Board Work for Lecture 15

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1 Relational Contracts

Throughout, we will assume that the worker takes the job when indifferent.

1.1 Guessing a Strategy

We guess two strategies, one for the firm and one for the worker. Our strategies must specify what each player does for any possible history of play. This is just one of infinitely many equilibria. It is easier to analyze because strategies are simple. But this simplicity comes at a cost: if you think about what the worker and the firm do when either deviates, it is rather extreme.

For the worker, we guess they take the job and exert high effort as long as the firm pays a high wage $w_t = w_H$. The first time they observe any wage other than w_H, this period they either take the job and exert low effort (if $w_t > \bar{u}$) or they take the outside option (if $w_t \leq \bar{u}$). In the future, they never take the job regardless of what the firm does in the future.

For the firm, we guess they first post a high wage w_H. They continue to post this wage as long as they observe the worker providing high effort in the prior period, $e_{t-1} = H$. The first time the worker does not exert high effort (takes the outside option or exerts low effort) the firm posts a low wage w_L forever after regardless of what the worker does in the future.

If the firm and the worker do this strategy, the outcome of the game is simple: the firm posts a high wage, the worker exerts high effort, and this repeats forever. This is called "on path" because it is what actually happens on the equilibrium path. This is sustained by "off path" consequences: if either party fails to uphold their end, both sides start "punishing" the other: the firm posts low wages and the worker never takes the job forever. Notice that these are extreme punishments: the worker gets their outside option $\bar{u}$ forever, while the firm gets 0 profit forever.

1.2 Notation

Under this guessed strategy, we can write the present value of an action today to the worker as $U(action, state)$ and the firm $\Pi(action, state)$. I will be a little loose with this notation, but it is helpful to distinguish the payoff from a single time period, u, from the payoff from an entire strategy which includes all of future time periods as well as today.

Under our guessed strategy, the "state" is whether or not trust has ever been broken. Either side can break trust: the firm can do so by not paying w_H and the worker can do so by not exerting high effort. In our strategy who broke trust in the past is irrelevant, all that matters is whether it has happened. To check one shot deviations, we only need to check single deviations under the two possible states: (1) trust has been broken at some point in the past (2) trist has never been broken.

1.3 No Incentive to Deviate: Checking One-Shot Deviations

To show our guessed strategies form a subgame perfect Nash equilibrium, we must check all one shot deviations. We will check the worker's one shot deviations thoroughly, and only briefly consider the firm. This is mainly to make this model less of a burden for tests/homework. In graduate school if you were to use this model you would want to check the firm's side as well. The deviations we need to check are as follows:

1. Taking the job and exerting low effort when the worker is supposed to be not taking the job (because trust is broken and are being offered w_L).
2. Not taking the job when the worker is supposed to take the job (because trust has never been broken and they are being offered w_H)
3. Taking the job and exerting low effort when they are supposed to take the job and exert high effort (because trust has never been broken and they are being offered w_H).

Deviations 2 and 3 have the same consequence tomorrow: trust is now broken and the low wage is offered forever. However, deviation 3 is slightly better because the worker gets the high wage for one period instead of the outside option. Since the high wage must be at least $\bar{u}+c$, this is always the more attractive deviation. So we if 3 is not profitable, 2 will also not be profitable. Therefore we do not need to check deviation 2, and we are left with only two deviations to check:

1. Taking the job and exerting low effort when the worker is supposed to be not taking the job (because trust has been broken in the past and they are being offered w_L).
4. Taking the job and exerting low effort when they are supposed to take the job and exert high effort (because trust has never been broken and they are being offered w_H).

Before we check for deviations, we need to think about what low wage w_L the firm will choose. Remember that whenever w_L is being played the worker is never exerting high effort. Thus, if the wage w_L is above $\bar{u}$ the worker takes the wage and their payoff is just w_L. However, if the wage is less than $\bar{u}$ the worker takes the outside option and gets $\bar{u}$. Will the firm offer a wage above $\bar{u}$? Well, a wage above $\bar{u}$ only makes exerting low effort more tempting, and it also costs the firm money. So it will not offer a wage above $\bar{u}$. Will it offer exactly $w_L = \bar{u}$? Well, this gets the worker to take the job, but the worker exerts low effort, so the firm gets 0 revenue from this and then has to pay out a positive wage. This generates negative profit, so the firm will not do it. Thus the firm wants the worker to take the outside option.

