Lecture 2: The Toolkit

Econ 490: Compensation in Organizations

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Discussion: Hartzell, Parsons, Yermack (2010)

What Should You Know Already?

- Single variable derivatives.

- Inequalities.

- Very basic probability.

What Will I Teach You Today?

- ▶ The concept of risk aversion and a useful formula.

- ▶ A decision problem.

- ▶ Simple game theory, static.

- ▶ Simple game theory, dynamic.

Caveats

- ▶ I will try to cover just what is needed for the class.

- ▶ I will not cover Nash equilibrium in depth.

- ▶ I will not cover risk aversion in depth.

- ▶ If you want to go beyond this or want more practice, see my notes and practice problems from an old course: https://github.com/jakekohlhepp/Econ101.

Table of Contents

Compensation as Lotteries

- Your performance in a job is often effort + noise/luck/chance/circumstance

- Effort is in your control, but the rest is random.

- If you get paid based on performance, your compensation is a lottery!

A Survey

Suppose a company offered you three compensation schemes:

 a. X_a: \$100,000 with probability 50%, \$0 with probability 50%.

 b. X_b: \$49,000 with probability 100%

 c. X_c: \$200,000 with probability 24%, \$0 with probability 76%.

Which would you choose? (will tally on board)

Risk Attitudes

▶ Before looking at the results, notice some facts:
  ▶ b has lower average than a: $E[X_a] = 50 > 49 = E[X_b]$.
  ▶ But a has higher variance than b: $Var(X_a) = 2500 > 0 = Var(X_b)$.
  ▶ So if you dislike variance or uncertainty, you will prefer a.
  ▶ $E[X_c] = 48 < 49 < 50$: in terms of expected money, c is worse than both a and b.
  ▶ However, the variance of c is higher: $Var(X_c) \approx 7296 > 2500 > 0$.
  ▶ You will only choose c if you like uncertainty/risk.

▶ A rough interpretation of the results:
  ▶ If you chose b you are *risk averse*: you dislike uncertainty/risk, and are willing to pay to reduce it
  ▶ If you chose a you are either *risk neutral*
  ▶ If you chose c you are *risk loving*

▶ We will assume people are risk averse or risk neutral.

Expected Utility Theory

▶ We analyze uncertainty using **Expected Utility Theory.**

▶ This theorem justifies the tools we will use in this class:

Theorem 1

*Under a set of axioms (which you do not need to know), we can represent an individual's preferences over lotteries using an **expected utility function** u where $E[u(X_a)] \geq E[u(X_b)]$ means that lottery a is preferred to lottery b.*

Risk Attitudes As Functions

A person with expected utility function u is...

- **risk averse** if u is concave.

- **risk neutral** if u is linear.

- **risk loving** if u is convex.

Certainty Equivalent

▶ We often want to rank lotteries/gambles. The following concept is useful for ranking lotteries:

Definition 2

The amount of money for sure a decision maker is willing to pay for lottery a is the **certainty equivalent** (d_a). Mathematically:

$$u(d_a) = E[u(X_a)]$$

▶ Given a lottery that gives me d dollars for sure and X_a, it is the value of d where I am indifferent.

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▶ Gut check: What is the certainty equivalent of a lottery with $E[X_a] = 10$ when the decision maker is risk neutral?

Certainty Equivalent

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Definition 3

The amount of money for sure a decision maker is willing to pay for lottery a is the **certainty equivalent** (d_a). Mathematically:

$$u(d_a) = E[u(X_a)]$$

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▶ Gut check: What is the certainty equivalent of a lottery with $E[X_a] = 10$ when the decision maker is risk neutral?

Exponential Utility

▶ We will use the exponential utility function in this class:

$$u(x) = \frac{1 - e^{-rx}}{r}$$

▶ r captures risk aversion:
  ▶ When $r > 0$ the decision maker is risk averse

  ▶ When $r < 0$ they are risk loving.

▶ What happens when $r \to 0$? Using L'Hopital's rule:

$$\lim_{r \to 0} \frac{1 - e^{-rx}}{r} = x$$

So we have risk neutrality!

Exponential Utility

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Theorem 4

When a person has risk preference given by an exponential utility function $u(x) = \frac{1-e^{-rx}}{r}$, the certainty equivalent of a normal lottery with mean μ and variance σ^2 is given by:

$$d = \mu - r\frac{\sigma^2}{2}$$

▶ In this class, you can apply this formula directly.

