

2025_teaching_pylibs

November 5, 2025

1 2025 Python libraries

- Presenter: Jake Krol
- Notebook contents inspired by Erik Johnson and Michael Bradshaw

1.1 Purpose

- Gain hands-on experience with NumPy, Pandas, Matplotlib, and other libraries

1.2 Methods

- Regression
- Hypothesis testing
- Gini Coefficient and Lorenz curve

1.3 Topics

- Climate change
- Population equilibrium
- Rice production “wealth”

1.4 Related work

- Lecture slides: https://docs.google.com/presentation/d/1eq1OidZWZSggBQHWibpLE_m3-_NNYqYcCeVR95GhkMc/edit?usp=sharing

1.5 Setup

- Upload agrofood_co2_emission.csv file

1.6 Scenario

You work for a data science consulting firm, and your supervisor sends you some data with minimal context. Your task is to understand what the data encodes and what insights are attainable purely from the dataset.

```
[36]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import sklearn as skl
```

```
import re
import scipy.stats as stats
```

1.7 Basic inspection

```
[37]: # read
      # view shape
      df = pd.read_csv('agrofood_co2_emission.csv')
      print('# shape ', df.shape)
```

```
# shape (6965, 31)
```

```
[38]: # preview
      df.head()
```

```
[38]:
```

| | Area | Year | Savanna fires | Forest fires | Crop Residues \ |
|---|-------------|------|---------------|--------------|-----------------|
| 0 | Afghanistan | 1990 | 14.7237 | 0.0557 | 205.6077 |
| 1 | Afghanistan | 1991 | 14.7237 | 0.0557 | 209.4971 |
| 2 | Afghanistan | 1992 | 14.7237 | 0.0557 | 196.5341 |
| 3 | Afghanistan | 1993 | 14.7237 | 0.0557 | 230.8175 |
| 4 | Afghanistan | 1994 | 14.7237 | 0.0557 | 242.0494 |

| | Rice Cultivation | Drained organic soils (CO2) | Pesticides Manufacturing \ |
|---|------------------|-----------------------------|----------------------------|
| 0 | 686.00 | 0.0 | 11.807483 |
| 1 | 678.16 | 0.0 | 11.712073 |
| 2 | 686.00 | 0.0 | 11.712073 |
| 3 | 686.00 | 0.0 | 11.712073 |
| 4 | 705.60 | 0.0 | 11.712073 |

| | Food Transport | Forestland ... | Manure Management | Fires in organic soils \ |
|---|----------------|----------------|-------------------|--------------------------|
| 0 | 63.1152 | -2388.803 ... | 319.1763 | 0.0 |
| 1 | 61.2125 | -2388.803 ... | 342.3079 | 0.0 |
| 2 | 53.3170 | -2388.803 ... | 349.1224 | 0.0 |
| 3 | 54.3617 | -2388.803 ... | 352.2947 | 0.0 |
| 4 | 53.9874 | -2388.803 ... | 367.6784 | 0.0 |

| | Fires in humid tropical forests | On-farm energy use | Rural population \ |
|---|---------------------------------|--------------------|--------------------|
| 0 | 0.0 | NaN | 9655167.0 |
| 1 | 0.0 | NaN | 10230490.0 |
| 2 | 0.0 | NaN | 10995568.0 |
| 3 | 0.0 | NaN | 11858090.0 |
| 4 | 0.0 | NaN | 12690115.0 |

| | Urban population | Total Population - Male | Total Population - Female \ |
|---|------------------|-------------------------|-----------------------------|
| 0 | 2593947.0 | 5348387.0 | 5346409.0 |
| 1 | 2763167.0 | 5372959.0 | 5372208.0 |
| 2 | 2985663.0 | 6028494.0 | 6028939.0 |

| | | | |
|---|-----------|-----------|-----------|
| 3 | 3237009.0 | 7003641.0 | 7000119.0 |
| 4 | 3482604.0 | 7733458.0 | 7722096.0 |

| | total_emission | Average Temperature °C |
|---|----------------|------------------------|
| 0 | 2198.963539 | 0.536167 |
| 1 | 2323.876629 | 0.020667 |
| 2 | 2356.304229 | -0.259583 |
| 3 | 2368.470529 | 0.101917 |
| 4 | 2500.768729 | 0.372250 |

[5 rows x 31 columns]

```
[39]: # count column data types
df.dtypes.value_counts()
```

```
[39]: float64    29
object       1
int64        1
Name: count, dtype: int64
```

checkpoint - Agricultural data - For various (area,year) combinations, there exists numeric features - e.g., Afghan 1990 rice cultivation questions - How many distinct countries exist in df? - What is the year span? - What are the units of measurement for these columns? - What are mean/expected values in each numeric column?

```
[40]: # distinct areas
print(df['Area'].nunique())
print(df['Area'].unique()[:10])
```

236

```
['Afghanistan' 'Albania' 'Algeria' 'American Samoa' 'Andorra' 'Angola'
 'Anguilla' 'Antigua and Barbuda' 'Argentina' 'Armenia']
```

```
[41]: # year span
print(
    df['Year'].min(),
    df['Year'].max()
)
```

1990 2020

```
[42]: df[df.columns.drop(['Area', 'Year'])].describe().T['mean'].round(2)
```

```
[42]: Savanna fires          1188.39
Forest fires              919.30
Crop Residues            998.71
Rice Cultivation         4259.67
Drained organic soils (C02) 3503.23
Pesticides Manufacturing  333.42
```

| | |
|----------------------------------|-------------|
| Food Transport | 1939.58 |
| Forestland | -17828.29 |
| Net Forest conversion | 17605.64 |
| Food Household Consumption | 4847.58 |
| Food Retail | 2043.21 |
| On-farm Electricity Use | 1626.68 |
| Food Packaging | 1658.63 |
| Agri-food Systems Waste Disposal | 6018.44 |
| Food Processing | 3872.72 |
| Fertilizers Manufacturing | 3035.72 |
| IPPU | 19991.50 |
| Manure applied to Soils | 923.23 |
| Manure left on Pasture | 3518.03 |
| Manure Management | 2263.34 |
| Fires in organic soils | 1210.32 |
| Fires in humid tropical forests | 668.45 |
| On-farm energy use | 3008.98 |
| Rural population | 17857735.39 |
| Urban population | 16932296.97 |
| Total Population - Male | 17619629.63 |
| Total Population - Female | 17324469.29 |
| total_emission | 64091.24 |
| Average Temperature °C | 0.87 |

Name: mean, dtype: float64

questions - why is forestland negative? is change in forestland measured? - given this data, what hypotheses could we form? - let's get a frame of reference by inspecting one area/country

```
[43]: df_usa=df[df['Area'] == 'United States of America']
df_usa.head()
```

```
[43]:
```

| | Area | Year | Savanna fires | Forest fires | \ |
|------|--------------------------|------|---------------|--------------|---|
| 6591 | United States of America | 1990 | 1391.1481 | 1999.5617 | |
| 6592 | United States of America | 1991 | 1391.1481 | 1999.5617 | |
| 6593 | United States of America | 1992 | 1391.1481 | 1999.5617 | |
| 6594 | United States of America | 1993 | 1391.1481 | 1999.5617 | |
| 6595 | United States of America | 1994 | 1391.1481 | 1999.5617 | |

| | Crop Residues | Rice Cultivation | Drained organic soils (CO2) | \ |
|------|---------------|------------------|-----------------------------|---|
| 6591 | 19193.1314 | 11195.520 | 50713.3546 | |
| 6592 | 17574.1126 | 11029.312 | 50713.3546 | |
| 6593 | 20964.8025 | 12421.402 | 50713.3546 | |
| 6594 | 16761.5327 | 11235.602 | 50566.2744 | |
| 6595 | 21370.6468 | 13151.110 | 50418.1314 | |

| | Pesticides Manufacturing | Food Transport | Forestland | ... | \ |
|------|--------------------------|----------------|--------------|-----|---|
| 6591 | 13772.0 | 45410.2673 | -520573.1593 | ... | |
| 6592 | 13139.0 | 44566.9408 | -520573.1593 | ... | |

| | | | | |
|------|---------|------------|--------------|-----|
| 6593 | 14109.0 | 46198.5933 | -520573.1593 | ... |
| 6594 | 12948.0 | 45175.6740 | -520573.1593 | ... |
| 6595 | 14401.0 | 46739.0889 | -520573.1593 | ... |

| | Manure Management | Fires in organic soils | \ |
|------|-------------------|------------------------|---|
| 6591 | 47024.7024 | 0.0 | |
| 6592 | 48206.0451 | 0.0 | |
| 6593 | 48416.2439 | 0.0 | |
| 6594 | 48426.0899 | 0.0 | |
| 6595 | 49165.7227 | 0.0 | |

| | Fires in humid tropical forests | On-farm energy use | Rural population | \ |
|------|---------------------------------|--------------------|------------------|---|
| 6591 | 9.8513 | 54454.7092 | 62373717.0 | |
| 6592 | 9.8513 | 54565.6091 | 61957131.0 | |
| 6593 | 9.8513 | 57469.9273 | 61539241.0 | |
| 6594 | 9.8513 | 54689.3180 | 61136396.0 | |
| 6595 | 9.8513 | 55539.8937 | 60759735.0 | |

| | Urban population | Total Population - Male | Total Population - Female | \ |
|------|------------------|-------------------------|---------------------------|---|
| 6591 | 190156233.0 | 121451448.0 | 126632284.0 | |
| 6592 | 193017688.0 | 123229931.0 | 128330258.0 | |
| 6593 | 195915032.0 | 125081499.0 | 130093840.0 | |
| 6594 | 198883790.0 | 126914111.0 | 131865642.0 | |
| 6595 | 201981831.0 | 128685441.0 | 133588147.0 | |

| | total_emission | Average Temperature °C |
|------|----------------|------------------------|
| 6591 | 463050.9394 | 0.733583 |
| 6592 | 473285.7816 | 0.706333 |
| 6593 | 486026.3425 | 0.253000 |
| 6594 | 484238.6197 | 0.153500 |
| 6595 | 509412.4984 | 0.470250 |

[5 rows x 31 columns]

questions - we have savannas in the USA? - does avg. temp support climate warming hypotheses?
 - are urban/rural populations shrinking/growing?

```
[44]: # graph avg temp over time in the US
      # plt.plot(df_usa['Year'], df_usa['AvgTemp'])
      # plt.show()
```

- oh, that's very annoying. the temperature column uses the '°' character ...
- what's a simple way to resolve this?

```
[45]: # let's clean up column names
def cln(x):
    x = x.replace('°', '')
    # whitespace
```

```

x = re.sub(r"\s+", "_", x)
# hyphens
x = x.replace('-', '_')
# parentheses
x = x.replace('(', '')
x = x.replace(')', '')
# redundant '_' removal
x = re.sub(r"_+", "_", x)
# lower
return x.lower()
df.columns = [cln(x) for x in df.columns]
df.head()

```

```

[45]:
      area  year  savanna_fires  forest_fires  crop_residues  \
0  Afghanistan  1990      14.7237      0.0557      205.6077
1  Afghanistan  1991      14.7237      0.0557      209.4971
2  Afghanistan  1992      14.7237      0.0557      196.5341
3  Afghanistan  1993      14.7237      0.0557      230.8175
4  Afghanistan  1994      14.7237      0.0557      242.0494

      rice_cultivation  drained_organic_soils_co2  pesticides_manufacturing  \
0           686.00           0.0           11.807483
1           678.16           0.0           11.712073
2           686.00           0.0           11.712073
3           686.00           0.0           11.712073
4           705.60           0.0           11.712073

      food_transport  forestland  ...  manure_management  fires_in_organic_soils  \
0           63.1152  -2388.803  ...           319.1763           0.0
1           61.2125  -2388.803  ...           342.3079           0.0
2           53.3170  -2388.803  ...           349.1224           0.0
3           54.3617  -2388.803  ...           352.2947           0.0
4           53.9874  -2388.803  ...           367.6784           0.0

      fires_in_humid_tropical_forests  on_farm_energy_use  rural_population  \
0                0.0                NaN          9655167.0
1                0.0                NaN          10230490.0
2                0.0                NaN          10995568.0
3                0.0                NaN          11858090.0
4                0.0                NaN          12690115.0

      urban_population  total_population_male  total_population_female  \
0          2593947.0          5348387.0          5346409.0
1          2763167.0          5372959.0          5372208.0
2          2985663.0          6028494.0          6028939.0
3          3237009.0          7003641.0          7000119.0
4          3482604.0          7733458.0          7722096.0

