Problem Set 2

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The purpose of this homework is to work through a multitasking problem and a relative performance evaluation problem. There are only minor differences between these problems and the ones we did in class, so your notes should be very helpful in completing this problem set.

1 Meaning and Performance

2 Multitasking

There are two key differences in the setup of this problem vs. what we did in class:

1. Task 1 effort is measured in different "units."
2. I am telling you specific values for a, b in the last part of the problem.

2.1 Setup

- Output is $y = ae_1 + be_2$
- Cost of effort is:
$$c(e_1, e_2) = \begin{cases} 0 & \text{if } e_1 + e_2 \leq 2\bar{e} \\ (e_1 + e_2 - 2\bar{e})^2/2 & \text{if } e_1 + e_2 > 2\bar{e} \end{cases}$$
- We assume that without incentives the worker supplies all 0 cost effort and splits effort evenly:
$$e_1 = e_2 = \bar{e}$$
- Only task 1 effort is measured: $m = k \cdot e_1$, where $k > 0$.
- The firm can only pay based on task 1: $w(m) = \alpha + \beta m = \alpha + \beta k e_1$
- The firm's and worker's outside options are 0.

2.2 Questions

1. Setup the firm's problem in the first-best, that is when the firm can just choose effort directly and we do not care about wages.
2. Solve for the first-best e_1, e_2 when $a > b, a > 0$. Only assume that $a > b$ for this problem.
3. From now on we are solving for equilibrium, meaning the firm cannot choose effort directly but just chooses a compensation scheme. Setup the worker's effort choice problem.
4. Solve for worker's choice of effort assuming for now until told otherwise that $\beta > 0$.
5. Write down the inequality that determines whether the worker takes the job. Argue that it must be an equality.

6. Setup the firm's profit maximization problem. Substitute past work in so that it is only a function of β.
7. Solve for β, e_1, e_2.
8. Now, solve for e_1, e_2 when $\beta = 0$. You may use the same steps we just did or do it your own way.
9. From now until I say otherwise assume that $a = -1, b = 2, \bar{e} = 1$. Provide an interpretation for a being negative.
10. Using the work you have already done, should the firm set $\beta = 0$ or $\beta > 0$? Find β, e_1, e_2.
11. Now assume that $a = 2, b = 1, \bar{e} = 1$. Using the work you have already done, should the firm set $\beta = 0$ or $\beta > 0$? Find β, e_1, e_2.
12. Do your answers to any of these questions depend on k? Interpret your answer.

3 Relational Contracts

3.1 Setup

- A firm and a worker both have discount rate δ and interact for many periods ($t = 1, ..., \infty$)
- At each period t the following occur:
  - First the firm offers a flat wage w_t
  - Second the worker chooses high (H) or low (L) effort e_t
- High effort has cost c, low effort has cost 0.
- High effort yields revenue v, low effort yields revenue 0.
- Firm outside option is 0, worker outside option is $\bar{u} > 0$.
- Assume the firm wants to motivate high effort.

3.2 Questions

1. Guess an equilibrium strategy for the firm in words. Guess an equilibrium strategy for the worker in words. (Hint: guess the same strategy as in class)
2. Call the high wage w_H and the low wage w_L. Assume the strategy we guessed is being played. What value of w_L will the firm choose and why?
3. What is the worker's payoff in any period where the firm posts a wage of w_L? Justify your answer.
4. Consider the case when trust was already broken in the past. Write down the worker's present value utility from not deviating, that is following our guessed strategy. Write down the worker's present value utility from deviating from a one shot deviation from our guessed strategy that involves taking the job and exerting low effort.
5. Consider the case when trust was already broken in the past. Write down an inequality for when there are no incentives for the worker to deviate in this case. Make sure to simplify. When does this inequality hold?
6. Consider the case when trust has never been broken. Write down the worker's present value utility from not deviating and following our guessed strategy. Write down the worker's utility from a one shot deviation of exerting low effort today.

7. Consider the case when trust has never been broken. Argue that the worker would always prefer to take the job and exert low effort rather than not take the job.

8. Consider the case when trust has never been broken. Write down (and simplify) an inequality for when there is no incentive for the worker to deviate. When is it satisfied?

   Assume w_H in equilibrium is the lowest possible wage such that the inequality is satisfied.

9. Suppose $\delta = 0.4, v = 3, \bar{u} = 1, c = 1$. Is the relational contract we derived profitable for the firm?

10. Suppose $\delta = 0.6, v = 3, \bar{u} = 1, c = 1$. Is the relational contract we derived profitable for the firm?

11. Interpret the difference between your prior two answers.