Because they differ only in vertical skill, conditional on being hired, any firm will assign the same distribution of time across tasks. Therefore the impact of hiring m compared to m' on profit comes only through wage differences and skill level differences. It can be shown that firms maximize wage-adjusted skills. Therefore firm j hires worker m over worker m' if: $w_m - w_{m'} \geq \rho^{-1}(\bar{\theta}_m - \bar{\theta}_{m'})$. Notice that this inequality is the same for all firms. This implies that labor markets do not clear unless wages are such that firms are indifferent: $w_m - w_{m'} = \rho^{-1}(\bar{\theta}_m - \bar{\theta}_{m'})$. Back

Fact 3: Specialized salons are more productive

Dependent Variable:	Revenue per Minute			
Model:	(1)	(2)	(3)	(4)
Task Specialization	2.469*	2.375	3.150	3.263*
	(1.256)	(1.223)	(1.648)	(1.578)
Task Mix Controls	Yes	Yes	Yes	Yes
Quarter-Year		Yes	Yes	Yes
County			Yes	Yes
Firm Size				Yes
R^2	0.06231	0.08746	0.23591	0.25913

Back

Service Sales Tax Elimination (4.5% to 0%)

Firm Choices

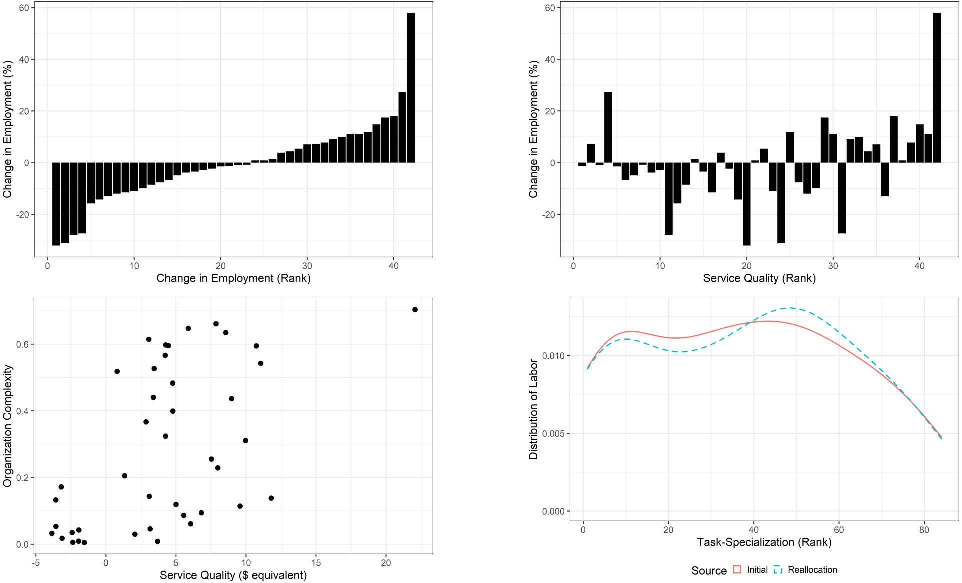
Statistic	Total
Avg. Price	8.68%
Avg. Complexity	5.53%
Avg. Quality	10.03%
Task Specialization	1.83%

Welfare

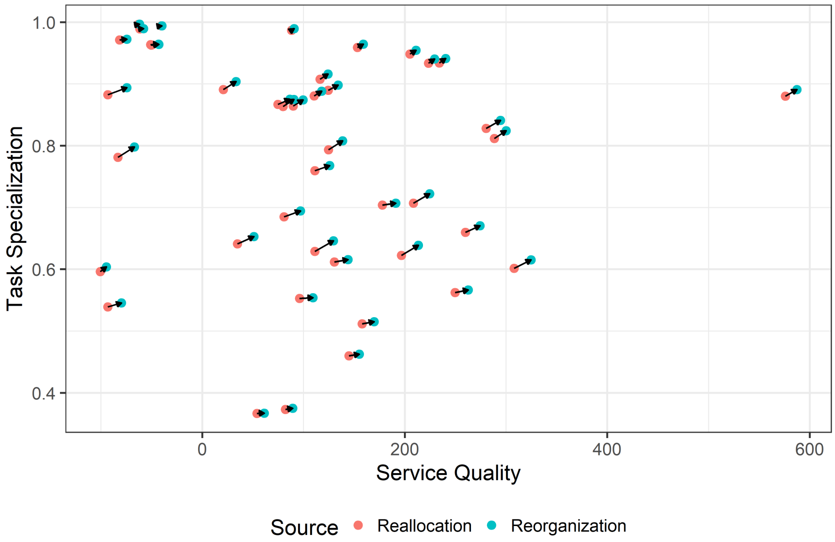
Source	Change	Percent Change
Salon Profit	$942,740	0.58%
Consumer Welfare	-$494,199	-0.30%
Wages	$11,603,777	7.12%
Tax Revenue	-$11,739,300	-7.20%
Total Welfare	$313,017	0.19%