```

| | total_emission | average_temperature_c |
|---|----------------|-----------------------|
| 0 | 2198.963539 | 0.536167 |
| 1 | 2323.876629 | 0.020667 |
| 2 | 2356.304229 | -0.259583 |
| 3 | 2368.470529 | 0.101917 |
| 4 | 2500.768729 | 0.372250 |

[5 rows x 31 columns]

```
[46]: # clean usa subset
df_usa.columns = [cln(x) for x in df_usa.columns]
df_usa.head()
```

```
[46]:
```

| | | area | year | savanna_fires | forest_fires | \ |
|------|--------------------------|------|-----------|---------------|--------------|---|
| 6591 | United States of America | 1990 | 1391.1481 | 1999.5617 | | |
| 6592 | United States of America | 1991 | 1391.1481 | 1999.5617 | | |
| 6593 | United States of America | 1992 | 1391.1481 | 1999.5617 | | |
| 6594 | United States of America | 1993 | 1391.1481 | 1999.5617 | | |
| 6595 | United States of America | 1994 | 1391.1481 | 1999.5617 | | |

| | crop_residues | rice_cultivation | drained_organic_soils_co2 | \ |
|------|---------------|------------------|---------------------------|---|
| 6591 | 19193.1314 | 11195.520 | 50713.3546 | |
| 6592 | 17574.1126 | 11029.312 | 50713.3546 | |
| 6593 | 20964.8025 | 12421.402 | 50713.3546 | |
| 6594 | 16761.5327 | 11235.602 | 50566.2744 | |
| 6595 | 21370.6468 | 13151.110 | 50418.1314 | |

| | pesticides_manufacturing | food_transport | forestland | ... | \ |
|------|--------------------------|----------------|--------------|-----|---|
| 6591 | 13772.0 | 45410.2673 | -520573.1593 | ... | |
| 6592 | 13139.0 | 44566.9408 | -520573.1593 | ... | |
| 6593 | 14109.0 | 46198.5933 | -520573.1593 | ... | |
| 6594 | 12948.0 | 45175.6740 | -520573.1593 | ... | |
| 6595 | 14401.0 | 46739.0889 | -520573.1593 | ... | |

| | manure_management | fires_in_organic_soils | \ |
|------|-------------------|------------------------|---|
| 6591 | 47024.7024 | 0.0 | |
| 6592 | 48206.0451 | 0.0 | |
| 6593 | 48416.2439 | 0.0 | |
| 6594 | 48426.0899 | 0.0 | |
| 6595 | 49165.7227 | 0.0 | |

| | fires_in_humid_tropical_forests | on_farm_energy_use | rural_population | \ |
|------|---------------------------------|--------------------|------------------|---|
| 6591 | 9.8513 | 54454.7092 | 62373717.0 | |
| 6592 | 9.8513 | 54565.6091 | 61957131.0 | |
| 6593 | 9.8513 | 57469.9273 | 61539241.0 | |
| 6594 | 9.8513 | 54689.3180 | 61136396.0 | |

```
6595          9.8513          55539.8937          60759735.0
```

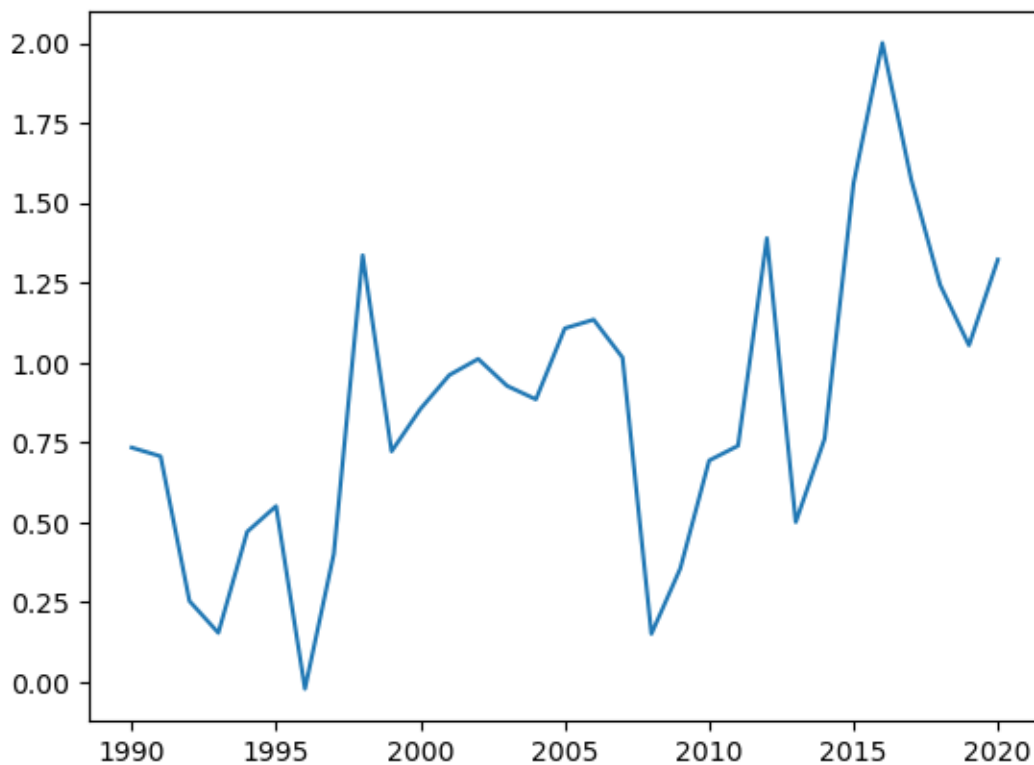
```
      urban_population  total_population_male  total_population_female  \  
6591      190156233.0          121451448.0          126632284.0  
6592      193017688.0          123229931.0          128330258.0  
6593      195915032.0          125081499.0          130093840.0  
6594      198883790.0          126914111.0          131865642.0  
6595      201981831.0          128685441.0          133588147.0
```

```
      total_emission  average_temperature_c  
6591      463050.9394          0.733583  
6592      473285.7816          0.706333  
6593      486026.3425          0.253000  
6594      484238.6197          0.153500  
6595      509412.4984          0.470250
```

```
[5 rows x 31 columns]
```

```
[47]: # is climate warming hypothesis supported by USA data?  
x = df_usa['year']  
y = df_usa['average_temperature_c']  
plt.plot(x,y)
```

```
[47]: [<matplotlib.lines.Line2D at 0x7f8171643d10>]
```



interesting, there's an upward trend.

Q: how could we simply, numerically quantify this trend?

Click to reveal spoiler

linear regression

1.8 Climate change regression

```
[48]: # regression with OLS objective
# using scipy
slope, intercept, r_value, p_value, std_err = stats.linregress(
    x,
    y,
    alternative='greater'
)
print(
    'slope: ', slope,
    '\nintercept: ', intercept,
    '\nr_value: ', r_value,
    '\np_value: ', p_value,
    '\nstd_err: ', std_err
)
```

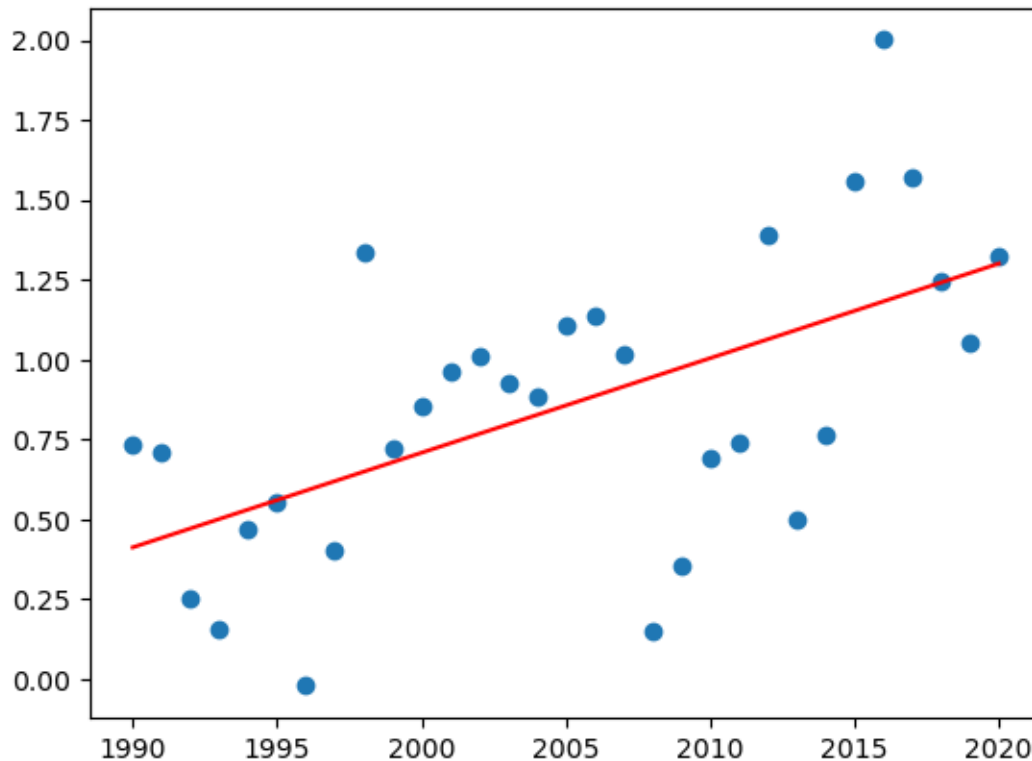
```
slope: 0.02962479838709677
intercept: -58.5416320564516
r_value: 0.5784210865537276
p_value: 0.0003266897078220169
std_err: 0.007758239185112271
```

checkpoint: regression analysis - how can we visualize our regression line - why did we get a p-value? how do we interpret it in this context? (hint: what did we talk about in monte carlo hypothesis testing?)

Click to reveal spoiler

- slope and intercept provide the linear equation; therefore, we can plug in arbitrary inputs in the domain $x \in R$. we're interested

```
[49]: # plot fit regression line and data
plt.plot(x, y, 'o')
plt.plot(x, intercept + slope*x, 'r')
plt.show()
```



Yes, from the data, we can **reject the null hypothesis** ($p = .00032$) that average temperature is non-increasing over time ... in the USA.

