Final Exam: Econ 490 Compensation in Organizations

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Name: ____ PID: ____________________________

You have 3 hours to complete this exam. Please stop writing when told to do so. Write all answers in the space provided, and show work where possible. If you run out of room, make a note and use the additional pages attached at the end of the exam. This is a closed book exam. The only materials you may use are a pen and paper. By taking this exam, you agree to follow the UNC Chapel Hill honor code, in particular the standards of academic integrity. All academic dishonesty will be reported to the Office of Student and the Student Attorney General. Each individual question (both reading and models) is worth 5 points, for a total of 170 points.

1 Readings

Answer these questions in 3 sentences or less.

1. In Alexander (2020), what were doctor's bonuses based on?

   "I shed new light on the importance of experimental design in healthcare using the New Jersey Gainsharing Demonstration, a pilot program in which hospitals paid doctors bonuses for reducing the total treatment costs for Medicare admissions."

2. In Bandiera et. al. (2005), what two incentive schemes are compared, and which results in higher productivity?

   "We present evidence on whether workers have social preferences by comparing workers' productivity under relative incentives, where individual effort imposes a negative externality on others, with their productivity under piece rates, where it does not. We find that the productivity of the average worker is at least 50 percent higher under piece rates than under relative incentives."

3. Friebel, Heinz, Krueger, and Zubanov (2017) randomly assign a team bonus to bakeries. What impact did the team bonus have on treated bakeries, and what happened when the bonus was rolled out across the firm?

   "The bonus increases both sales and number of customers dealt with by 3 percent...After rolling out the bonus scheme, the performance of the treatment and control shops converges, suggesting long-term stability of the treatment effect."

4. Oyer and Schaefer (2005) argue that incentives are unlikely to be the reason why stock options are offered to employees by firms. Describe their argument.

   Argument ad absurdem is enough. Alternatively: The authors calibrate a model of performance pay like in class, and show that for it to justify stock options, risk premiums have to be very large relative to the cost of effort. Essentially, the primitives would need to be absurd values.

5. In Blair and Chung (2022) "Job Market Signaling through Occupational Licensing," in what types of states does occupational licensing reduce the racial wage gap the most? Use one sentence.

   States with ban the box laws.

6. How does Cullen (2024) summarize the effect of horizontal pay transparency laws on wage levels AND gender wage gaps?

   "In the cases where transparency achieved greater pay equalization between men and women—those in the lower left quadrant of the graph—the reduction in pay gap was accompanied by an overall reduction in wages."

2 Teamwork

2.1 Setup

- There are N workers, indexed by $i = 1, .., N$ that are risk neutral.
- Each worker can exert effort e_i at cost $c_i(e_i) = e_i^2/2$
- Output is the sum of everyone's effort: $y(e) = \sum_{i=1}^{N} e_i$
- The firm can pay a wage to each worker based only on team output $w_i(y(e))$. The firm is also risk neutral.
- You are given an explicit cost function, so please give answers as explicitly as possible. When possible, provide a number.

2.2 Questions

1. Find the first-best effort for each worker (the amount of effort which maximizes total surplus).

$$\max_{e} \sum_{=1}^{N} (e_i - c_i(e_i))$$

   Taking the FOC for a single worker:

$$1 - c_i'(e_i) = 0 \implies c'(e_i) = e_i^{FB} = 1 \tag{1}$$

   Consider a wage scheme $w_1(y(e)), ..., w_N(y(e))$ that is a partnership (see the definition from class). You may assume that the wage is differentiable.

2. Setup the worker's utility maximization problem. Also write down the budget-balance condition for partnerships.

   The worker's utility maximization problem:

$$\max_{e_i} w_i(y(e)) - c_i(e_i)$$

   Budget balance:

$$\sum_{i=1}^{N} w_i(y(e)) = \sum_{i=1}^{N} e_i$$

   Or:

$$\sum_{i=1}^{N} w_i(y(e)) = y(e) \tag{2}$$

3. Find and simplify the worker's effort first-order condition.

$$\frac{\partial w_i(y(e))}{\partial e_i} - c_i'(e_i) = 0$$

We have that $\frac{\partial w_i(y(e))}{\partial e_i} = w_i'(y(e))\frac{\partial y(e)}{\partial e_i} = w_i'(y(e))$ by the chain rule so:

$$w_i'(y(e)) = c_i'(e_i) = e_i \tag{3}$$

4. Use your answers from (1) and (3) and the fact that in partnerships all money must be paid out to prove that we cannot get first-best effort if $N > 1$.