Effects by Worker Type

Sales Tax Elimination Reallocation Effect



Sales Tax Elimination Reorganization Effect



Characterizing the Firm's Problem

Theorem

An organizational structure (B_j^*) is profit-maximizing if and only if it solves:

$$\min_{B_j \in \mathbb{B}} \gamma_j I(B_j) + W(B_j) - \rho^{-1} \xi(B_j)$$

Proof

Characterizing the Firm's Problem

Theorem

An organizational structure (B_j^*) is profit-maximizing if and only if it solves:

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i,k)(w_i - \rho^{-1}\theta_{i,k})$$

- Rate-distortion problem (information theory)

Proof

Characterizing the Firm's Problem

Theorem

An organizational structure (B_j^*) is profit-maximizing if and only if it solves:

$$\max_{B_j \in \mathbb{B}} \sum_{i,k} B_j(i,k)(\rho^{-1}\theta_{i,k} - w_i) - \gamma_j I(B_j)$$

- Rate-distortion problem (information theory)
- Rational inattention problem with MI costs (behavioral econ)
  - Org. frictions make the firm act as if it is run by a manager with limited attention

Proof

Characterizing the Firm's Problem

Theorem

An organizational structure (B_j^*) is profit-maximizing if and only if it solves:

$$\min_{B_j \in \mathbb{B}} \gamma_j I(B_j) + W(B_j) - \rho^{-1}\xi(B_j)$$

- Rate-distortion problem (information theory)
- Rational inattention problem with MI costs (behavioral econ)
  - Org. frictions make the firm act as if it is run by a manager with limited attention
- Internal organizations are connected only via wages

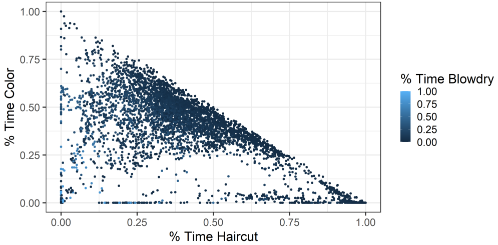
Proof

Firm-Quarter Statistics

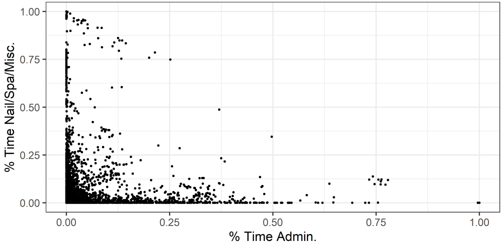
Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Revenue	4,558	213,201.30	248,359.90	5	58,912.5	271,236.5	2,559,703
Price	4,558	199.73	135.16	0.20	111.71	261.88	3,180.44
Employees	4,558	13.38	10.79	1	6	17	92
Customers	4,558	1,159.23	1,098.45	1	397	1,619	16,768
Task Categories	4,558	4.45	0.86	1	4	5	5
Labor per. Customer	4,558	2.15	1.63	0.10	1.52	2.57	61.33

Back Task-Mix Variation

Task-Mix Variation



(a) Cut, Color, Blowdry



(b) Admin.,Misc.

Back

Organization Costs As Average Task-Specialization

Define the generalist job as the job as: $b_j^G(k) = \alpha_k$

Proposition

Complexity ($I(B_j)$) is the weighted-average Kullback-Leibler divergence between the jobs at a firm and the firm's generalist job $b_j^G(k)$, where the weights are the share of each worker type.

Proof. Using the definition of mutual information, we can write out complexity as:

$$
\begin{aligned}
I(B_j) &= \sum_{i,k} B(i,k) log\left(\frac{B(i,k)}{\sum_{k'} B(i,k') \sum_{i'} B(i',k)}\right) = \sum_{i,k} E_i \frac{B(i,k)}{E_i} log\left(\frac{B(i,k)}{E_i \alpha_k}\right) \\
&= \sum_i E_i \sum_k b_i(k) log\left(\frac{b_i(k)}{\alpha_k}\right) = \sum_i E_i \sum_k b_i(k) log\left(\frac{b_i(k)}{b_j^G(k)}\right) \\
&= \sum_i E_i D_{KL}(b_i || b_j^G)
\end{aligned}
$$

Back

Managerial Attention

- X is the task type, with prior α. Y is assigned worker type. Manager's payoff from the assignment of workers to tasks is $m(X, Y)$.
- Manager chooses any signal Z with info about the task type and an assignment function $\delta(Z)$ mapping signal to an assignment.
- Cost of signal is γ_j multiplied by the mutual information between the signal and the task type:

$$\max_{\delta, Z} \mathbb{E}[m(X, \delta(Z))] - \gamma_j I(X, Z)$$

- Jung et al. (2019) show it is WLOG to choose joint distribution directly:

$$\max_{B_j \in \mathbb{B}_j} \mathbb{E}[m(X, Y)] - \gamma_j I(X, Y)$$

Back

Revenue Regressed on Complexity

Model:	(1)	(2)	(3)	(4)	(5)	(6)
Organization Complexity	456571.3***	440904.1***	485026.4***	486995.5***	271694.6**	261697**
	(100394.8)	(108427.1)	(116918.9)	(125004.8)	(87031.1)	(80920.6)
Staff Request Rate						-94370.7
						(89112.9)
Task Mix Control				Yes	Yes	Yes
Fixed-effects						
Quarter-Year		Yes	Yes	Yes	Yes	Yes
County			Yes	Yes	Yes	Yes
Firm Size					Yes	Yes
Fit statistics						
Observations	5,116	5,116	5,116	5,116	5,116	5,116
R^2	0.01475	0.01915	0.3104	0.31047	0.34273	0.34365