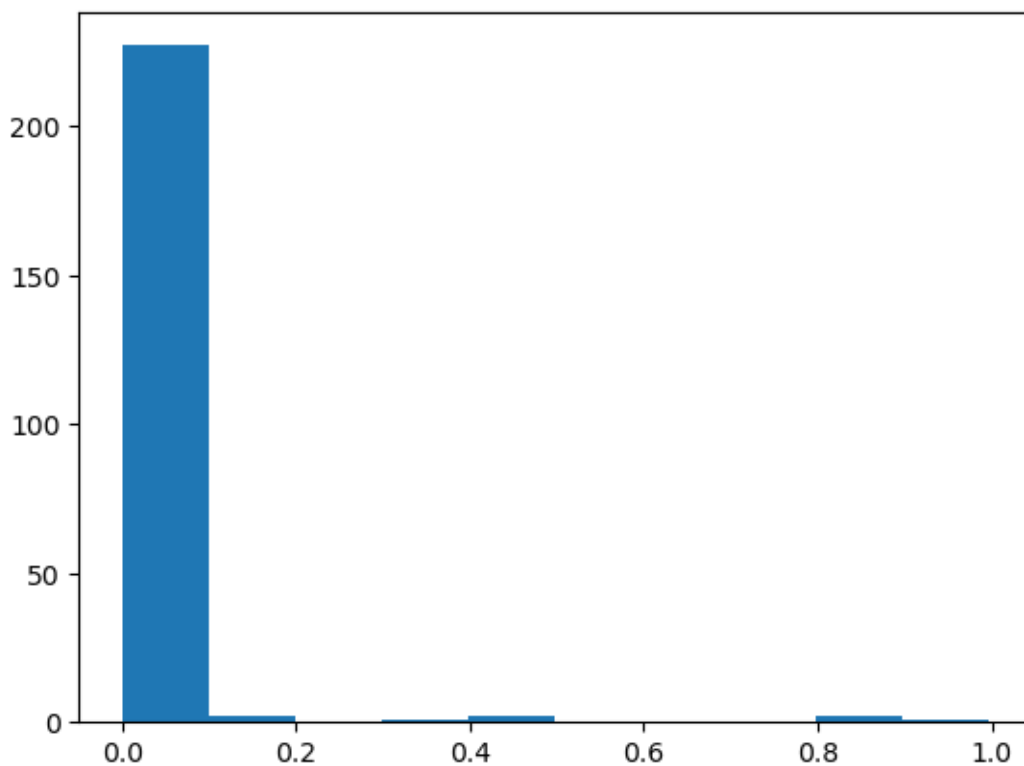
What about other countries? How would you design and implement this experiment on a larger scale?

```
[50]: def gwarm_exp(years, avg_temps):
    slope, _, _, p_value, _ = stats.linregress(
        years,
        avg_temps,
        alternative='greater'
    )
    return p_value, slope
# apply this experiment to all areas
p_values = []
slopes = []
for group, df_group in df.groupby('area'):
    p_value, slope = gwarm_exp(df_group['year'],
    ↪df_group['average_temperature_c'])
    p_values.append(p_value)
    slopes.append(slope)
# plot distribution of p-values
plt.hist(p_values)
```

```

/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10729: RuntimeWarning: invalid value
encountered in scalar divide
    slope = ssxym / ssxm
/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10743: RuntimeWarning: invalid value
encountered in sqrt
    t = r * np.sqrt(df / ((1.0 - r + TINY)*(1.0 + r + TINY)))
/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10749: RuntimeWarning: invalid value
encountered in scalar divide
    slope_stderr = np.sqrt((1 - r**2) * ssym / ssxm / df)
[50]: (array([227.,  2.,  0.,  1.,  2.,  0.,  0.,  0.,  2.,  1.]),
      array([0.          , 0.09962106, 0.19924212, 0.29886318, 0.39848424,
            0.49810531, 0.59772637, 0.69734743, 0.79696849, 0.89658955,
            0.99621061])),
      <BarContainer object of 10 artists>)

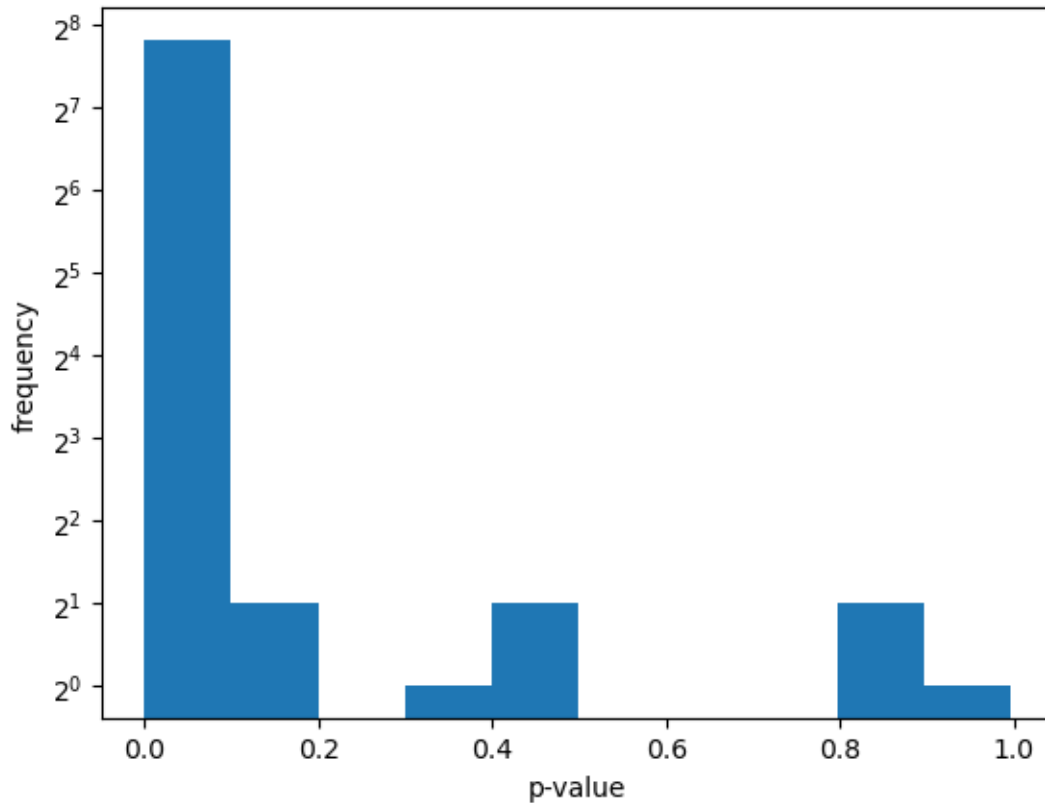
```



most p-values are close to zero. But, \exists some $p \gg 0$.

How can we make this plot better?

```
[51]: plt.hist(p_values)
plt.yscale("log", base = 2)
plt.xlabel("p-value")
plt.ylabel("frequency")
plt.show()
```



now, high p-values can easily be counted (cases where climate warming hyp was not supported).
i counted 8.

let's be more precise

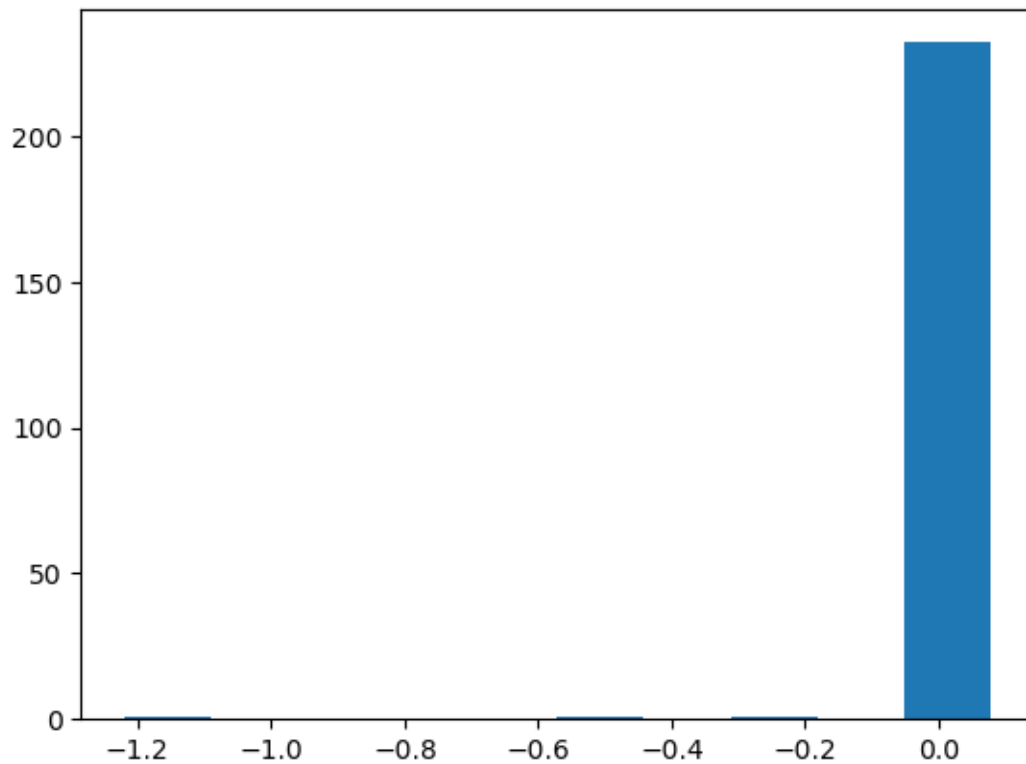
```
[52]: # what fraction of areas support climate warming hypothesis
# try diff alpha and see how fraction changes
alpha = 0.05
n = len(p_values)
num = [p < alpha for p in p_values].count(True)
print(num/n)
```

0.940677966101695

conclusion: 94% of areas in our dataset support a climate warming hypothesis
we showed that temperature is rising, but to what degree and in which places?

```
[53]: plt.hist(slopes)
```

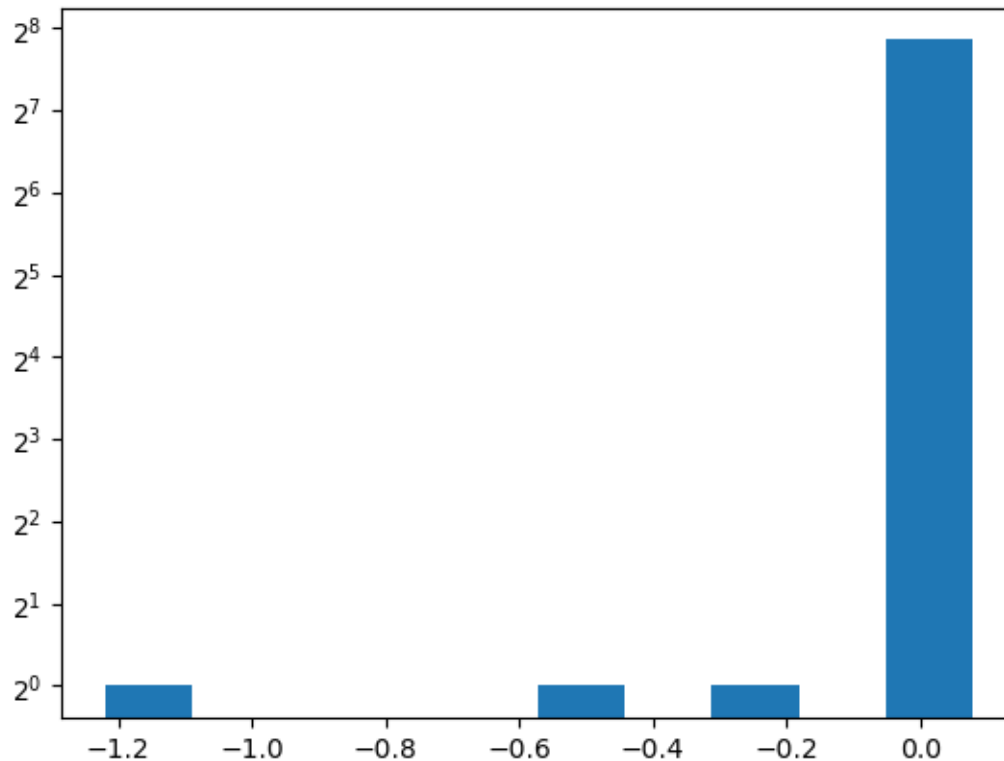
```
[53]: (array([ 1.,  0.,  0.,  0.,  0.,  1.,  0.,  1.,  0., 232.]),  
      array([-1.21991667, -1.09021346, -0.96051025, -0.83080704, -0.70110383,  
            -0.57140062, -0.44169741, -0.3119942 , -0.18229099, -0.05258778,  
            0.07711544]),  
      <BarContainer object of 10 artists>)
```



clearly many slopes are close to zero, ... however, the tail of this distribution is interesting.

some places have a strong decrease in temp over the years

```
[54]: # again we scale y-axis for better view of dist. tail  
plt.hist(slopes)  
plt.yscale("log", base = 2)  
plt.show()
```



two paths 1. i want to show evidence against climate warming 2. i want to show evidence supporting climate warming

```
[55]: # path 1
# goal: find outlier areas where avg temp is decreasing sharply
p_values_cherry = []
slopes_cherry = []
groups_cherry = []
for group, df_group in df.groupby('area'):
    p_value, slope = gwarm_exp(df_group['year'],
    ↪df_group['average_temperature_c'])
    if slope < -0.1:
        p_values_cherry.append(p_value)
        slopes_cherry.append(slope)
        groups_cherry.append(group)
groups_cherry
```

```
/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10729: RuntimeWarning: invalid value
encountered in scalar divide
    slope = ssxym / ssxm
/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10743: RuntimeWarning: invalid value
```

```

encountered in sqrt
    t = r * np.sqrt(df / ((1.0 - r + TINY)*(1.0 + r + TINY)))
/data/jake/miniconda3/lib/python3.12/site-
packages/scipy/stats/_stats_py.py:10749: RuntimeWarning: invalid value
encountered in scalar divide
    slope_stderr = np.sqrt((1 - r**2) * ssym / ssxm / df)

```

```
[55]: ['Ethiopia PDR', 'USSR', 'Yugoslav SFR']
```

```

[56]: # path 1 cont.
df_cherry = df[df['area'].isin(groups_cherry)]
df_cherry.head()

```

```

[56]:
          area  year  savanna_fires  forest_fires  crop_residues  \
2152  Ethiopia PDR  1990      3851.3866      1509.6419      349.6276
2153  Ethiopia PDR  1991      3851.3866      1509.6419      286.8652
2154  Ethiopia PDR  1992      3851.3866      1509.6419      270.0245
6684          USSR  1990      8405.2264      7262.4148     14854.7660
6685          USSR  1991      8405.2264      7262.4148     11900.4386

          rice_cultivation  drained_organic_soils_co2  pesticides_manufacturing  \
2152          3391.507802                        5513.0102          12.850182
2153          3309.287814                        5513.0102          12.098396
2154          3586.696796                        5513.0102          11.606165
6684          4813.760000          131838.2352          1169.000000
6685          4699.296000          131838.2352          1301.000000

          food_transport  forestland  ...  manure_management  \
2152          100.5070      -107.2750  ...          1543.7229
2153           98.8585      -107.2750  ...          1554.3704
2154           33.8336      -107.2750  ...          1594.3863
6684        32821.6716 -605722.9991  ...          60407.5004
6685        32500.3786 -605722.9991  ...          59296.5706

          fires_in_organic_soils  fires_in_humid_tropical_forests  \
2152                        0.0          416.3368
2153                        0.0          416.3368
2154                        0.0          416.3368
6684                        0.0           0.0000
6685                        0.0           0.0000

          on_farm_energy_use  rural_population  urban_population  \
2152           80.8401      44542203.0      6657624.0
2153           82.1289      45925516.0      7022864.0
2154           41.6932      47359180.0      7407170.0
6684        248879.1769      99158922.0     188910774.0
6685        246785.4967      99725999.0     189602194.0