Assume that a partnership w_i does achieve the first-best. Then because it is a partnership, we have budget balance which is given by Equation 2. Taking the derivative of both sides wrt $y(e)$:

$$\frac{\partial}{\partial y(e)} \sum_{i=1}^{N} w_i(y(e)) = \frac{\partial}{\partial y(e)} y(e)$$

$$\sum_{i=1}^{N} \frac{\partial w_i(y(e))}{\partial y(e)} = 1$$

$$\sum_{i=1}^{N} w_i'(y(e)) = 1$$

This equation must be true at the first-best, so:

$$\sum_{i=1}^{N} w_i'(y(e^{FB})) = 1$$

But then from Equation 3 we have that:

$$\sum_{i=1}^{N} c_i'(e_i^{FB}) = 1$$

And from Equation 1 we have that:

$$\sum_{i=1}^{N} 1 = 1$$

But then this is a contradiction whenever $N > 1$:

$$\sum_{i=1}^{N} 1 = N \neq 1$$

Consider a wage scheme $w_1(y(e)), ..., w_N(y(e))$ that is a group bonus (see the definition from class) where the target is total first-best effort $\bar{y} = \sum_{i=1}^{N} e_i^{FB}$ and the bonus amount is more than the effort cost $b_i \geq c_i(e_i^{FB})$.

5. Argue that each worker does not want to exert too little effort ($e_i < e_i^{FB}$).

First, given all other workers are exerting first-best effort, exerting first-best effort yields a payoff that is weakly positive because $b_i - c_i(e_i^{FB}) \geq 0$. Consider the case where everyone but one worker i is exerting first-best effort. If worker i exerts less than first-best effort, target is not achieved ($y(e) < \bar{y}$) and the bonus is not paid out to everyone including worker i. Worker i receives 0 payment and pays an effort cost equal to whatever low effort $e_i' < e_i^{FB}$ he/she chooses. The "best" deviation of this type is then to exert 0 effort, in which case worker i receives $0 - c_i(0) = 0$. Given that the payoff from exerting first-best effort is at least 0, this deviation at best has no benefit to the worker.

6. Argue that each worker does not want to exert too much effort ($e_i > e_i^{FB}$).

   First, given all other workers are exerting first-best effort, exerting first-best effort yields a payoff that is weakly positive because $b_i - c_i(e_i^{FB}) \geq 0$. Consider the case where everyone but one worker i is exerting first-best effort. If worker i exerts more than first-best effort, target is achieved ($y(e) > \bar{y}$) but the worker's bonus does not increase. However, effort costs do increase. Thus worker i's utility strictly decreases and they are not better off from deviating in this way.

7. Find a group bonus that gives everyone the same bonus $b_i = b$ and that achieves first-best effort. Be as explicit as possible.

   A group bonus that achieves the first-best effort must cover everyone's effort cost, that is:

   $$b_i \geq c_i(e_i^{FB}) \text{ for all i}$$

   To be feasible, output must always be weakly larger than the total amount paid out, that is:

   $$\sum_i w_i(y(e)) \leq y(e) \text{ for all } y(e)$$

   There are only two cases under our group bonus: either we make target ($y(e) \geq \bar{y}$) and we pay out each person's bonus or we do not and we pay out 0 to everyone. Clearly, paying out 0 is always feasible so we need only think through the first case. In the first case, we know that output is always above $\bar{y}$ which we set to be first-best output $y(e^{FB}) = \sum_i 1 = N$. Our requirement for feasibility becomes:

   $$\sum_i b_i \leq \bar{y} = N$$

   We need to satisfy this and:

   $$b_i \geq c_i(e_i^{FB}) = 1/2 \text{ for all i}$$

   Imposing that everyone gets the same bonus, we can substitute $b_i = b$ and have the following two equations:

   $$b \geq 1/2 \qquad \sum_i b \leq N$$

   Depending on whether we want the bonus scheme to give all surplus to the workers or the firm, we can set either inequality to be an equality. The firm-generous bonus sets the first to be an equality, so that $b = 1/2$. This satisfies the second inequality because $\sum_i 1/2 = N/2 \leq N$. The worker-generous bonus sets the second to be an equality, so that $\sum_i b = N \implies b = 1$. This clearly satisfies the first inequality. Other answers include any bonus where $1/2 \leq b \leq 1$.

