Problem Set 3

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1 Relational Contracts

1.1 Setup

- A firm and a worker both have discount rate δ and interact for many periods ($t = 1, ..., \infty$)
- At each period t the following occur:
  - First the firm offers a flat wage w_t
  - Second the worker chooses high (H) or low (L) effort e_t
- High effort has cost c, low effort has cost 0.
- High effort yields revenue v, low effort yields revenue 0.
- Firm outside option is 0, worker outside option is $\bar{u} > 0$.
- Assume the firm wants to motivate high effort.

1.2 Questions

1. Guess an equilibrium strategy for the firm in words. Guess an equilibrium strategy for the worker in words. (Hint: guess the same strategy as in class)
2. Call the high wage w_H and the low wage w_L. Assume the strategy we guessed is being played. What value of w_L will the firm choose and why?
3. What is the worker's payoff in any period where the firm posts a wage of w_L? Justify your answer.
4. Write down the worker's present value utility if they slacked in the past but are now following our guessed strategy. Write down the worker's present value utility from two possible one shot deviations: taking the job and exerting low effort and taking the job and exerting high effort.
5. Write down inequalities for when there are no incentives for the worker to make these deviations. Make sure to simplify. When do they hold?
6. Write down the worker's present value utility from not deviating (never slacking). Write down the worker's utility from a one shot deviation of slacking today when he/she has never slacked before.
7. Write down (and simplify) an inequality for when there is no incentive for the worker to deviate. When is it satisfied?
8. Suppose $\delta = 0.4, v = 3, \bar{u} = 1, c = 1$. Is the relational contract we derived profitable for the firm?
9. Suppose $\delta = 0.6, v = 3, \bar{u} = 1, c = 1$. Is the relational contract we derived profitable for the firm?
10. Interpret the difference between your prior two answers.

2 Hold Up

2.1 Setup

- Players: Elijah Wood (EW), New Line Cinema (NLC)
- NLC decides whether or not to start LOTR, which entails simultaneously announcing EW as Frodo, and paying fixed cost of production $c > 0$.
- After this, EW proposes a wage w for all movies.
- If NLC rejects, EW gets outside option $\bar{u}$ and NLC makes 0 box office revenue.
- If they accept, NLC receives box office revenue from LOTR b.
- Assume indifference is broken in favor of making LOTR.

2.2 Questions

1. We solve using backwards induction, starting at the end of the game. For what wages w will NLC accept EW's offer? Your answer should be an inequality.
2. What wage w will EW offer? Justify your answer.
3. Write down an inequality under which NLC will start making LOTR.
4. Suppose the expected box office revenue doubles. How does this impact the decision to make LOTR?

   Suppose the timing changes: EW now offers a wage first. If NLC accepts, they start and finish making LOTR and incur the fixed cost c, pay the wage w, and receive box office revenue b. If they reject everyone gets their outside option.
5. For which wages does NLC accept? Your answer should be an inequality.
6. Which wage does EW offer? Justify your answer.
7. Is this outcome more or less efficient[1] than the outcome under sequential filming?

3 Teamwork

Note: it may be easiest to just do all math with a generic worker i.

3.1 Setup

- There are N workers, indexed by $i = 1, .., N$
- Each worker can exert effort e_i at cost $c_i(e_i) = e_i^2/2$
- Output is the sum of everyone's effort: $y(e) = \sum_{i=1}^{N} e_i$
- The firm can pay a wage to each worker based only on team output $w_i(y(e))$

[1] You can assume efficiency means maximizes total surplus of both EW and NLC, not distribution.

3.2 Questions

1. Find the first-best effort for each worker (the amount of effort which maximizes total surplus).

   Consider a wage scheme $w_i(y(e)), ..., w_N(y(e))$ that is a partnership (see the definition from class). You may assume that the wage is differentiable.

2. Setup the worker's utility maximization problem. Also write down the budget-balance condition for partnerships.

3. Find and simplify the worker's effort first-order condition.

4. Use your answers from (1) and (3) and the fact that in partnerships all money must be paid out to prove that we cannot get first-best effort.

   Consider a wage scheme $w_1(y(e)), ..., w_N(y(e))$ that is a group bonus (see the definition from class) where the target is total first-best effort $\bar{y} = \sum_{i=1}^{N} e_i^{FB}$ and the bonus amount is more than the effort cost $b_i \geq c_i(e_i^{FB})$.

5. Argue that each worker does not want to exert too little effort ($e_i < e_i^{FB}$).

6. Argue that each worker does not want to exert too much effort ($e_i > e_i^{FB}$).

7. Find a group bonus that gives everyone the same bonus $b_i = b$ and that achieves first-best effort.

8. Give a situation (an effort choice of each worker) where money is burned under this group bonus. Note that this situation does not need to be an equilibrium.