Therefore the firm is indifferent between offering any wage where $w_L < \bar{u}$ because they are all rejected. The one I suggest specifying is $w_L^* = 0$. Any time the firm offers such a low wage, the worker will want to take the outside option. As a result, the worker's payoff in a single period whenever the firm posts a low wage is $\bar{u}$.

With this in hand, let's consider deviation (1): Taking the job and exerting low effort when the worker is supposed to be not taking the job (because trust has been broken in the past and they are being offered $w_L^* = 0 < \bar{u}$). Not deviating in this case means never taking the job, which gives the worker $\bar{u}$ forever:

$$U(\textit{don't deviate,trust broken}) = \sum_{t=0}^{\infty} \bar{u} = \frac{\bar{u}}{1-\delta}$$

Deviating and taking the job and exerting low effort in this case yields the low wage today ($w_L = 0$) and no effort cost, and then $\bar{u}$ starting tomorrow until the end of time:

$$U(\textit{deviate to accept and low effort, trust broken}) = 0 + \delta \sum_{t=0}^{\infty} \delta^t \bar{u} = \frac{\delta \bar{u}}{1-\delta}$$

Comparing deviating to not deviating we have that:

$$U(\textit{don't deviate, trust broken}) \geq u(\textit{deviate to accept and low effort, trust broken})$$

$$\leftrightarrow \frac{\bar{u}}{1-\delta} \geq \frac{\delta \bar{u}}{1-\delta} \leftrightarrow \bar{u} \geq 0$$

The outside option is always weakly positive, so this is always true. There is no incentive to deviate in this way.

Now consider deviation (3): taking the job and exerting low effort when they are supposed to take the job and exert high effort (because trust has been broken in the past and they are being offered w_H). Let's consider first the worker's payoff from doing what they are supposed to do (never slacking off). The worker receives w_H forever and pays the effort cost c:

$$U(\textit{don't deviate, trust never broken}) = \sum_{t=0}^{\infty}(w_H - c) = \frac{w_H - c}{1-\delta}$$

The worker compares this to a one-shot-deviation: exerting low effort today (at 0 cost), still getting paid w_H today, but then getting offered w_L forever after. Remember, the worker changes action only in one period and is assumed to follow through on our guessed strategy everywhere else. The gain today from this is w_H without any effort cost. The loss is that the firm now posts w_L forever. Putting this all together, we have:

$$U(\textit{deviate, trust never broken}) = w_H + \delta\sum_{t=0}^{\infty} w_L = w_H + \frac{\delta\bar{u}}{1-\delta}$$

The worker has no incentive to deviate if:

$$U(\textit{don't deviate, trust never broken}) \geq u(\textit{deviate, trust never broken}) \leftrightarrow \frac{w_H - c}{1-\delta} \geq w_H + \frac{\delta\bar{u}}{1-\delta}$$

This is a substantive inequality: it will not always be true for reasonable parameter values $(\delta, c, v, \bar{u})$. Also, it involves a choice variable: w_H.

1.4 Finding the High Wage

First, simplify the inequality we derived:

$$\begin{aligned}\frac{w_H - c}{1-\delta} &\geq w_H + \frac{\delta\bar{u}}{1-\delta}\\ w_H - c &\geq (1-\delta)w_H + \delta\bar{u}\\ \delta w_H &\geq c + \delta\bar{u}\\ w_H &\geq \frac{c}{\delta} + \bar{u}\end{aligned}$$

We see that this inequality is more likely to be satisfied the higher w_H is. However, the firm maximizes profit by paying the lowest wage that satisfies the inequality, changing the inequality into an equality:

$$w_H^* = \frac{c}{\delta} + \bar{u}$$

The argument is the same as always: paying a high w_H does not change the worker's behavior, but it costs the firm profit. The firm's profit is the revenue less the high wage, because on path the worker never slacks:

$$\Pi = \sum_{t=0}^{\infty}\delta^t(v - w_H^*) = \frac{v - \frac{c}{\delta} - \bar{u}}{1-\delta}$$

If you stare at this you will notice that profit is increasing in δ. We can also see that a relational contract is profitable if:

$$\delta(v - \bar{u}) \geq c$$

The more patient the worker and the firm are the more likely this is to hold. The higher the value of working together the more likely this is to hold. The higher the cost of effort and the higher the outside option the more unlikely this is to hold.