

Lab 7: Transient Response of a 2nd Order Circuit

Josh Ortiz & James Robinson

ECEN 214 - 502

TA: Yichi Zhang

Date Performed: 4/2/19

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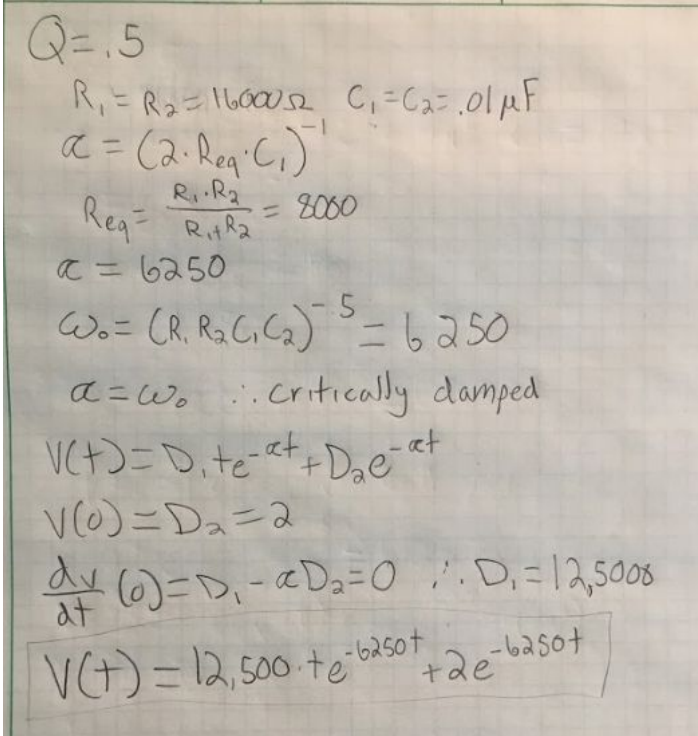
Procedure

1. Build the Sallen-Key circuit depicted in Figure 7.1.
2. Use a 100 Hz square wave with 2 Volts (peak to peak) as your input.
3. Produce the 5 cases that were discussed in the prelab with the correct component values
4. Save the waveforms.

Data and Results

Theoretical

$$Q = .5$$



Handwritten mathematical derivation on a piece of paper:

$$Q = .5$$
$$R_1 = R_2 = 16000 \Omega \quad C_1 = C_2 = .01 \mu F$$
$$\alpha = (2 \cdot R_{eq} \cdot C_1)^{-1}$$
$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 8000$$
$$\alpha = 6250$$
$$\omega_0 = (R_1 R_2 C_1 C_2)^{-.5} = 6250$$
$$\alpha = \omega_0 \quad \therefore \text{critically damped}$$
$$V(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$
$$V(0) = D_2 = 2$$
$$\frac{dV}{dt}(0) = D_1 - \alpha D_2 = 0 \quad \therefore D_1 = 12,500$$
$$V(t) = 12,500 \cdot t e^{-6250t} + 2 e^{-6250t}$$

$$Q = .25$$

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$$R_1 = 13000 \Omega \quad R_2 = 5600 \Omega \quad C_1 = .01 \mu F$$

$$C_2 = .033 \mu F$$

$$\alpha = (2R_{eq}C_1)^{-1}$$

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 3913.9$$

$$\alpha = 12774.7$$

$$\omega_0 = (R_1 R_2 C_1 C_2)^{-.5} = 6451.7$$

$$|\alpha| > |\omega_0| \quad \therefore \text{overdamped}$$

$$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 = -1748.9 \quad s_2 = -23806.5$$

$$V(0) = A_1 + A_2 = 2$$

$$\frac{dV}{dt}(0) = s_1 A_1 + s_2 A_2 = 0$$

$$A_1 = 2.16 \quad A_2 = -.16$$

$$V(t) = 2.16 e^{-1748.9 t} - .16 e^{-23806.5 t}$$

$$Q = .1$$

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$$R_1 = 3300 \Omega \quad R_2 = 33 \Omega \quad C_1 = C_2 = .47 \mu$$

$$\alpha = (2 \cdot R \cdot C_1)^{-1}$$

$$R = R_1 \parallel R_2 = R_1 \cdot R_2 \cdot (R_1 + R_2)^{-1} = 32.673$$

$$\alpha = 32559.639$$

$$\omega_0 = (R_1 R_2 C_1 C_2)^{-.5} = 6447.453$$

$$|\alpha| > |\omega_0|$$

\therefore Overdamped

$$s_{1,2} = -\alpha \pm (\alpha^2 - \omega_0^2)^{.5}$$

$$(\alpha^2 - \omega_0^2)^{.5} = 31914.894$$

$$s_1 = -644.745 \quad s_2 = -64474.53$$

$$V = A_1 e^{-s_1 t} + A_2 e^{-s_2 t}$$

$$V(0) = A_1 + A_2 = 2$$

$$\frac{dV}{dt}(0) = -A_1 \cdot (644.745) - A_2 (64474.53) = 0$$

$$A_1 = 2.02 \quad A_2 = -.02$$

$$V = 2.02 e^{644.745t} - .02 e^{64474.53t}$$

$$Q = 1$$

$$Q = 1$$

$$R_1 = 820 \Omega \quad R_2 = 820 \Omega \quad C_1 = .4 \mu F \quad C_2 = .1 \mu F$$