8. Give a situation (an effort choice of each worker) where money is burned under the group bonus you designed. Note that this situation does not need to be an equilibrium.

   There are many possible answers. One possibility is that everyone except one worker (let's just say worker 1) exerts first-best effort. Worker 1 exerts 0 effort. Then total output is $\sum_{i \neq 1} 1 = N - 1$. This is less than first-best total output N, therefore no one receives a bonus and the firm has to "burn" $N - 1$ output.

9. For this question only assume there are 2 workers. Worker 1 has our typical effort cost function $c_1(e_1) = e_1^2/2$. Worker 2's effort cost is no longer the typical cost function, and is now $c_2(e_2) = e_2^2/4$. Write down a group bonus that achieves first-best effort for these two workers.

   We are now told that $N = 2$ and the two workers have different cost functions. The group bonus must continue to satisfy two conditions: (1) $b_i \geq c_i(e_i^{FB})$ and (2) $\sum_i w_i(y(e)) \leq y(e)$. First-best effort for worker 1, who has our typical cost function, is $e_i^{FB} = 1$. Therefore, their cost of effort at the first-best is $c_i(e_i^{FB}) = 1/2$, the same as in the last question. Their bonus must satisfy $b_1 \geq 1/2$.

   Worker 2 has a new cost function. To find their first-best effort, simply solve:$\max_{e_2} e_2 - c_2(e_2)$. Taking the FOC, we have that $1 - c_2'(e_2) = 1 - e_2/2 = 0$. Solving this equation, first-best effort is then $e_2^{FB} = 2$. The cost of first-best effort for this worker is then $c_2(e_2^{FB}) = (2)^2/4 = 1$. Worker 2's bonus must satisfy $b_2 \geq 1$. Feasibility means that the bonus must not exceed first-best output: $b_1 + b_2 \leq e_2^{FB} + e_1^{FB} = 1 + 2 = 3$. A worker-generous bonus is then $b_1 = 1, b_2 = 2$. A firm-generous bonus is then $b_1 = 1/2, b_2 = 1$. There are also many bonuses in between. A bonus of $b_1 = 1, b_2 = 1$ would actually work for both this problem and the last problem.

3 Relative Performance Evaluation

This problem is mathematically identical to the one we did in class and in the problem set, except that the noise terms have an explicit interpretation. This interpretation should not impact how you solve for equilibrium, but it will impact how you interpret the results.

3.1 Setup

- Suppose there are two workers labeled 1 and 2 with the same cost of effort $c(e_i)$.
- Output for each $y_1 = e_1 + \epsilon_1$, $y_2 = e_2 + \epsilon_2$
- ϵ_1 represents the unknown skill of worker 1, and ϵ_2 represents the unknown skill of worker 2. Workers do not know their own skill, and neither do firms.
  - $\epsilon_1 = v_s + v_1$
  - $\epsilon_2 = v_s + v_2$
  - where $v_s \sim N(0, \sigma_s^2)$, $v_1 \sim N(0, \sigma_1^2)$ and $v_2 \sim N(0, \sigma_2^2)$[1]
- v_s represents the shared skill level of workers, due to something like sorting into the firm from the same college. v_1, v_2 represent random skill differences across workers.
- Let's focus just on worker 1 (so do all questions for worker 1 but not 2)
- The firm can offer linear wages:
  - $w(y_1, y_2) = \alpha + \beta(y_1 - \gamma y_2)$

3.2 Questions

1. Derive the certainty equivalent of the worker's wage, and subtract effort costs to get an expression for the worker's utility.

   We will apply our certainty equivalent formula:

$$d = E[w(y_1, y_2)] - r\frac{Var[w(y_1, y_2)]}{2}$$

   The mean of the wage is:

$$\begin{aligned}
E[w(y_1, y_2)] &= E[\alpha + \beta(y_1 - \gamma y_2)] && (4)\\
&= \alpha + \beta E[y_1 - \gamma y_2] && (5)\\
&= \alpha + \beta E[e_1 + v_1 + v_s - \gamma(e_2 + v_2 + v_s)] && (6)\\
&= \alpha + \beta(e_1 - \gamma e_2) && (7)\\
& && (8)
\end{aligned}$$

[1] Technical note: All are also jointly independent.