```

| | total_population_male | total_population_female | total_emission \ |
|------|-----------------------|-------------------------|------------------|
| 2152 | 24890395.0 | 25137639.0 | 65914.320714 |
| 2153 | 25886400.0 | 26090250.0 | 61115.468490 |
| 2154 | 26890139.0 | 27042568.0 | 61654.073430 |
| 6684 | 136777703.0 | 153126995.0 | 524473.945541 |
| 6685 | 137641632.0 | 153853429.0 | 520480.833615 |

| | average_temperature_c |
|------|-----------------------|
| 2152 | 0.625000 |
| 2153 | 0.342667 |
| 2154 | 0.048444 |
| 6684 | 1.158250 |
| 6685 | 0.609000 |

[5 rows x 31 columns]

```
[57]: # path 2 cont.
x_eth = df_cherry[df['area'] == 'Ethiopia PDR']['year']
y_eth = df_cherry[df['area'] == 'Ethiopia PDR']['average_temperature_c']
plt.plot(x_eth, y_eth, 'r')
x_ussr = df_cherry[df['area'] == 'USSR']['year']
y_ussr = df_cherry[df['area'] == 'USSR']['average_temperature_c']
plt.plot(x_ussr, y_ussr, 'b')
x_yugo = df_cherry[df['area'] == 'Yugoslav SFR']['year']
y_yugo = df_cherry[df['area'] == 'Yugoslav SFR']['average_temperature_c']
plt.plot(x_yugo, y_yugo, 'g')
plt.show()
```

/tmp/ipykernel_3250460/3954201872.py:2: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

```
x_eth = df_cherry[df['area'] == 'Ethiopia PDR']['year']
```

/tmp/ipykernel_3250460/3954201872.py:3: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

```
y_eth = df_cherry[df['area'] == 'Ethiopia PDR']['average_temperature_c']
```

/tmp/ipykernel_3250460/3954201872.py:5: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

```
x_ussr = df_cherry[df['area'] == 'USSR']['year']
```

/tmp/ipykernel_3250460/3954201872.py:6: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

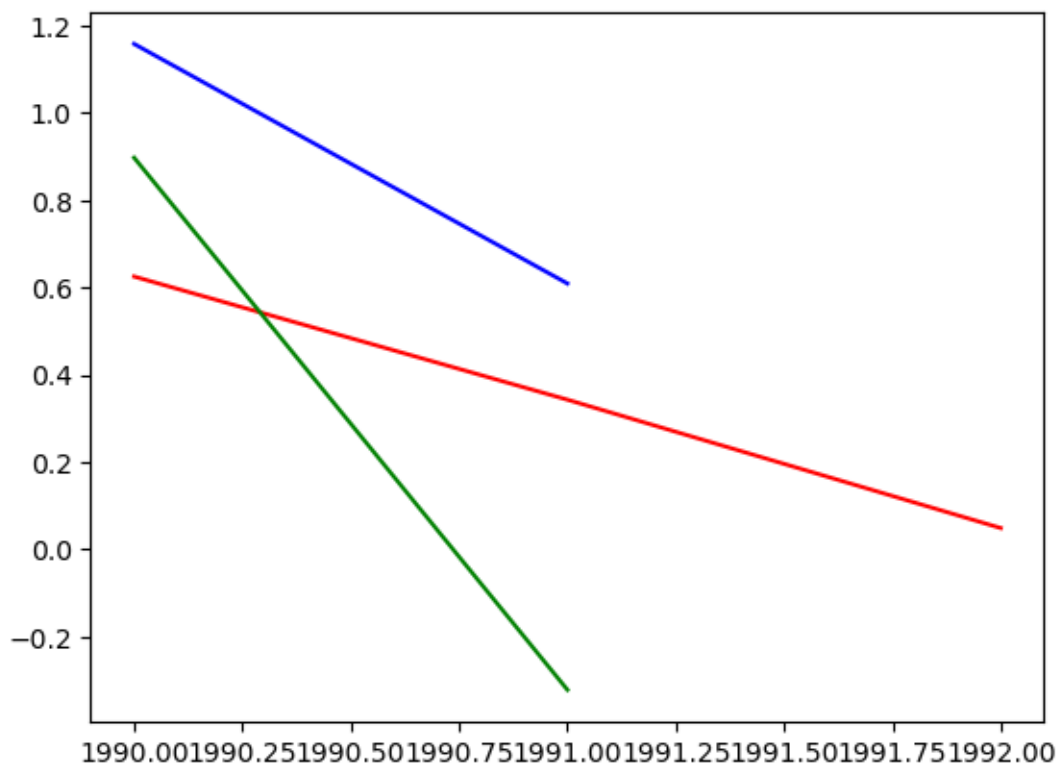
```
y_ussr = df_cherry[df['area'] == 'USSR']['average_temperature_c']
```

/tmp/ipykernel_3250460/3954201872.py:8: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

```
x_yugo = df_cherry[df['area'] == 'Yugoslav SFR']['year']
```

/tmp/ipykernel_3250460/3954201872.py:9: UserWarning: Boolean Series key will be reindexed to match DataFrame index.

```
y_yugo = df_cherry[df['area'] == 'Yugoslav SFR']['average_temperature_c']
```

the data available for these countires is only from a few years

```
[58]: # observe year span of 1990-1991/1992
df_cherry
```

```
[58]:
```

| | area | year | savanna_fires | forest_fires | crop_residues | \ |
|------|--------------|------|---------------|--------------|---------------|---|
| 2152 | Ethiopia PDR | 1990 | 3851.3866 | 1509.6419 | 349.6276 | |
| 2153 | Ethiopia PDR | 1991 | 3851.3866 | 1509.6419 | 286.8652 | |
| 2154 | Ethiopia PDR | 1992 | 3851.3866 | 1509.6419 | 270.0245 | |
| 6684 | USSR | 1990 | 8405.2264 | 7262.4148 | 14854.7660 | |
| 6685 | USSR | 1991 | 8405.2264 | 7262.4148 | 11900.4386 | |
| 6901 | Yugoslav SFR | 1990 | 17.5124 | 27.8984 | 844.4150 | |
| 6902 | Yugoslav SFR | 1991 | 17.5124 | 27.8984 | 1047.9798 | |

| | rice_cultivation | drained_organic_soils_co2 | pesticides_manufacturing | \ |
|------|------------------|---------------------------|--------------------------|---|
| 2152 | 3391.507802 | 5513.0102 | 12.850182 | |
| 2153 | 3309.287814 | 5513.0102 | 12.098396 | |
| 2154 | 3586.696796 | 5513.0102 | 11.606165 | |
| 6684 | 4813.760000 | 131838.2352 | 1169.000000 | |
| 6685 | 4699.296000 | 131838.2352 | 1301.000000 | |
| 6901 | 69.619200 | 276.0991 | 40.000000 | |
| 6902 | 68.145300 | 276.0991 | 34.000000 | |

| | food_transport | forestland | ... | manure_management | \ |
|------|----------------|--------------|-----|-------------------|---|
| 2152 | 100.5070 | -107.2750 | ... | 1543.7229 | |
| 2153 | 98.8585 | -107.2750 | ... | 1554.3704 | |
| 2154 | 33.8336 | -107.2750 | ... | 1594.3863 | |
| 6684 | 32821.6716 | -605722.9991 | ... | 60407.5004 | |
| 6685 | 32500.3786 | -605722.9991 | ... | 59296.5706 | |
| 6901 | 1874.2445 | -10843.0573 | ... | 4584.6630 | |
| 6902 | 1727.6882 | -10843.0573 | ... | 4502.9731 | |

| | fires_in_organic_soils | fires_in_humid_tropical_forests | \ |
|------|------------------------|---------------------------------|---|
| 2152 | 0.0 | 416.3368 | |
| 2153 | 0.0 | 416.3368 | |
| 2154 | 0.0 | 416.3368 | |
| 6684 | 0.0 | 0.0000 | |
| 6685 | 0.0 | 0.0000 | |
| 6901 | 0.0 | 0.0000 | |
| 6902 | 0.0 | 0.0000 | |

| | on_farm_energy_use | rural_population | urban_population | \ |
|------|--------------------|------------------|------------------|---|
| 2152 | 80.8401 | 44542203.0 | 6657624.0 | |
| 2153 | 82.1289 | 45925516.0 | 7022864.0 | |
| 2154 | 41.6932 | 47359180.0 | 7407170.0 | |
| 6684 | 248879.1769 | 99158922.0 | 188910774.0 | |
| 6685 | 246785.4967 | 99725999.0 | 189602194.0 | |
| 6901 | 836.4433 | 11929250.0 | 11445927.0 | |
| 6902 | 776.2953 | 11824752.0 | 11526722.0 | |

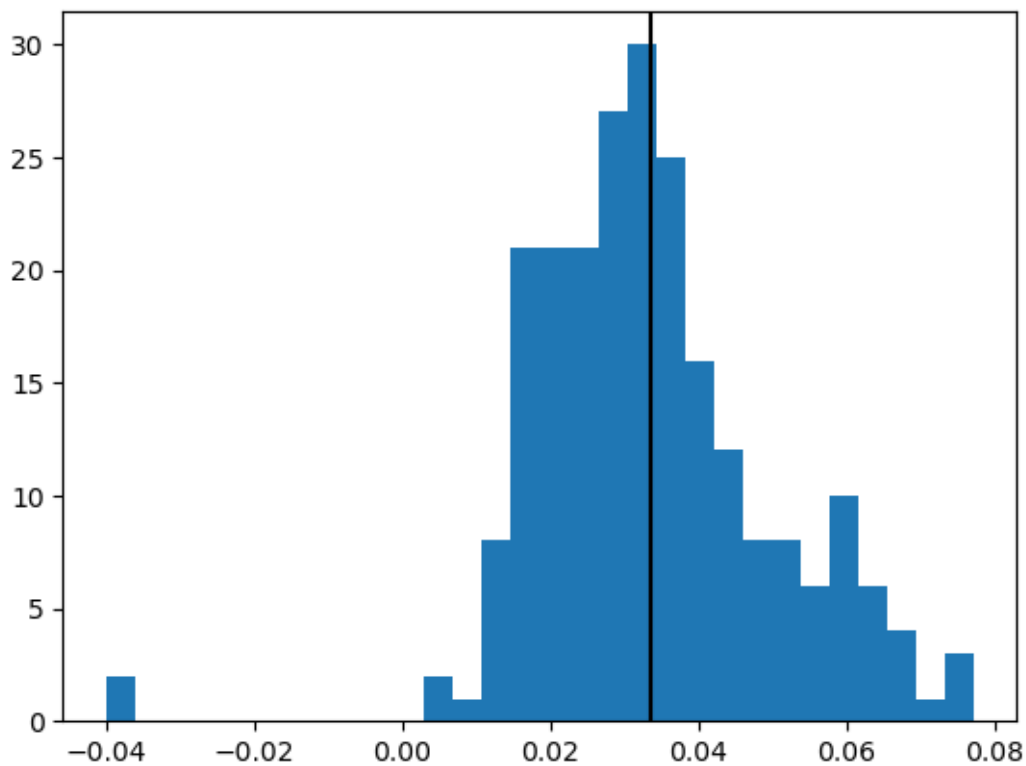
| | total_population_male | total_population_female | total_emission | \ |
|------|-----------------------|-------------------------|----------------|---|
| 2152 | 24890395.0 | 25137639.0 | 65914.320714 | |
| 2153 | 25886400.0 | 26090250.0 | 61115.468490 | |
| 2154 | 26890139.0 | 27042568.0 | 61654.073430 | |
| 6684 | 136777703.0 | 153126995.0 | 524473.945541 | |
| 6685 | 137641632.0 | 153853429.0 | 520480.833615 | |
| 6901 | 10758632.0 | 11248553.0 | 28161.400700 | |
| 6902 | 10733004.0 | 11229798.0 | 24271.147832 | |

| | average_temperature_c |
|------|-----------------------|
| 2152 | 0.625000 |
| 2153 | 0.342667 |
| 2154 | 0.048444 |
| 6684 | 1.158250 |
| 6685 | 0.609000 |
| 6901 | 0.897500 |
| 6902 | -0.322417 |

