Board Work for Lecture 9: Multitasking

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1 Model

- Output is $y = ae_1 + be_2, a > 0, b > 0$
- Cost of effort is:

$$c(e_1, e_2) = \begin{cases} 0 & \text{if } e_1 + e_2 \leq 2\bar{e} \\ (e_1 + e_2 - 2\bar{e})^2/2 & \text{if } e_1 + e_2 > 2\bar{e} \end{cases}$$

- Only task 1 effort is measured: $m = e_1$
- Only what is measured is rewarded: $w(m) = \alpha + \beta m = \alpha + \beta e_1$
- We assume that without incentives the worker supplies all "free effort" (all total effort up to $2\bar{e}$) and splits total effort evenly across the two tasks:

$$e_1 = e_2 = \bar{e}$$

2 Solution: First-Best

What would the firm do if it could do everything directly? Setup surplus, which is just output less cost:

$$\max_{e_1, e_2} ae_1 + be_2 - c(e_1, e_2)$$

The cost function makes this problem tricky: cost is zero up to some threshold, but effort is always valuable. So the firm definitely wants to set $e_1 + e_2$ to be at least $2\bar{e}$. Notice that effort is "together" inside the square. This means that the marginal cost of both types of effort above $2\bar{e}$ is the same! it also means that the marginal cost is increasing (harder to do 1 more hour after 24 hours than after 2 hours).

But the marginal benefit is constant and different! $MB_1 = a, MB_2 = b$. Since the costs are the same but benefits are different, the firm chooses the task with the higher benefit and has the agent perform only that task. if $a > b$ this is task 1. How much do they perform? Well, set $e_2 = 0$ and take the FOC for e_1:

$$a - (e_1 - 2\bar{e}) = 0 \leftrightarrow e_1^{FB} = 2\bar{e} + a, e_2^{FB} = 0$$

So have the worker do the intrinsic motivation effort level plus the marginal benefit! Remember that if $a < b$ we get the reverse:

$$b - (e_2 - 2\bar{e}) = 0 \leftrightarrow e_2^{FB} = 2\bar{e} + b, e_1^{FB} = 0$$

3 Solution: What Actually Happens

In the actual model, we only measure task 1. As always we start with the last stage and ask what effort the worker chooses given some wage $w(e_1) = \alpha + \beta e_1$.

$$\max_{e_1, e_2} \alpha + \beta e_1 - c(e_1, e_2)$$

There are two cases. if $\beta = 0$ (no incentives) the worker provides the bare minimum $e_1 = e_2 = \bar{e}$. If $\beta > 0$, the worker is getting a benefit from task 1 but no benefit from task 2. Since both types of effort are costly, the worker will set $e_2 = 0$. That is, incentives cause e_2 to be crowded out!

What about the first task? Well, the agent will supply all of the free effort to task 1 ($2\bar{e}$) and a little more. How much more? Well, once again we know that $e_1 > 2\bar{e}$ so we are above the pointy part and calculus works again. So we can setup the FOC with $e_2^* = 0$:

$$\max_{e_1,e_2} \alpha + \beta e_1 - (e_1 - 2\bar{e})^2/2$$

$$[e_1 :]\beta - (e_1 - 2\bar{e}) = 0 \leftrightarrow e_1^* = \beta + 2\bar{e}$$

The worker's utility from accepting is given by:

$$u(accept) = \alpha + \beta e_1 - c(e_1, e_2) = \alpha + \beta(\beta + 2\bar{e}) - \beta^2/2 = \alpha + \beta^2/2 + 2\beta\bar{e}$$

as always the firm sets the worker's utility equal to the outside option of 0 using α:

$$\alpha = -\beta^2/2 - 2\beta\bar{e}$$

Continuing with the assumption that $\beta > 0, e_2 = 0$, let's maximize profit:

$$\max_{\beta} ae_1 + be_2 - \alpha - \beta e_1 = \max_{\beta} a(\beta + 2\bar{e}) + \beta^2/2 + 2\beta\bar{e} - \beta(\beta + 2\bar{e})$$

$$= \max_{\beta} a2\bar{e} + a\beta - \beta^2/2$$

FOC:

$$[\beta :]a - \beta = 0 \leftrightarrow \beta = a$$

$$e_1 = \beta + 2\bar{e} = a + 2\bar{e}, e_2 = 0$$

Profit is then:

$$\pi_{HIGH} = a(a + 2\bar{e}) - a^2/2 = a^2/2 + 2a\bar{e}$$

However, we must compare this to profit from $\beta = 0$, which generates $e_1 = e_2 = \bar{e}$. Note that in this case $\alpha = 0$ too, so profit is just revenue:

$$\pi_{LOW} = a\bar{e} + b\bar{e}$$

Now we ask: when does $\pi_{HIGH} > \pi_{LOW}$?

$$a^2/2 + 2a\bar{e} \geq a\bar{e} + b\bar{e}$$

$$a^2/2 \geq b\bar{e} - a\bar{e}$$

$$a^2/2 \geq \bar{e}(b - a)$$

$$a^2 \geq 2\bar{e}(b - a)$$

$$a \geq 2\bar{e}\frac{b - a}{a}$$

So we use a high-powered incentives ($\beta > 0$) if "intrinsic motivation" is low ($\bar{e} \approx 0$). This is because the cost of incentives is crowding out the intrinsic motivation. We use low-powered incentives if the task we do not measure is very profitable/important relative to the task we measure ($b - a >> 0$). We use high-powered incentives if the measured task is important enough ($a >> 0$).