Download the template Jupyter notebook `HW3.ipynb` from Canvas and work from that template. For Problem 1, you can write your solutions down either 1) in Jupyter using markdown and Latex or 2) on handwritten on paper and scanned/submitted to Canvas. You should be able to complete all problems with only `numpy`, `scipy`, and `matplotlib`, but as usual feel free to use whatever you are comfortable with.

1. **CODING:** (10 pts) Students in PHYS150 were tasked to measure the local acceleration of gravity g by dropping an object from height H, measuring the time t it takes to reach the ground, and using the formula,

$$H = \frac{1}{2} g t^2. \tag{1}$$

The datafile `GravityMeasurements.dat` contains measurements of the value of H in meters, its uncertainty σ_{H_i}, time t in seconds, and its uncertainty σ_{t_i} made independently by $N = 350$ students. The file has five columns – 1) index $i = 0, ..., N-1$, 2) measurement $\{H_i\}$, 3) its corresponding uncertainty $\{\sigma_{H_i}\}$, 4) measurement $\{t_i\}$, and 5) its corresponding uncertainty $\{\sigma_{t_i}\}$ from each student.

```
0   20.066  1.120   2.092  0.084
1   20.363  0.389   2.023  0.050
2   21.498  0.706   1.939  0.115
3   19.709  0.791   1.963  0.118
4   21.192  1.225   2.132  0.097
5   16.631  1.479   1.877  0.112
...
```

(a) Calculate the values of acceleration of gravity $\{g_i\}$.

(b) Calculate the corresponding uncertainties $\{\sigma_{g_i}\}$ using the proper error propagation formulae.

(c) In this part and the next one, ignore the uncertainties $\{\sigma_{g_i}\}$, but rather estimate it directly from the measurements by computing the standard deviation of $\{g_i\}$. What is the best estimate of this value $\tilde{\sigma}_g$?

(d) Using this value of $\tilde{\sigma}_g$, calculate the maximum-likelihood estimate (MLE) of the mean of $\{g_i\}$ and the uncertainty in the mean from the measurements.

(e) Now compute the inverse-variance weighted MLE of the mean of $\{g_i\}$ using the actual uncertainty $\{\sigma_{g_i}\}$ on each measurement. Also compute the uncertainty in the mean to show that it is smaller than the value $\tilde{\sigma}_g$ estimated in a).

2. **CODING:** (10 pts) Using the same dataset as above:

(a) Calculate the χ^2 value defined as:

$$\chi^2 = \sum_{i=0}^{N-1} \left(\frac{g_i - \mu'}{\sigma_{g_i}} \right)^2 \tag{2}$$

where μ' is the inverse-variance weighted MLE of the mean of $\{g_i\}$ from 1c).

(b) Now calculate the χ^2 value as a function of μ' from $\mu' = 9.600$ to 10.000 in steps of 0.001 and show that the value of μ' that minimizes the χ^2 is indeed given by the answer from 1c).

(c) Finally, determine the lower and upper values of μ' at which the χ^2 is larger than the minimum value by 1.00.

(d) **EXTRA CREDIT:** (2 pts) Make a single plot that shows all of these facts with appropriate labels to show the important features.