Download the template Jupyter notebook `HW2_Template.ipynb` from Canvas and work from that template. For Problem 1, you can write your solutions down either 1) in Jupyter using markdown and Latex or 2) on handwritten on paper and scanned/submitted to Canvas. You should be able to complete Problems 2 – 4 using only `numpy`, `scipy`, and `matplotlib`, but feel free to use whatever you are comfortable with.

1. (5pts) For the binomial distribution,

$$p_B(x; n, p) = \binom{n}{x} p^x\, q^{n-x} = \frac{n!}{x!\,(n-x)!}\, p^x\,(1-p)^{n-x} \tag{1}$$

   for observing x of n items, where p and $q = (1-p)$ are the success and failure probabilities, respectively,

   (a) show that the average number of successes $\mu = np$.

   (b) show that the variance of the distribution $\sigma^2 = np(1-p)$.

2. **CODING:** (5 pts) *Simulating coin flips.* Write a function that simulates a series of coin flips. The coin could be biased. The function should take three arguments, 1) p, the probability of getting a head, 2) the number of coins per trial `nCoins`, and 3) the number of trials `nTrials`, and return the mean number of heads per trial and the variance. Confirm that the results are consistent with the expression in Problem 1 for large values of `nTrials`.

3. **CODING:** (10 pts) *Drawing random deviates from arbitrary functions.* Although `np.random` has numerous functions from which you can draw random deviates, in real life you will encounter cases that are not in `numpy`. This problem will guide you through the steps of writing your own routine that samples from an arbitrary distribution function. Consider an asymmetric gaussian distribution function given by the following:

$$p_{2G}(x; \sigma_L, \sigma_R) \propto \begin{cases} e^{-x^2/2\sigma_L^2} & x < 0 \\ e^{-x^2/2\sigma_R^2} & x \geq 0 \end{cases} \tag{2}$$

   The function is continuous across $x = 0$.

   (a) Write a function that returns the value of $p_{2G}(x; \sigma_L, \sigma_R)$ normalized to unity, i.e., such that

$$\int_{-\infty}^{\infty} p_{2G}(x; \sigma_L, \sigma_R)dx = 1 \tag{3}$$

   The function should take three arguments 1) σ_L, 2) σ_R, and 3) `x`, a vector of x values.

   (b) Write a function that returns the cumulative distribution function (CDF) of the normalized $p_{2G}(x; \sigma_L, \sigma_R)$, i.e.,

$$CDF(x; \sigma_L, \sigma_R) = \int_{-\infty}^{x} p_{2G}(x'; \sigma_L, \sigma_R)dx' \tag{4}$$

   This function should also take three arguments 1) σ_L, 2) σ_R, and 3) `x`, a vector of x values.

   (c) Using the results from above, write a function that draws random deviates from p_{2G}. The sampler should take three arguments: 1) σ_L, 2) σ_R, and 3) `size`, the number of random numbers to return.

   (d) Draw 10^6 random numbers from this distribution for $\sigma_L = 0.4$ and $\sigma_R = 1.2$ and plot a histogram of the values from $-2 \leq x \leq 5$. Overplot the analytic function on the same graph.