

# Lecture 3: The Principal-Agent Model and Effort-Based Pay

Compensation in Organizations

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Discussion: Loyalka (2019)

## Why a Model? (Clarity of Thought)

- ▶ “All models are wrong, but some are useful.” -George Box
- ▶ Maps are useful precisely because they ignore some features.
- ▶ A good model has what we need to study one question, and nothing more.
- ▶ Under the model's assumptions, we know the truth.
- ▶ If the outcome is unrealistic, we can argue about which precise assumptions led to it.

## Why THIS Model?

- ▶ The model we study in this lecture will be the main model of this class.
- ▶ We will slightly change it to study different questions.
- ▶ It was developed in the 1970s and has framed theoretical and empirical research for decades.

# The Principal-Agent Model

## Players

- ▶ There is a firm (the principal) who is risk neutral (exponential utility with parameter  $r = 0$ ).
- ▶ There is a worker (the agent) who is risk averse (exponential utility with parameter  $r = r \geq 0$ ).

## Actions

- ▶ Firm chooses a linear wage which depends on effort ( $w(e)$ ) or output ( $w(y)$ )
- ▶ After seeing the wage, the worker either accepts or rejects the job.
- ▶ If they accept, worker chooses effort  $e$  at an increasing, convex cost  $c(e)$

# The Principal-Agent Model

## Output

- ▶ Output is effort ( $e$ ) plus noise/luck ( $\epsilon$ ):  $y = e + \epsilon$  where  $\epsilon \sim N(0, \sigma^2)$
- ▶ This implies output is normal with mean  $e$  and variance  $\sigma^2$

## Payoffs

- ▶ If accepted, firm's payoff  $\pi$  is expected output minus expected wages:  $E[y - w|e]$
- ▶ If accepted, worker's payoff is expected utility of the wage minus effort cost:  $E[u(w) - c(e)|e]$
- ▶ If rejected, worker has “outside option” of  $\bar{u}$  and firm has “outside option” of 0.<sup>1</sup>

1. We will assume throughout that the firm prefers to hire the worker ex-ante.

Timing

See the board!

## Effort-Based Pay

- ▶ Suppose the firm can pay based on the worker's effort.
- ▶ Then wage is a linear function of effort:  $w(e) = \alpha + \beta e$
- ▶ We now go to the board to solve!



# Effort-Based Pay

## Theorem 1

*When wages depend directly on effort, effort is  $e^*$  which solves  $c'(e^*) = 1$  and  $\beta^* = 1, \alpha^* = \bar{u} + c(e^*) - 1$*

- ▶ We reward the worker for more effort.
- ▶ Question: What is the worker's expected payoff?
- ▶ Question: Why is the marginal cost set equal to 1?
- ▶ Let's compute the firm's payoff and total surplus on the board.

## Effort-Based Pay

- ▶ We showed that this is the first-best: the firm could not do better even if they did the work themselves!
- ▶ The firm gets everything, but if the worker proposed they would get everything
- ▶ Question: What assumption did we bake in that got us here?
- ▶ Question: Is this assumption realistic?

# Selling the Firm

## Theorem 2

*When wages depend directly on effort, effort is  $e^*$  which solves  $c'(e^*) = 1$  and  $\beta^* = 1, \alpha^* = \bar{u} + c(e^*) - 1$*

- ▶ Consider our result:  $\beta = 1$  and the average return to effort is 1
- ▶ The firm gives the worker everything they produce.
- ▶ This is called selling the firm: the worker pays  $\alpha$  but keeps everything they make!
- ▶ These are extremely strong incentives (why?)

## Paying to Work

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- ▶ This means the worker is paying to work!
- ▶ Does this ever happen?
- ▶ Question: Can you give some examples?