Problem Set 2

Jacob Kohlhepp

November 21, 2025

The purpose of this homework is to work through a multitasking problem and a relative performance evaluation problem. There are only minor differences between these problems and the ones we did in class, so your notes should be very helpful in completing this problem set.

1 Relative Performance Evaluation

This problem is exactly the same as what we did in class on the board during the relative performance pay lectures.

1.1 Setup

- Suppose there are two workers labeled 1 and 2.
- Output for each $y_1 = e_1 + \epsilon_1$, $y_2 = e_2 + \epsilon_2$
- The noise terms are distributed:
  - $\epsilon_1 = v_s + v_1$
  - $\epsilon_2 = v_s + v_2$
  - where $v_s \sim N(0, \sigma_s^2)$, $v_1 \sim N(0, \sigma_1^2)$ and $v_2 \sim N(0, \sigma_2^2)$[1]
- Let's focus just on worker 1 (so do all questions for worker 1 but not 2)
- The firm can offer linear wages:
  - $w(y_1, y_2) = \alpha + \beta(Y_1 - \gamma Y_2)$

1.2 Questions

1. Derive the certainty equivalent of the worker's wage, and subtract effort costs to get an expression for the worker's utility.
2. Stare at the expression you obtained. Argue that γ does not impact the worker's choice of effort at all, either mathematically or verbally.
3. Argue as in class that γ only impacts the variance, so to find the optimal γ we only need to minimize the variance of the wage.
4. Minimize the variance of the wage to find the profit maximizing γ. Call it γ_{rel} and do not ever plug it into anything for the rest of this problem.
5. Interpret your expression for γ in terms of the informativeness principle.
6. Write wages as three parts as in lecture: constant objects, effort of worker 1 times bonus, and bonus times random objects.

[1] Technical note: All are also jointly independent.

7. Find the variance of the random part of wages and call it σ^2_{tot}. (Hint: β should not be in this formula at all because it is multiplying the random objects.)

8. Find the profit-maximizing β by using our formula from the performance pay lecture $\beta = c'(e) = \frac{1}{1+r\sigma^2 c''(e)}$ with σ^2_{tot} replacing σ^2.

2 Multitasking

The important difference between this and what we did in class is that I am telling you specific values for a, b in the last part of the problem. So up until I tell you what a, b are this is EXACTLY what we did in class!

2.1 Setup

- Output is $y = ae_1 + be_2$
- Cost of effort is:
$$c(e_1, e_2) = \begin{cases} 0 & \text{if } e_1 + e_2 \leq 2\bar{e} \\ (e_1 + e_2 - 2\bar{e})^2/2 & \text{if } e_1 + e_2 > 2\bar{e} \end{cases}$$
- We assume that without incentives the worker supplies all 0 cost effort and splits effort evenly:
$$e_1 = e_2 = \bar{e}$$
- Only task 1 effort is measured: $m = e_1$
- The firm can only pay based on task 1: $w(m) = \alpha + \beta m = \alpha + \beta e_1$
- The firm's outside option is 0, the worker is $\bar{u}$

2.2 Questions

1. Setup the firm's problem in the first-best, that is when the firm can just choose effort directly and we do not care about wages.
2. Solve for the first-best e_1, e_2 when $a > b, a > 0$. Only assume that $a > b$ for this problem.
3. From now on we are solving for equilibrium, meaning the firm cannot choose effort directly but just chooses a compensation scheme. Setup the worker's effort choice problem.
4. Solve for worker's choice of effort assuming for now until told otherwise that $\beta > 0$.
5. Write down the inequality that determines whether the worker takes the job. Argue that it must be an equality.
6. Setup the firm's profit maximization problem. Substitute past work in so that it is only a function of β.
7. Solve for β, e_1, e_2.
8. Now, solve for e_1, e_2 when $\beta = 0$. You may use the same steps we just did or do it your own way.
9. From now until I say otherwise assume that $a = -1, b = 2, \bar{e} = 1$. Provide an interpretation for a being negative.
10. Using the work you have already done, should the firm set $\beta = 0$ or $\beta > 0$? Find β, e_1, e_2.
11. Now assume that $a = 2, b = 1, \bar{e} = 1$. Provide a real life example where a would be positive.
12. Using the work you have already done, should the firm set $\beta = 0$ or $\beta > 0$? Find β, e_1, e_2.