Clustered standard-errors in parentheses

*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

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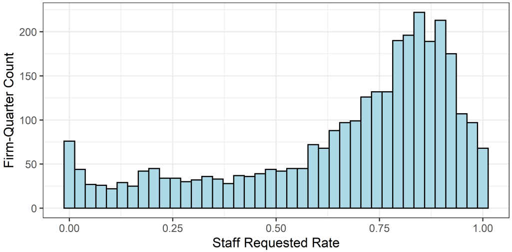
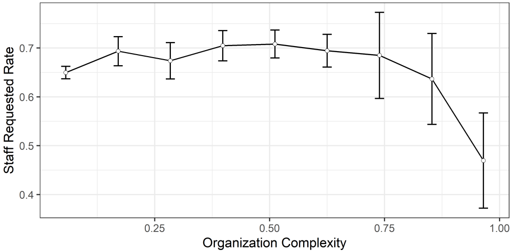
Was Staff Requested?



(a) Histogram



(b) Request Rate and specialization

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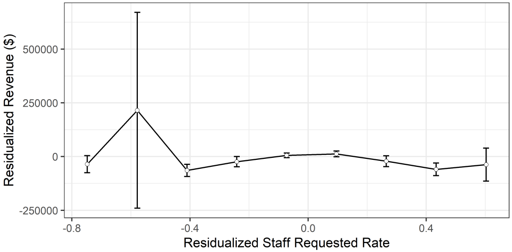
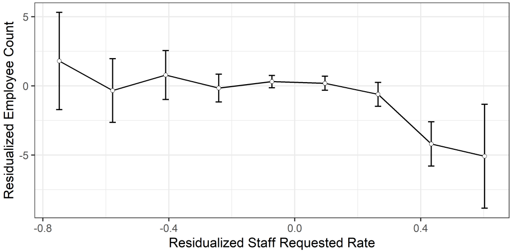
Was Staff Requested?



(a) Revenue



(b) Employees

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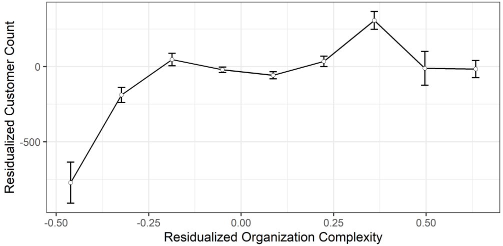
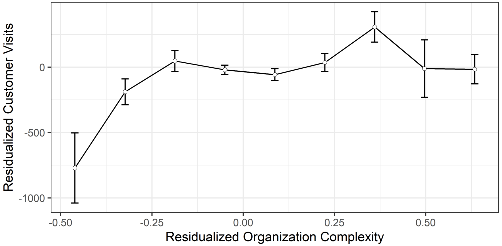
Relationship Between specialization and Customers/Visits



(a) Customers



(b) Visits

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Manhattan Firm Size and specialization Regressions

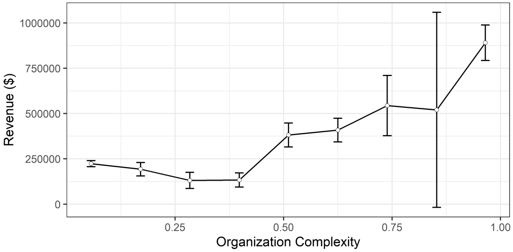
Dependent Variables:	Revenue	Employees	Utilized Labor	Customers	Visits
Model:	(1)	(2)	(3)	(4)	(5)
Variables					
Org. Complexity	430406.6*	12.55	-17733.9	277.2	876.9
	(179977.4)	(6.531)	(70765.2)	(600)	(907.1)
Fixed-effects					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
Fit statistics					
Observations	595	595	595	595	595
R^2	0.33485	0.21039	0.20359	0.44164	0.48831

Clustered standard-errors in parentheses

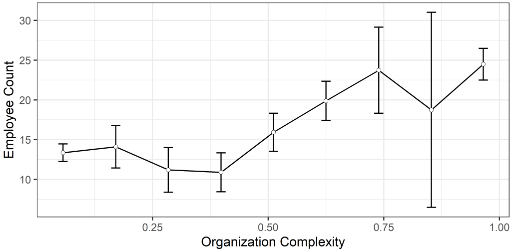
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Back