[7 rows x 31 columns]

```
[59]: # path 2
# slope distribution excluding outliers
slopes_filt = [i for i in slopes if i > -0.1]
plt.hist(slopes_filt, bins=30)
plt.axvline(x=np.mean(slopes_filt), color='black')
```

[59]: <matplotlib.lines.Line2D at 0x7f8167a5e780>



using regression analysis,

average temperature is rising in most areas after we filter out areas with small year span

what's next? we have many other dimensions to understand and formulate hypotheses with

```
[60]: df.head()
```

```
[60]:
```

| | area | year | savanna_fires | forest_fires | crop_residues | \ |
|---|-------------|------|---------------|--------------|---------------|---|
| 0 | Afghanistan | 1990 | 14.7237 | 0.0557 | 205.6077 | |
| 1 | Afghanistan | 1991 | 14.7237 | 0.0557 | 209.4971 | |
| 2 | Afghanistan | 1992 | 14.7237 | 0.0557 | 196.5341 | |
| 3 | Afghanistan | 1993 | 14.7237 | 0.0557 | 230.8175 | |
| 4 | Afghanistan | 1994 | 14.7237 | 0.0557 | 242.0494 | |

| | rice_cultivation | drained_organic_soils_co2 | pesticides_manufacturing | \ |
|---|------------------|---------------------------|--------------------------|---|
| 0 | 686.00 | 0.0 | 11.807483 | |
| 1 | 678.16 | 0.0 | 11.712073 | |
| 2 | 686.00 | 0.0 | 11.712073 | |
| 3 | 686.00 | 0.0 | 11.712073 | |
| 4 | 705.60 | 0.0 | 11.712073 | |

| | food_transport | forestland | ... | manure_management | fires_in_organic_soils | \ |
|---|----------------|------------|-----|-------------------|------------------------|---|
| 0 | 63.1152 | -2388.803 | ... | 319.1763 | 0.0 | |
| 1 | 61.2125 | -2388.803 | ... | 342.3079 | 0.0 | |
| 2 | 53.3170 | -2388.803 | ... | 349.1224 | 0.0 | |
| 3 | 54.3617 | -2388.803 | ... | 352.2947 | 0.0 | |
| 4 | 53.9874 | -2388.803 | ... | 367.6784 | 0.0 | |

| | fires_in_humid_tropical_forests | on_farm_energy_use | rural_population | \ |
|---|---------------------------------|--------------------|------------------|---|
| 0 | 0.0 | NaN | 9655167.0 | |
| 1 | 0.0 | NaN | 10230490.0 | |
| 2 | 0.0 | NaN | 10995568.0 | |
| 3 | 0.0 | NaN | 11858090.0 | |
| 4 | 0.0 | NaN | 12690115.0 | |

| | urban_population | total_population_male | total_population_female | \ |
|---|------------------|-----------------------|-------------------------|---|
| 0 | 2593947.0 | 5348387.0 | 5346409.0 | |
| 1 | 2763167.0 | 5372959.0 | 5372208.0 | |
| 2 | 2985663.0 | 6028494.0 | 6028939.0 | |
| 3 | 3237009.0 | 7003641.0 | 7000119.0 | |
| 4 | 3482604.0 | 7733458.0 | 7722096.0 | |

| | total_emission | average_temperature_c |
|---|----------------|-----------------------|
| 0 | 2198.963539 | 0.536167 |
| 1 | 2323.876629 | 0.020667 |
| 2 | 2356.304229 | -0.259583 |
| 3 | 2368.470529 | 0.101917 |
| 4 | 2500.768729 | 0.372250 |

[5 rows x 31 columns]

regression experiment showed the year span varies for diff areas.

these outliers may confound our analysis

what is the distribution of year span for each area?

1.9 Exercise 1

- goal: make a single figure to visualize and understand the year spans in the dataset
- time: 5m
- hint: you can use either 1D or 2D input for your plot

1.10 Population equilibrium

questions: - are male/female populations in equilibrium? - what is the rate of change of urban and rural populations?

```
[61]: df.head()
```

```
[61]:
```

| | area | year | savanna_fires | forest_fires | crop_residues | \ |
|---|-------------|------|---------------|--------------|---------------|---|
| 0 | Afghanistan | 1990 | 14.7237 | 0.0557 | 205.6077 | |
| 1 | Afghanistan | 1991 | 14.7237 | 0.0557 | 209.4971 | |
| 2 | Afghanistan | 1992 | 14.7237 | 0.0557 | 196.5341 | |
| 3 | Afghanistan | 1993 | 14.7237 | 0.0557 | 230.8175 | |
| 4 | Afghanistan | 1994 | 14.7237 | 0.0557 | 242.0494 | |

| | rice_cultivation | drained_organic_soils_co2 | pesticides_manufacturing | \ |
|---|------------------|---------------------------|--------------------------|-----------|
| 0 | 686.00 | | 0.0 | 11.807483 |
| 1 | 678.16 | | 0.0 | 11.712073 |
| 2 | 686.00 | | 0.0 | 11.712073 |
| 3 | 686.00 | | 0.0 | 11.712073 |
| 4 | 705.60 | | 0.0 | 11.712073 |

| | food_transport | forestland | ... | manure_management | fires_in_organic_soils | \ |
|---|----------------|------------|-----|-------------------|------------------------|-----|
| 0 | 63.1152 | -2388.803 | ... | 319.1763 | | 0.0 |
| 1 | 61.2125 | -2388.803 | ... | 342.3079 | | 0.0 |
| 2 | 53.3170 | -2388.803 | ... | 349.1224 | | 0.0 |
| 3 | 54.3617 | -2388.803 | ... | 352.2947 | | 0.0 |
| 4 | 53.9874 | -2388.803 | ... | 367.6784 | | 0.0 |

| | fires_in_humid_tropical_forests | on_farm_energy_use | rural_population | \ |
|---|---------------------------------|--------------------|------------------|---|
| 0 | 0.0 | NaN | 9655167.0 | |
| 1 | 0.0 | NaN | 10230490.0 | |
| 2 | 0.0 | NaN | 10995568.0 | |
| 3 | 0.0 | NaN | 11858090.0 | |
| 4 | 0.0 | NaN | 12690115.0 | |

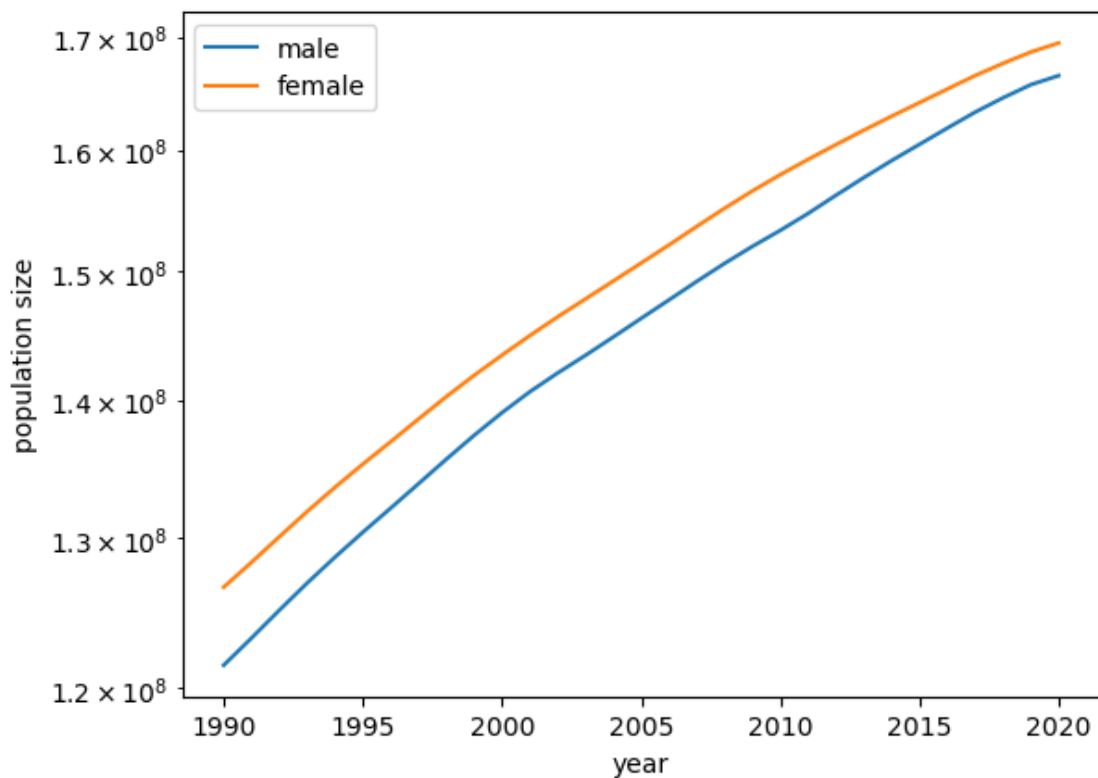
| | urban_population | total_population_male | total_population_female | \ |
|---|------------------|-----------------------|-------------------------|---|
| 0 | 2593947.0 | 5348387.0 | 5346409.0 | |
| 1 | 2763167.0 | 5372959.0 | 5372208.0 | |
| 2 | 2985663.0 | 6028494.0 | 6028939.0 | |
| 3 | 3237009.0 | 7003641.0 | 7000119.0 | |
| 4 | 3482604.0 | 7733458.0 | 7722096.0 | |

| | total_emission | average_temperature_c |
|---|----------------|-----------------------|
| 0 | 2198.963539 | 0.536167 |
| 1 | 2323.876629 | 0.020667 |
| 2 | 2356.304229 | -0.259583 |
| 3 | 2368.470529 | 0.101917 |
| 4 | 2500.768729 | 0.372250 |

[5 rows x 31 columns]

```
[62]: # plot male and female populations over time in us
x = df_usa['year']
y_male = df_usa['total_population_male']
y_female = df_usa['total_population_female']
plt.plot(x,y_male,label='male')
plt.plot(x,y_female,label='female')
plt.yscale("log",base=10)
plt.xlabel('year')
plt.ylabel('population size')
plt.legend()
```

[62]: <matplotlib.legend.Legend at 0x7f81727c21b0>

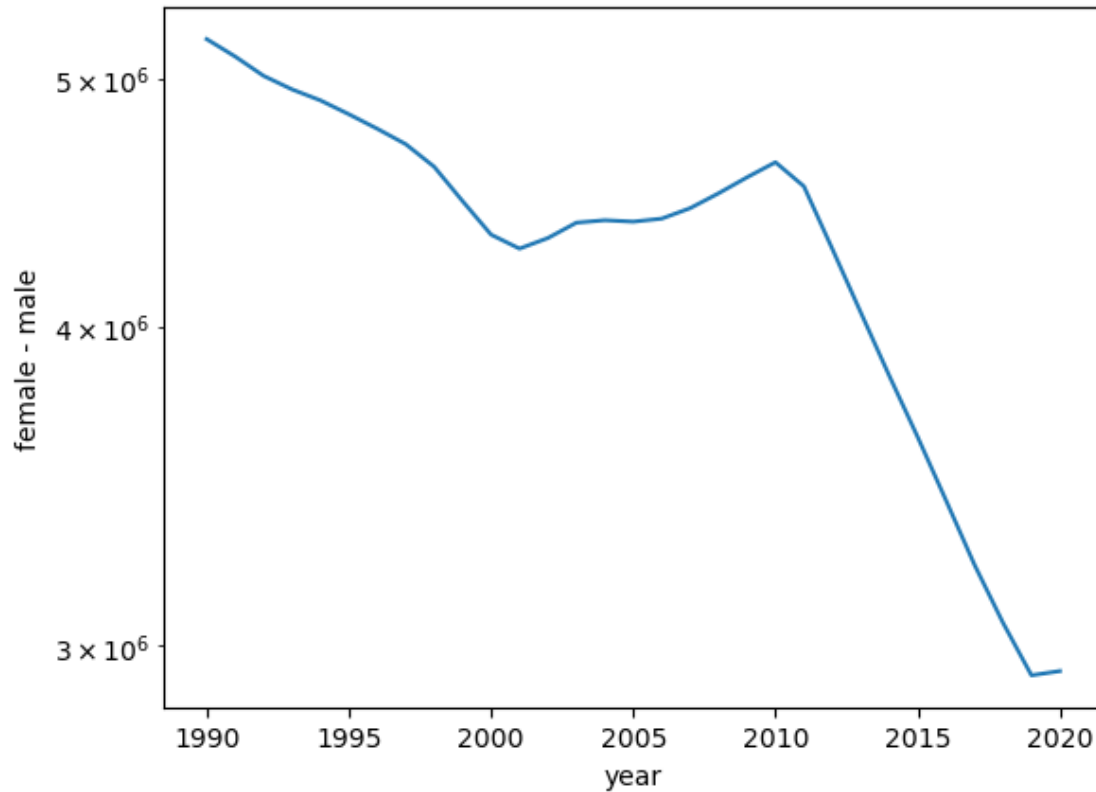


more females than males, but what is the gap here? hard to tell by this plot

```
[63]: # plot difference of males and females
y_diff = y_female - y_male
plt.plot(x,y_diff)
plt.yscale("log",base=10)
```

```
plt.ylabel('female - male')
plt.xlabel('year')
```

```
[63]: Text(0.5, 0, 'year')
```