$$\alpha = (2 \cdot R \cdot C_1)^{-1}$$

$$R = R_1 \cdot R_2 \cdot (R_1 + R_2)^{-1} = 672400 / 1640 = 410 \Omega$$

$$\therefore \alpha = (2 \cdot 410 \cdot .4 \cdot 10^{-6})^{-1} = 3048.78$$

$$\omega_0 = (R_1 \cdot R_2 \cdot C_1 \cdot C_2)^{-.5} = (672400 \cdot .04 \cdot 10^{-12})^{-.5} \\ = 6097.56$$

$$|\omega_0| > |\alpha| : \text{Underdamped}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 5280.64$$

$$V(0) = A = 2V$$

$$V(t) = B_1 \cdot e^{-3048.78t} \cos(5280.64t) + B_2 \cdot e^{-3048.78t} \cdot \sin(5280.64t)$$

$$\frac{dV}{dt} = 0 = B_1 \cdot (-3048.78) + B_2(5280.64)$$

↓

$$V(t) = .00021e^{-3048.78t} \cdot \sin(5280.64t) + 1.99864e^{-3048.78t} \cdot \cos(5280.64t)$$

$$Q = 2.5$$

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$$R_1 = 2000 \Omega = R_2 \quad C_1 = .4 \mu F \quad C_2 = .015 \mu F$$

$$\alpha = (2 \cdot R \cdot C_1)^{-1}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = 1000$$

$$\alpha = 1250$$

$$\omega_0 = (R_1 R_2 C_1 C_2)^{-.5} = 6250$$

$$|\omega_0| > |\alpha| \quad \therefore \text{underdamped}$$

$$\omega_d = (\omega_0^2 - \alpha^2)^{.5} = 6123.72$$

$$V = B_1 e^{-\alpha t} \cos(\omega_d t) + B_2 e^{-\alpha t} \sin(\omega_d t)$$

$$V(0) = B_1 = 2$$

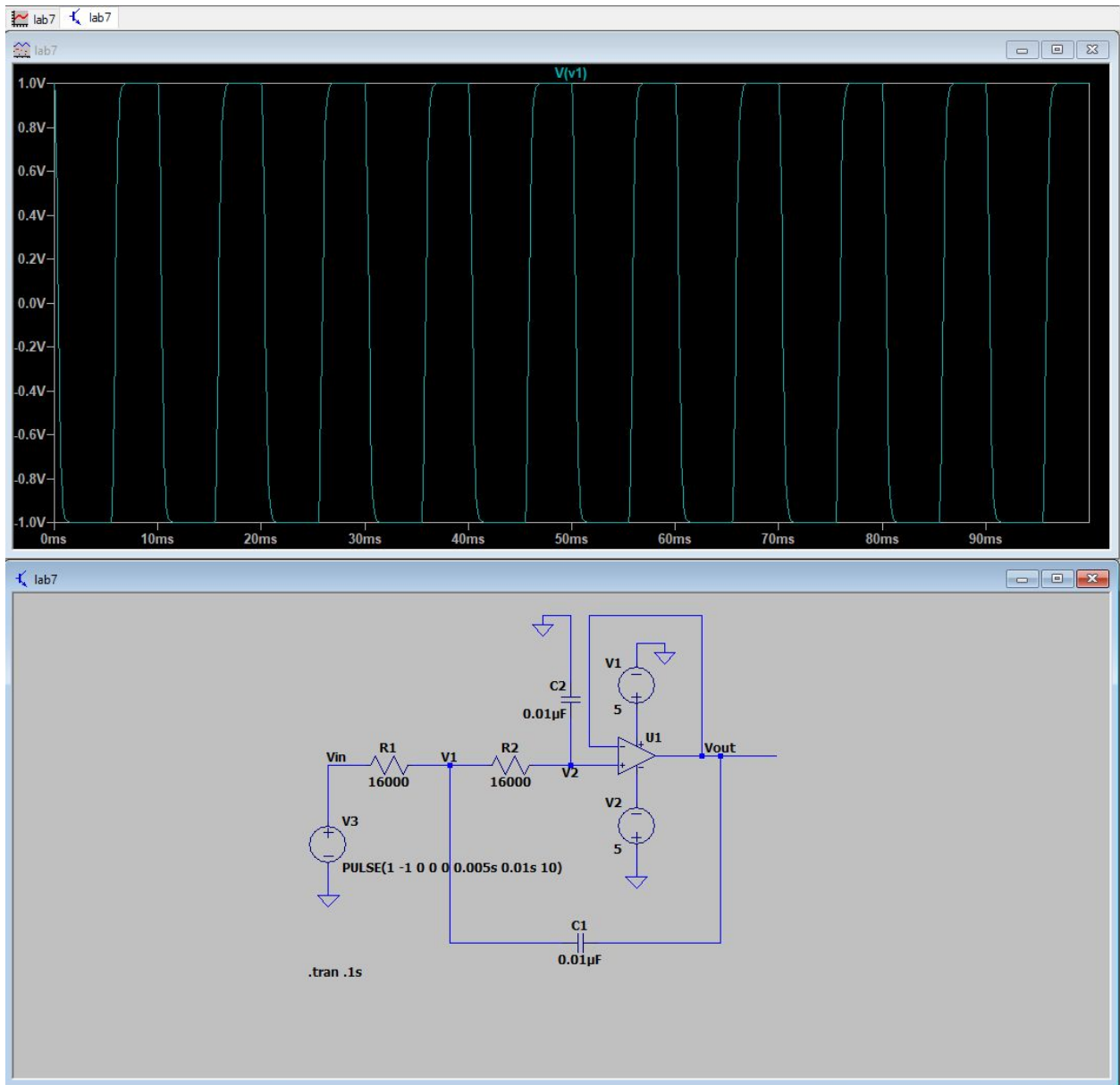
$$\frac{dV}{dt}(0) = B_1(-1250) + B_2(6123.72) = 0$$

