Problem Set 1

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The purpose of this homework is to work through our main principal-agent model with uncertainty. The only difference between the model you will solve in this homework and the one in class is that output is not mean 0. You can use the work we did in class as a reference when you get stuck, but please show all steps.

You may solve this two ways: either derive everything assuming a general cost function ($c(e)$) and plug in the explicit forms of $c(e), c'(e), c''(e)$ at the end, or plug in $c(e) = e^2/2$ from the beginning. Both are acceptable.

Setup

Players

- There is a firm (the principal) who is risk neutral (exponential utility with parameter $\theta = 0$).
- There is a worker (the agent) who is risk averse (exponential utility with parameter $\theta = r \geq 0$).

Actions

- Firm chooses a linear wage which depends on effort ($w(e)$) or output ($w(y)$)
- After seeing the wage, the worker either accepts or rejects the job.
- If they accept, worker chooses effort e at an increasing, convex cost $c(e) = e^2/2$

Output

- Output is effort (e) plus noise/luck (ϵ): $y = e + \epsilon$ where $\epsilon \sim N(1, \sigma^2)$
- Notice that output is now not mean 0.
- This implies output is normal with mean $1 + e$ and variance σ^2

Payoffs

- If accepted, firm's payoff π is expected output minus expected wages: $E[y - w|e]$
- If accepted, worker's payoff is expected utility of the wage minus effort cost: $E[u(w) - c(e)|e]$
- If rejected, worker has "outside option" of $\bar{u}$ and firm has "outside option" of 0.[1]

Timing

The same as in class. The firm proposes a wage schedule, the worker accepts or rejects, the worker exerts effort, output occurs, and then the wage is paid out.

[1] We will assume throughout that the firm prefers to hire the worker ex-ante.

1 Effort Based Pay

Suppose the firm can pay based on the workers effort: $w(e) = \alpha + \beta e$.

1.1 Questions

1. Is the worker's wage uncertain? Why or why not.
2. Write the worker's certainty equivalent for a wage with fixed (α, β) and fixed effort e. Hint: use the answer to question 1.
3. For a fixed wage (α, β) what level of effort does the agent choose? Hint: your answer should be a function of β
4. For what α, β does the worker accept the job?
5. What α will the firm choose, and why? Hint: use the answer to the previous question.
6. Write the firm's profit. DO NOT simplify or plug anything in.
7. Simplify the firm's profit. You should get an expression that is a function of only β or e (either is fine)
8. What is the profit-maximizing β? What is the profit maximizing effort e?
9. What is the profit-maximizing α?

2 Performance-Based Pay

Suppose the firm can pay based ONLY on output: $w(y) = \alpha + \beta y$ (effort is not observed).

2.1 Questions

1. Is the worker's wage uncertain? Why or why not.
2. Write the worker's certainty equivalent for a wage with fixed (α, β) and fixed effort e. Hint: use the answer to question 1.
3. For a fixed wage (α, β) what level of effort does the agent choose? Hint: your answer should be a function of β
4. For what α, β does the worker accept the job? I suggest that you not substitute away β yet.
5. What α will the firm choose, and why? Hint: use the answer to the previous question.
6. Write the firm's profit. DO NOT simplify or plug anything in.
7. Simplify the firm's profit. You should get an expression that is a function of only β or e (either is fine)
8. What is the profit-maximizing β? What is the profit maximizing effort e?
9. What is the profit-maximizing α?

3 Relative Performance Evaluation

This problem is exactly the same as what we did in class on the board during the relative performance pay lectures.

3.1 Setup

- Suppose there are two workers labeled 1 and 2 with the same cost of effort $c(e_i)$.
- Output for each $y_1 = e_1 + \epsilon_1$, $y_2 = e_2 + \epsilon_2$
- The noise terms are distributed:
  - $\epsilon_1 = v_s + v_1$
  - $\epsilon_2 = v_s + v_2$
  - where $v_s \sim N(0, \sigma_s^2)$, $v_1 \sim N(0, \sigma_1^2)$ and $v_2 \sim N(0, \sigma_2^2)$[2]
- Let's focus just on worker 1 (so do all questions for worker 1 but not 2)
- The firm can offer linear wages:
  - $w(y_1, y_2) = \alpha + \beta(y_1 - \gamma y_2)$

3.2 Questions

1. Derive the certainty equivalent of the worker's wage, and subtract effort costs to get an expression for the worker's utility.
2. Stare at the expression you obtained. Argue that γ does not impact the worker's choice of effort at all, either mathematically or verbally.
3. Argue as in class that γ only impacts the variance, so to find the optimal γ we only need to minimize the variance of the wage.
4. Minimize the variance of the wage to find the profit maximizing γ. Call it γ_{rel} and do not ever plug it into anything for the rest of this problem.
5. Interpret your expression for γ in terms of the informativeness principle.
6. Write wages as three parts as in lecture: constant objects, effort of worker 1 times bonus, and bonus times random objects.
7. Find the variance of the random part of wages and call it σ_{tot}^2. (Hint: β should not be in this formula at all because it is multiplying the random objects.)
8. Find the profit-maximizing β by using your answers from the performance pay questions, with σ_{tot}^2 replacing σ^2.

[2] Technical note: All are also jointly independent.