- 1990: 5M more females than males
- 2020: only 3M more females than males
- the difference (females - males) is decreasing over time

1.11 Exercise 2

- goal: quantify the balance and rate of change between rural and urban populations in the USA
 - produce two line plots
1. Urban and rural population sizes over time
 2. The difference urban and rural populations over time

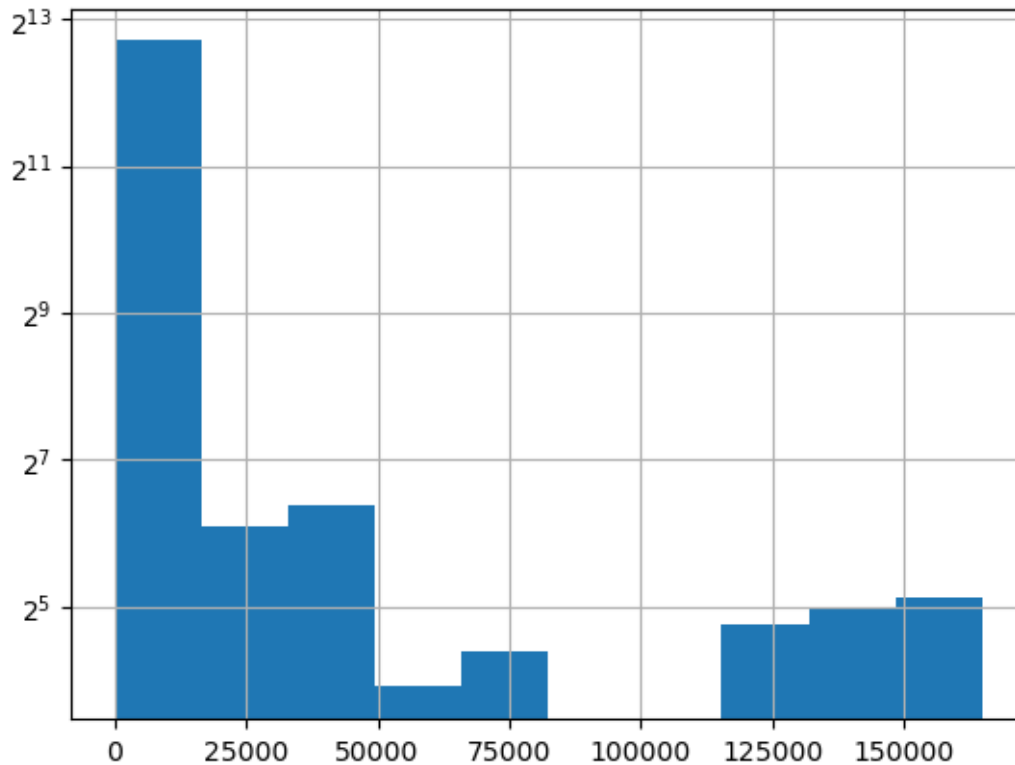
Write a 1 sentence hypothesis based on your plots

1.12 Rice cultivation dominated by few areas

the majority of human wealth and power is held by a small fraction of people

what about rice?

```
[64]: df['rice_cultivation'].hist()  
plt.yscale('log',base=2)
```



most areas produce a small amount of rice while relatively few produce a lot of rice

heavy-tailed distributions are infamous in social sciences and empirical network dataset

in fact, many non-negative count data sets (natural numbers \mathbb{N}_0) are heavy-tailed

let's quantify how important the areas are on the right-hand tail of distribution

The **Gini coefficient** and **Lorenz curve** will quantify the degree of inequality in a distribution.

Lorenz curve

sort x_1, \dots, x_n from least to most “wealthy” - x-axis: cumulative fraction of the population - y-axis: cumulative fraction of total wealth

Gini coefficient - $x \in \mathbb{R}^n := \text{data}$ - $\bar{x} := \text{empirical mean}$

$$G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n^2 \bar{x}}$$

- numerator is sum of pairwise differences
- divide by 2 bc we double count
- divide by n^2 to average

- I feel like we should use $\binom{n}{2}$ instead ...
- $G = \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i - x_j|}{2n(n-1)\bar{x}}$
- divide by \bar{x} s.t. $\text{Range}(G) \in [0, 1]$ (normalization)
- interpretation
 - $G = 0 \rightarrow$ wealth is equally distributed (one for all)
 - $G = 1 \rightarrow$ maximal inequality (all for one)

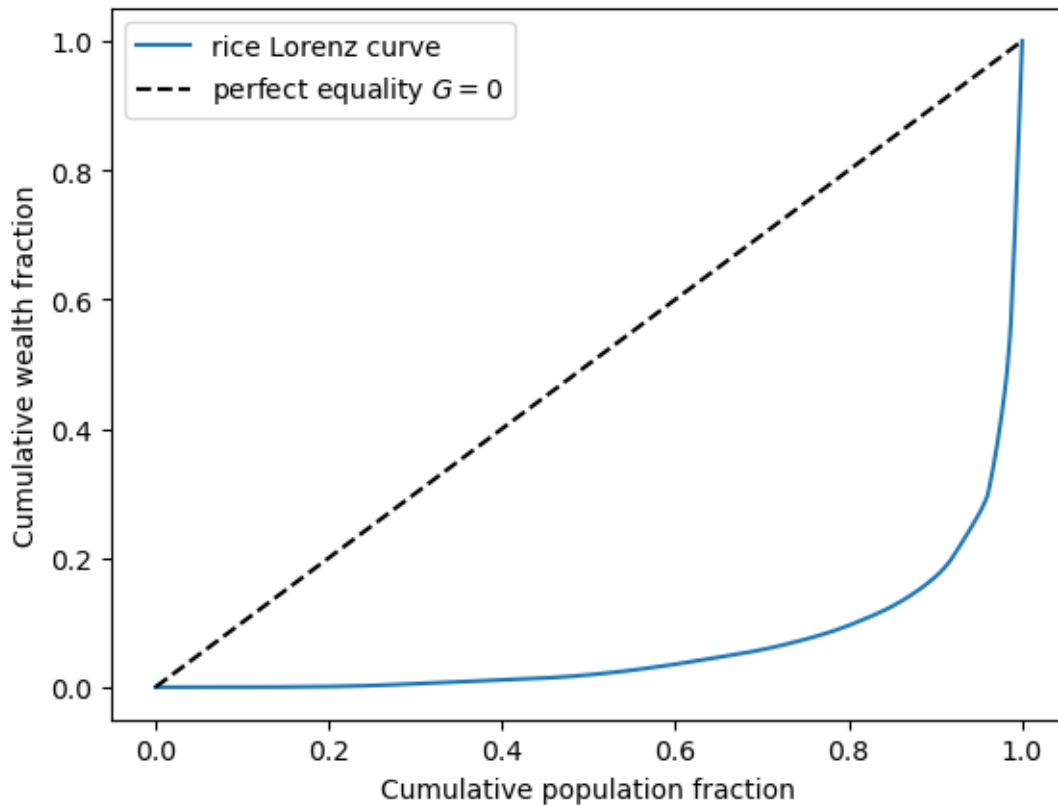
```
[65]: def gini(x, avg='nchoose2'):
    # there are more efficient ways to compute this
    # but the form used above is most interpretable
    n = len(x)
    num = 0
    mean = np.mean(x)
    for i in range(n):
        for j in range(n):
            num += np.abs(x[i] - x[j])
    if avg == 'nchoose2':
        denom = 2*n*(n-1) * mean
    elif avg == 'quad':
        denom = 2*(n**2)*mean
    else:
        raise ValueError('use "nchoose2" or "quad" for denominator scale')
    return num / denom
# takes about 1 min
gini(df['rice_cultivation'].values)
```

[65]: 0.8707302318270593

```
[66]: # plot lorenz curve
x = np.sort(df['rice_cultivation'].values)
n = len(x)
# cumulative sum vector
cs = np.cumsum(x)
# add 0,0
cs = np.insert(cs, 0, 0)
# normalize cs to [0,1]
cs = cs / cs[-1]
# space our population along the line segment [0,1]
pop = np.linspace(0,1,n+1) # +1 because we added 0,0
# plot
# lorenz curve
plt.plot(pop, cs, label='rice Lorenz curve')
# perfect equality line (G=0)
plt.plot([0,1], [0,1], label="perfect equality $G=0$", color = 'black',
         linestyle='--')
plt.xlabel('Cumulative population fraction')
plt.ylabel('Cumulative wealth fraction')
```

```
plt.legend()
```

[66]: <matplotlib.legend.Legend at 0x7f81678aee70>



interpretation of rice Lorenz curve - 20% of areas produce \$ 90y 0.1\$ of Lorenz curve)

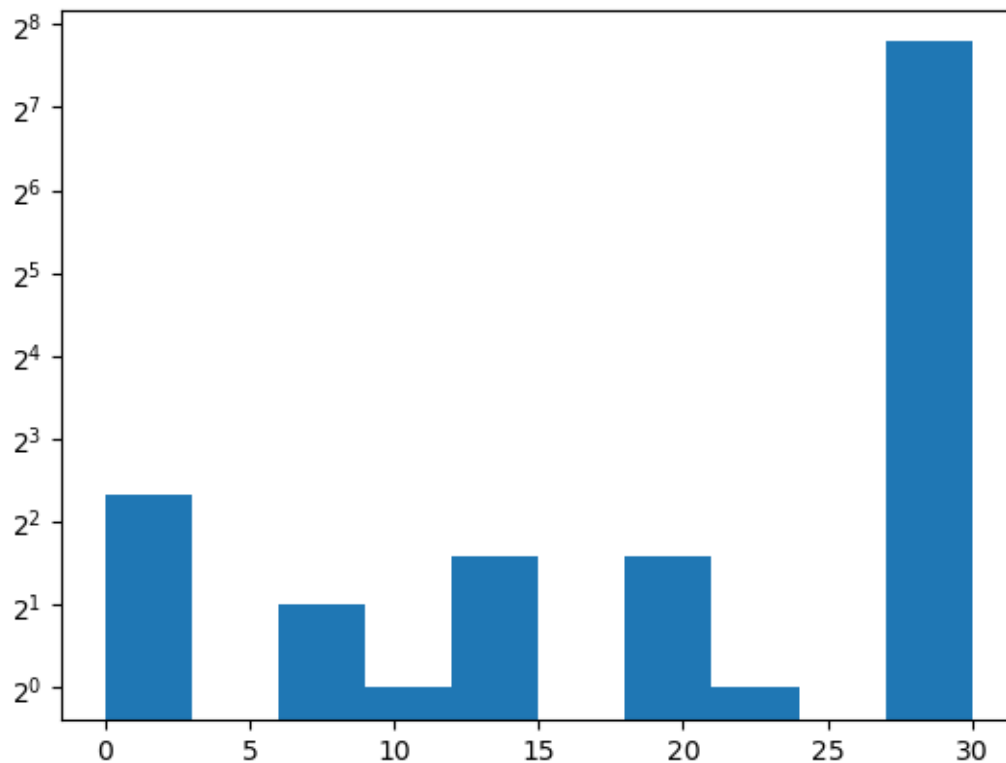
1.13 Exercise 3

- goal: compute Gini coef. for simulated data
- approach:
 - for zipf and powerlaw distributions (hint: use np.random)
 - * sample 1000 values
 - * compute Gini coef. and plot Lorenz curve
- time: 10m

2 Exercise solutions

2.1 Excerise 1

```
[67]: # histogram solution
spans = []
for group, df_group in df.groupby('area'):
    spans.append(df_group['year'].max() - df_group['year'].min())
plt.hist(spans)
plt.yscale('log',base=2)
```



histogram collapses year data for each area into a single integer: span length.