Fact 2: Complex salons have higher revenue and employment



(a) Revenue



(b) Employees

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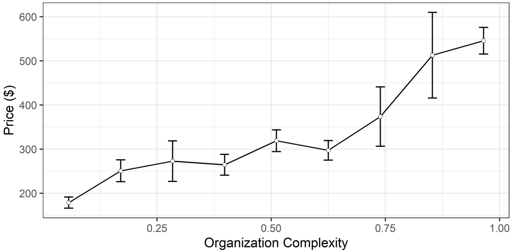
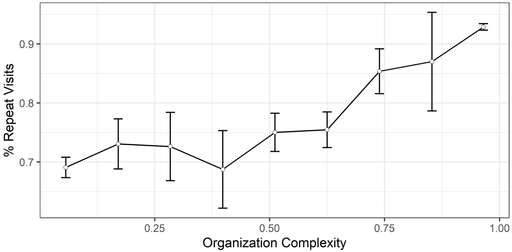
Fact 3: Complex salons have higher prices and repeat customers



(a) Prices



(b) Repeat Customers

Back

Quantity Model Implies Price $\downarrow$ specialization

$$F_{\alpha,B}(a_j) = \min\left\{\frac{a_1}{\alpha_1 \sum_i \theta_{i,1} B_j(i,1)}, \cdots \frac{a_k}{\alpha_k \sum_i \theta_{i,k} B_j(i,k)}, \cdots, \frac{a_K}{\alpha_K \sum_i \theta_{i,K} B_j(i,K)}\right\}$$

Given any fixed organizational structure, the efficient way to produce a single unit of output is to set $a_k = \alpha_k \sum_i \theta_{i,k} B_j(i,k)$. Thus marginal costs are constant and consist of the per-unit wage bill and organization costs:

$$MC_j = \sum_i w_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k) + \gamma_j I(B_j)$$

Proposition

Under these assumptions, prices are decreasing with organizational specialization.

Back

Proof of Theorem: Only if Direction 1/2

- Consider any feasible (p', B_j') where price is higher than marginal cost.[1]
- There always exists B_j^* which solves the equivalent problem.[2]
- Construct $p_j = p_j' + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B_j') - W(B_j')$. This price is feasible b/c $p_j' - \gamma_j I(B_j') - W(B_j')$ is price less MC and $\gamma_j I(B_j^*) + W(B_j^*)$ is positive.
- By construction, price less marginal cost is equal under (p_j, B_j^*) and (p', B_j').
- To show profit is higher under (p_j, B_j^*) we need only show demand is higher.

Back

1. When $p < MC$ profit is always negative.
2. b/c it is an RI problem (convex objective over compact set).

Proof of Theorem: Only If Direction 2/2

To show demand is higher we need only show the quality-price index is higher:

$$
\begin{aligned}
&= \xi(B_j^*) - \rho[p_j' + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B_j') - W(B_j')] && (1)\\
&= \xi(B_j^*) - \rho[p_j' + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B_j') - W(B_j')] + \xi(B_j') - \xi(B_j') && (2)\\
&= \xi(B_j') - \rho[p_j' + \gamma_j I(B_j^*) + W(B_j^*) - \gamma_j I(B_j') - W(B_j') - \rho^{-1}\xi(B_j^*) + \rho^{-1}\xi(B_j')] && (3)\\
&= \xi(B_j') - \rho p_j' - \rho[\underbrace{\gamma_j I(B_j^*) + W(B_j^*) - \rho^{-1}\xi(B_j^*) - \{\gamma_j I(B_j') + W(B_j') - \rho^{-1}\xi(B_j')\}}_{\leq 0 \text{ because } B_j^* \text{ minimizes}}] && (4)\\
&\geq \xi(B_j') - \rho p_j' && (5)
\end{aligned}
$$

Back

Proof of Theorem: If Direction

- Suppose there exists B_j' which maximizes profit but does not solve the RI problem.
- As before, there exists B_j^* which does solve.
- Construct p_j as before.
- Because B_j' does not solve the RI problem, we have that $\xi(B_j^*) - \rho p_j > \xi(B_j') - \rho p_j'$
- This implies B_j^* does not maximize profit, a contradiction.

Back

Proof of Frontier Shape and Profit/Complexity Relationship 1/2

- Denote Q as quality-adjusted wages. Denote $I^*(Q)$ as optimal complexity as a function of quality-adjusted wages.

- RD equivalence $\implies$ $I^*(Q)$ is continuous, convex and decreasing. Also strictly decreasing above some threshold $\bar{Q}$ (Chen, n.d.).

- The firm's choice of quality-adjusted wages solves:

$$V := \min_{Q} \gamma I^*(Q) + Q$$

- Envelope theorem implies the index and thus profit are increasing in γ:

$$\frac{\partial V}{\partial \gamma} = I^*(Q) \geq 0$$

Back

Proof of Frontier Shape and Profit/Complexity Relationship 1/2

- Examining the FOC:

$$\frac{dI^*(Q) + \gamma^{-1}Q}{dQ} = \frac{dI^*(Q)}{dQ} + \gamma^{-1} = 0 \implies \frac{dI^*(Q)}{dQ} = -\gamma^{-1}$$

- Because I^* is decreasing and convex, its derivative is negative and increasing.
- Therefore Q which solves is increasing in γ.
- Thus profit and complexity will be positively correlated via γ.

