Problem Set 3

Jacob Kohlhepp

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The purpose of this homework is to work through three models we solved in class. There are only minor differences between these problems and the ones we did in class, so your notes should be very helpful in completing this problem set.

1 Teamwork

Note: it may be easiest to just do all math with a generic worker i.

1.1 Setup

- There are N workers, indexed by $i = 1, .., N$
- Each worker can exert effort e_i at cost $c_i(e_i) = 1/2 + e_i^2/2$
- Output is the sum of everyone's effort: $y(e) = \sum_{i=1}^{N} e_i$
- The firm can pay a wage to each worker based only on team output $w_i(y(e))$

1.2 Questions

1. Find the first-best effort for each worker (the amount of effort which maximizes total surplus).

   Consider a wage scheme $w_i(y(e)), ..., w_N(y(e))$ that is a partnership (see the definition from class). You may assume that the wage is differentiable.

2. Setup the worker's utility maximization problem. Also write down the budget-balance condition for partnerships.

3. Find and simplify the worker's effort first-order condition.

4. Use your answers from (1) and (3) and the fact that in partnerships all money must be paid out to prove that we cannot get first-best effort.

   Consider a wage scheme $w_1(y(e)), ..., w_N(y(e))$ that is a group bonus (see the definition from class) where the target is total first-best effort $\bar{y} = \sum_{i=1}^{N} e_i^{FB}$ and the bonus amount is more than the effort cost $b_i \geq c_i(e_i^{FB})$.

5. Argue that each worker does not want to exert too little effort ($e_i < e_i^{FB}$).

6. Argue that each worker does not want to exert too much effort ($e_i > e_i^{FB}$).

7. Find a group bonus that gives everyone the same bonus $b_i = b$ and that achieves first-best effort.

8. Give a situation (an effort choice of each worker) where money is burned under this group bonus. Note that this situation does not need to be an equilibrium.

9. Interpret the change in the cost function in this problem relative to our "normal" cost function $c(e_i) = e_i^2/2$. How does this change impact the solution?

10. If workers had an outside option and were allowed to choose to take the job prior to choosing effort, how would the change to the cost function impact how the firm design's compensation?

2 Career Concerns

Setup

- There are two firms and one worker.
- The worker has a skill level a that no one knows.
- However, everyone knows that skills are distributed uniformly between $[0, A]$. That is, $a \sim U[0, A]$
- The worker exerts unobserved, costly effort: $c(e) = e^2/2$.
- Revenue is equal to effort plus skill: $y = e + a$
- The worker is hired and exerts effort in two periods.
- The worker is hired in each period by the firm that posts the highest wage, and if there is a tie they randomly pick a firm (Bertrand style)
- All outside options are 0.

Questions

1. What is the first-best level of effort for a single period? That is, the e_{FB} that maximizes output less the cost of effort?
2. How much effort will the worker exert in period 2? Justify your answer.
3. Denote the effort the firm believes the worker exerts in period 1 $\tilde{e}_1$. How can the firm recover the worker's skill using $\tilde{e}_1$ and output y_1?
4. What output levels y_1 will the firm never observe if the worker does the effort that is expected ($\tilde{e}_1$)?
5. Suppose the firms believe skill is a in period 2. What wage will they bid in period 2? Justify your answer.
6. Solve for the worker's effort in period 1.
7. What wage do the firms in period 1 bid? Justify your answer.
8. How does this effort compare to the effort in sub question 1? Why is the worker working hard?
9. Suppose $A = 100$. If a worker has skill 50, by what amount does their wage change from period 1 to period 2?
10. Explain, either verbally or mathematically, what effort and wages in each period would be if the worker's skill was just a fixed number, a, that everyone knew from the very beginning. How does this help explain why the worker exerts effort in the main model where skill is unknown?

11. Explain, either verbally or mathematically, what effort and wages in each period would be if skill was just a fixed number, a, that everyone knew from the very beginning AND both firms could use performance pay (i.e. a wage where $w(y) = \alpha + \beta y$). Assume that each firm "bids" a performance pay $w_1(y), w_2(y)$ and the worker chooses the performance pay that they expect to give them the highest utility. Hint: the firms use α to "get" the worker and they use β to get the right effort once the worker is hired.