$n \quad 2n \quad 2n \quad 3n$

10. Prove or disprove that if $f(n) \in \Theta(g(n))$ and $g(n) \in \Theta(h(n))$, then $h(n) \in \Theta(f(n))$.

$3n \quad n$ ✓ simple check works

Transitive property!

$f(n) \in \Theta(h(n))$

$f(n) \le c_1(h(n))$ ie $\frac{1}{c_1} f(n) \le h(n) \le \frac{1}{c_2} f(n)$

$\therefore h(n) \in \Theta(f(n))$

11. Give an example of a built-in Java operation that does not take constant time in the size of it's input(s). What time does it take?

int[] var = new int[100]; $O(n)$

12. Compute the following limit $\lim_{n\to\infty} \log_n n$.

$$\lim_{n\to\infty} \frac{\cancel{\ln n}}{\cancel{\ln n}} \rightarrow 1$$

13. Prove or disprove that $\lfloor n \rfloor \lceil n \rceil \in \Theta(n^2)$?

$\lfloor n \rfloor \lceil n \rceil \le cn^2$

Rememer

$n-1 < \lfloor n \rfloor \le n \le \lceil n \rceil < n+1$

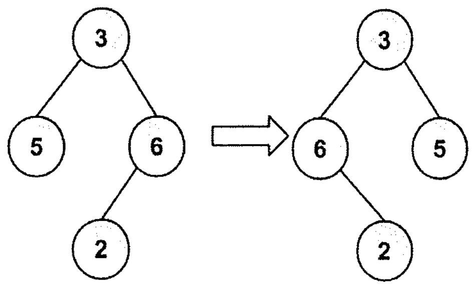
$\therefore$

At least ✓ $\lfloor n \rfloor \lceil n \rceil < (n-1)(n)$

A most $\lfloor n \rfloor \lceil n \rceil > n(n+1)$

$\lim_{n\to\infty} \frac{n(n-1)}{n^2} \rightarrow 1 \qquad \lim_{n\to\infty} \frac{n(n+1)}{n^2} \rightarrow 1$

E. Now that you've done this attempt to write a function which will invert a binary tree using recursion.



see attachment!

C. Benin is a fisherman who is simply good at fishing. One day, he finds a nice place to go fishing with two ponds. Moving from the $i-th$ fish-pond (the one he starts at) to the $j-th$ fishpond would cost $|i-j|$ units of time. Initially Benin can get F_i fish in the $i-th$ fishpond. In the next turn at the same fishpond, the amount of fish he can get is decreased by D_i. Notice that Benin will not get negative amount of fish. Each turn of fishing takes Benin 1 unit of time if Benin is at that pond and $|i-j|$ units of time to switch.

For example, if $F_1 = 10$, $F_2 = 5$, $D_1 = 2$, $D_2 = 3$ and Benin can fish for up to eight units of time, then he will get $10+8+6+5+4=33$. Washington Department of Fish and Wildlife (WDFW) requires that Benin switch to the adjacent pond when it has more fish and he cannot fish for "negative" fish. Write a recursive algorithm to see how many fish Benin can fish for!

see attachment!

9. Prove that $\mathscr{F}(n) < 2^n$ where $\mathscr{F}(n) = \mathscr{F}(n-1) + \mathscr{F}(n-2)$ and $\mathscr{F}(1) = 1$ and $\mathscr{F}(2) = 1$ $\forall n \geq 2$.

BC :

$n = 2 \quad \mathcal{F}(2) = 1 + 1 = 2$

$2^2 = 4 \qquad 2 < 4$ ✓

IH :

$T(K) : \mathcal{F}(K) = \mathcal{F}(K-1) + \mathcal{F}(K-2) < 2^K$

IS

$T(K+1) : \quad \mathcal{F}(K+1) = \mathcal{F}(K) + \mathcal{F}(K-1) < 2^{K+1}$

$\mathcal{F}(K+1) = \mathcal{F}(K) + \mathcal{F}(K-1)$

$< \mathcal{F}(K) + \underbrace{\mathcal{F}(K-1) + \mathcal{F}(K-2)}_{\mathcal{F}(K)}$

$< 2\mathcal{F}(K)$

$< 2(2^K)$

$< 2^{K+1}$ $\blacksquare$

$$\frac{1}{\ln 2}\int_1^{2^{16}} \ln i \, di + 4$$

integration by parts!