The variance is:

$$\begin{aligned}
Var[w(y_1,y_2)] &= Var[\alpha+\beta(y_1-\gamma y_2)] \\
&= Var[\alpha]+Var[\beta(y_1-\gamma y_2)] \\
&= 0+Var[\beta(y_1-\gamma y_2)] \\
&= \beta^2 Var[y_1-\gamma y_2] \\
&= \beta^2 Var[e_1+v_1+v_s-\gamma(e_2+v_2+v_s)] \\
&= \beta^2 Var[v_1+v_s-\gamma(v_2+v_s)] \\
&= \beta^2 Var[v_1+(1-\gamma)v_s-\gamma v_2] \\
&= \beta^2 (Var[v_1]+Var[(1-\gamma)v_s]+Var[\gamma v_2]) \\
&= \beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)
\end{aligned}$$

We can now plug this into the certainty equivalent formula:

$$d(w)=\alpha+\beta(e_1-\gamma e_2)-r\frac{\beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)}{2}$$

Remember to get the utility in CE units from the job we must subtract the cost of effort:

$$u_{worker}=\alpha+\beta(e_1-\gamma e_2)-r\frac{\beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)}{2}-c(e)$$

2. Stare at the expression you obtained. Argue that γ does not impact the worker's choice of effort at all, either mathematically or verbally.

   If we stare at the expression, we see that γ does not multiply e_1 anywhere, so it will not enter the first-order condition. We can also show this by just taking the FOC:

   $$\frac{du_{worker}}{de_1}=\beta-c'(e)=0$$

   Economically, γ is the weight placed on the performance of someone else. Since I cannot impact someone else's effort and therefore their performance in this model, γ will not impact my choice of effort.

3. Argue as in class that γ only impacts the variance, so to find the optimal γ we only need to minimize the variance of the wage.

   We just showed that γ does not impact the worker's choice of effort. Therefore it will only impact whether the worker takes the job. Looking at the worker's utility we see that γ only impacts the variance of the worker's wage. We know there is a bias-variance trade-off, and that greater variance reduces profit. So the firm will use γ to minimize the variance of the worker's wage for fixed β.

4. Minimize the variance of the wage to find the profit maximizing γ. Call it γ_{rel}.

   We use this equation to obtain the variance term:

   $$u_{worker}=\alpha+\beta(e_1-\gamma e_2)-r\underbrace{\frac{\beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)}{2}}_{\text{variance term}}-c(e)$$

   We want to minimize the variance of the wage. It does not matter if we multiply by r, but we need to make sure we do not include the negative sign.

   $$\min_{\gamma}\frac{\beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)}{2}$$

FOC:

$$\beta^2/2\left(-2(1-\gamma)\sigma_s^2+2\gamma\sigma_2^2\right)=0$$

Simplify:

$$\left(-2(1-\gamma)\sigma_s^2+2\gamma\sigma_2^2\right)=0$$

$$2\left(-(1-\gamma)\sigma_s^2+\gamma\sigma_2^2\right)=0$$

$$-(1-\gamma)\sigma_s^2+\gamma\sigma_2^2=0$$

$$-\sigma_s^2+\gamma\sigma_s^2+\gamma\sigma_2^2=0$$

$$\sigma_s^2=\gamma\sigma_s^2+\gamma\sigma_2^2$$

$$\sigma_s^2=(\sigma_s^2+\sigma_2^2)\gamma$$

$$\gamma_{rel}=\frac{\sigma_s^2}{\sigma_s^2+\sigma_2^2}$$

5. Derive the variance of wages when $\gamma=\gamma_{rel}$, simplifying as time permits. Exclude the β^2 term. Call it σ_{Tot}^2.

