TCSS 343 - Week 5 - Wednesday

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Dynamic Programming

"Perhaps thinking should be measured not by what you do
but by how you do it".
...
Richard Hamming

"For the last sixty five years(speaking in 2018), due to Moore's law, with a clockwork precision, computer capability has been doubling every year and a half. Without fast algorithms you cannot bring to bare Moore's Law. A dramatic increase in computer speed needs to be coupled with efficient algorithms".
...
Christos Papadimitriou

"Nobody expects plumbers to have a physics degree but they do have to know some things about water physics, and that can be learned in a way that doesn't necessarily involve getting a physics degree. And that is super cool, totally valid, and not a problem. But that doesn't mean that physics degrees are bullshit or not useful to plumbers".
...
Steve Klabnik

0. Consider the Job Selection problem from lecture. Compute the optimal set of jobs and the maximum earnings for the following input.

$$P = \{9, 8, 14, 14, 5, 10, 9, 12, 13, 14, 7, 13, 12, 5, 4, 10, 17, 8, 11, 6, 14, 6, 10, 13, 16\}$$

1. Recover your solution to the prior part.

2. Given two string A and B of lengths n and m, respectively over an alphabet Σ determine the length of the longest subsequence that is common to both A and B. A subsequence of $A = a_1, a_2, \ldots, a_n$ is a string of the form $a_{i1}, a_{i2}, \ldots, a_{ik}$ where each i and j is between 1 and n and $1 \leqslant i_1 < i_2 < \cdots < i_k \leqslant n$, Below is the self-reduction so you needn't look at your notes. What is the alphabet Σ shared among $A =$ "xyxxzxyzxy" and $B =$ "zxzyyzxxyxxz"?

$$L(i,j) = \begin{cases} 0 & \text{if } i = 0 \vee j = 0 \\ \text{L[i-1,j-1]} + 1 & \text{if } i > 0, j > 0 \wedge a_i = b_j \\ \max\{\text{L[i,j-1],L[i-1,j]}\} & \text{if } i > 0, j > 0 \wedge a_i \neq b_j \end{cases}$$

3. Show the result of applying the algorithm LCS on the instance A and B above by drawing the grid and keeping track of the length at each entry.

4. Backtrack through your solution you made on the prior page recovering the solution.

5. How would you modify the Algorithm LCS to require $min\{m, n\}$ space?

6. Give a reduction of the coin change problem developed in class for the U.S. coin system: $\{1, 5, 10, 25\}$.

7. Attempt to prove this reduction correct by contradiction. Reminder that proof by contradiction is an indirect proof technique. To prove a proposition p assume $\neg p$. This involves deriving a contradiction q such that $\neg p \implies \neg q$ which proves the proposition p we wish to show is true.