$$\therefore B_2 = .408$$

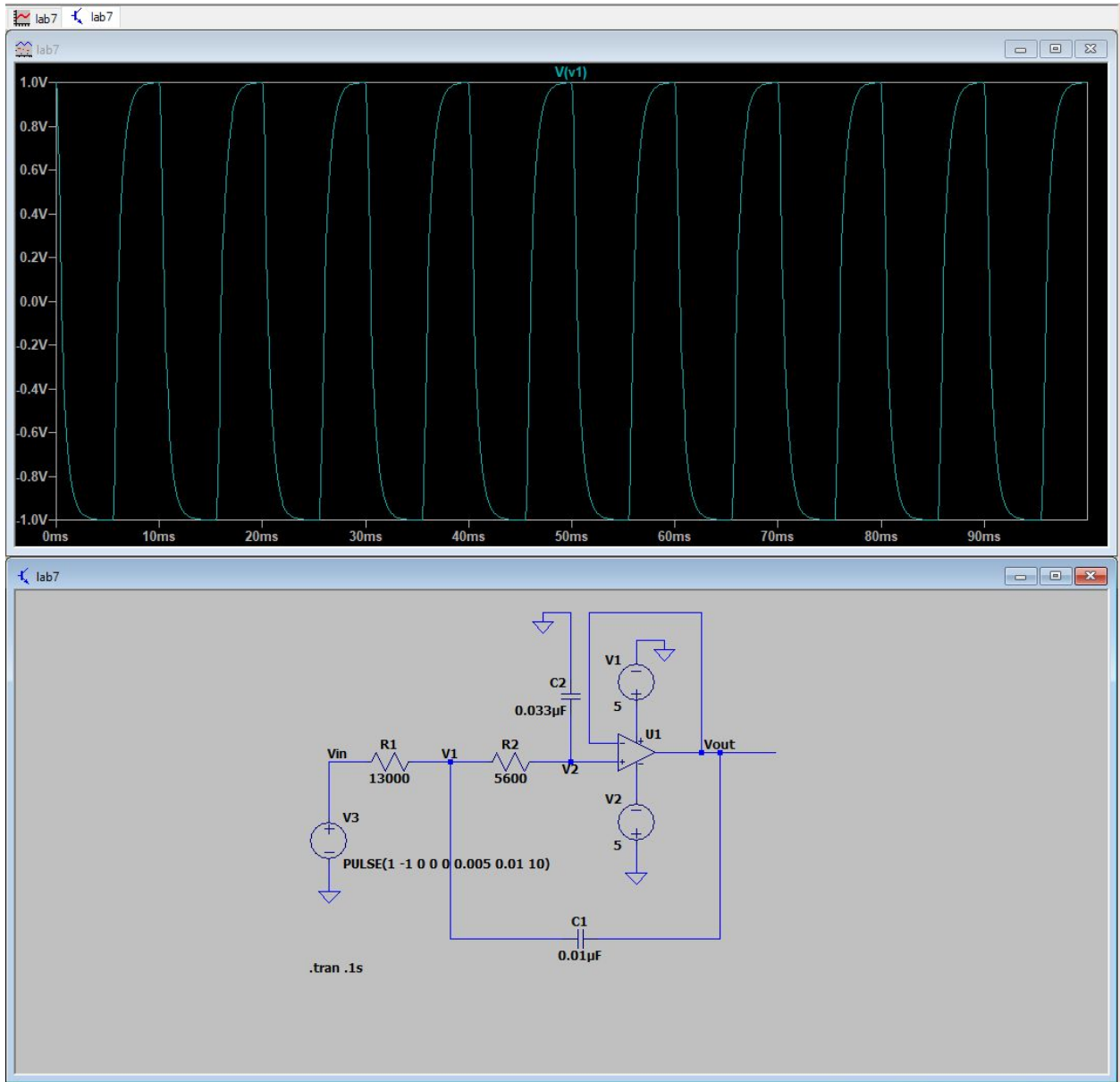
$$V(t) = 2e^{-1250t} \cos(6123.72t) + .408e^{-1250t} \sin(6123.72t)$$

