

# Lecture 4: Performance Pay

Compensation in Organizations

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Discussion: Loyalka et. al. (2019)

# The Principal-Agent Model

## Players

- ▶ There is a firm (the principal) who is risk neutral (exponential utility with parameter  $r = 0$ ).
- ▶ There is a worker (the agent) who is risk averse (exponential utility with parameter  $r \geq 0$ ).

## Actions

- ▶ Firm chooses a linear wage which depends on effort ( $w(e)$ ) or output ( $w(y)$ )
- ▶ After seeing the wage, the worker either accepts or rejects the job.
- ▶ If they accept, worker chooses effort  $e$  at an increasing, convex cost  $c(e)$

# The Principal-Agent Model

## Output

- ▶ Output is effort ( $e$ ) plus noise/luck ( $\epsilon$ ):  $y = e + \epsilon$  where  $\epsilon \sim N(0, \sigma^2)$
- ▶ This implies output is normal with mean  $e$  and variance  $\sigma^2$

## Payoffs

- ▶ If accepted, firm's payoff  $\pi$  is expected output minus expected wages:  $E[y - w|e]$
- ▶ If accepted, worker's payoff is expected utility of the wage minus effort cost:  $E[u(w) - c(e)|e]$
- ▶ If rejected, worker has “outside option” of  $\bar{u}$  and firm has “outside option” of 0.<sup>1</sup>

1. We will assume throughout that the firm prefers to hire the worker ex-ante.

## Timing

See the board!

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Recap: Effort-Based Pay

Performance-Based Pay

## Recap: Effort-Based Pay

- ▶ Suppose the firm can pay based on the worker's effort.
- ▶ Then wage is a linear function of effort:  $w(e) = \alpha + \beta e$
- ▶ We now go to the board to solve!

## Recap: Effort-Based Pay

### Theorem 1

*When wages depend directly on effort, effort is  $e^*$  which solves  $c'(e^*) = 1$  and  $\beta^* = 1, \alpha^* = \bar{u} + c(e^*) - 1$*

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Recap: Effort-Based Pay

Performance-Based Pay

## Performance-Based Pay

- ▶ Suppose the firm can pay based ONLY on output  $y$
- ▶ Then wage is a linear function of output:  $w(y) = \alpha + \beta y$
- ▶ We now go to the board to solve!

# Performance-Based Pay

## Theorem 2

When wages depend only on output, effort is  $e_p$  which solves

$$c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

and  $\beta_p = c'(e_p)$ ,  $\alpha_p = \bar{u} - \beta_p e_p + r\beta^2 \sigma^2 / 2 + c(e_p)$ .

- ▶ Notice that  $\frac{1}{1 + r\sigma^2 c''(e_p)} < 1$ .
- ▶ Therefore we are getting less than surplus maximizing effort:  $e_p < e^*$
- ▶ Performance pay generates inefficiency relative to effort based pay!

## Performance-Based Pay: Explicit Cost Function

- ▶ Suppose that  $c(e) = e^2/2$
- ▶ Let's work it out on the board!

## Performance-Based Pay: Explicit Cost Function

- ▶ Suppose that  $c(e) = e^2/2$
- ▶ Let's work it out on the board!
- ▶ Under this quadratic effort cost:

$$e_p = \beta_p = \frac{1}{1 + r\sigma^2}$$

$$\alpha_p = \frac{r\sigma^2 - 1}{2} \left( \frac{1}{1 + r\sigma^2} \right)^2 - \bar{u}$$

## Interpreting Results

- ▶  $\beta$  is the average amount of money paid to the worker per unit of effort.
- ▶  $\beta$  represents the strength of incentives (question: why?)
- ▶ We say incentives are high-powered when  $\beta$  is high (close to 1)
- ▶ Because  $\beta = c'(e)$  we have that:

$$\beta_p = c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

## Interpreting Results

- ▶ Because  $\beta = c'(e)$  we have that:

$$\beta_p = c'(e_p) = \frac{1}{1 + r\sigma^2 c''(e_p)}$$

- ▶ The strength of incentives rises when:
  - ▶ risk-aversion decreases  $\downarrow r$  (question: what if  $r = 0$ ?)
  - ▶ noise/luck becomes less important  $\downarrow \sigma^2$  (question: what if  $\sigma^2 = 0$ ?)
  - ▶ the marginal-marginal cost of effort decreases  $\downarrow c''(e_p)$ 
    - ▶ a high  $c''(e_p)$  means working one more hour after already working 8 hours is much harder than working one more hour after working 1 hour.

## Discussion: Drago and Garvey (1998)