Lecture 2: The Toolkit

Compensation in Organizations

Jacob Kohlhepp

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Discussion: Hartzell, Parsons, Yermack (2010)

What Should You Know Already?

- Single variable derivatives.
- Inequalities.
- Very basic probability.

What Will I Teach You Today?

- The concept of risk aversion and a useful formula.
- A decision problem.
- Simple game theory, static.
- Simple game theory, dynamic.

Caveats

- I will try to cover just what is needed for the class.
- I will not cover Nash equilibrium in depth.
- I will not cover risk aversion in depth.
- If you want to go beyond this or want more practice, see my notes and practice problems from an old course: https://github.com/jakekohlhepp/Econ101.

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Compensation as Lotteries

- Your performance in a job is often effort + noise/luck/chance/circumstance
- Effort is in your control, but the rest is random.
- If you get paid based on performance, your compensation is a lottery!

A Survey

Suppose a company offered you three compensation schemes:

a. X_a: \$100,000 with probability 50%, \$0 with probability 50%.

b. X_b: \$49,000 with probability 100%

c. X_c: \$200,000 with probability 24%, \$0 with probability 76%.

Which would you choose? (will tally on board)

Risk Attitudes

- Before looking at the results, notice some facts:
  - b has lower average than a: $E[X_a] = 50 > 49 = E[X_b]$.
  - But a has higher variance than b: $Var(X_a) = 2500 > 0 = Var(X_b)$.
  - So if you dislike variance or uncertainty, you will prefer a.
  - $E[X_c] = 48 < 49 < 50$: in terms of expected money, c is worse than both a and b.
  - However, the variance of c is higher: $Var(X_c) \approx 7296 > 2500 > 0$.
  - You will only choose c if you like uncertainty/risk.
- A rough interpretation of the results:
  - If you chose b you are *risk averse*: you dislike uncertainty/risk, and are willing to pay to reduce it
  - If you chose a you are either *risk neutral*
  - If you chose c you are *risk loving*
- We will assume people are risk averse or risk neutral.

Expected Utility Theory

- We analyze uncertainty using **Expected Utility Theory.**
- This theorem justifies the tools we will use in this class:

Theorem 1

Under a set of axioms (which you do not need to know), we can represent an individual's preferences over lotteries using an ***expected utility function*** *u where $E[u(X_a)] \geq E[u(X_b)]$ means that lottery a is preferred to lottery b.*

Risk Attitudes As Functions

A person with expected utility function u is...

- **risk averse** if u is concave.
- **risk neutral** if u is linear.
- **risk loving** if u is convex.

Certainty Equivalent

- We often want to rank lotteries/gambles. The following concept is useful for ranking lotteries:

Definition 2

The amount of money for sure a decision maker is willing to pay for lottery a is the **certainty equivalent** (d_a). Mathematically:

$$u(d_a) = E[u(X_a)]$$

- Given a lottery that gives me d dollars for sure and X_a, it is the value of d where I am indifferent.

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- Given a lottery that gives me d dollars for sure and X_a, it is the value of d where I am indifferent.
- Gut check: What is the certainty equivalent of a lottery with $E[X_a] = 10$ when the decision maker is risk neutral?

Certainty Equivalent

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Exponential Utility

- We will use the exponential utility function in this class:

$$u(x) = \frac{1 - e^{-rx}}{r}$$

- r captures risk aversion:
  - When $r > 0$ the decision maker is risk averse
  - When $r < 0$ they are risk loving.
- What happens when $r \to 0$? Using L'Hopsital's rule:

$$\lim_{r \to 0} \frac{1 - e^{-rx}}{r} = x$$

So we have risk neutrality!

Exponential Utility

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Theorem 4

When a person has risk preference given by an exponential utility function $u(x) = \frac{1-e^{-rx}}{r}$, the certainty equivalent of a normal lottery with mean μ and variance σ^2 is given by:

$$d = \mu - r\frac{\sigma^2}{2}$$

- In this class, you can apply this formula directly.
- Talk to me if you are interested in the derivation!

