Peer-to-peer Accommodations and Congestion Externalities

Working Paper

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September 19, 2018

Abstract

As urban tourism has been in the rise, and tourists look for a more authentic experience outside of the typically tourist district the peer-to-peer (P2P) market for accommodations has emerged as an alternative to standard hotel services. These P2P accommodations are not subject to the same zoning regulations as traditional tourist accommodations, and as a result tourists are able to stay in areas that were previously inaccessible. The effects of redistribution tourists into areas not built for large amounts of traffic or accustom to servicing tourists is largely unexplored. Using a Hotelling model with bundling and a congestion externality, this paper explores how the redistributive effect of P2P accommodation services like Airbnb impacts the firm pricing decisions and consumer welfare. The results of the model suggest firms in the area where tourists are being redistributed to will increase their price. This leads to a decline in the welfare of the residents in the area due to the increase in prices as well as higher levels of congestion.

JEL Codes: D62, L10, Z32

Key Words: P2P platforms, Airbnb, Externality, Urban Development

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Introduction

Over the past few decades urban tourism has increased in popularity. Tourist visiting a city desire to explore the cultural institutions, main business districts, shopping districts, and urban parks, which are typically concentrated in the city's innermost parts. (Burtenshaw et al., 1991). Furthermore, tourists look for accommodations close to these attractions (Arben and Pizam, 1977), and therefore, most accommodations are concentrated around the city center (Egan and Nield, 2000). Additionally, research by Shoval et al. (2011) shows tourists spend most of their time and money within the vicinity of their accommodation.

These factors create highly concentrated areas of urban tourism, which have led to the development of tourist districts. Areas marked by high concentrations of shops, restaurants, and accommodations geared towards tourism. Though these districts benefit from the increased economic activity it comes at a cost. Tourist districts are frequently subject to high levels of congestion, high prices, and tourism gentrification. However, the spread of these districts, and their accompanying problems, have been able to be limited to a city's downtown or commercial district(s) with the help of zoning regulations.

Zoning laws allow cities to maintain separate commercial and residential districts.[1] By keeping these two districts separate the residential districts are protected from the congestion, high prices, and tourism gentrification associated with the central business district. However, the growing demand for accommodations by tourist along with the increasing popularity of internet services has lead to accommodations being offered on peer-to-peer (P2P) platforms.

The most successful P2P accommodation platform has been Airbnb, which allows private individuals (hosts) the opportunity to rent spare space to people (guests) looking for a place to stay. Since its launch in 2008 Airbnb has more than three million listings in over 190 countries, and is valued at over $10 billion, which exceeds that of most well-established hotel chains (Zervas et al., 2017). The success of Airbnb can be attributed to its business model centered on cost savings and providing a more authentic local experience (Guttentag, 2015).

[1]US zoning laws were created with the intent to regulate the use and development of real estate by dividing municipalities into residential, commercial, and industrial districts. Furthermore, the Supreme Court deemed zoning constitutional citing the mixing of factories with residences are detrimental to families' "health, safety, and welfare" (euc, 1926; Shlay and Rossi, 1981). Though within the last two decades there has been a push for mixed use zoning (Walker, 1997; Grant, 2002; Hoppenbrouwer and Louw, 2005); however, most of the research argues for mixed use in already commercial districts. Therefore, residential areas will still remain insulated from the problems associated with excessive commercial development. It should be noted as well that many European cities do not have the same restrictive zoning regulations as the US, and therefore, mixed-use development is much more common (Hirt, 2012).

Airbnb is able to offer cheaper accommodations and create a "more authentic local experience" because the listings consist of houses and apartment buildings that already exist. As a result, Airbnb is not limited to the same zoning requirements as hotels, which are restricted to commercial districts, and therefore can more easily expand its supply (Zervas et al., 2017). Work by Guttentag (2015) and Gutiérrez et al. (2017) show Airbnb listings are more scattered throughout a city than hotels, which allows Airbnb guests to stay in neighborhoods that do not typically receive many tourists. The result of redistributing tourists into these neighborhoods could be the introduction of congestion and tourism gentrification (Russo, 2002; Neuts and Nijkamp, 2012). Problems that were previously restricted to tourist districts.

For example, in Barcelona a popular market, La Boqueria, became so congested with tourists vendors were loosing customers. In response, city officials enacted a policy banning tourist groups from the market between the hours of 8am and 3pm on Friday and Saturday.[2] The increase in tourism also has the potential to effect both the residential and commercial landscape of the neighborhood. Residents are physically displaced by the transformation of residential apartments into short term rental flats (Russo and Domínguez, 2014; Stors and Kagermeier, 2015; Sans and Domínguez, 2016). Furthermore, Cócola-Grant (2015) argues the influx of tourists into a neighborhood will replace residential services with commercial entertainment and tourist venues.

Though P2P accommodations clearly have an effect on residential neighborhoods most of the research, specifically on Airbnb, has focused on the impact it has had on the hotel industry. Guttentag (2015) identified Airbnb as a disruptive innovation. Much of the research that followed focuses determining the effect Airbnb has had on hotel revenues (Choi et al., 2015; Zervas et al., 2017). However, the impact of Airbnb in not limited to the hotel industry. The goal of this paper is to begin filling the gap within the literature by analyzing the externalities P2P accommodations like Airbnb impose on residents. While both congestion and tourism gentrification are important areas of study, for simplicity this paper chooses to focus only on the problem of congestion.

To model the consumption pattern of tourists and residents as well as capture the congestion externality imposed on residents by tourists this paper extends the Hotelling model with bundling developed by Kim and Serfes (2006). The city is represented by the unit interval with the left and right extreme points representing the tourist and residential districts, respectively. There at two firms producing either entertainment goods or residential goods with locations fixed at the end

[2] *Source:* `www.thelocal.es/20150408/barcelona-bans-tourists-from-famous-market`

points, and two types of consumers, residents and tourists. By allowing consumers to bundle the model is able to capture a consumers desire for both entertainment and residential goods. This paper uses the model to conduct a theoretical analysis of the impact redistributing tourists has on firm prices and the welfare of consumers.