Simulations

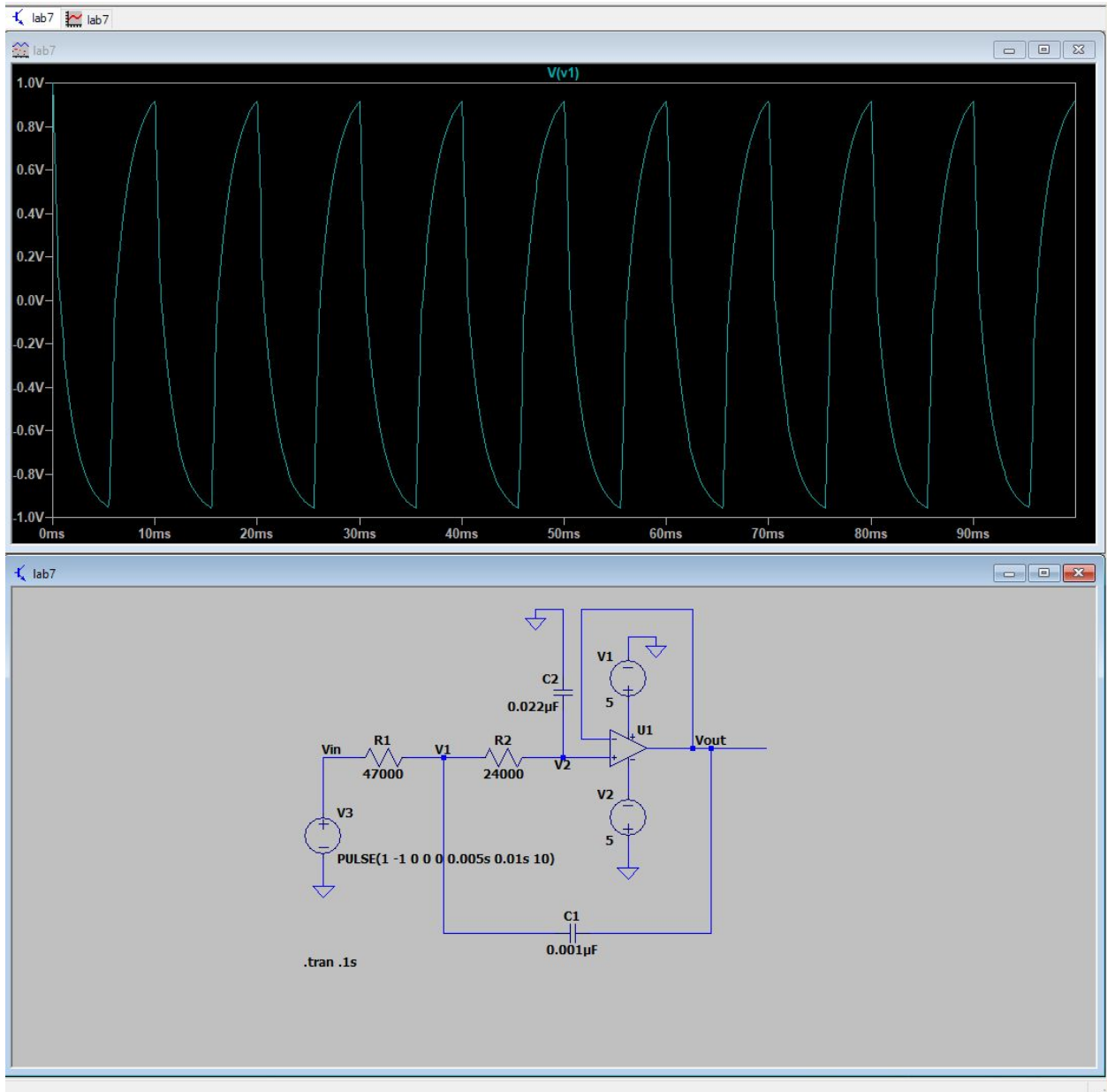
$$Q = .5$$



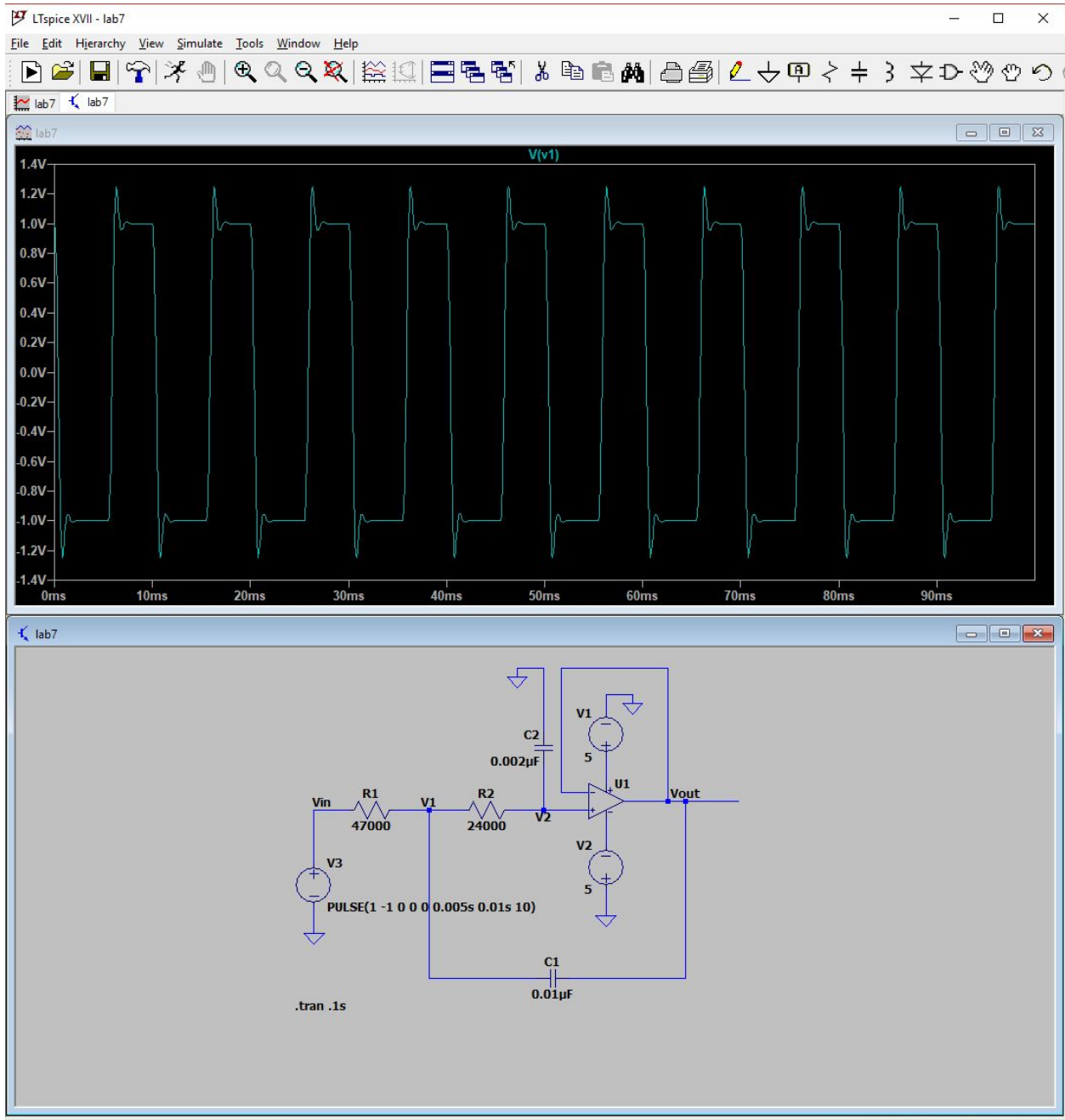
$$Q = .25$$



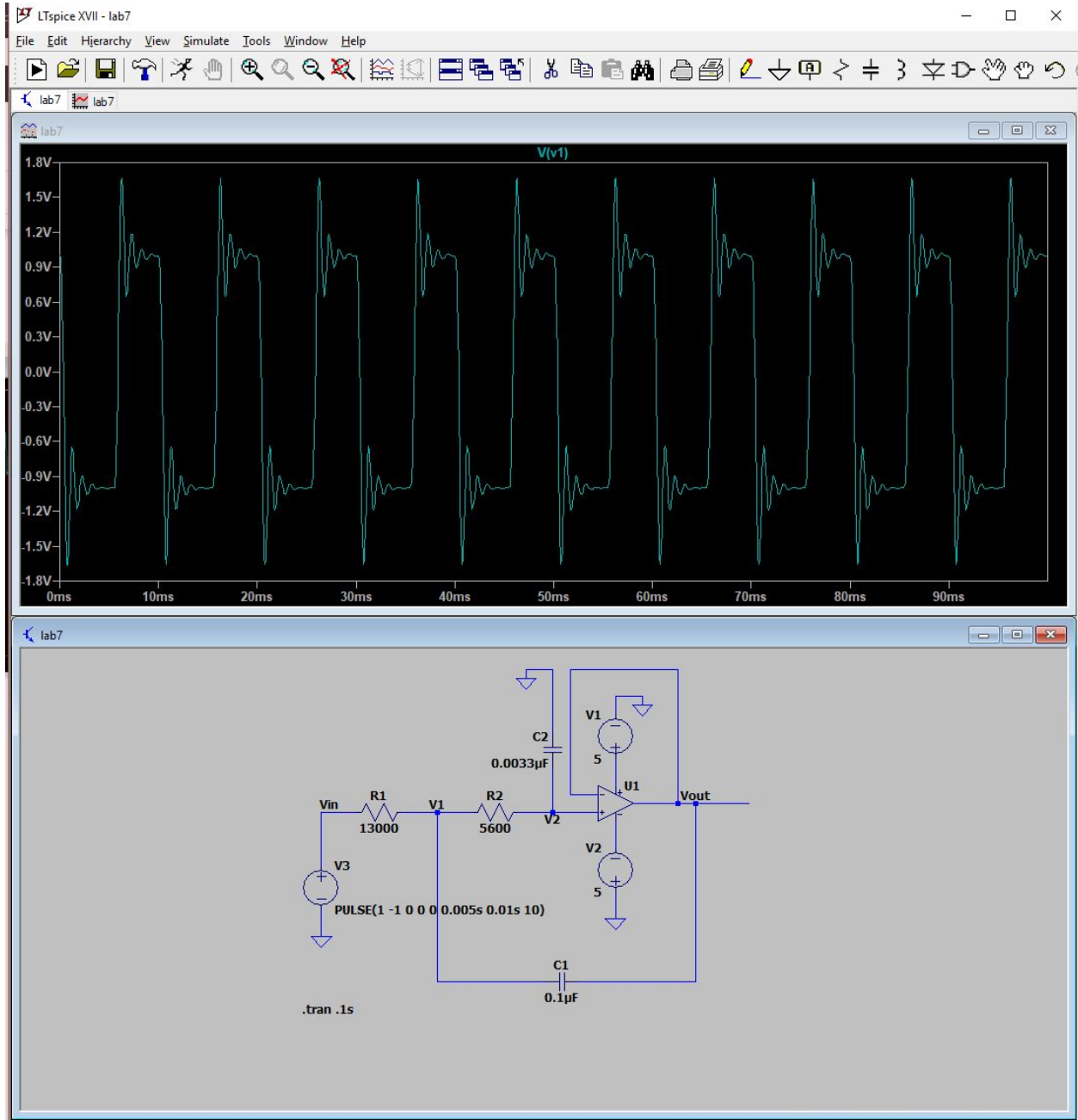
$$Q = .1$$



$$Q = 1$$

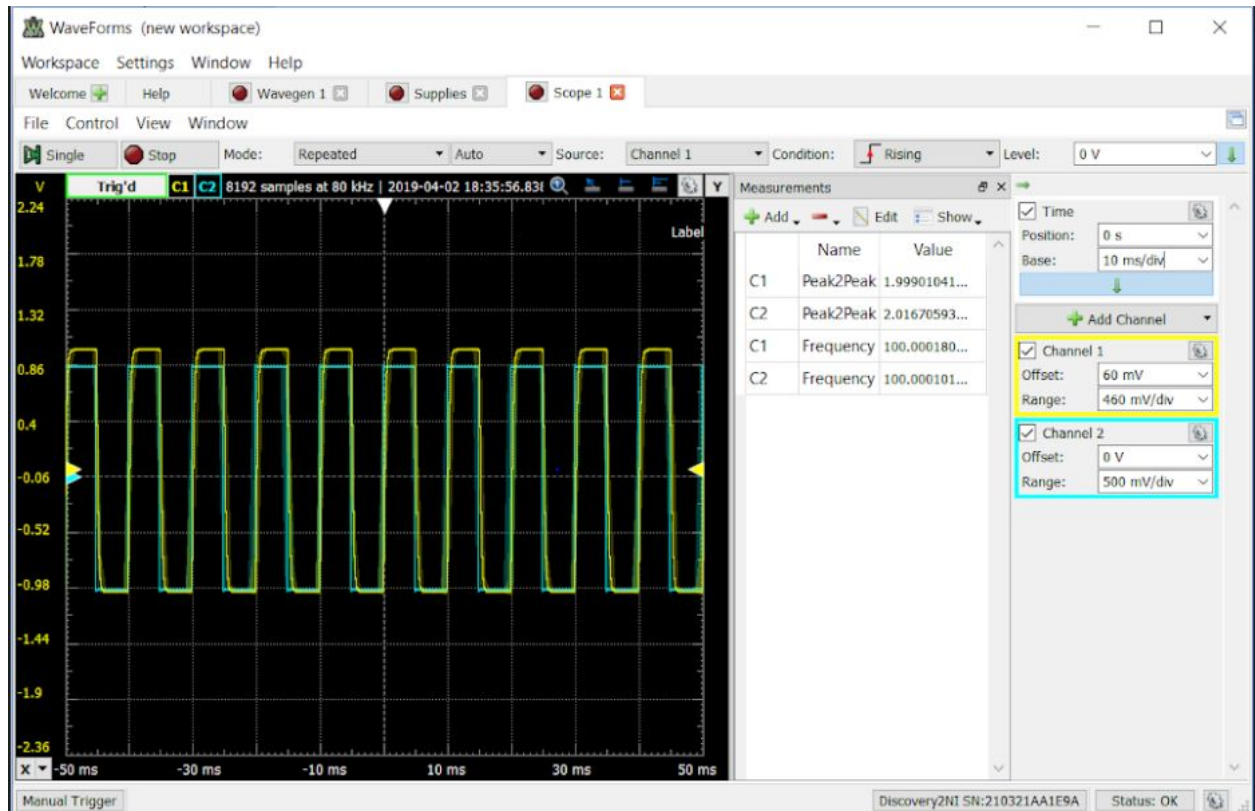


