TCSS 343 - Week 2 - Thursday

Jake McKenzie

January 3, 2019

Asymptotics with Divide and Conquer

"The object of life is not to be on the side of the majority, but to escape finding oneself in the ranks of the insane".
...
Marcus Aurelius

"Trust no friend without faults, and love a woman, but no angel."
...
Doris Lessing

0. Prove the theorem below using the techniques of binding the term and splitting the sum to find a tight bound for the sum. Make sure your proof is complete, concise, clear and precise.
   Theorem 0:
   $$\sum_{i=1}^{\sqrt{n}} i^2 \in \Theta(n^{\frac{3}{2}})$$

Consider the task of detecting whether a given array has an element that is repeated in more than half of the positions of the array. For example. the value 2 is the majority element in the array $\{3, 2, 5, 2, 3, 2, 7, 2, 2\}$, while the array $\{5, 8, 8, 3, 10, 8, 5, 8\}$ has no majority element. In the present task you will develop a divide and conquer algorithm for solving this problem, and you will analyze its running time.

1. Write a formal statement of this problem. That is, state the input and output criteria as precisely as possible.

2. Suppose that an array $a[1 \ldots n]$ has an element v_L that occurs in strictly more than half of the first floor($\frac{n}{2}$) positions of a, and an element v_H that occurs in strictly more than half of the remaining ceiling($\frac{n}{2}$) positions. Argue carefully why if there is an element v of a that occurs in over half of all n positions of a, then v must be either v_L or v_H. Note that it would not be enough for v to occur more times in a than either v_L or v_H. We explicitly require that v occur in strictly more than half of all positions of the entire array.

3. Using the information you came to in the prior pages, design a divide and conquer algorithm that returns the majority element of an array or the value -1 if no such element exists. Give detailed tidy psuedocode.

4. Describe an algorithm to determine in $O(n)$ time whether an arbitrary array $A[1 \ldots n]$ contains more than $\frac{n}{4}$ copies of any value.