In addition to the policy implications, the model developed in this paper contributes to the Hotelling literature in three ways. First, this paper extends the model developed by Kim and Serfes (2006) to allow consumers to be differentiated over multiple dimensions.[3] Consumers are differentiated based on their location within the city (horizontal location) and their type, tourist or resident. Second, congestion is included within the residents' utility. By incorporating congestion within the utility of residents the model captures the consumption externality tourists impose on residents when consuming a particular firm's product.[4] Finally, I diverge from the Hotelling literature by allowing consumers to be distributed non-uniformly, which is necessary to study how changes in the distribution of tourists effects the equilibrium pricing decisions and welfare. Traditionally, the uniform distribution assumption is made for simplicity, but it is not necessary (Hotelling, 1929). Furthermore, work by Neven (1986), Tabuchi and Thisse (1995), and Anderson and Goeree (1997) all show if the distribution of consumers satisfies certain conditions a pricing equilibrium will still exit.

The welfare analysis is broken down into three cases based on the level of additional utility a consumer receives from buying from both firms. In the first case firms compete head-to-head over both residents and tourists regardless of how tourists are distributed. The resulting welfare function is convex, and therefore, welfare is largest when distribution of tourists is at the extreme points, either uniformly distributed or clustered near the tourist district.

In the second case the firms will have a local monopoly over both tourists and residents when the tourists are distributed asymmetrically. As tourists become more uniformly distributed firms switch from having a local monopoly to competing head-to-head. Welfare in this case decreases as tourists become more uniformly distributed. Finally, in the third case firms have a local monopoly over both tourists and residents regardless of how tourists are distributed. Welfare is maximized when tourists are uniformly distributed.

The policy implications that can been gleaned from these welfare analysis depends on the focus of

[3]Work by Economides (1986); Irmen (1998); Larralde et al. (2009); Elizalde (2013) also allow consumers to be differentiated along multiple dimentsions.

[4]Work by Grilo et al. (2001) Laussel et al. (2004), and Lamertini and Orsini (2005) also look at congestion externalities in a differentiated product model.

the policy maker. The effects of the redistribution of tourists resulting from P2P accommodations on total welfare depends the level of additional benefit consumers receive from purchasing both types of goods. However the effect on particular agents within the model are more consistent. The firm in the residential neighborhood receives higher profits as a result of an increase in demand. Conversely, residents buying from this firm are consistently made worse off due to increased prices and tourist congestion. Based on the additional revenues brought in policy makers should not impose regulations on P2P accommodations. At the same time, the negative effects on residents from rising prices and congestion suggest imposing regulations to limit P2P accommodation.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 solves for and analyzes the Nash equilibrium assuming the tourists are distributed uniformly. Section 4 provides results of the welfare analysis. Section 5 discusses the results and Section 6 concludes.

Model

There are two firms, A and B, who produce differentiated products, and are located at the endpoints of the unit interval $[0, 1]$, respectively. The unit interval can be thought of as a linear representation of a city with the left extreme point representing the tourist district and the right extreme point representing the residential district. Firm A, which occupies the tourist district, represents a firm aimed at providing entertainment and other tourist services.[5] Firm B, located in the residential district, represents a firm designed to meet residential needs.[6]

There are two types of consumers, tourists (T) and residents (R), distributed independently and each with a mass normalized to one.[7] The residents are assumed to be distributed uniformly, and the tourists are assumed to be distributed according to the beta distribution, $B(1, \gamma)$.[8] Holding the first shape parameter fixed at 1 allows the distribution to be monotonically decreasing along the unit interval. Therefore, when the tourists are distributed asymmetrically they will be concentrated in the tourist district.

[5]Specific examples of theses tourist oriented businesses include chain restaurants, bars, or shopping districts.

[6]Examples of residential firms include local markets, firms providing home services, and local restaurants.

[7]Assuming equal mass of tourists and residents implies the population of residents and tourists are equal which is not necessarily true. However, the model can be extended allow firms to weight the revenues generated from each consumer type. A model with equal weights is equivalent to a scaled version of the model discussed in this paper. If the weights are assumed to be unequal weights then the model is equivalent to assuming different masses of residents and tourists. For example, placing a larger weight on residents is equivalent to assuming the population of residents is larger than the population of tourists.

[8]Other functional forms can be used. However, for an equilibrium to exists the distribution cannot be "too asymmetric" or "too concave." Anderson and Goeree (1997) provide conditions which the distribution needs to meet in order for an equilibrium to exist.

$$f_T(x,\gamma) = \gamma(1-x)^{\gamma-1} \qquad f_R(x) = 1$$

As Airbnb is adopted throughout a city tourists have more opportunities to stay in accommodations outside the main tourist district, which results in a more uniform distribution of tourists across the city. Therefore, the effect Airbnb has on the distribution of tourists can be represented by reducing the shape parameter γ. When $\gamma = 1$ the tourists are distributed uniformly. Additionally, the distribution of residents is assumed to be fixed and unaffected by the redistribution of tourists. Many cities have imposed regulations in order to control the spread of the short-term rental market, and its redistributive effects. Some cities including Nashville, Philadelphian, and Chicago, have imposed regulations requiring hosts to obtain a short-term license, which allows the city to control the number of listings in an area. Others like New York, Portland, and San Francisco have gone even further restricting hosts to listing only their primary residence.[9]

Each consumer can buy at most one unit from a given firm. Additionally, the market is assumed to be covered. Tourists have the following valuation: $V = \alpha q_i + \theta^T q_A q_B$, where $i = A, B$ and q_i represents the quantity bought from Firm i, with $q_i \in \{0,1\}$. Residents have a similar valuation defined as: $V = \eta q_i + \theta^R q_A q_B$. The valuation does not depend on a consumer's location within the unit interval. Hence, if a tourist (resident) only buys from one of the firms, his valuation is $V(1) = \alpha > 0$ $\big(V(1) = \eta > 0\big)$, while if he buys from both firms, his valuation is $V(2) = \alpha + \theta^T$ $\big(V(2) = \eta + \theta^R\big)$, where $\theta^j \geq 0$ and $j = T, R$. Furthermore, diminishing marginal utility is assumed, i.e, $\theta^T \leq \alpha$ and $\theta^R \leq \eta$. The incremental utility from purchasing a second product will be denoted by $\theta^j = V(2) - V(1)$, with $\theta^j = [0, V(1)]$. For simplicity, it will be assumed $\alpha = \eta$ and $\theta^T = \theta^R$.[10]

Consumers will also incur disutility from having to travel to purchase a product. A consumer located at $x \in [0,1]$ will incur a disutility equal to cx if he buys from Firm A, $c(1-x)$ if he buys from Firm B, and $cx + c(1-x)$ if he buys from both. The literature on Hotelling models typically assumes quadratic transportation costs because in the standard Hotelling model a pricing equilibrium fails to exist when firms are not sufficiently far apart (d'Aspermont et al., 1979). However, Kim and Serfes (2006) state in a footnote that when consumers are allowed to bundle a pricing equilibrium exists even when linear transportation costs are assumed.