Back

Minimum Wage Counterfactual Details

- Counterfactuals assume the utility of not getting a service remains fixed.
- Ruling out Multiple Equilibria
  - Assume beforehand which wages bind (i.e. which wages are $20)
  - 0 excess labor supply for all types except binding types.
  - Check that assumed binding types have excess labor supply.
  - If yes, count as an equilibria. If not exclude.
- I do this for all 2^5 combinations.
- This results in only one equilibrium.

Back

Consumer Welfare

Therefore expected utility of consumer i has the well-known closed form:

$$V_i = \mathbb{E}[\max_j\{\xi_j - \rho p_j + \epsilon_{i,j}\}] = ln\left[\sum_{j=1}^{J} exp\left(\xi_j - \rho p_j\right)\right] + C$$

where C is Euler's Constant. There are a mass M of consumers, therefore total consumer expected utility is $M \cdot V_i$. We then can denominate this in dollar terms by dividing by the coefficient on price, ρ. Our measure of total consumer welfare in dollar terms is:

$$CS = \frac{M}{\rho}\left\{ln\left[\sum_{j=1}^{J} exp\left(\xi_j - \rho p_j\right)\right] + C\right\}$$

With a sales tax τ, it is:

$$CS = \frac{M}{\rho}\left\{ln\left[\sum_{j=1}^{J} exp\left(\xi_j - \rho(1+\tau) p_j\right)\right] + C\right\}$$

Back

Essential Equilibrium Uniqueness

Proposition

There exists a unique Nash equilibrium in prices (p_j) and organizations (B_j) for every fixed-w subgame.

Proof Sketch:

- Bertrand oligopoly with logit demand has unique NE — Caplin and Nalebuff (1991)
- Profit is strictly incr. in quality-adjusted cost — Equivalence to RI
- Pointwise exponential of PD matrix is PD — Schur Product Thm
- PD exp. payoffs implies unique org. — Matějka and McKay (2015)

Back

A Sufficient Condition for the Uniqueness of B_j

Assumption

Define the wage-quality vector of a worker of type i at firm j as $v_{i,j} = \{exp(\gamma_j^{-1}(\rho^{-1}\theta_{i,k} - w_i))\}_{k=1}^{K}$. Each firm's wage-quality vector $\{v_{i,j}\}_{i\in\mathcal{I}}$ is affinely independent.

Source: Matêjka and McKay (2015)

Back

Minimum Wage Counterfactual Employment and Wages

	Initial			Reallocation			Counterfactual
Worker Type	Hours	Wage	Hours	Wage	Hours	Wage	
Haircut/Shave	537550	$16.96	506090	$20.00	502152	$20.00	
Color/Highlight/Wash	997053	$37.75	997053	$37.33	997053	$37.52	
Blowdry/Style/Treatment/Extension	444040	$20.91	444040	$21.88	444040	$21.64	
Administrative	41860	$26.99	41860	$28.40	41860	$28.12	
Nail/Spa/Eye/Misc.	34844	$81.16	34844	$81.63	34844	$81.71	

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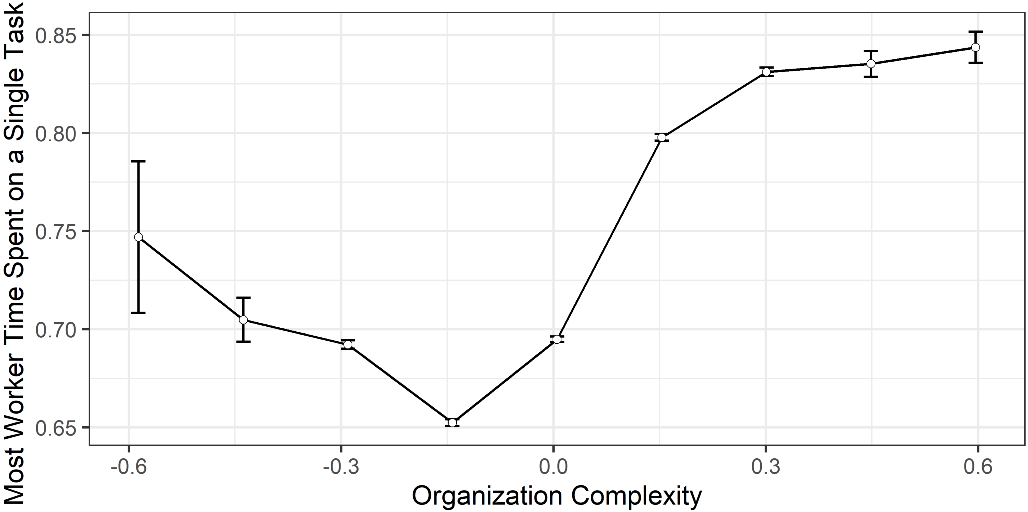
Regressions of Worker Specialization on Organization Complexity



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Why Aggregation?

