TCSS 343 - Week 4

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Dynamic Programming

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

...

Richard Bellman's **Principle of Optimality**

"What we choose means more than what was handed to us by chance."

...

Ada Palmer

" If 'dynamic programming' didn't have such a cool name, it would be known as 'populating a table'".

...

Mark Dominus

1. Today we're going to explore dynamic programming. Below are three implementations of the fibonacci algorithm that I wrote in python. I want you to draw the **"tree"** for each then reflect on how the "bottom up" apprach is different from the other two? (Hint: They are all trees but also different types of trees. This is a key insight in my opinion in idea in understanding dynamic programming)

```
# recusive fibonacci
def F(n):
    if n == 0: return 0
    elif n == 1: return 1
    else: return F(n-1) + F(n-2)
```

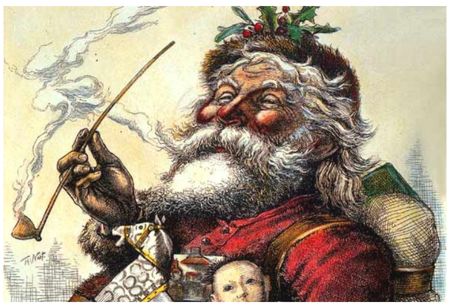
```
# "top down" memoized recursive fibonacci
memo = {}
def Fib(n):
    if n in memo: return memo[n]
    if n == 0: f = 0
    elif n == 1: f = 1
    else: f = Fib(n-1) + Fib(n-2)
    memo[n] = f
    return f
```

```
# "bottom up" iterative fibonacci
def fib(n):
    fn = [0,1]
    for i in range(n >> 1):
        fn[0] += fn[1]
        fn[1] += fn[0]
    return fn[n % 2]
```

```python
# Rules: X,Y > 0
def game_of_Stanley_Gill(X,Y):
    x = X
    u = X
    y = Y
    v = Y
    while x != y:
        if x > y:
            x = x - y
            u = v + u
        elif y > x:
            y = y - x
            v = u + v
    print((x + y) >> 1) #GCD(X,Y)
    print((u + v) >> 1) #LCM(X,Y)
```

2. This is a game shown to Edsgar Dijkstra(pictured right) when he was still an undergraduate, attributed to Stanley Gill(pictured left), an early computer scientist. Now it is true that $2XY = xv + yu$, which can be seen when the variables are initialized. But it is always true given that $X, Y > 0$? Show why this is true. (Hint: You can use the comments to help you along. Remember that the $>> 1$ operation is the same as $/2$.)

i	w_i	h_i	n_i
1	1	1	3
2	2	4	2
3	3	6	2
4	4	5	1
5	5	7	1
6	6	8	1



3. Suppose Santa has 6 kinds of toys, each kind of toy has its own weight w_i in tons, happiness rating h_i in ... joy, and quantity n_i. Santa would like to maximize the total hapiness of the children but the total weight of his bag cannot exceed 17 tons. Their weight, hapiness rating and quantity are defined above. Please help Santa by filling in the DP table below, where dp[i][j] indicates the maximum value you can get with weight less or equal to j using toys 1 to i. What is the final solution to this problem and briefly explain how you came to this solution. To help you get started, 23 was generated by solving the equation $i_1 + 2i_2 + 3i_3 \leqslant 15$ which gives you the most value. That value was found by $3 \cdot 1 + 2 \cdot 4 + 2 \cdot 6 = 23$. 12 was found by solving the equation $i_1 + 2i_2 + 3i_3 + 4i_4 + 5i_5 + 6i_6 \leqslant 6$ which gives you the most value. That value was found by $2 \cdot 6 = 12$

i\w	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0																	
2	0																	
3	0															23		
4	0																	
5	0																	
6	0						12											