[9] See `www.Airbnb.com/help/article/1376/responsible-hosting-in-the-united-states` for a full list of city regulations.

[10] There is no theoretical foundation to motivate why one consumer type would receive more or less valuation than the other. However, one could argue different consumer types have different valuations. This extension is left for future research.

Firm prices are denoted by p_A and p_B. A tourists located at x who buys from Firm A will have an indirect utility of $V(1) - cx - p_A$. If the tourist buys from Firm B his indirect utility is $V(1) - c(1-x) - p_B$. If he buys from both firms his indirect utility is $V(2) - cx - c(1 - x) - p_A - p_B$. There will be two marginal consumers denoted by x_1^T and x_2^T. x_1^T represents the consumer indifferent between buying from only Firm A and buying from both firms. x_2^T identifies the consumer indifferent between buying from both firms and only buying from Firm B. The first marginal consumer is located at

$$V(1) - cx_1^T - p_A = V(2) - cx_1^T - c(1 - x_1^T) - p_A - p_B$$

$$x_1^T = \frac{c - \theta + p_B}{c} \tag{1}$$

The second marginal consumers is located at

$$V(1) - c(1 - x_2^T) - p_B = V(2) - cx_2^T - c(1 - x_2^T) - p_A - p_B$$

$$x_2^T = \frac{\theta - p_A}{c} \tag{2}$$

Note $x_1^T \leq x_2^T$ when $p_A + p_B \leq 2\theta - c$. Additionally, $x_1^T \geq 0$ when $p_B \geq \theta - c$, and $x_2^T \leq 1$ when $p_A \geq \theta - c$.[11] When $x_1^T = x_2^T$, the marginal consumers occur at the same location, the consumer is indifferent between buying one unit from Firm A and one unit from Firm B. In this case, no consumers will purchase from both firms. When this case occurs the marginal consumer is located at

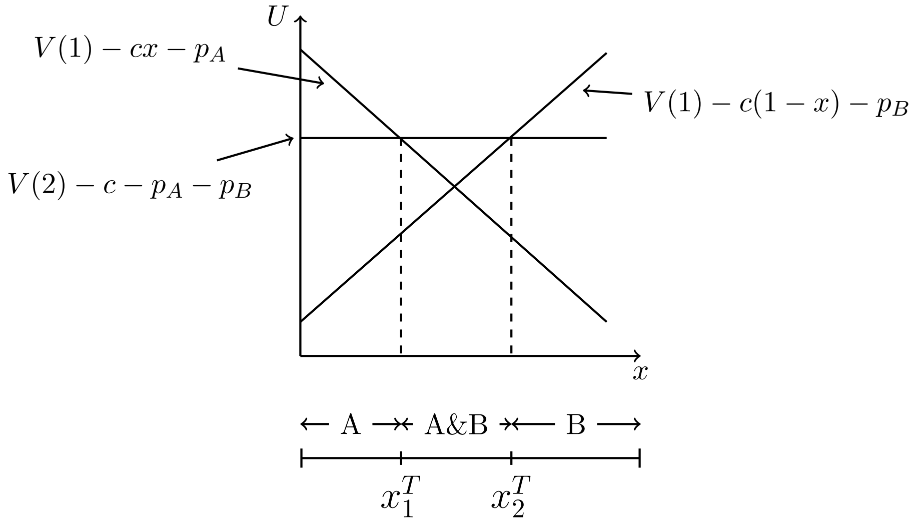
$$V(1) - c\hat{x}^T - p_A = V(1) - c(1 - \hat{x}^T) - p_B$$

$$\hat{x}^T = \frac{p_B - p_A + c}{2c} \tag{3}$$

Therefore, tourists located in the interval $[0, x_1^T]$ purchase exclusively from Firm A, tourists in (x_1^T, x_2^T) purchase from both firms, and tourists in the interval $[x_2^T, 1]$ purchase solely from Firm B (see Figure 1).

[11]It is also the case $x_1^T \leq 1$ when $\theta \geq p_B$, and $x_2^T \geq 0$ when $\theta \geq p_A$. In general, it is the case that $x_A \leq b$ and $x_B \geq a$. A proof is provided in the Appendix.

Figure 1



The residents' indirect utility equations are similar to those of the tourists with an added term for congestion when purchasing from residential firms, Firm B.[12] This congestion term captures the consumption externality impose on residents by tourists when they consume the residential firms product. These firms are located in areas designed to handle the limited residential traffic. Consequently, the presence of tourists increases congestion, and makes it more difficult for residents to purchase from Firm B. The congestion externality will be measured by tourist's demand for Firm B's product.

A resident located at x who buys from Firm A has indirect utility is $V(1)-cx-p_A$. If the resident buys from Firm B his indirect utility is, $V(1)-c(1-x)-p_B+\beta D_{T,B}$. $D_{T,B}$ represents the demand for Firm B by tourists, $1 - F_T(x_1^T)$, and $\beta < 0$ represents the weight of the congestion externality. If the resident buys from both firms his indirect utility is, $V(2) - cx - c(1-x) - p_A - p_B + \beta D_{T,B}$. As before, there are two indifferent consumers, x_1^R and x_2^R. These terms represent the consumer indifferent between buying only from Firm A and both firms and the consumer indifferent between buying only from Firm B and both firms, respectively. The equation for the marginal consumer for Firm A is given by

$$x_1^R = \frac{c - \theta + p_B - \beta D_{T,B}}{c} \tag{4}$$

[12]No congestion externality is included in the indirect utility from Firm A because Firm A is located in the tourist district. This district is designed to handle more traffic. Additionally, there is a certain level of congestion that is expected when visiting the tourist district.

