Comparison of GLMM Estimation Methods

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A Motivating Example

- Owlet begging data from Roulin and Bersier (2007).
  - 599 observations from 27 barn owl nests in western Switzerland.
  - Response: number of calls in a 30 sec. interval.
  - Covariates used: brood size, food treatment, and arrival time of parent.

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  - Response: number of calls in a 30 sec. interval.
  - Covariates used: brood size, food treatment, and arrival time of parent.
- Observations are correlated at the nest level due to repeated measurements and the count data likely follow a Poisson distribution.

Generalized Linear Mixed Models

- In ecological and evolutionary biology, ordinary linear models are not always well suited for data analysis.
  - Presence/absence and count data are common.
  - Dependence between observations occur through repeated measures, clustered observations, or within spatial elements.
- With both non-normal responses and correlated observations, we can use generalized linear mixed models (GLMMs) for estimation and inference.

Issues with Estimation

Problem:

The likelihood functions of GLMMs involve high-dimensional integrals that lack closed form solutions, making evaluation of the exact likelihood function essentially impossible.

- $f(Y_{ij}|b_i)$ and $f(b_i)$ are not normal distributions.

$$L(\beta, D) = \prod_i \int f(Y_{ij}|b_i) f(b_i) db_i$$

- Because of that, approximations of the integration or of the likelihood function need to be used.

GLMM Estimation Methods

- In place of closed form solutions, a number of approximation methods have been developed since the first major introduction of GLMMs by McCullagh and Nelder (1989). Commonly used approximations include:
  - Laplace Approximation
  - (Adaptive) Gaussian Hermite Quadrature (AGHQ)
  - Penalized Quasi-likelihood
- Bayesian approaches to GLMMs are also commonly used as it sidesteps the issues integration issues involved with the exact likelihood.
- Non-Bayesian Monte Carlo methods are available, but their implementation is not easy and are poorly documented.

Laplace Approximation and AGHQ

- Laplace approximation relies on a Taylor series approximation of the likelihood function to give an integrable result.
- Adaptive Gaussian Hermite Quadrature is a generalization of Laplace approximation, and AGHQ with one quadrature point is Laplacian approximation.

Penalized Quasi-likelihood

- PQL for GLMMs was introduced by Breslow and Clayton and relies on a Laplace approximation of the likelihood.
  - Quasi-likelihoods are not related to a true probability distribution, only specifying the relationship between the mean and variance.
- Implementing PQL relies on weighted least-squares for estimating the mean parameters and uses an approximating normal model for estimation of the variance components.
- One issue with PQL is that it can result in downwardly biased variance component estimates.

Bayesian Approach

Need to flesh out the model/priors here.

- Adapted JAGS code from Zuur.
- Relied on weakly informative priors.
- Focused on posterior means for comparisons.

Data Generation and Study Design

- The `simr` package was used to generate new responses based on an original Poisson GLMM fit to the owlet data, fixing all covariates and random effects.
- For each iteration of the simulation, models using the four estimation models were fit.
- Estimates of the fixed effects, random effects, and the variance of the random effects were recorded.

Results

- Pretty empirical sampling distributions of selected components.
- Will be comparing bias, spread, and distribution across methods.

Conclusions

- It's likely that `glmer` works just fine in most cases, especially when using AGHQ, and is adequate in most cases.
- PQL to show biased variance component estimates for small cluster sizes, but with large samples it doesn't matter much.
- Extensions: using `simr` to change aspects of the model to see how performance differs for other cases.