$$Q = 2.5$$

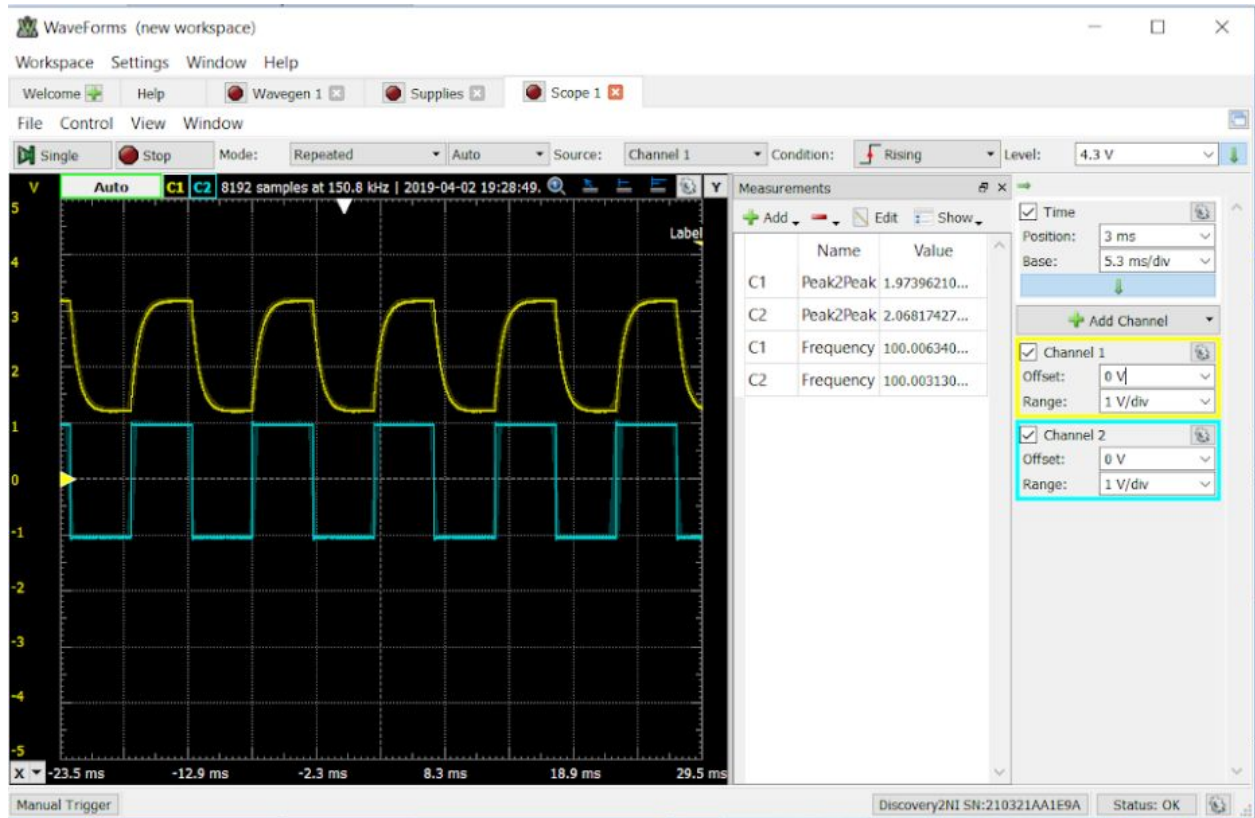


Measured

$$Q = .5$$



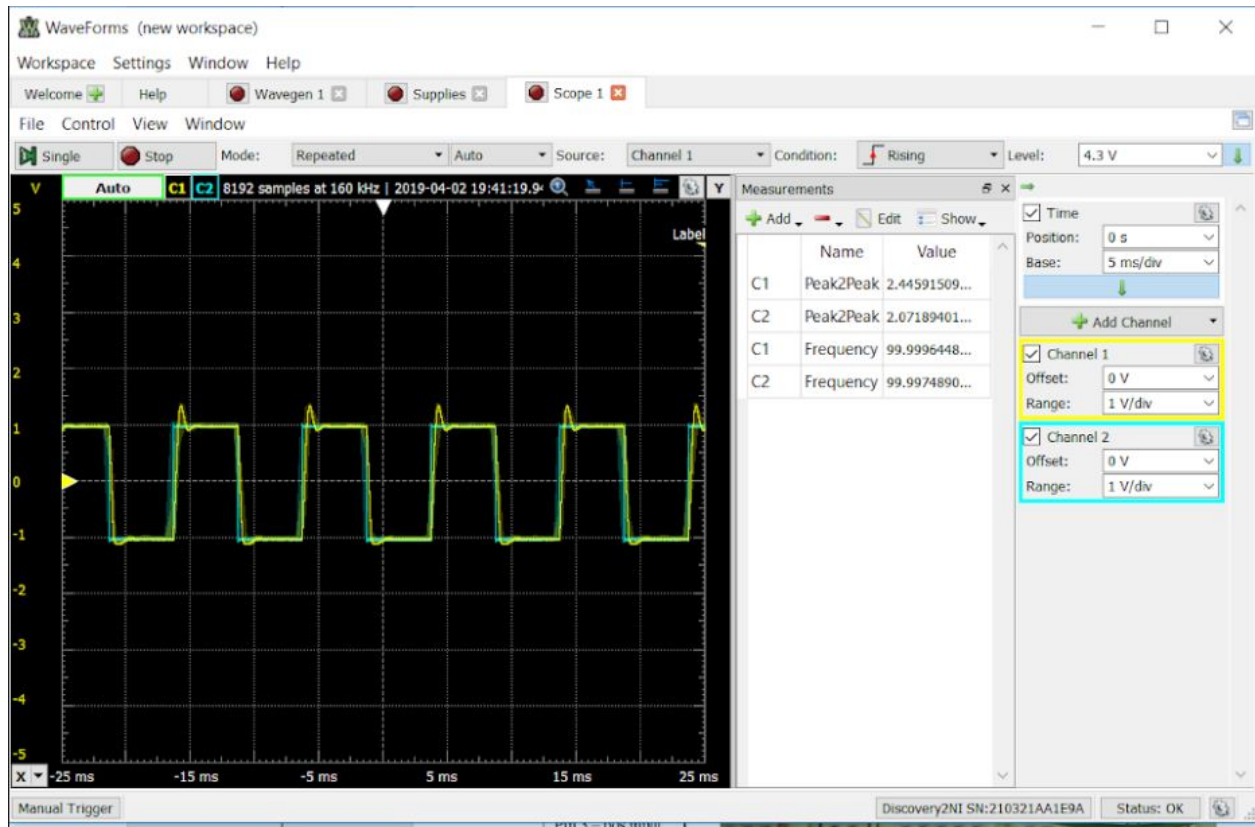
$$Q = .25$$



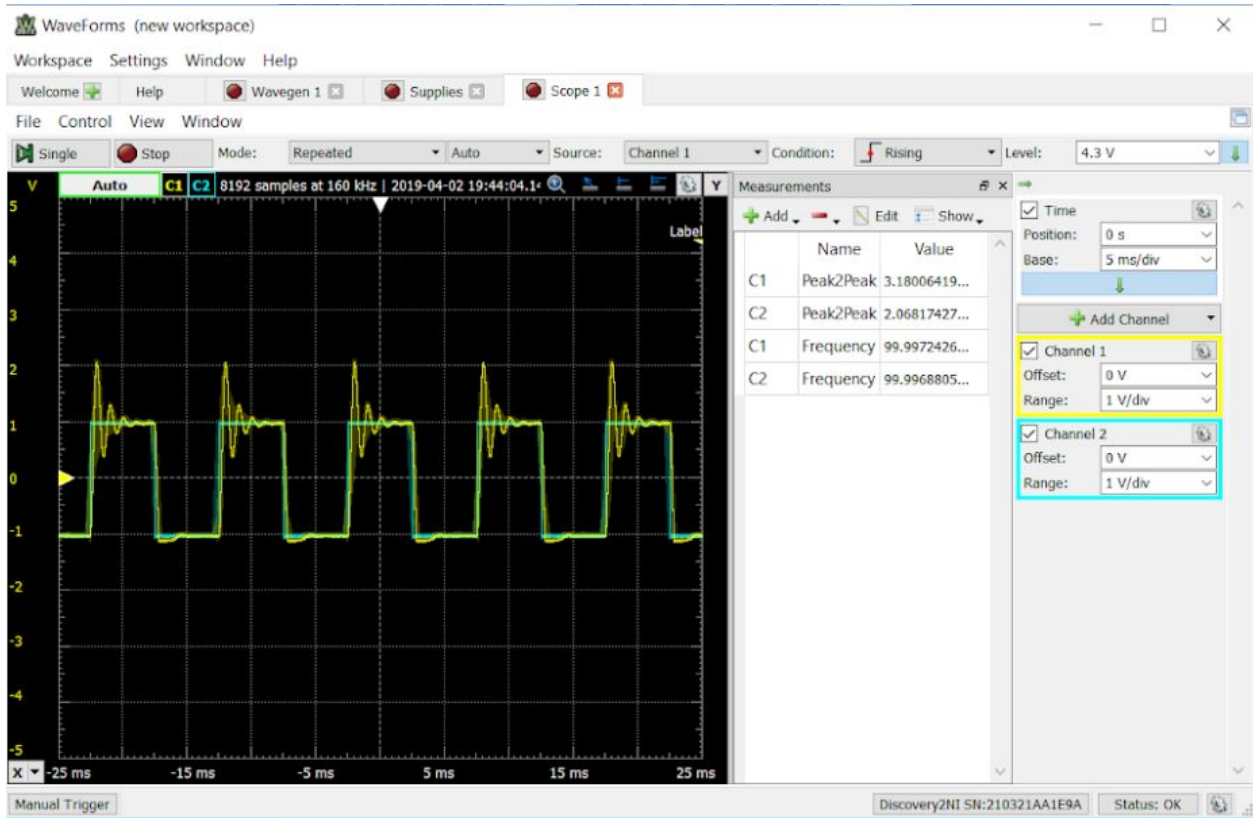
$$Q = .1$$



$$Q = 1$$



$$Q = 2.5$$



Discussion

The results of our theoretical, simulated and measured circuits all matched up. The lab proceeded exactly as it was intended to. Therefore there would be no changes that we would need to make in order for our results to come out better.

Conclusion

This lab was very complicated and forced us to use all of the things we had learned thus far in ECEN 214. As discussed in the overview, there were several different components that were required to function together. The calculations were cumbersome at times, but that was the toughest part of the experiment, once we understood all the concepts in the lab, implementing the circuits and taking the measurements were very straightforward.