From the derivation of the certainty equivalent we have that the variance of wages is:

$$\beta^2(\sigma_1^2+(1-\gamma)^2\sigma_s^2+\gamma^2\sigma_2^2)$$

Excluding β^2 and plugging in $\gamma=\gamma_{rel}$:

$$\begin{aligned}
\sigma_{tot}^2 &= \sigma_1^2+(1-\gamma_{rel})^2\sigma_s^2+\gamma_{rel}^2\sigma_2^2\\
&= \sigma_1^2+\left(1-\frac{\sigma_s^2}{\sigma_s^2+\sigma_2^2}\right)^2\sigma_s^2+\left(\frac{\sigma_s^2}{\sigma_s^2+\sigma_2^2}\right)^2\sigma_2^2\\
&= \sigma_1^2+\frac{\sigma_s^2\sigma_2^4}{(\sigma_s^2+\sigma_2^2)^2}+\frac{\sigma_s^4\sigma_2^2}{(\sigma_s^2+\sigma_2^2)^2}\\
&= \sigma_1^2+\frac{\sigma_s^2\sigma_2^4+\sigma_s^4\sigma_2^2}{(\sigma_s^2+\sigma_2^2)^2}\\
&= \sigma_1^2+\frac{\sigma_2^2\sigma_s^2}{\sigma_s^2+\sigma_2^2}\\
&= \sigma_1^2+\frac{\sigma_s^2}{\frac{\sigma_s^2}{\sigma_2^2}+1}
\end{aligned}$$

We see from the final expression that as σ_2^2 falls, the denominator increases, therefore the entire variance falls. This will be helpful later.

6. Derive the profit-maximizing β as a function of σ_{Tot}^2. If you remember the theorem for β from lecture you may use it without proof. Otherwise, you can maximize profit given $\gamma=\gamma_{rel}$.

The theorem from class states that:

$$\beta=\frac{1}{1+r\sigma^2c''(e)}$$

Given that we have our typical cost function, $c''(e)=1$. Under relative performance pay with $\gamma=\gamma_{rel}$, $\sigma^2=\sigma_{tot}^2$. Thus:

$$\beta_{rel}=\frac{1}{1+r\sigma_{tot}^2}$$

7. Suppose the firm hires workers that are more similar in skill, so that random differences in skill across workers becomes less important. Which primitives in the model could we decrease to represent this?

   The worker skills are captured by ϵ_1, ϵ_2. If workers are some similar in skill, random skill differences across individuals become less important. This corresponds to the variance of v_1, v_2 decreasing. So the answer is that σ_1^2, σ_2^2 fall.

8. Suppose the firm hires workers that are more similar in skill, so that random differences in skill across workers becomes less important. How does this impact γ and β.

   Given our previous answers, we know that this means σ_2^2 falls. From our expression for γ_{rel}, we see that γ_{rel} increases as σ_2^2 falls. From our expression for σ_{tot}^2 that we derived from plugging in γ_{rel}, we can see that σ_{tot}^2 falls as σ_2^2 falls which implies β rises from subquestion 6. Intuitively, the risk-incentive trade-off is alleviated because relative performance pay becomes more effective as sorting into the firm improves.

   This was not asked, but the firm can therefore impose greater incentives without imposing as much risk, increasing effort and total surplus. The firm obtains all surplus, so this increases profit.

4 Career Concerns

Setup

- There are two firms and one worker.
- The worker has a skill level a that no one knows.
- However, everyone knows that skills are distributed uniformly between $[0, A]$. That is, $a \sim U[0, A]$
- The worker exerts unobserved, costly effort: $c(e) = e^2/2$.
- Revenue is equal to effort plus skill: $y = e + a$
- The worker is hired and exerts effort in two periods.
- The worker is hired in each period by the firm that posts the highest wage, and if there is a tie they randomly pick a firm (Bertrand style)
- All outside options are 0.

Questions

1. What is the first-best level of effort for a single period? That is, the e_{FB} that maximizes output less the cost of effort?

   The first-best level of effort is the one which solves:

   $$\max_e E[e + a] - e^2/2$$

   Skill is additive so it does not impact optimal effort. Thus we can just take the FOC to get:

   $$[e:] 1 - e = 0 \implies e_{FB} = 1$$

2. How much effort will the worker exert in period 2? Justify your answer.

   In period 2 there is no future and there is no performance pay, so the worker exerts 0 effort (because effort is costly and has no benefit). Thus $e_2^* = 0$.

3. Denote the effort the firm believes the worker exerts in period 1 $\tilde{e}_1$. How can the firm recover the worker's skill using $\tilde{e}_1$ and output y_1?

   The firm knows that $y_1 = a + e_1$. If the firm believes $e_1 = \tilde{e}_1$ for all workers regardless of skill, they can figure out a by subtracting effort from output: $y_1 - \tilde{e}_1 = a$.