▶ Talk to me if you are interested in the derivation!

Table of Contents

Making a Decision

- Suppose there is a single person (A) making a decision.

- A takes an action which we will call e.

- e can be a **discrete action**:
  - accept or reject

  - work hard or slack off

- e can be a **continuous action**:
  - exert e units of effort

  - drive e miles

- The utility or payoff of an action e is $u(e)$

Table of Contents

Game Theory in This Class

▶ Game theory lets us model strategic interaction.

▶ Therefore it is a tool, but not the point, of this class.

▶ I do not require you to learn the definitions of Nash equilibrium, best responses, etc.

▶ However, if you want to go to econ. grad school this can be useful.

▶ I do require you to make either mathematical and/or verbal arguments.

▶ For tests, I will only ask you to solve models we solved in class (sometimes with slight modifications).

Competing for a Worker (Bertrand Game)

We will use the basic ideas of this game often:

► **Players.** Two identical firms, numbered $i = 1, 2$, and one worker.

► **Actions.** Firms choose wages *continuously*: $0 \leq w_i < \infty$

► **Payoffs.**
1. Worker payoff is the wage of the firm they choose.
2. When firms set the same wage the worker chooses randomly.
3. The firm which hires the worker gets productivity p and pays the wage.
4. If a firm does not hire they get 0.

See the board (we will solve this almost entirely verbally)!

The Company Call List (Tragedy of the Commons)

- **Players.** Two sales workers, $i = 1, 2$, share a common list of company sales contacts.

- **Actions.** Each chooses a number of people on the list to call, q_i

- **Payoffs.**
    1. The cost to the worker of making a call on the list is 0.
    2. The amount of commission the worker makes per call is $120 - q_1 - q_2$

See the board!

Sequential Games

▶ We will consider several models in this class where a firm moves first (usually to set up a compensation plan)

▶ Then a worker reacts to this compensation plan.

▶ Unlike the last examples these are sequential games.

▶ Two key differences between static and dynamic:
  ▶ In sequential games, future players take past player actions as fixed.

  ▶ Earlier players anticipate future players will react to their current choices.

▶ To deal with this, we use subgame perfect Nash equilibrium.

▶ For this class, that just means we use backwards induction.

The Pirate Riddle

There is a tricky riddle that we will use to illustrate backwards induction.

Five pirates, numbered 1 through 5, must decide how to divide 100 gold coins. Their decision process is as follows. Starting with pirate 1, each pirate proposes a split consisting of a number of coins for each of the pirates on the ship. Then all pirates vote. If a strict majority approve, the allocation happens. If it does not the proposer is thrown off the ship and the remaining pirates repeat the process. Assume pirates value 2 coins more than 1, etc and that getting thrown off is worse than getting 0 coins. Assume pirates vote no when indifferent (they get a little bit of enjoyment from watching someone walk the plank). What is the maximum number of coins P1 can obtain and not get thrown off?

Pirate Riddle: Verbal Solution

▶ Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!

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- ▶ Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving P4 0 coins. P3 proposes 0 for all other pirates and 100 for themselves.

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▶ Roll back. P2 needs to get two votes. All pirates except P3 get 0 coins if the game continues, so P2 proposes 1 coin to P1 and 1 coin to P4 and keeps 98.

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▶ Roll back. P2 needs to get two votes. All pirates except P3 get 0 coins if the game continues, so P2 proposes 1 coin to P1 and 1 coin to P4 and keeps 98.

▶ Roll back. P1 needs two other votes. P3 is the cheapest to convince. P1 and P4 are next cheapest, and P1 need only convince one. So P1 proposes 0 for P2 and P5, P3 1, and P4 2 and keeps 97!

What can we learn from the pirate riddle? What is surprising about the outcome/solution?

The Company Call List - Sequential Version

We now modify our static company call list game to be sequential.

- **Players.** Two sales workers, $i = 1, 2$, share a common list of company sales contacts.

- **Actions.** Each chooses a number of people on the list to call, q_i

- **Timing.** Worker 1 calls first, then worker 2.

- **Payoffs.**
  1. The cost to the worker of making a call on the list is 0.
  2. The amount of commission the worker makes per call is $120 - q_1 - q_2$

See the board!