Question 1a.
Conditions:

- a, b, q, r are positive integers
- $0 < a < b$
- $r < a$ and $0 \leq r < 1$

Rearrange

$$\begin{aligned}
\frac{a}{b} &= \frac{1}{q + \frac{r}{a}} \\
\frac{a}{b} &= (q + ra)^{-1} \\
\frac{a}{b}(q + \frac{r}{a}) &= (q + \frac{r}{a})^{-1}(q + \frac{r}{a}) \\
\frac{a}{b}(q + \frac{r}{a}) &= 1 \\
\frac{qa}{b} + \frac{ra}{ab} &= 1 \\
\frac{qa}{b} + \frac{r}{b} &= 1 \\
\frac{qa + r}{b} &= 1
\end{aligned}$$

Solve for q

$$\begin{aligned}
\frac{qa + r}{b} &= 1 \\
qa + r &= b \\
qa &= b - r \\
q &= \frac{b - r}{a} \\
q &= \frac{b}{a} - \frac{r}{a}
\end{aligned}$$

Since $r < a$ and a is an integer greater than 0 this implies $\frac{r}{a} < r$. Of which, r ranges from $(0, 1)$. Implying $\frac{r}{a} \leq r \leq 1$.

Given, $q = \frac{b}{a} - \frac{r}{a}$ and that q is an integer while $\frac{r}{a}$ is a fraction between 0 and 1. q

Solve for r

$$\begin{aligned}
\frac{qa + r}{b} &= 1 \\
qa + r &= b \\
r &= b - qa \\
r &= b - a(\lfloor \frac{b}{a} \rfloor)
\end{aligned}$$

Question 2