TCSS 343 - Week 2 - Thursday

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Recurrences and Tracing

"To tell the truth is an act of love. To withhold the truth is an act of hate. Or worse, apathy".

...

Gene Kim

"All claims of education notwithstanding, the pupil will accept only that which his mind craves".

...

Emma Goldman

"Power concedes nothing without a demand. It never did and it never will".

...

Frederick Douglass

0. The instructions for this problem are in the comments lines 1 through 4.

```
1  // Show each value of i, j and k for each line executed
2  // in the while loop (lines:11, 13, 15, 16, 18) and the
3  // final values of i, j and k for when a = -1, b = 1 and
4  // a = 1, and b = -1 and finally when a = 0 and b = 0.
5  #include <stdio.h>
6  int main(void) {
7      int i = 1;
8      int j = 0;
9      int k = -1;
10     int a = 1;
11     int b = -1;
12     while (i > j) {
13         i = i + a - 2 * j;
14         if (j >= k) {
15             i = i + 2;
16             k = k - b + 2 * j;
17         }
18         j++;
19     }
20 }
```

i	j	k
1	0	-1
2	0	-1
4	0	-1
4	0	0
4	1	0
3	1	0
5	1	0
5	1	3
5	2	3

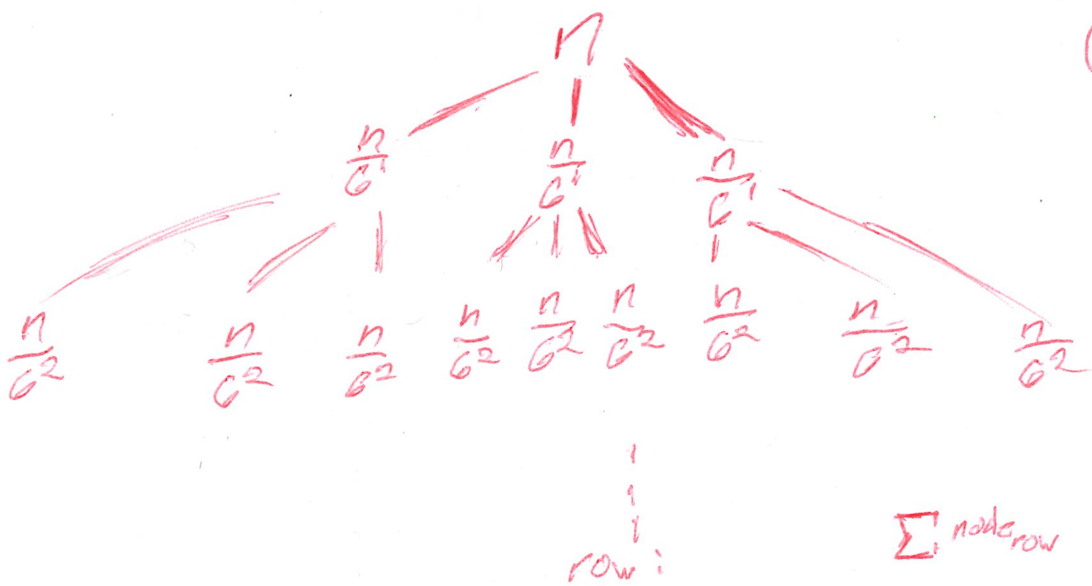
i	j	k
5	2	3
2	2	3
2	3	3

$Z(1)$ $\quad Z(\frac{n}{6^i})$ $\quad 1$ $\quad n = 6^i$

1. Consider the following recurrence:

$$Z(n) = \begin{cases} 1, & \text{if } n = 1 \quad (1) \\ 3Z(\frac{n}{6}) + n, & n > 1 \quad (2) \end{cases}$$

Draw out a visualization of what this recurrences looks like as a tree.



$(\frac{3}{6})^0 n$

$(\frac{3}{6})^1 n$

$(\frac{3}{6})^2 n$

$\sum_i \text{node}_{row} = (\frac{3}{6})^i n$

2. How much work is done on level i?

$(\frac{3}{6})^i n$ amantof work with 3^i nodes per level where each node does $\frac{n}{6^i}$ amout of work each!