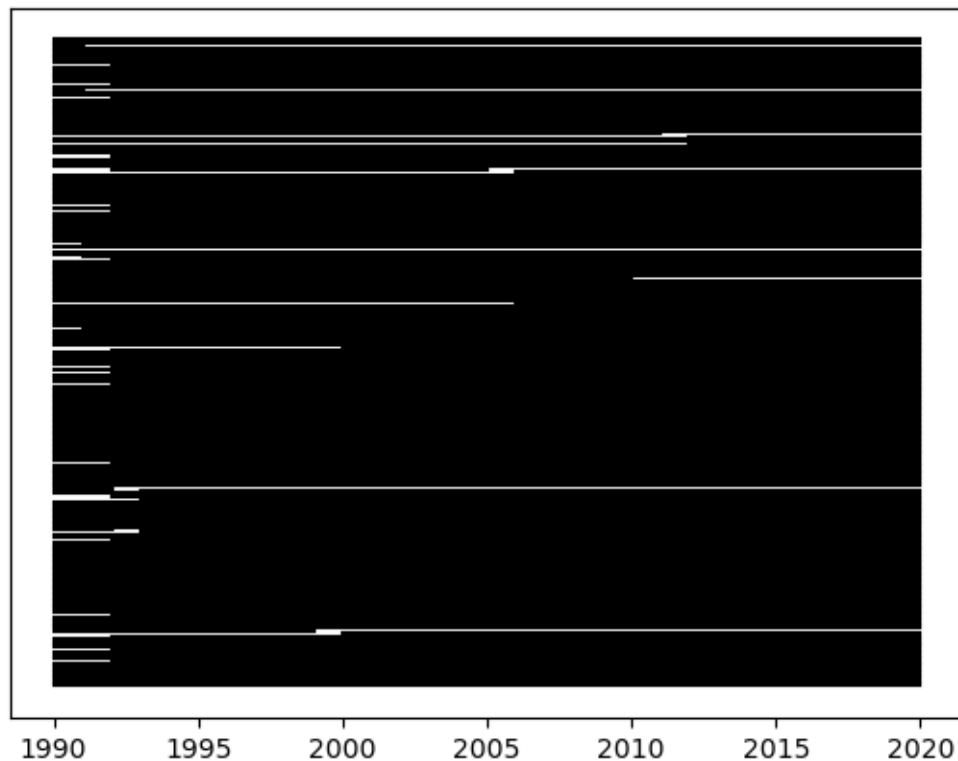
does not indicate which years are missing

```
[71]: # interval line solution
fig,ax = plt.subplots()
ticks = []
spacing = 1
i=-2
i_prev=0
for group, df_group in df.groupby('area'):
    i = i_prev + spacing
```

```

years = df_group['year'].values
ax.plot(years, np.repeat(i, len(years)),color='black')
ticks.append(group)
i_prev = i
# hide yticks
ax.yaxis.set_visible(False)
# set y axis label
ax.set_ylabel('Countries')
fig.show()

```



interval plot has fine grain resolution on which years are missing

2.2 Exercise 2

```
[72]: df_usa.head()
```

```

[72]:
   area  year  savanna_fires  forest_fires  \
6591  United States of America  1990      1391.1481      1999.5617
6592  United States of America  1991      1391.1481      1999.5617
6593  United States of America  1992      1391.1481      1999.5617
6594  United States of America  1993      1391.1481      1999.5617
6595  United States of America  1994      1391.1481      1999.5617

```

| | crop_residues | rice_cultivation | drained_organic_soils_co2 | \ |
|------|---------------|------------------|---------------------------|---|
| 6591 | 19193.1314 | 11195.520 | 50713.3546 | |
| 6592 | 17574.1126 | 11029.312 | 50713.3546 | |
| 6593 | 20964.8025 | 12421.402 | 50713.3546 | |
| 6594 | 16761.5327 | 11235.602 | 50566.2744 | |
| 6595 | 21370.6468 | 13151.110 | 50418.1314 | |

| | pesticides_manufacturing | food_transport | forestland | ... | \ |
|------|--------------------------|----------------|--------------|-----|---|
| 6591 | 13772.0 | 45410.2673 | -520573.1593 | ... | |
| 6592 | 13139.0 | 44566.9408 | -520573.1593 | ... | |
| 6593 | 14109.0 | 46198.5933 | -520573.1593 | ... | |
| 6594 | 12948.0 | 45175.6740 | -520573.1593 | ... | |
| 6595 | 14401.0 | 46739.0889 | -520573.1593 | ... | |

| | manure_management | fires_in_organic_soils | \ |
|------|-------------------|------------------------|---|
| 6591 | 47024.7024 | 0.0 | |
| 6592 | 48206.0451 | 0.0 | |
| 6593 | 48416.2439 | 0.0 | |
| 6594 | 48426.0899 | 0.0 | |
| 6595 | 49165.7227 | 0.0 | |

| | fires_in_humid_tropical_forests | on_farm_energy_use | rural_population | \ |
|------|---------------------------------|--------------------|------------------|---|
| 6591 | 9.8513 | 54454.7092 | 62373717.0 | |
| 6592 | 9.8513 | 54565.6091 | 61957131.0 | |
| 6593 | 9.8513 | 57469.9273 | 61539241.0 | |
| 6594 | 9.8513 | 54689.3180 | 61136396.0 | |
| 6595 | 9.8513 | 55539.8937 | 60759735.0 | |

| | urban_population | total_population_male | total_population_female | \ |
|------|------------------|-----------------------|-------------------------|---|
| 6591 | 190156233.0 | 121451448.0 | 126632284.0 | |
| 6592 | 193017688.0 | 123229931.0 | 128330258.0 | |
| 6593 | 195915032.0 | 125081499.0 | 130093840.0 | |
| 6594 | 198883790.0 | 126914111.0 | 131865642.0 | |
| 6595 | 201981831.0 | 128685441.0 | 133588147.0 | |

| | total_emission | average_temperature_c |
|------|----------------|-----------------------|
| 6591 | 463050.9394 | 0.733583 |
| 6592 | 473285.7816 | 0.706333 |
| 6593 | 486026.3425 | 0.253000 |
| 6594 | 484238.6197 | 0.153500 |
| 6595 | 509412.4984 | 0.470250 |

[5 rows x 31 columns]

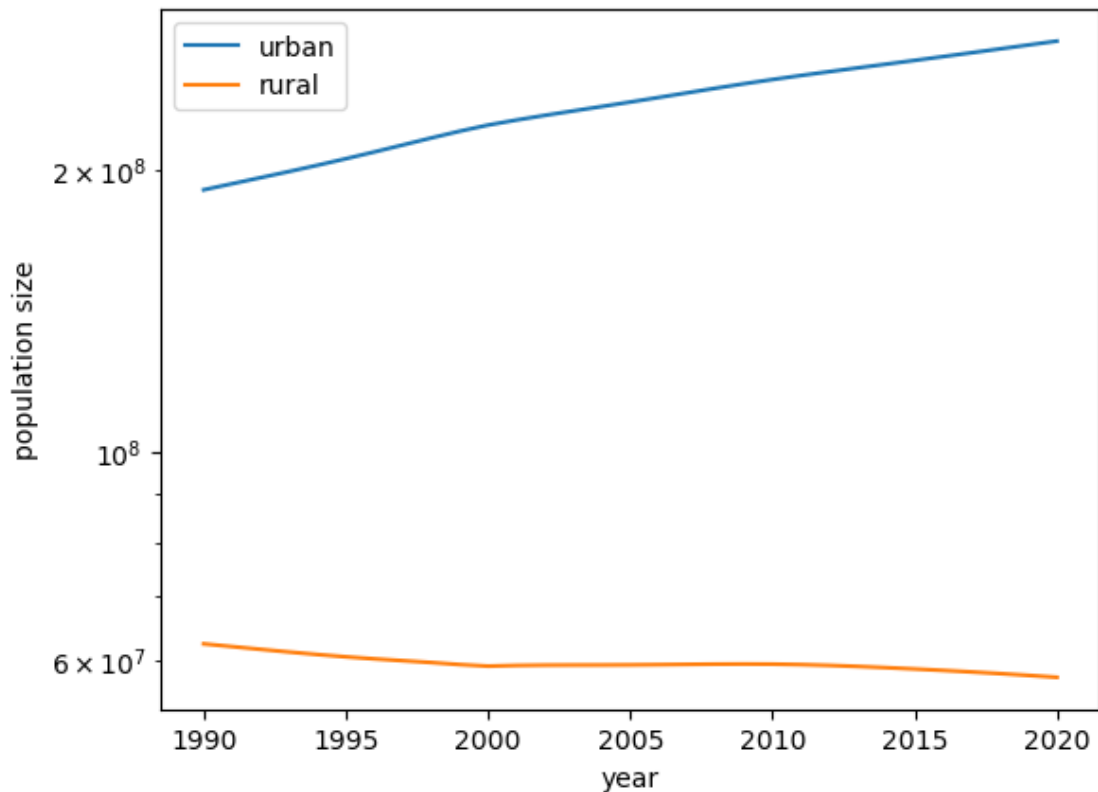
```
[73]: # plot urban and rural populations over time in us
x = df_usa['year']
```

```

y_urban = df_usa['urban_population']
y_rural = df_usa['rural_population']
plt.plot(x,y_urban,label='urban')
plt.plot(x,y_rural,label='rural')
plt.yscale("log",base=10)
plt.xlabel('year')
plt.ylabel('population size')
plt.legend()

```

[73]: <matplotlib.legend.Legend at 0x7f8171481d00>

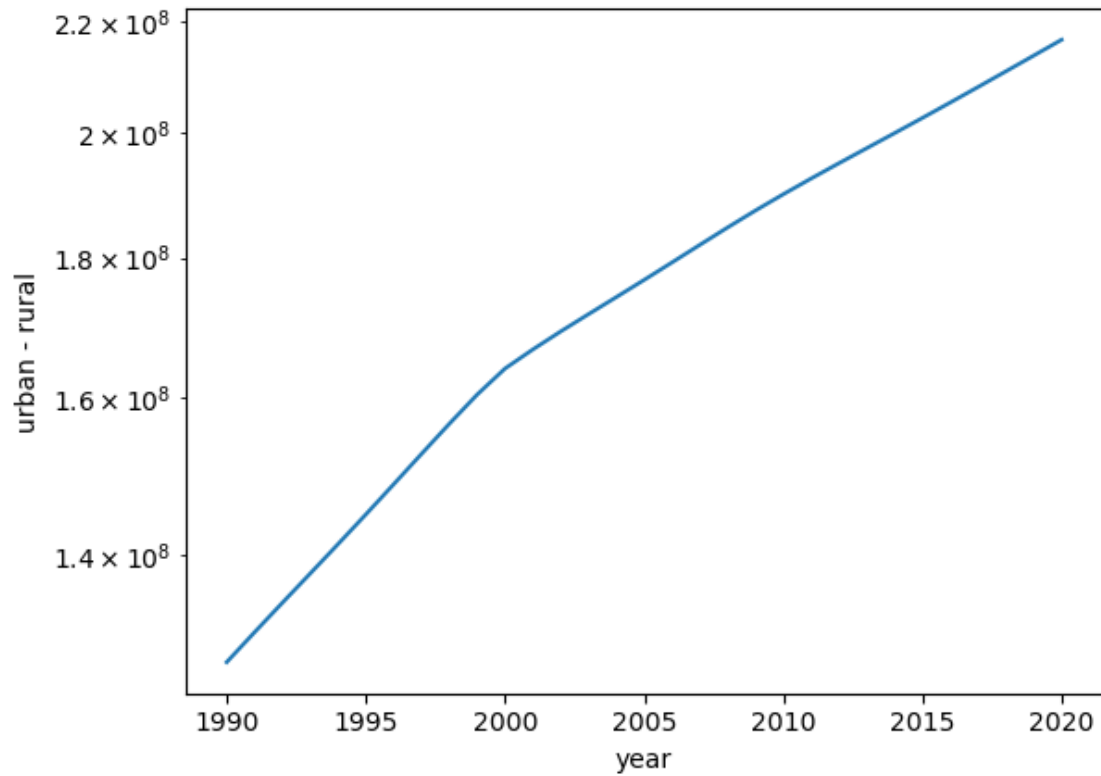


```

[74]: # plot difference of males and females
y_diff = y_urban - y_rural
plt.plot(x,y_diff)
plt.yscale("log",base=10)
plt.ylabel('urban - rural')
plt.xlabel('year')

