

# Problem Set 1

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The purpose of this homework is to work through our main principal-agent model with uncertainty. The only difference between the model you will solve in this homework and the one in class is that output is not mean 0. You can use the work we did in class as a reference when you get stuck, but please show all steps.

## Setup

### Players

- There is a firm (the principal) who is risk neutral (exponential utility with parameter  $\theta = 0$ ).
- There is a worker (the agent) who is risk averse (exponential utility with parameter  $\theta = r \geq 0$ ).

### Actions

- Firm chooses a linear wage which depends on effort ( $w(e)$ ) or output ( $w(y)$ )
- After seeing the wage, the worker either accepts or rejects the job.
- If they accept, worker chooses effort  $e$  at an increasing, convex cost  $c(e) = e^2/2$

### Output

- Output is effort ( $e$ ) plus noise/luck ( $\epsilon$ ):  $y = e + \epsilon$  where  $\epsilon \sim N(\bar{y}, \sigma^2)$
- Notice that output is now not mean 0.
- This implies output is normal with mean  $\bar{y} + e$  and variance  $\sigma^2$

### Payoffs

- If accepted, firm's payoff  $\pi$  is expected output minus expected wages:  $E[y - w|e]$
- If accepted, worker's payoff is expected utility of the wage minus effort cost:  $E[u(w) - c(e)|e]$
- If rejected, worker has "outside option" of  $\bar{u}$  and firm has "outside option" of 0

### Timing

The same as in class. The firm proposes a wage schedule, the worker accepts or rejects, the worker exerts effort, output occurs, and then the wage is paid out.

## 1 Effort Based Pay

Suppose the firm can pay based on the workers effort:  $w(e) = \alpha + \beta e$ .

## 1.1 Questions

1. Is the worker's wage uncertain? Why or why not.
2. Write the worker's certainty equivalent for a wage with fixed  $(\alpha, \beta)$  and fixed effort  $e$ . Hint: use the answer to question 1.
3. For a fixed wage  $(\alpha, \beta)$  what level of effort does the agent choose? Hint: your answer should be a function of  $\beta$
4. For what  $\alpha, \beta$  does the worker accept the job?
5. What  $\alpha$  will the firm choose, and why? Hint: use the answer to the previous question.
6. Write the firm's profit. DO NOT simplify or plug anything in.
7. Simplify the firm's profit. You should get an expression that is a function of only  $\beta$  or  $e$  (either is fine)
8. What is the profit-maximizing  $\beta$ ? What is the profit maximizing effort  $e$ ?
9. What is the profit-maximizing  $\alpha$ ?

## 2 Performance-Based Pay

Suppose the firm can pay based ONLY on output:  $w(y) = \alpha + \beta y$  (effort is not observed).

### 2.1 Questions

1. Is the worker's wage uncertain? Why or why not.
2. Write the worker's certainty equivalent for a wage with fixed  $(\alpha, \beta)$  and fixed effort  $e$ . Hint: use the answer to question 1.
3. For a fixed wage  $(\alpha, \beta)$  what level of effort does the agent choose? Hint: your answer should be a function of  $\beta$
4. For what  $\alpha, \beta$  does the worker accept the job? I suggest that you not substitute away  $\beta$  yet.
5. What  $\alpha$  will the firm choose, and why? Hint: use the answer to the previous question.
6. Write the firm's profit. DO NOT simplify or plug anything in.
7. Simplify the firm's profit. You should get an expression that is a function of only  $\beta$  or  $e$  (either is fine)
8. What is the profit-maximizing  $\beta$ ? What is the profit maximizing effort  $e$ ?
9. What is the profit-maximizing  $\alpha$ ?

## 3 Conclusion

### 3.1 Questions

1. How did allowing for a non-zero average output change the results (hint: we solved the zero mean version in class)?
2. Provide an intuition or interpretation. Hint: this is deliberately open ended.