# TCSS 343 - Week 3

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**Master Theorem Practice**

## Master Theorem

The Master Theorem applies to recurrences of the following form:

$$T(n) = aT(n/b) + f(n)$$

where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function.

There are 3 cases:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a} \log^k n)$ with[1] $k \geq 0$, then $T(n) = \Theta(n^{\log_b a} \log^{k+1} n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ with $\epsilon > 0$, and $f(n)$ satisfies the regularity condition, then $T(n) = \Theta(f(n))$.
   Regularity condition: $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large $n$.

For the following problems show which case of the master theorem each problem goes to. The master theorem is applicable for each problem.

0.

```c
#include <stdio.h>

int main(void) {
    int n = 8;
    while (foo(n) != 0){
        n = n - foo(n);
    }
    printf("%d",n);
}

int foo(int n){
    int sum = 1;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            sum = sum + i + n / i;
        }
    }
    if (sum == n && n != 1) {
        return 0;
    }
    return 1;
}
```

What is the final value of $n$? Use the tracing technique we used in last class where you keep track of each line.

1. $T(n) = 3T(\frac{n}{2}) + n^2$

2. $T(n) = 4T(\frac{n}{2}) + n^2 \log n$

3. $T(n) = 3T(\frac{n}{4}) + n \log n$

4. $T(n) = 2T(\frac{n}{4}) + 2$

5. $T(n) = T(\frac{n}{4}) + \log n$

6. $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$

7. $T(n) = 2T(\frac{n}{4}) + n^{0.51}$

8. $T(n) = 3T(\frac{n}{2}) + n$

9. $T(n) = 4T(\frac{n}{2}) + n$

A. $T(n) = 3T(\frac{n}{3}) + \frac{n}{2}$

B. $T(n) = 4T(\frac{n}{2}) + \frac{n}{\log n}$

C. $T(n) = T(\frac{n}{3}) + n^2$

D. $T(n) = 8T(\frac{n}{3}) + 2^n$

E. $T(n) = 16T(\frac{n}{4}) + n$

F. $T(n) = 2T(\frac{n}{4}) + n!$

10. $T(n) = 0.5T(\frac{n}{2}) + \frac{1}{n}$

11. $T(n) = 16T(\frac{n}{4}) + n!$

12. $T(n) = 9T(\frac{n}{3}) + n^2$

13. $T(n) = 7T(\frac{n}{3}) + \cos n$

14. $T(n) = 8T(\frac{n}{3}) + 1$

15. $T(n) = T(\frac{n}{2}) + n^3$