- A single product allows me to focus on the effects of organization on overall salon quality.
- Consumers buy a bundle of services at salons.
- It buys significant numerical/theoretical tractability.
- Nocke and Schutz (2018): any pricing game with multi-product firms and MNL demand can be represented as a single product firm game with transformed qualities and costs:

$$\tilde{q}_j = \rho log\left(\sum_k exp((q_k - c_k)/\rho)\right) + 1 \qquad \tilde{c}_j = 1$$

Back

Sales Tax Elimination Effects by Worker Type

Type	Wage Change	Task-Spec. Change
Haircut/Shave	31.99%	0.29%
Color/Highlight/Wash	20.09%	2.57%
Blowdry/Style/Treatment/Extension	6.06%	3.01%
Administrative	17.99%	1.03%
Nail/Spa/Eye/Misc.	12.74%	2.39%

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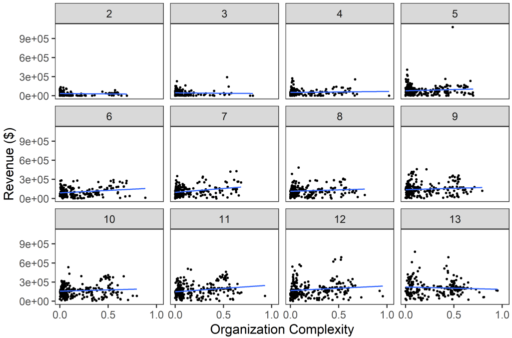
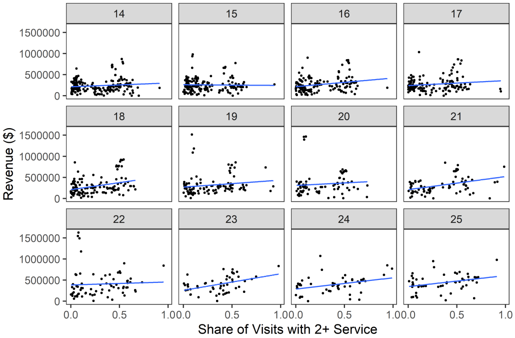
Minimum Wage Welfare Effects

Source	Change	Percent Change
Salon Profit	-$714,413	-0.472%
Consumer Welfare	-$2,528,784	-1.671%
Employed Wages	$1,689,600	1.116%
Unemployed Wages	-$600,240	-0.397%
Total Welfare	-$2,153,838	-1.423%

Revenue and Complexity Among Similar Size Firms



(a) 2-13 Employees



(b) 14-25 Employees

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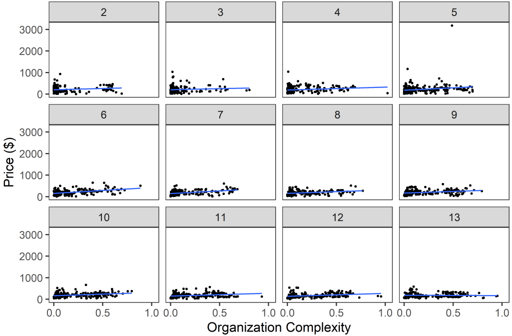
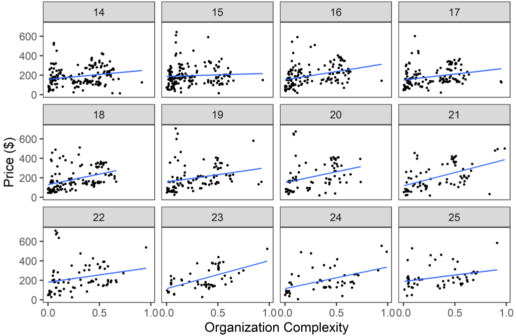
Price and Complexity Among Similar Size Firms



(a) 2-13 Employees



(b) 14-25 Employees

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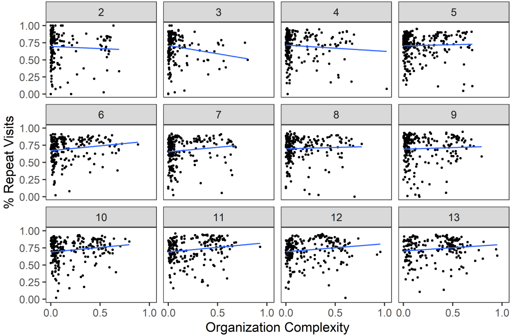
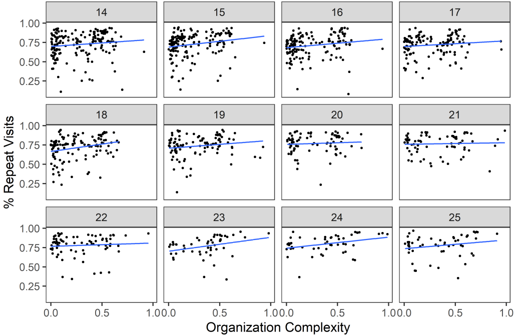
Repeat Visits and Complexity Among Similar Size Firms