The marginal consumer for Firm B is located at

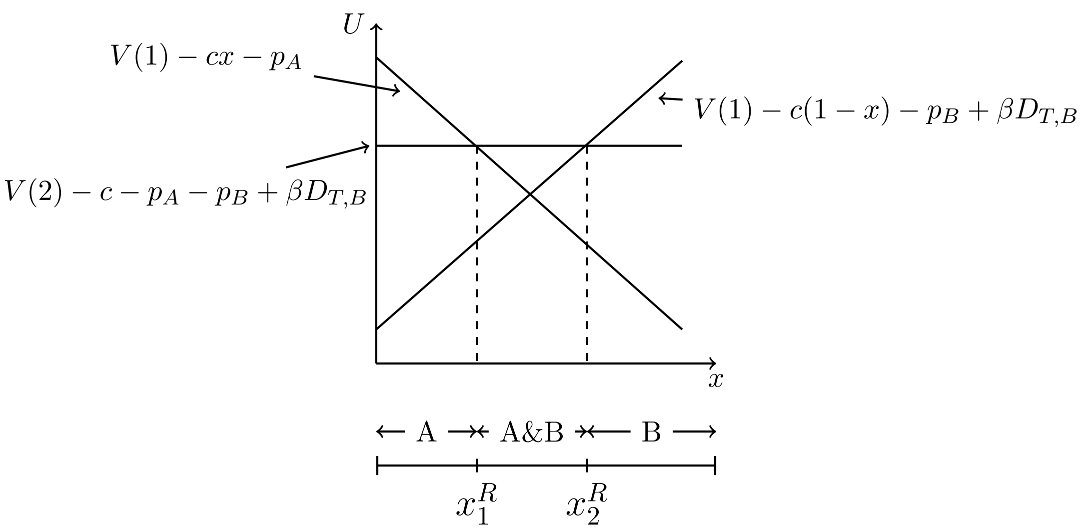
$$x_2^R = \frac{\theta - p_A}{c} \tag{5}$$

Similar to before, $x_1^R \leq x_2^R$ when $p_A + p_B \leq 2\theta - c + \beta D_{T,B}$. Additionally, $x_1^R \geq 0$ when $p_B \geq \theta_R - c + \beta D_{T,B}$ and $x_2^R \leq 1$ when $p_A \geq \theta_R - c$.[13] When $x_1^R = x_2^R$ the marginal consumer occurs at

$$\hat{x}^R = \frac{p_B - p_A + c - \beta D_{T,B}}{2c} \tag{6}$$

Figure 2 plots the indirect utilities for residential consumers. The figure is similar to the indirect utility graph for the tourists expect now the interval boundaries depend also on the congestion effect. Residents in the interval $[0, x_1^R]$ buy from Firm A; those in $(x_1^R, x_2^R$ buy from both firms; and residents in $[x_2^R, 1]$ buy only from Firm B.

Figure 2



It follows the demand for each firm will be given by the sum of the demands by each consumer type. When the model includes only one type of consumer two cases can occur (Kim and Serfes, 2006). Either there is a set of consumers who by from both firms, $x_1 < x_2$ or consumers buy from either Firm A or Firm B, $x_1 = x_2$. When the model allows two consumer types there exists analogs to these two case, but also a third, intermediate case.

The analog to the first case occurs when both tourists and residents have a subset that buys

[13] Like with the tourist indiffierent consumers, $x_1^R \leq 1$ when $\theta \geq p_B - \beta D_{T,B}$, and $x_2^R \geq 0$ when $\theta \geq p_A$.

from both firms, $x_1^T < x_2^T$ and $x_1^R < x_2^R$. The parallel to the second case occurs when $x_1^T = x_2^T$ and $x_1^R = x_2^R$, and no consumers buy from both firms. Finally, the intermediate case will occur when only one consumer type has a subset that buys from both firms. This will occur when $x_1^T < x_2^T$ and $x_1^R = x_2^R$ or $x_1^T = x_2^T$ and $x_1^R < x_2^R$. Firm A's demand function is

$$d_A = \begin{cases} 1 - \left(1 - \frac{\theta - p_A}{c}\right)^\gamma + \frac{\theta - p_A}{c}, & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & \quad p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ 1 - \left(1 - \frac{\theta - p_A}{c}\right)^\gamma + \frac{p_B - p_A + c - \beta D_{T,B}}{2c}, & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & \quad p_A + p_B > 2\theta - c + \beta D_{T,B} \\ 1 - \left(1 - \frac{p_B - p_A + c}{2c}\right)^\gamma + \frac{\theta - p_A}{c}, & \text{if } p_A + p_B > 2\theta - c \ \& \\ & \quad p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ 1 - \left(1 - \frac{p_B - p_A + c}{2c}\right)^\gamma + \frac{p_B - p_A + c - \beta D_{T,B}}{2c}, & \text{if } p_A + p_B > 2\theta - c \ \& \\ & \quad p_A + p_B > 2\theta - c + \beta D_{T,B}. \end{cases}$$

For the intermediate case to occur either $p_A + p_B > 2\theta - c + \beta D_{T,B}$ or $p_A + p_B > 2\theta - c$. However, based of the assumptions $\theta^T = \theta^R$ and $\beta < 0$ it will never be the case that $p_A + p_B > 2\theta - c$ and $p_A + p_B \leq 2\theta - c + \beta D_{T,B}$. Therefore, the only intermediate case that will occur is when $x_1^T < x_2^T$ and $x_1^R = x_2^R$, $p_A + p_B \leq 2\theta - c$ and $p_A + p_B > 2\theta - c + \beta D_{T,B}$. A subset of tourists will buy from both firms and residents buy only from Firm A or Firm B. Since the other intermediate case will not occur results will not be reported.

Additionally, the assumptions $\theta^T = \theta^R$ and $\beta < 0$ further imply $2\theta - c + \beta D_{T,B} < 2\theta - c$. Therefore, if $p_A + p_B \leq 2\theta - c + \beta D_{T,B}$ then it must be the case that $p_A + p_B \leq 2\theta - c$. Additionally, if $p_A + p_B > 2\theta - c$ then $p_A + p_B > 2\theta - c + \beta D_{T,B}$. Therefore, the conditions that define the cases can be simplified.