```

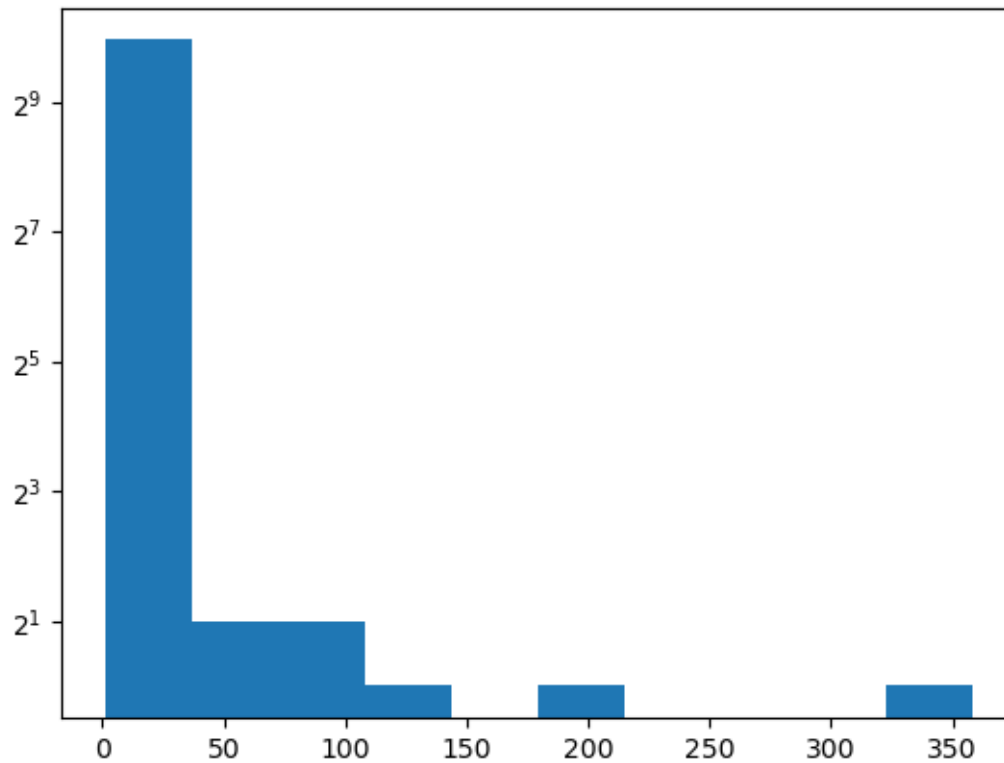
[74]: Text(0.5, 0, 'year')



hypothesis: in the last three decades of the USA, people have been migrating from rural to urban areas

2.3 Exercise 3

```
[75]: x_zipf = np.random.zipf(a=2,size=1000)
      plt.hist(x_zipf)
      plt.yscale("log",base=2)
```



```
[76]: # zipf Gini
gini_zipf = gini(x_zipf)
print("zipf gini: ", gini_zipf)
```

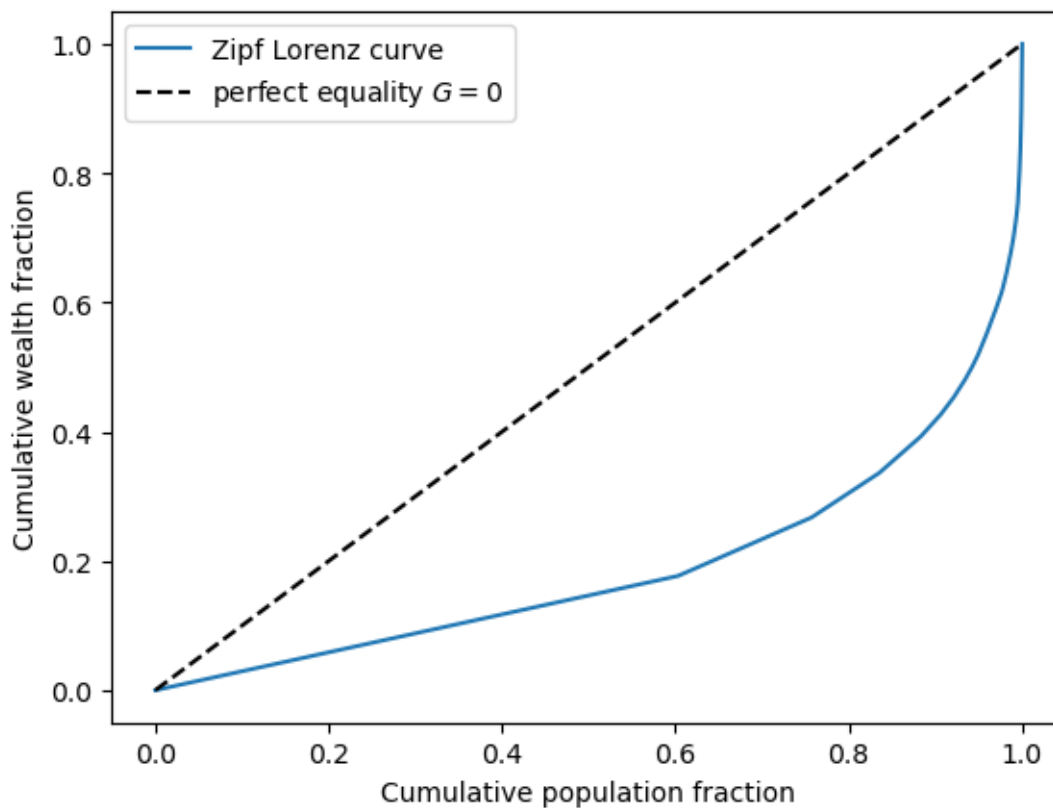
zipf gini: 0.6202736694539973

```
[77]: # zipf Lorenz
x_zipf_sort = np.sort(x_zipf)
n = len(x_zipf_sort)
# cumulative sum vector
cs = np.cumsum(x_zipf_sort)
# add 0,0
cs = np.insert(cs, 0, 0)
# normalize cs to [0,1]
cs = cs / cs[-1]
# space our population along the line segment [0,1]
pop = np.linspace(0,1,n+1) # +1 because we added 0,0
# plot
# lorenz curve
plt.plot(pop, cs, label='Zipf Lorenz curve')
# perfect equality line (G=0)
```

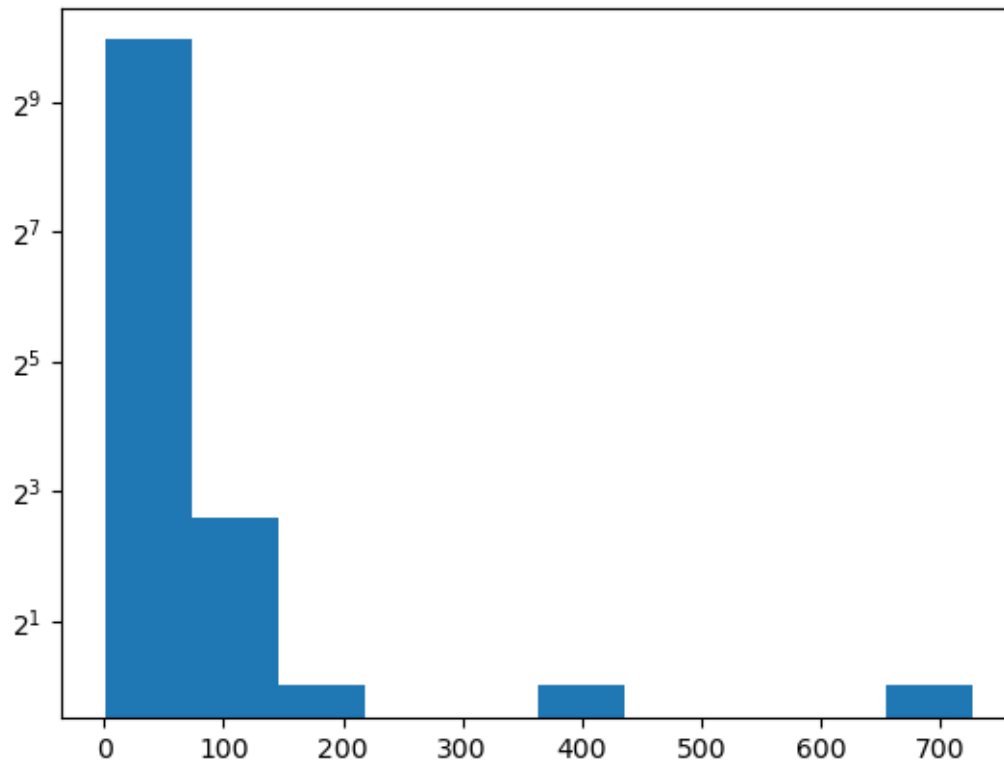


```
plt.plot([0,1], [0,1], label="perfect equality  $G=0$ ", color = 'black',  
         linestyle='--')  
plt.xlabel('Cumulative population fraction')  
plt.ylabel('Cumulative wealth fraction')  
plt.legend()
```

[77]: <matplotlib.legend.Legend at 0x7f8171735430>



```
[78]: # zipf a = 2  
x_zipf = np.random.zipf(2,1000)  
plt.hist(x_zipf)  
plt.yscale("log",base=2)
```



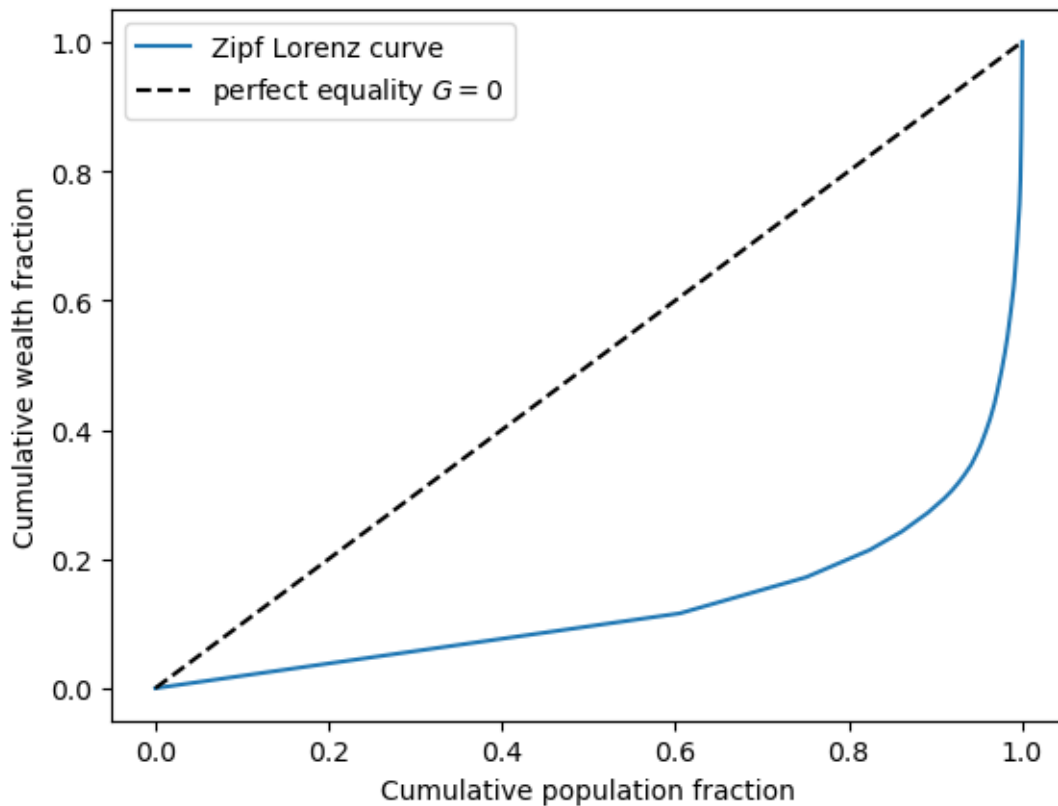
```
[79]: # zipf Gini
gini_zipf = gini(x_zipf)
print("zipf gini: ", gini_zipf)
```

zipf gini: 0.739825818556948

```
[80]: # zipf Lorenz
x_zipf_sort = np.sort(x_zipf)
n = len(x_zipf_sort)
# cumulative sum vector
cs = np.cumsum(x_zipf_sort)
# add 0,0
cs = np.insert(cs, 0, 0)
# normalize cs to [0,1]
cs = cs / cs[-1]
# space our population along the line segment [0,1]
pop = np.linspace(0,1,n+1) # +1 because we added 0,0
# plot
# lorenz curve
plt.plot(pop, cs, label='Zipf Lorenz curve')
# perfect equality line (G=0)
```

```
plt.plot([0,1], [0,1], label="perfect equality  $G=0$ ", color = 'black',  
         linestyle='--')  
plt.xlabel('Cumulative population fraction')  
plt.ylabel('Cumulative wealth fraction')  
plt.legend()
```

[80]: <matplotlib.legend.Legend at 0x7f8167279640>



increasing the a parameter increased the Gini Coefficient and changed the shape of Lorenz curve.

3 Instructor notes

from command-line

jupyter server

copy link into web browser