(a) 2-13 Employees



(b) 14-25 Employees

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Within-Visit Specialization

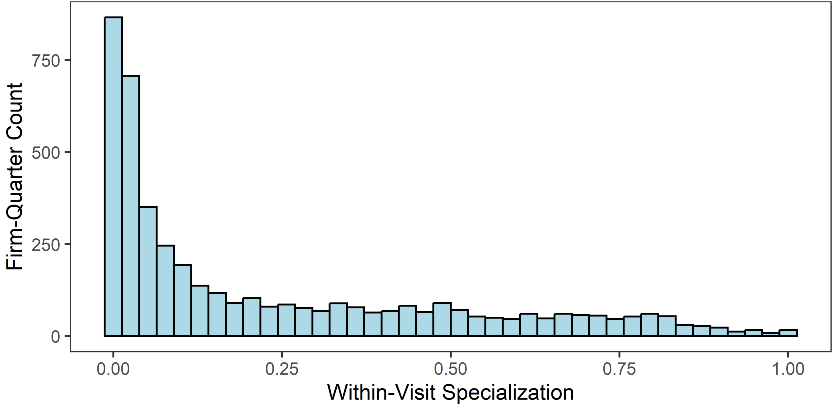
- Within-visit specialization: the number of customer visits[3] with two or more employees assigned divided by the number of customer visits with two or more services performed.

- R-squared of complexity regressed on within-visit specialization is 0.5

- Two firm-quarters are drawn randomly their ordering according to complexity and within-visit specialization will be the same 74.4% of the time.

3. Visits are the number of unique customer-date pairs in a quarter.

Within-Visit Specialization Histogram



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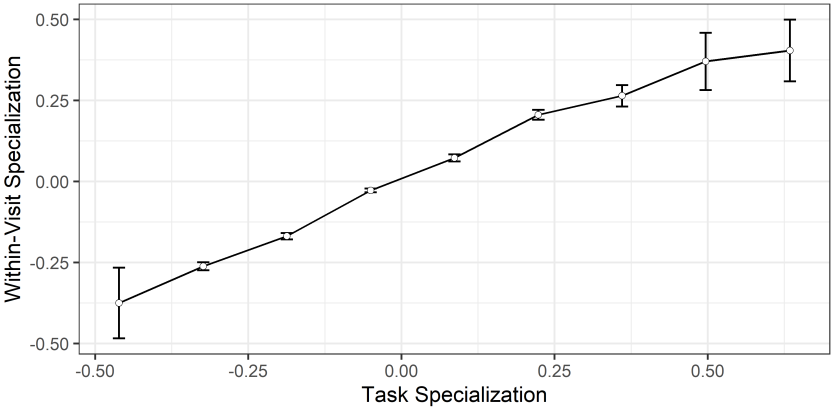
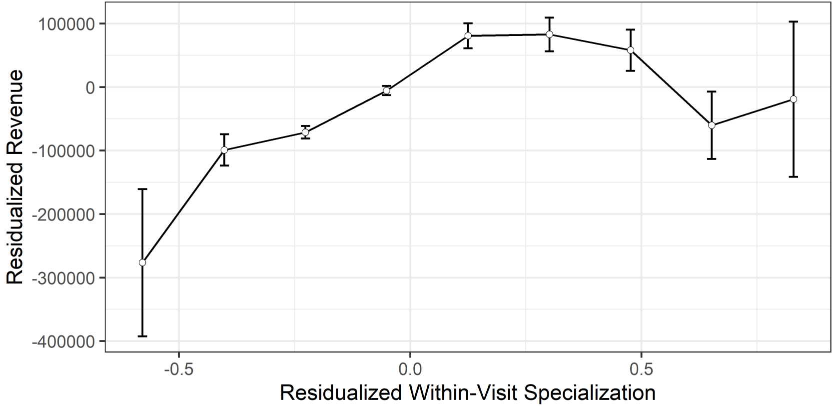
Within-Visit Specialization and Complexity



Within-Visit Specialization and Revenue



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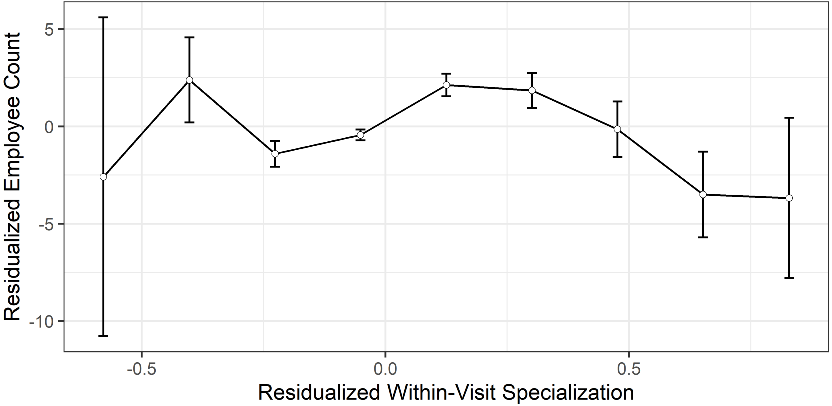
Within-Visit Specialization and Employees