Firm B's demand function is given by

$$d_B = \begin{cases} 2 - \left(1 - \left(1 - \frac{c-\theta+p_B}{c}\right)^\gamma\right) - \frac{c-\theta+p_B-\beta D_{T,B}}{c} = \left(\frac{\theta-p_B}{c}\right)^\gamma + \frac{\theta-p_B+\beta D_{T,B}}{c}, & \text{if } p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ 2 - \left(1 - \left(1 - \frac{c-\theta+p_B}{c}\right)^\gamma\right) - \frac{p_B-p_A-\beta D_{T,B}+c}{2c} = \left(\frac{\theta-p_B}{c}\right)^\gamma + \frac{p_A-p_B+\beta D_{T,B}+c}{2c}, & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & p_A + p_B > 2\theta - c + \beta D_{T,B} \\ 2 - \left(1 - \left(1 - \frac{p_B-p_A+c}{2c}\right)^\gamma\right) - \frac{p_B-p_A-\beta D_{T,B}+c}{2c} = \left(\frac{p_A-p_B+c}{2c}\right)^\gamma + \frac{p_A-p_B+\beta D_{T,B}+c}{2c}, & \text{if } p_A + p_B > 2\theta - c \end{cases}$$

Note the demand function exhibits to two kinks when $\beta < 0$. If $\beta = 0$ then the demand curve will have only one kink, because the conditions for the intermediate case will never be met. When prices are sufficiently low demand depends only on the firm's own price, and a reduction in price does not cause consumers to switch between firms. Therefore, this case can be thought of as firms having a local monopoly. As prices increase past the first kink firms compete head-to-head over residents, but still have a local monopoly over tourists. Firm competition is referred as head-to-head because a reduction in price by one firm will reduced the demand for the other firm. Once prices increase past the second kink firms will compete head-to-head on both residents and tourists.[14]

Assuming firms have constant and equal marginal costs, which are normalized to zero, yields profit functions for the firms given by[15]

$$\pi_A = \begin{cases} p_A\left(1 - \left(1 - \frac{\theta-p_A}{c}\right)^\gamma + \frac{\theta-p_A}{c}\right), & \text{if } p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ p_A\left(1 - \left(1 - \frac{\theta-p_A}{c}\right)^\gamma + \frac{p_B-p_A+c-\beta D_{T,B}}{2c}\right), & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & p_A + p_B > 2\theta - c + \beta D_{T,B} \\ p_A\left(1 - \left(1 - \frac{p_B-p_A+c}{2c}\right)^\gamma + \frac{p_B-p_A+c-\beta D_{T,B}}{2c}\right), & \text{if } p_A + p_B > 2\theta - c. \end{cases} \tag{7}$$

[14] "Local monopoly" and "head-to-head" are adopted from Kim and Serfes (2006).

[15] One effect of tourism gentrification is the shutdown of residential firms and entry of tourist firms, i.e. tourism gentrification (Cócola-Grant, 2015). By relaxing the equal marginal cost assumption a theoretical model of tourism gentrification could be developed. However, the focus of this paper is to gain an initial understanding of the effects redistributing tourists has on the equilibrium, and relaxing this assumption would unnecessarily complicate the problem.

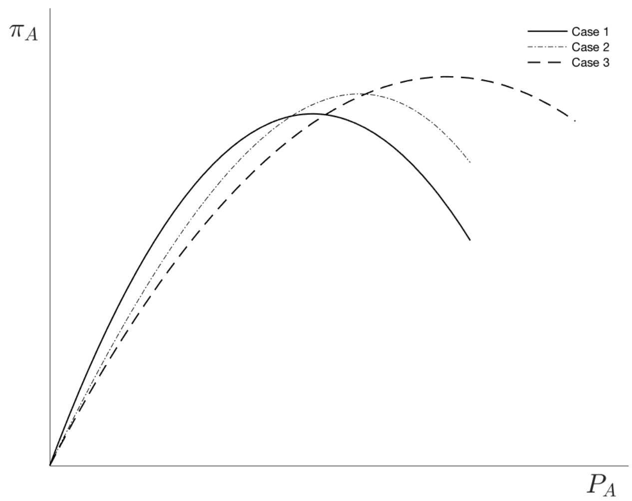
and

$$\pi_B = \begin{cases} p_B\left(\left(\frac{\theta-p_B}{c}\right)^\gamma + \frac{\theta-p_B+\beta D_{T,B}}{c}\right), & \text{if } p_A+p_B \leq 2\theta - c + \beta D_{T,B} \\ p_B\left(\left(\frac{\theta-p_B}{c}\right)^\gamma + \frac{p_A-p_B+\beta D_{T,B}+c}{2c}\right), & \text{if } p_A+p_B \leq 2\theta - c \text{ \& } \\ & \quad p_A+p_B > 2\theta - c + \beta D_{T,B} \\ p_B\left(\left(\frac{p_A-p_B+c}{2c}\right)^\gamma + \frac{p_A-p_B+\beta D_{T,B}+c}{2c}\right), & \text{if } p_A+p_B > 2\theta - c. \end{cases} \tag{8}$$

Pricing Subgame

Figure 3 depicts the functions in equation (7) when $p_B = 0.4$, $\theta = 1$, $c = 1$, and $\beta = -0.5$.[16] The profit function for Firm A is the upper envelope of Figure 3. Since the profit function is not quasi-concave the assumptions of Kakutani's fixed point theorem are not satisfied; however, the game is supermodular.[17] Therefore, the best reply functions are increasing, and an equilibrium in pure strategies must exist (Topkis, 1968). The following analysis characterizes the equilibrium of the game.

Figure 3 : Firm A's Profits



As long as γ is assumed to be variable no closed form solutions for prices exists, and any further analysis of the equilibrium would need to be conducted as a numerical exercise. However, if γ is assumed to be equal to one, the tourists are distributed uniformly, then closed form solutions exist. For that reason, this section will continue under the assumption $\gamma = 1$, which allows for a the

[16] A similar plot can be drawn for Firm B.
[17] Proof of supermodularity is provided in the Appendix.

basic properties of the equilibrium to be understood. A discussion of how relaxing this assumption affects the equilibrium is included at the end of this section.