4. What output levels y_1 will the firm never observe if the worker does the effort that is expected ($\tilde{e}_1$)?

   If all workers exert $\tilde{e}_1$ the firm will only observe outputs between $\tilde{e}_1 + 0$ (the lowest skill worker) and $\tilde{e}_1 + A$ (the highest skill worker). This means the firm will never observe output below $\tilde{e}_1$ or above $\tilde{e}_1 + A$.

5. Suppose the firms believe skill is a in period 2. What wage will they bid in period 2? Justify your answer.

   The firms then compete Bertrand style for the worker. They know skill is a and they know $e_2^* = 0$, so revenue from the worker is just skill: $y_2 = a + e_2^* = a$. Both firms bid a wage of $w_2^* = a$. To see why, consider any deviation from this strategy, keeping the other firm bidding at a. If either firm deviates to $w > a$, they win the worker but lose money because $a - w < 0$. Suppose a firm bids a wage below a. Then they lose for sure and still get 0.

6. Solve for the worker's optimal effort.

   The period 1 wage w_1 is already decided by this point and it is a flat wage, so it does not impact the worker's choice of effort. When finding the worker's effort choice in period 1, we must be careful, because there are two efforts floating around. There is actual effort e_1, which the worker controls and the firm does not see, and there is $\tilde{e}_1$, the effort the firm expects the worker to put in. The worker can make the firm think that he/she is higher skill by exerting effort that is different than $\tilde{e}_1$. But the worker CANNOT impact the firm's belief about effort $\tilde{e}_1$. Thus when we take derivatives to find e_1, $\tilde{e}_1$ will behave as a constant. Thus the worker maximizes:

   $$\max_{e_1} w_1 + E[w_2(e_1, \tilde{e}_1)] - e_1^2/2$$

   The expected wage in period 2 given effort e_1 and expected effort $\tilde{e}_1$ is:

   $$E[w_2(e_1, \tilde{e}_1)] = E[y_1 - \tilde{e}_1] = E[a + e_1 - \tilde{e}_1] = E[a] + e_1 - \tilde{e}_1$$

   Thus the maximization problem becomes:

   $$\max_{e_1} w_1 + E[a] + e_1 - \tilde{e}_1 - e_1^2/2$$

   assuming e_1 is in the range $(\tilde{e}_1, \tilde{e}_1 + A)$. Taking derivative with respect to only e_1 (not $\tilde{e}_1$):

   $$[e_1] : 1 - e_1 = 0 \leftrightarrow e_1^* = 1$$

7. What wage do the firms in period 1 bid? Justify your answer.

   In equilibrium what workers do must match beliefs, so $\tilde{e}_1 = e_1^* = 1$. In period 1, no one knows true skill a. So the value to the firms of the worker in period 1 is just expected skill plus the effort everyone exerts in period 1. By the same argument we used in period 2, firms bid exactly $E[a] + \tilde{e}_1$. The mean of a uniform random variable is just the middle of the range: $(A + 0)/2$ so the wage in period 1 will be $w_1^* = A/2 + \tilde{e}_1 = A/2 + 1$.

8. How does this effort compare to the effort in sub question 1? Why is the worker working hard?

   They are the same. The worker exerts first-best effort in order to get a better paying job on the market in the future (career concerns). Alternatively, to make sure they are perceived to be high skill in the future.

9. Suppose $A = 100$. If a worker has skill 50, by what amount does their wage change from period 1 to period 2?

   We know that the wage for all types of workers is the same in the first-period and equal to $w_1 = A/2 + 1 = 100/2 + 1 = 51$. In the second period, firms observe output and therefore learn skill is a. Wages in the second period are then $w_2 = a + 0 = 50$. Thus wages fall by 1 for a worker with skill 50.

10. Explain, either verbally or mathematically, what effort and wages in each period would be if the worker's skill was just a fixed number, a, that everyone knew from the very beginning. How does this help explain why the worker exerts effort in the main model where skill is unknown?

    This question can be answered either verbally or mathematically. Let's start with a verbal argument. In the original model, the only reason the worker exerts effort is to try and make the market believe they have high skill (or to make sure no one thinks they are low skill). If everyone knows skill, this motive no longer exists, and because there is no performance pay, the worker exerts no effort in both periods.

