TCSS 343 - Week 3 - Monday

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Stating Problems Formally and Master Method

"The purpose of a storyteller is not to tell you how to think, but to give
you questions to think upon."
. . .
Brandon Sanderson

"Writing is nature's way of letting you know how sloppy your thinking is."
. . .
Guindon

"Master method will never be sufficient for the detailed 'knuthian' analysis
of algorithms, but they can free algorithm designers from mundane analyses
to let them work on more interesting problems."
. . .
Jon Bentley, Dorothea Blostein et al (creators of the master method)

For this problem consider the algorithm "drawCantorCircles", on a specified canvas (square painting surface), draws a nested circle pattern, where the largest circle is centered a (x, y) with a given radius and filled with colour1. There are two nested subpatterns, each with half the radius, internally tangent along the main horizontal diameter, with colours alternating between colour2 and colour1. No circle whose radius is less than a minimum radius is drawn.
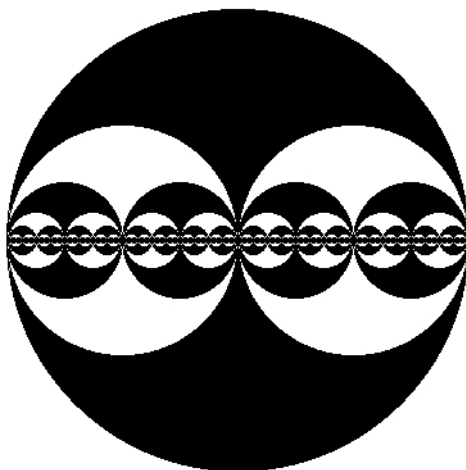
0. Express this problem formally with input and output conditions.

1. Describe a simple brute force algorithm to compute this picture assuming that it takes some constant amount of work to place each circle. How many circles does this algorithm draw asymptotically in the worst case?

2. Assume that the radius of the first circle is greater than zero and divisible by 2. What type of picture will this algorithm output? Draw the picture.

3. Assume that the radius of the first circle is greater than zero and is **NOT** divisible by 2. What type of picture will this algorithm output? Draw the picture.

4. State a self-reduction for your problem.

5. State a recursive algorithm that solves the problem.

6. State a tight asymptotic bound on the act of drawing a circle used by your algorithm in the worst case. (HINT: develop a recurrence relation for your algorithm then use the Master Theorem to solve it)

7. In this problem use the Master Theorem to find and prove tight bounds for the following recurrence.

$$T(n) = \begin{cases} c & \text{if } n \leqslant 1 \\ 9T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 3 & \text{if } n > 1 \end{cases}$$

8. In this problem use the Master Theorem to find and prove tight bounds for the following recurrence.

$$T(n) = \begin{cases} c & \text{if } n \leqslant 1 \\ 3T\left(\left\lfloor \frac{n}{9} \right\rfloor\right) + 3n & \text{if } n > 1 \end{cases}$$

9. In this problem use the Master Theorem to find and prove tight bounds for the following recurrence.

$$T(n) = \begin{cases} c & \text{if } n \leqslant 1 \\ 3T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 3n & \text{if } n > 1 \end{cases}$$