Firm prices can be found by differentiating equations (7) and (13) with respect to p_A and p_B respectively, and solving for the firm's own price. Doing this yields[18]

$$p_A = \begin{cases} \frac{\theta}{2}, & \text{if } p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ \frac{p_B(1+\frac{\beta}{c})+\theta(2-\frac{\beta}{c})+c}{6}, & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & \qquad p_A + p_B \geq 2\theta - c + \beta D_{T,B} \\ \frac{p_B(2+\frac{\beta}{2c})-\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}, & \text{if } p_A + p_B \geq 2\theta - c \end{cases}$$

and

$$p_B = \begin{cases} \frac{\theta}{2}, & \text{if } p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ \frac{p_A+\theta(2+\frac{\beta}{c})+c}{2(3+\frac{\beta}{c})}, & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & \qquad p_A + p_B \geq 2\theta - c + \beta D_{T,B} \\ \frac{p_A(2+\frac{\beta}{2c})+\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}, & \text{if } p_A + p_B \geq 2\theta - c \end{cases}$$

Let us focus on Firm A. Firm B has a comparable problem. Assume p_B is fixed then Firm A has three choices. It can set $p_A = \frac{\theta}{2}$ if $p_A + p_B \leq 2\theta - c + \beta D_{T,B}$, $p_A = \frac{p_B(1+\frac{\beta}{c})+\theta(2-\frac{\beta}{c})+c}{6}$ if $p_A + p_B > 2\theta - c + \beta D_{T,B}$ and $p_A + p_B \leq 2\theta - c$, or $p_A = \frac{p_B(2+\frac{\beta}{2c})-\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}$ if $p_A + p_B \geq 2\theta - c$. The first case is valid when

$$x_2^T \geq x_1^T \text{ and } x_2^R \geq x_1^R \iff \frac{\theta}{2} + p_B \leq 2\theta - c + \beta(1 - \frac{c - \theta + p_B}{c})$$

$$p_B \leq \frac{\theta(\frac{3}{2} + \frac{\beta}{c}) - c}{1 + \frac{\beta}{c}}. \tag{9a}$$

The second case is valid when

$$x_2^T \geq x_1^T \iff \frac{p_B(1 + \frac{\beta}{c}) + \theta(2 - \frac{\beta}{c}) + c}{6} + p_B \leq 2\theta - c$$

[18] When $\gamma = 1$ and $\beta = 0$ the model reduces to the model discussed in Kim and Serfes (2006) with a mass of consumers normalized to two.

$$p_B \leq \frac{\theta(10+\frac{\beta}{c})-7c}{7+\frac{\beta}{c}} \tag{9b}$$

and

$$x_2^R = x_1^R \Leftrightarrow \frac{p_B(1+\frac{\beta}{c})+\theta(2-\frac{\beta}{c})+c}{6} + p_B \geq 2\theta - c + \beta(1-\frac{c-\theta+p_B}{c})$$

$$p_B \geq \frac{\theta(10+\frac{7\beta}{c})-7c}{7(1+\frac{\beta}{c})}. \tag{9c}$$

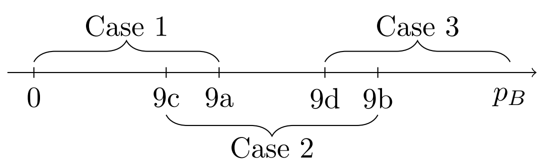
Finally, the third case is valid when

$$x_2^T = x_1^T \text{ and } x_2^R = x_1^R \Leftrightarrow \frac{p_B(2+\frac{\beta}{2c})-\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})} + p_B \geq 2\theta - c$$

$$p_B \geq \frac{4\theta(2+\frac{\beta}{2c})-2c(3+\frac{\beta}{2c})+\frac{\beta}{2}}{3(2+\frac{\beta}{2c})}. \tag{9d}$$

The regions defined by the bounds above may overlap. Figure 4 provides an illustration of how the regions may overlap.[19] The figure shows the first and second cases are both valid when $p_B \in \left[\frac{\theta(10+\frac{\beta}{c})-7c}{7+\frac{\beta}{c}}, \frac{\theta(\frac{3}{2}+\frac{\beta}{c})-c}{1+\frac{\beta}{c}}\right]$, and cases two and three are both valid when $p_B \in \left[\frac{4\theta(2+\frac{\beta}{2c})-2c(3+\frac{\beta}{2c})+\frac{\beta}{2}}{3(2+\frac{\beta}{2c})}, \frac{\theta(10+\frac{7\beta}{c})-7c}{7(1+\frac{\beta}{c})}\right]$.

Figure 4



Since the profits for each case are increasing with respect to p_B there exists a threshold within each interval, $thresh_1 \in \left[\frac{\theta(10+\frac{\beta}{c})-7c}{7+\frac{\beta}{c}}, \frac{\theta(\frac{3}{2}+\frac{\beta}{c})-c}{1+\frac{\beta}{c}}\right]$ and $thresh_2 \in \left[\frac{4\theta(2+\frac{\beta}{2c})-2c(3+\frac{\beta}{2c})+\frac{\beta}{2}}{3(2+\frac{\beta}{2c})}, \frac{\theta(10+\frac{7\beta}{c})-7c}{7(1+\frac{\beta}{c})}\right]$, such that if $p_B \leq thresh_1$ the first case yields the highest profits, if $thresh_1 \leq p_B \leq thresh_2$ the second case yields the highest profits, and if $p_B \geq thresh_2$ the third choice yields the highest profits.[20] Therefore, the best response functions are defined as

[19] The regions may overlap in other ways, and are discussed in the Appendix.