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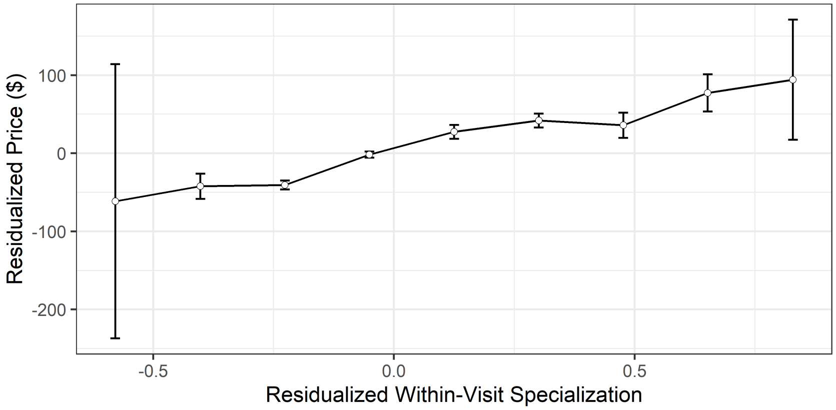
Within-Visit Specialization and Price



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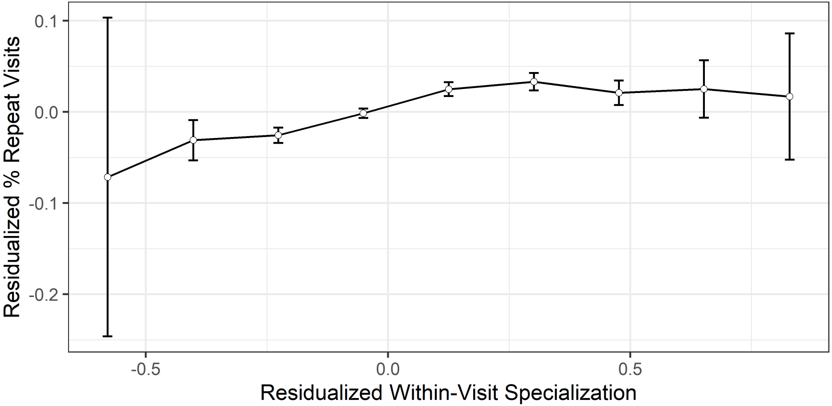
Within-Visit Specialization and Repeat Visits



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Complexity of the Two Structures

Specialist Salon

Tasks

Employee	Cut	Color	Dry	
A	1/2	0	0	1/2
B	0	1/4	0	1/4
C	0	0	1/4	1/4
Tot.	1/2	1/4	1/4	

Exactly match tasks and workers

If cut send "0" assign to A

If color send "01" assign to B

If dry send "10" assign to C

$$\frac{1}{2}(1bit) + \frac{1}{4}(2bit) + \frac{1}{4}(2bit) = 1.5$$

Generalist Salon

Tasks

Employee	Cut	Color	Dry	
A	1/6	1/12	1/12	1/3
B	1/6	1/12	1/12	1/3
C	1/6	1/12	1/12	1/3
Tot.	1/2	1/4	1/4	

Randomly match tasks and workers

If cut send nothing roll dice

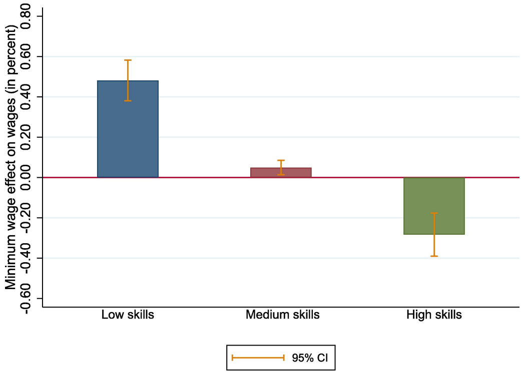
If color send nothing roll dice

If dry send nothing roll dice

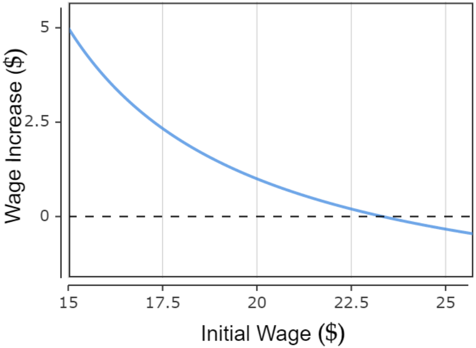
$$\frac{1}{2}(0bit) + \frac{1}{4}(0bit) + \frac{1}{4}(0bit) = 0$$

Minimum Wage Increases In Models with Distance Dependent Substitution

Wage Increase by Skill Level



Wage Changes by Initial Wage Percentile



Left is from Gregory and Zierahn (2022), right is stylized example