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Making a Decision

- Suppose there is a single person (A) making a decision.
- A takes an action which we will call e.
- e can be a **discrete action**:
  - accept or reject
  - work hard or slack off
- e can be a **continuous action**:
  - exert e units of effort
  - drive e miles
- The utility or payoff of an action e is $u(e)$

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Game Theory in This Class

- Game theory lets us model strategic interaction.
- Therefore it is a tool, but not the point, of this class.
- I do not require you to learn the definitions of Nash equilibrium, best responses, etc.
- However, if you want to go to econ. grad school this can be useful.
- I do require you to make either mathematical and/or verbal arguments.
- For tests, I will only ask you to solve models we solved in class (sometimes with slight modifications).

Competing for a Worker (Bertrand Game)

We will use the basic ideas of this game often:

- **Players.** Two identical firms, numbered $i = 1, 2$, and one worker.
- **Actions.** Firms choose wages *continuously*: $0 \leq w_i < \infty$
- **Payoffs.**
  1. Worker gets the wage of the firm they choose.
  2. When firms set the worker chooses randomly.
  3. The firm which hires the worker gets productivity p and pays the wage.
  4. If a firm does not hire they get 0.

Competing for a Worker: Solution

See the board (we will solve this almost entirely verbally)!

The Company Call List (Tragedy of the Commons)

- **Players.** Two sales workers, $i = 1, 2$, share a common list of company sales contacts.
- **Actions.** Each chooses a number of people on the list to call, q_i
- **Payoffs.**
  1. The cost to the worker of making a call on the list is 0.
  2. The amount of commission the worker makes per call is $120 - q_1 - q_2$

The Company Call List (Tragedy of the Commons) - Solution

See the board!

Sequential Games

- We will consider several models in this class where a firm moves first (usually to set up a compensation plan)
- Then a worker reacts to this compensation plan.
- Unlike the last examples these are sequential games.
- Two key differences between static and dynamic:
  - In sequential games, future players take past player actions as fixed.
  - Earlier players anticipate future players will react to their current choices.
- To deal with this, we use subgame perfect Nash equilibrium.
- For this class, that just means we use backwards induction.

The Pirate Riddle

There is a very famous and tricky riddle that we will use to illustrate backwards induction.

> *Five pirates, numbered 1 through 5, must decide how to divide 100 gold coins. Their decision process is as follows. Starting with pirate 1, each pirate proposes a split consisting of a number of coins for each of the pirates on the ship. Then all pirates vote. If a strict majority approve, the allocation happens. If it does not the proposer is thrown off the ship and the remaining pirates repeat the process. Assume pirates value 2 coins more than 1, etc and that getting thrown off is worse than getting 0 coins. Assume pirates vote no when indifferent (they get a little bit of enjoyment from watching someone walk the plank). What is the maximum number of coins P1 can obtain and not get thrown off?*

Pirate Riddle: Verbal Solution

- Start from the end of the game. If P5 gets to make a proposal, then they are the last pirate left. They propose 100 coins for themselves and get it!

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- Roll back. P3 needs 1 other vote to get a majority. The easiest person to convince is pirate 4, since pirate 4 gets thrown off if the game continues. To get P4's vote P3 can get away with giving him/her 0 coins. P3 proposes

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- Roll back. Pirate 2 needs to get two votes. P4 and P5 are the cheapest to convince because they get 0 next round. So P2 gives P4 and P5 1, P3 0, and keeps 98.

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- Roll back. Pirate 2 needs to get two votes. P4 and P5 are the cheapest to convince because they get 0 next round. So P2 gives P4 and P5 1, P3 0, and keeps 98.
- Roll back. P1 needs two other votes. P3 is the cheapest to convince. P4 and P5 are next cheapest, and P1 need only convince one. So P1 proposes 0 for P2 and P5, P3 1, and P4 2 and keeps 97!

Pirate Riddle: Open Discussion

What can we learn from the pirate riddle?
What is surprising about the outcome/solution?

The Company Call List - Sequential Version

We now modify our static company call list game to be sequential.

- **Players.** Two sales workers, $i = 1, 2$, share a common list of company sales contacts.
- **Actions.** Each chooses a number of people on the list to call, q_i
- **Timing.** Worker 1 calls first, then worker 2.
- **Payoffs.**
  1. The cost to the worker of making a call on the list is 0.
  2. The amount of commission the worker makes per call is $120 - q_1 - q_2$

The Company Call List - Sequential Version Solution

See the board!