[20] A proof the profit functions increase with respect to the other firm's price is included in the Appendix.

$$p_A = \begin{cases} \frac{\theta}{2}, & \text{if } p_B \leq thresh_1 \\ \frac{p_B(1+\frac{\beta}{c})+\theta(2-\frac{\beta}{c})+c}{6}, & \text{if } p_B \geq thresh_1 \ \& \\ & \quad p_B \leq thresh_2 \\ \frac{p_B(2+\frac{\beta}{2c})-\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}, & \text{if } p_B \geq thresh_2 \end{cases} \tag{10}$$

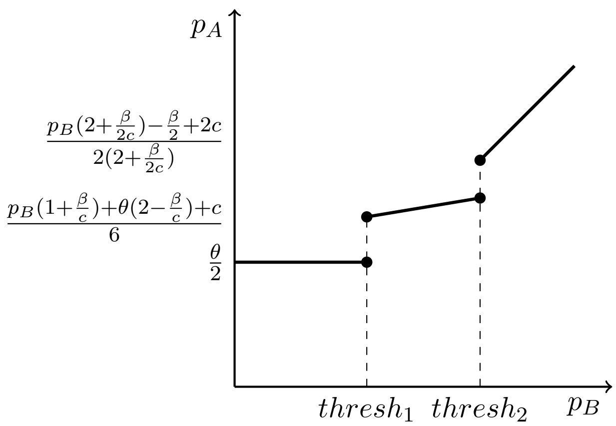
and

$$p_B = \begin{cases} \frac{\theta}{2}, & \text{if } p_B \leq thresh_1 \\ \frac{p_A+\theta(2+\frac{\beta}{c})+c}{2(3+\frac{\beta}{c})}, & \text{if } p_B \geq thresh_1 \ \& \\ & \quad p_B \leq thresh_2 \\ \frac{p_A(2+\frac{\beta}{2c})+\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}, & \text{if } p_B \geq thresh_2 \end{cases} \tag{11}$$

It is also possible for t $thresh_1 \geq thresh_2$. Consequently, the second case would not occur. When the second case does not occur the best response functions reduce to ones similar to those found in Kim and Serfes (2006). There will exist a threshold, $thresh_3 \in \left[\frac{4\theta(2+\frac{\beta}{2c})-2c(3+\frac{\beta}{2c})+\frac{\beta}{2}}{3(2+\frac{\beta}{2c})}, \frac{\theta(\frac{3}{2}+\frac{\beta}{c})-c}{1+\frac{\beta}{c}}\right]$, such that when $p_B \leq thresh_3$ the first choice yields higher profits, and when $p_B \geq thresh_3$ the third choice yields higher profits.

Figure 5 presents Firm A's best response function as it is defined in (10). When p_B is less than $thresh_1$ Firm A's best response is to charge $p_A = \frac{\theta}{2}$. This region corresponds to Firm A having a local monopoly over both residents and tourists, and a subset of both consumer types will buy from both products. This case will be referred to as the local monopoly (LM) case. When $thresh_1 < p_B < thresh_2$ Firm A's best response is to charge $p_A = \frac{p_B(1+\frac{\beta}{c})+\theta(2-\frac{\beta}{c})+c}{6}$. In this region Firm A has a local monopoly over tourists and competes head-to-head (H2H) with Firm B over residents. Only a subset of tourists will buy both products. This case will be referred to as the mixed market (M) case. Finally, Firm A charges $p_A = \frac{p_B(2+\frac{\beta}{2c})-\frac{\beta}{2}+2c}{2(2+\frac{\beta}{2c})}$ when $p_B > thresh_2$. At these prices the firms compete head-to-head for over both consumer types, and no consumers buys from both firms.

Figure 5



At each threshold Firm A has two equally profitable strategies. When $p_B = thresh_1$ Firm A can change the lower local monopoly price, and sell to more residents whose preferences for its product are not so strong; or it can charge the higher mixed market price, and sell only to a more loyal group of residents. At $thresh_1$ Firm A is only deciding on how to price with respect to residents since some tourists ill by from both firms until $p_B = thresh_2$. When $p_B = thresh_2$ then Firm A has a similar strategy choice as before except now it is with respect to tourists. Firm A can charge the lower mixed market price, and sell to more tourists whose preferences for its product are not so strong; or it can change the higher head-to-head price, and sell only to a more loyal group of tourists. Firm B's best response function can be determined using an equivalent strategy.

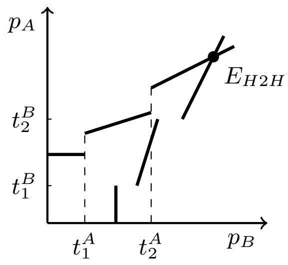
Figure 6 displays both firms' best response function, the thresholds, and the Nash equilibria for the five cases that can occur. Other forms of the best response functions can occur, but all possible sets of Nash equilibria are depicted in Figure 6. The Nash equilibria for the local monopoly, mixed market, and head-to-head cases are identified as E_{LM}, E_M, and E_{H2H} respectively.[21] In three of the case there are multiple Nash equilibria which can be Pareto ranked.[22] It is assumed firms coordinate strategies on the equilibrium yielding highest profits.

[21] Closed form solutions for the Nash equilibrium prices are

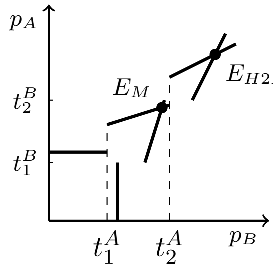
$$E_{LM} = \left(\frac{\theta}{2}, \frac{\theta}{2}\right) \quad E_M = \left(\frac{7c^3 + 3\beta c^2 + 14\theta c^2 + \beta\theta c - \beta^2\theta}{c(11\beta + 35c)}, \frac{7c^2 + 14\theta c + 5\beta\theta}{11\beta + 35c}\right) \quad E_{H2H} = \left(\frac{12c^2 - \beta c}{3(\beta + 4c)}, \frac{12c^2 + \beta c}{3(\beta + 4c)}\right)$$

[22] Theorem 7 in Milgrom and Roberts (1990) states if the payoffs to the firms are increasing with respect to other firms strategy, which is shown in the Appendix, then the largest and smallest equilibrium correspond to the most and least preferred.

