

# Board Work for Lecture 9: Multitasking

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## 1 Model

- Output is  $y = ae_1 + be_2, a > 0, b > 0$
- Cost of effort is:

$$c(e_1, e_2) = \begin{cases} 0 & \text{if } e_1 + e_2 \leq 2\bar{e} \\ (e_1 + e_2 - 2\bar{e})^2/2 & \text{if } e_1 + e_2 > 2\bar{e} \end{cases}$$

- Only task 1 effort is measured:  $m = e_1$
- Only what is measured is rewarded:  $w(m) = \alpha + \beta m = \alpha + \beta e_1$
- We assume that without incentives the worker supplies all “free effort” (all total effort up to  $2\bar{e}$ ) and splits total effort evenly across the two tasks:

$$e_1 = e_2 = \bar{e}$$

## 2 Solution: First-Best

What would the firm do if it could do everything directly? Setup surplus, which is just output less cost:

$$\max_{e_1, e_2} ae_1 + be_2 - c(e_1, e_2)$$

The cost function makes this problem tricky: cost is zero up to some threshold, but effort is always valuable. So the firm definitely wants to set  $e_1 + e_2$  to be at least  $2\bar{e}$ . Notice that effort is “together” inside the square. This means that the marginal cost of both types of effort above  $2\bar{e}$  is the same! it also means that the marginal cost is increasing (harder to do 1 more hour after 24 hours than after 2 hours).

But the marginal benefit is constant and different!  $MB_1 = a, MB_2 = b$ . Since the costs are the same but benefits are different, the firm chooses the task with the higher benefit and has the agent perform only that task. if  $a > b$  this is task 1. How much do they perform? Well, set  $e_2 = 0$  and take the FOC for  $e_1$ :

$$a - (e_1 - 2\bar{e}) = 0 \leftrightarrow e_1^{FB} = 2\bar{e} + a, e_2^{FB} = 0$$

So have the worker do the intrinsic motivation effort level plus the marginal benefit! Remember that if  $a < b$  we get the reverse:

$$b - (e_2 - 2\bar{e}) = 0 \leftrightarrow e_2^{FB} = 2\bar{e} + b, e_1^{FB} = 0$$

## 3 Solution: What Actually Happens

In the actual model, we only measure task 1. As always we start with the last stage and ask what effort the worker chooses given some wage  $w(e_1) = \alpha + \beta e_1$ .

$$\max_{e_1, e_2} \alpha + \beta e_1 - c(e_1, e_2)$$

There are two cases. if  $\beta = 0$  (no incentives) the worker provides the bare minimum  $e_1 = e_2 = \bar{e}$ . If  $\beta > 0$ , the worker is getting a benefit from task 1 but no benefit from task 2. Since both types of effort are costly, the worker will set  $e_2 = 0$ . That is, incentives cause  $e_2$  to be crowded out!

What about the first task? Well, the agent will supply all of the free effort to task 1 ( $2\bar{e}$ ) and a little more. How much more? Well, once again we know that  $e_1 > 2\bar{e}$  so we are above the pointy part and calculus works again. So we can setup the FOC with  $e_2^* = 0$ :

$$\max_{e_1, e_2} \alpha + \beta e_1 - (e_1 - 2\bar{e})^2/2$$

$$[e_1 :] \beta - (e_1 - 2\bar{e}) = 0 \leftrightarrow e_1^* = \beta + 2\bar{e}$$

The worker's utility from accepting is given by:

$$u(\text{accept}) = \alpha + \beta e_1 - c(e_1, e_2) = \alpha + \beta(\beta + 2\bar{e}) - \beta^2/2 = \alpha + \beta^2/2 + 2\beta\bar{e}$$

as always the firm sets the worker's utility equal to the outside option of 0 using  $\alpha$ :

$$\alpha = -\beta^2/2 - 2\beta\bar{e}$$

Continuing with the assumption that  $\beta > 0, e_2 = 0$ , let's maximize profit:

$$\begin{aligned} \max_{\beta} a e_1 + b e_2 - \alpha - \beta e_1 &= \max_{\beta} a(\beta + 2\bar{e}) + \beta^2/2 + 2\beta\bar{e} - \beta(\beta + 2\bar{e}) \\ &= \max_{\beta} a2\bar{e} + a\beta - \beta^2/2 \end{aligned}$$

FOC:

$$[\beta :] a - \beta = 0 \leftrightarrow \beta = a$$

$$e_1 = \beta + 2\bar{e} = a + 2\bar{e}, e_2 = 0$$

Profit is then:

$$\pi_{HIGH} = a(a + 2\bar{e}) - a^2/2 = a^2/2 + 2a\bar{e}$$

However, we must compare this to profit from  $\beta = 0$ , which generates  $e_1 = e_2 = \bar{e}$ . Note that in this case  $\alpha = 0$  too, so profit is just revenue:

$$\pi_{LOW} = a\bar{e} + b\bar{e}$$

Now we ask: when does  $\pi_{HIGH} > \pi_{LOW}$ ?

$$\begin{aligned} a^2/2 + 2a\bar{e} &\geq a\bar{e} + b\bar{e} \\ a^2/2 &\geq b\bar{e} - a\bar{e} \\ a^2/2 &\geq \bar{e}(b - a) \\ a^2 &\geq 2\bar{e}(b - a) \\ a &\geq 2\bar{e} \frac{b - a}{a} \end{aligned}$$

So we use a high-powered incentives ( $\beta > 0$ ) if “intrinsic motivation” is low ( $\bar{e} \approx 0$ ). This is because the cost of incentives is crowding out the intrinsic motivation. We use low-powered incentives if the task we do not measure is very profitable/important relative to the task we measure ( $b - a \gg 0$ ). We use high-powered incentives if the measured task is important enough ( $a > 0$ ).