

$$\int_1^n u dv = u \cdot v \Big|_1^n - \int_1^n v du$$

easy formula
I use,

$$\begin{array}{c} \text{I} \\ \hline \text{D} \end{array}$$

$\begin{array}{c} \uparrow v \rightarrow u \\ \downarrow dv \leftarrow du \end{array}$
 multiply

Example: let $f(x) = \frac{\ln x}{x}$

let $u = \ln x$
 $dv = \frac{1}{x} dx$

$$\begin{array}{c} \text{I} \\ \hline \text{D} \end{array}$$

$\begin{array}{c} \uparrow \ln x \rightarrow \frac{1}{\ln x} (\ln x) \\ \downarrow \frac{1}{x} dx \leftarrow \frac{1}{\ln x} \left(\frac{1}{x} \right) \end{array}$

$$\int_1^n \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} \Big|_1^n - \int_1^n \ln x \cdot \frac{1}{x} dx$$

$$\underbrace{\int_1^n \frac{\ln x}{x} dx}_A = \frac{(\ln x)^2}{2} \Big|_1^n - \underbrace{\int_1^n \frac{\ln x}{x} dx}_A$$

$$A = \frac{(\ln x)^2}{2} \Big|_1^n - A$$

$$2A = \frac{(\ln x)^2}{2} \Big|_1^n$$

$$\int_1^n \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2 \ln 2} \Big|_1^n = \frac{(\ln n)^2}{2 \ln 2} - 0$$

$$\boxed{= \frac{(\ln n)^2}{2 \ln 2}}$$