3. How many recursive levels are there in the tree?

$6^i = 3^{\#\text{ of rows}}$ $\therefore$ $6^i = n \Rightarrow \boxed{i = \log_6 n}$

note this is recursive levels! not total levels

w/ base case there are $\log_6 n + 1$ levels total

4. How much work is done at the leaf level?

$1 \cdot 3^{\log_6 n}$ amount of work at the

botom level since $i = \log_6 n$

we must count our levels staring at

0 not 1

5. Construct a non-recursive expression equivalent to the recurrence. Your solution may use a summation.

$$\underbrace{\sum_{i=0}^{\log n-1} n\left(\frac{3}{6}\right)^i}_{\text{All work done prior to the leaf}} + \underbrace{1 \cdot 3^{\log_6 n}}_{\text{leaf level work}}$$

6. Find the big-Θ bound for the recurrence.

geometric series

$$n\sum_{i=0}^{\log_6 n-1}\left(\frac{1}{2}\right)^i = n\left(\frac{\left(\frac{1}{2}\right)^{\log_6 n}-1}{\frac{1}{2}-1}\right)$$

$$n\left(2-\frac{2\cdot 3^{\log_6 n}}{n}\right) + 3^{\log_6 n}$$

$$2n - 2\cdot 3^{\log_6 n} + 3^{\log_6 n}$$

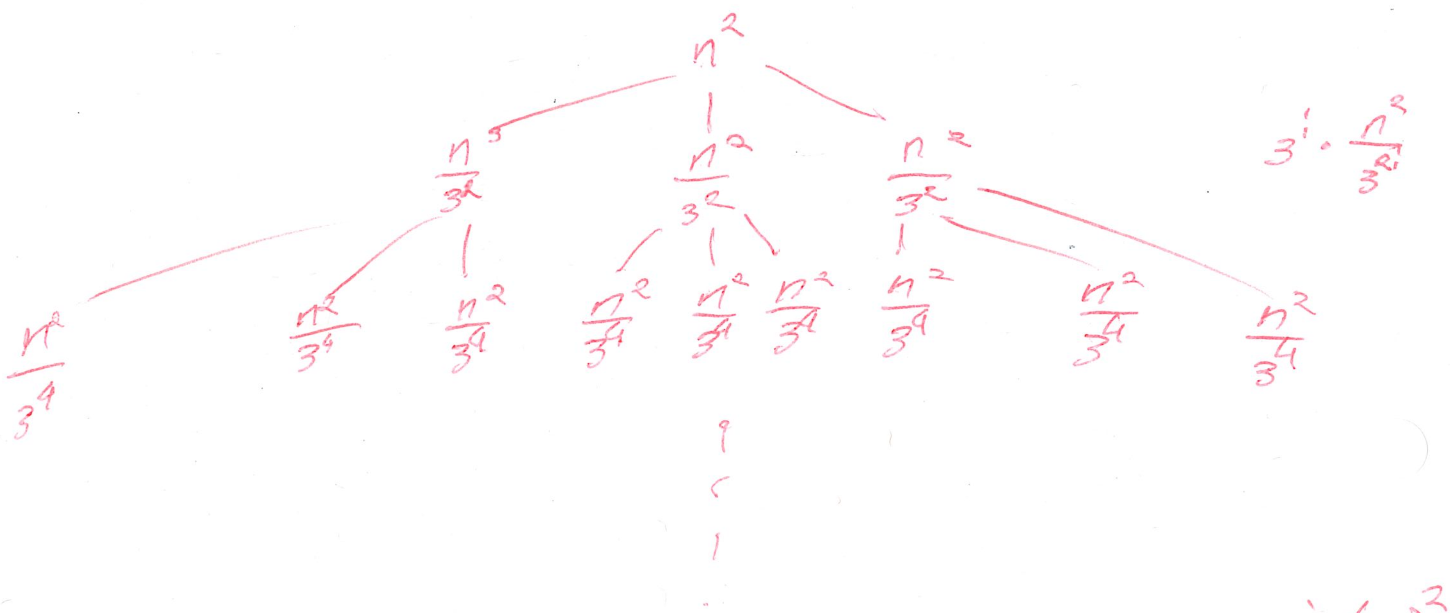
$$2n - 3^{\log_6 n}$$

$\therefore\ \Theta(n)$

7. Consider the following recurrence:

$$A(n) = \begin{cases} 1, & \text{if } n = 1 \quad (3) \\ 3A(\frac{n}{3}) + 3n^2, & n > 1 \quad (4) \end{cases}$$

Draw out a visualization of what this recurrences looks like as a tree.



row i

$\sum \text{nodes}_{row} = 3^i \cdot \left(\frac{n}{3^i}\right)^2$

8. How much work is done on level i?

$3^i \cdot \left(\frac{n}{3^i}\right)^2$

11. Construct a non-recursive expression equivalent to the recurrence. Your solution may use a summation.

12. Find the big-Θ bound for the recurrence.

13. Write an algorithm Brackets which takes a positive integer n that prints all combinations of well-formed brackets. For Brackets(3) the output would be ((())) (()()) (())() ()(()) ()()()