    Alternatively you could actually fully solve the model. The effort choice in the final period is the same because in both models there is no future. Thus it continues to be the case that $e_2 = 0$. What changes is that skill is known, so firms do not use output from the first period to learn skill. The benefit of hiring the worker is effort plus skill, and there is 0 effort so both firms bid just skill a. Now we consider effort in period 1. In period 1, workers understand that firms will not use their output to learn skill. Thus exerting more effort does not increase their wage but is costly, so $e_1 = 0$. Understanding this, firms expect to get a revenue of $a + 0$ from hiring the worker, so both firms bid a. Thus we have that wages are the same in both periods and effort is 0 in both periods.

11. Explain, either verbally or mathematically, what effort and wages in each period would be if skill was just a fixed number, a, that everyone knew from the very beginning AND both firms could use performance pay (i.e. a wage where $w(y) = \alpha + \beta y$). Assume that each firm "bids" a performance pay $w_1(y), w_2(y)$ and the worker chooses the performance pay that they expect to give them the highest utility. Hint: the firms use α to "get" the worker and they use β to get the right effort once the worker is hired.

    Let's begin with a verbal argument. This argument is a bit loose, but I will accept an answer close to it for full credit. We know from the last sub-question that when skill is known but there is no performance pay, workers exert 0 effort. The key to this question is realizing that since everyone knows skill, output is not random, and performance pay is essentially effort based pay. Therefore the first-best effort level is achieved in both periods: $e_1 = e_2 = 1$. One wrinkle is exactly what wages the firms will bid. To get first-best effort, it is necessary to set $\beta = 1$. Because workers get all the surplus in Bertrand-style models, it must be that each firm gets 0 profit. The firm's profit is given by: $a + 1 - \beta(a+1) - \alpha = -\alpha$. For the firm to get 0 surplus, it must be then that $\alpha = 0$. Again, this is a loose argument. I provide a formal one next.

    To fully solve the model, we can notice that each period is identical because skill is known from the very beginning. So by solving one period we solve both periods. We can start by noticing that because the firms can choose α, they will use β just to get the right effort. The β which yields the first best effort is $\beta = e_{FB} = 1$. Given a $\beta = 1$ and an effort of 1, firms obtain first-best effort plus skill less the bonus and the base pay: $a + e^{FB} - \beta(a + e^{FB}) - \alpha = a + 1 - 1 \cdot (a+1) = -\alpha$. Notice that the bonus includes a because skill is part of output. Both firm's receive this value from hiring the worker. They can then adjust α to get the worker. The logic from Bertrand continues to apply: firms bid using base pay α. The worker's utility from each job is given by the wage less the cost of effort:

    $$w(e^{FB}) = \alpha + \beta(a + e^{FB}) - c(e^{FB}) = \alpha + 1(a+1) - 1/2 = \alpha + a + 1/2$$

    The worker therefore chooses the firm that offers the highest base pay. Recall that both firms make negative profit if they offer positive α, so neither will do that. Both make money if they offer negative α, but only if they win the worker. Since both will want to undercut the other by offering a better wage, the only base pay that will work is $\alpha = 0$. If any firm deviates higher they win for sure but make negative profit. If any deviates lower they lose for sure and continue to make 0 profit. Thus both firms set $\alpha = 0, \beta = 1$. Notice that the worker makes positive utility because $\alpha + \beta(a + e^{FB}) - c(e^{FB}) = a + 1/2$. Further, they make more utility the higher their skill.

Putting this all together, we have that effort in both periods is the first-best effort, that is: $e_1 = e_2 = e^{FB} = 1$. Wages in both periods are $\alpha = 0, \beta = 1$. Formal performance pay has "restored" first-best effort after we eliminated career-concerns. Notice that formal performance pay does better than career concerns, because it obtains the first-best effort in both periods rather than just one.

A more rigorous answer would proceed in three steps:

(a) Prove that in all equilibria, the utility delivered to the worker must be $1/2 + a$. The proof is that if an equilibria delivers $u' < 1/2 + a$, we can construct a deviation that sets $\beta = 1$ and sets α slightly less than 0. Such an α always exists because $u' < 1/2 + a$.

(b) Prove that in all equilibria, profit of the firms is 0. This follows directly form Bertrand bidding in the space of α.

(c) This means that in all equilibria, the utility delivered to the worker is $1/2 + a$ and the utility delivered to the firm is 0. Because $1/2 + a$ is first-best total surplus, the only combination of α, β that does this is $\alpha = 0, \beta = 1$.