D	I
$\ln i$	1
$\frac{1}{i}$	i

$$\frac{1}{\ln(2)}\left(\left(\ln i\right)\cdot i\,\Big|_1^{2^{16}} - \int_1^{2^{16}} \frac{i}{i}\,di\right) + 4$$

$$(\log_2 2^{16})2^{16} - (\overset{0}{\log_2 1})1 \quad - \frac{i}{\ln 2}\Big|_1^{2^{16}} + 4$$

$$16(2^{16}) - \frac{2^{16}}{\ln 2} + \frac{1}{\ln 2} + 4 \;\boxed{\approx 954032.980495341231}$$

$\boxed{\text{Actual anwer } 954041}$

$\boxed{\text{ABS error} = 8.40583 \cdot 10^{-6}}$

4. Calculate the following anti-derivatives from 1 to n with respect to x. (Because I am a nice person. Remember that $\log_2 x = \frac{\ln x}{\ln 2}$). I have run into all of these integrals when computing runtimes of algorithms, I know you didn't expect this to be so much math but please, please don't hate me. I added these because I care.

a) $6 + 7x + 2x^2$

b) 2^n

c) $\log_2 x$

d) $\frac{1}{x^2}$

e) $\frac{1}{x}$

f) $\frac{\log_2 x}{x}$

a) $\Rightarrow \int_1^n 6+7x+2x^2\,dx = \left(6x+\frac{7}{2}x^2+\frac{2}{3}x^3\right)\Big|_1^n = 6n+\frac{7}{2}n^2+\frac{2}{3}n^3-6-\frac{7}{2}-\frac{2}{3}$

$\boxed{= 6n+\frac{7}{2}n^2+\frac{2}{3}n^3-\frac{11}{6}}$

c) $\Rightarrow \int_1^n \frac{\ln x\,dx}{\ln 2} = \frac{1}{\ln 2}\left[(\ln x)x\Big|_1^n - \int_1^n \frac{x}{x}dx\right]$

D I
$\ln x$ 1
$\frac{1}{x}$ x

$= \frac{1}{\ln 2}\left[(\ln n)n - \cancel{\ln 1} - x\Big|_1^n\right]$

$\boxed{= \frac{1}{\ln 2}\left[\ln n\, n - n + 1\right]}$

d) $\int_1^n x^{-2}dx = \frac{x^{-1}}{1-2}\Big|_1^n = -\frac{1}{n}+\frac{1}{1} \boxed{= 1-\frac{1}{n}}$

e) $\int_1^n \frac{dx}{x} = \ln n\Big|_1^n = \ln n - \cancel{\ln 1} \boxed{= \ln n}$

I like to make a substitution to reduce algebraic error

same thing

f) $\int_1^n \frac{\log_2 x}{x}dx = \frac{1}{\ln 2}\int_1^n \underbrace{\frac{1}{x}\ln x\,dx}_{\ominus} = (\ln x)^2\Big|_1^n - \underbrace{\int_1^n \frac{1}{x}\ln x\,dx}_{\ominus}$

D I
$\ln x$ $\frac{1}{x}$
$\frac{1}{x}$ $\ln x$

$\frac{1}{\ln 2}\ominus = (\ln x)^2\Big|_1^n - \ominus$
$+\ominus \qquad + \ominus$

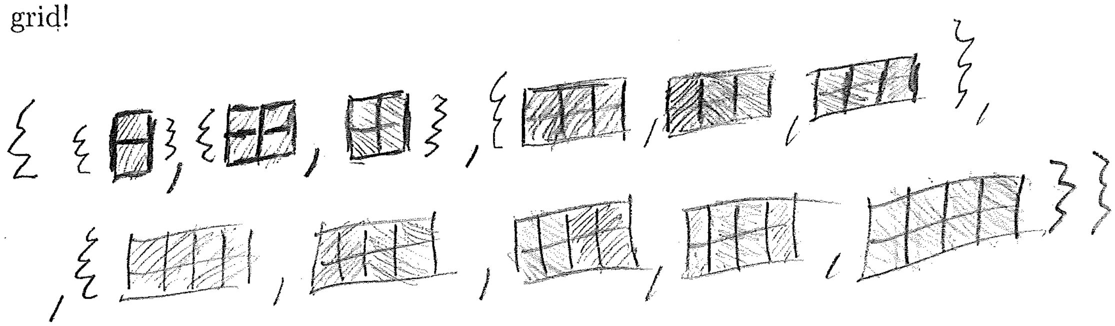
$\frac{2}{\ln 2}\ominus = (\ln x)^2\Big|_1^n$

$\frac{1}{\ln 2}\ominus = \frac{1}{2}(\ln x)^2\Big|_1^n$

$\therefore$

$\int_1^n \frac{\log_2 x}{x}dx \boxed{= \frac{1}{2}(\ln n)^2}$

0. Emily loves figuring out all the ways to arrange dominos. Help her find all the ways to arrange dominos in that are 2×1 in a 2×1,2×2,2×3 and 2×4 grid!



1. Now that you've helped Emily find how many ways to arrange the dominos in problem 0 she gets really philosophical. She starts pondering the nature of zero and wants you to help her find how many ways to arrange a 2×1 domino in a 2×0 grid. (You don't have to be too smart: Just find some justification from problem 0)

If I want to tile something does it exist?
we will avoid philosophical and say it is 1 way.

2. We've had a lot of fun arranging dominos but now Emily wants a recursive formula for the ways to arrange 2×1 dominos. The key to finding recursive definitions is to find the answer to larger problems by finding the answer to smaller problems.

D_n =# of tilings of a $2 \times n$ = $D_{n-1} + D_{n-2}$

= (# of tilings $2\times(n-1)$) + (# of tilings $2\times(n-2)$)