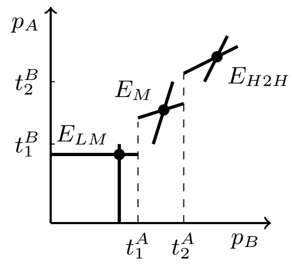
Figure 6



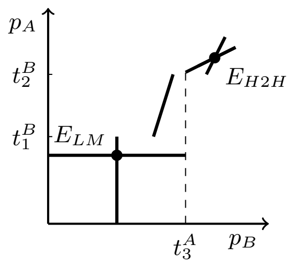
(a)



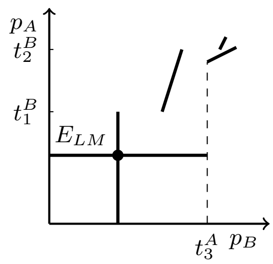
(b)



(c)



(d)



(e)

An equilibrium will arise only when the region for the respective case contains the equilibrium price. For example, in Figure 6a both Firm A's *thresh*$_1$ (t_1^A) and Firm B's *thresh*$_1$ (t_1^B) are less than the local monopoly Nash equilibrium price. So the local monopoly best response region does not contain E_{LM}. Furthermore, t_2^A and t_2^B are less than the mixed market Nash equilibrium price. Therefore the region where the mixed market occurs does not contain the equilibrium price, E_M, either. However, the region where firms compete heat-to-head does contain the equilibrium price, and so E_{H2H} is the only Nash equilibrium to occurs for this case.

Consequently, the thresholds determine which Nash equilibria occur, and therefore, which case in Figure 6 occurs. The thresholds are functions of θ, β, and c, and have the following properties.[23]

$$\frac{\partial t_1^A}{\partial \theta} > \frac{\partial t_2^A}{\partial \theta} > 0 \qquad \frac{\partial t_1^A}{\partial \beta} > \frac{\partial t_2^A}{\partial \beta} > 0 \qquad \frac{\partial t_1^A}{\partial c} < \frac{\partial t_2^A}{\partial c} < 0$$

$$\frac{\partial t_2^B}{\partial \theta} \approx \frac{\partial t_1^B}{\partial \theta} > 0 \qquad \frac{\partial t_1^B}{\partial \beta} > \frac{\partial t_2^B}{\partial \beta} > 0 \qquad \frac{\partial t_2^B}{\partial c} < \frac{\partial t_1^B}{\partial c} < 0$$

It should also be noted that for all of Firm A's derivatives the rate of change of t_1^A is greater than

[23]Signs of all derivatives were determined by conducting comparative statics exercises. **Should I report the plots of these numerical exercises in the Appendix?**

that of t_2^A. This means there will exists a point after which $t_1^A > t_2^A$. When this occurs the mixed market will not appear if Firm A's best response function, and the threshold will indicate the point at which head-to-head competition yields higher profits than the local monopoly, $thresh_3$ (t_3^A). The derivatives of t_3^A with respect to the parameters θ,β, and c have the same signs as the derivatives of t_1^A and t_2^A.

Movements between the graphs in Figure 6 can therefore be thought of as comparative statics exercises conducted on the thresholds. For brevity, the analysis to follow will focus on changes in θ relative to the other parameters, but a corresponding analysis can be conducted for the other parameters.

Beginning in Figure 6a, when θ is low relative to the other parameters, the unique equilibrium is E_{H2H}. Increasing θ results in the thresholds for both Firm A and Firm B to increase. When the thresholds increase enough the region where the mixed market occurs now includes E_M, and the case shown in Figure 6b occurs. Now the set of Nash equilibria insets both E_{H2H} and E_M. However, E_{H2H} yields the highest profits so firms will continue to compete heat-to-head.

Further increasing θ increases the thresholds and moves the firms to the case displayed in Figure 6c. The set of Nash equilibrium is now comprised of E_{H2H}, E_M, and E_{LM}, but like in the previous case, firms will coordinate on E_{H2H}. As θ is increased more eventually $t_1^A > t_2^A$.[24] As a result the mixed market price will never be a best response for Firm A. The best response functions are given by Figure 6d. Finally, as θ continues to increase Figure 6e shows E_{LM} becomes the unique Nash equilibrium.

For Figure 6a through 6d firms will coordinate on E_{H2H} since it yields the largest profits. It is not until θ becomes large enough, Figure 6e, that firms switch pricing strategies and have a local monopoly.[25] Therefore, the Nash equilibrium can be defined as a function of θ relative to the other parameters. Let $\tilde{\theta}$ represent the value of θ where the best response functions transition from Figure 6d to Figure 6e. Then when $\theta < \tilde{\theta}$ firms coordinate on E_{H2H}, and when $\theta > \tilde{\theta}$ firms coordinate on E_{LM}.

Finally, a corner solution for Firm B exists when $\theta \geq \frac{2c^2}{c-\beta}$. If Firm B was allowed to set its best response price the residential indifferent consumer would lie outside the unit interval, $x_1^R < 0$. Therefore, in order to keep the indifferent consumer within the unit interval, whenever θ exceeds

[24]Since the thresholds for Firm B increase at approximately the same rate, $\frac{\partial t_2^B}{\partial \theta} \approx \frac{\partial t_1^B}{\partial \theta}$, it will always be the case $t_2^B > t_1^B$.

[25]The mixed market Nash equilibrium never is the largest. Therefore, firms will never coordinate on E_M, which is a direct result of the assumption $\theta_R = \theta_T$. If this assumption were relaxed E_M could occur as the largest equilibrium.

this threshold Firm B sets its price such that all residents buy its product, $x_1^R = 0$. A corner solution also occurs for Firm A when $\theta \geq 2c$. When θ exceeds this threshold Firm A will set its price such that all consumers buy its product, $x_2^R = 1$ and $x_2^T = 1$.[26]

Vary γ

Now that a basic understanding of the model's equilibria has been established we can analyze how the model responds to changes in γ. Though it was not mentioned previously the thresholds are also functions of γ

$$\frac{\partial t_1^A}{\partial \gamma} < 0 \qquad \frac{\partial t_2^A}{\partial \gamma} < 0 \qquad \frac{\partial t_3^A}{\partial \gamma} < 0 \qquad \frac{\partial t_1^B}{\partial \gamma} > 0 \qquad \frac{\partial t_2^B}{\partial \gamma} > 0$$

Increasing γ reduces t_2^A and increases t_1^A, which shrinks the interval where the mixed market price is a best response for Firm A. Both t_1^B and t_2^B increase with respect to γ. Therefore, as long as $t_2^B > t_1^B$ increasing γ would result in the cases depicted in Figures 6a through 6c transitioning to the case depicted in Figure 6d. Increasing γ also reduces t_3^A suggesting the case depicted in Figure 6e would transition to Figure 6d as well. However, the equilibrium prices also respond to changes in the tourist distribution, and cause the best response functions to transition from Figure 6d to Figure 6e.

The local monopoly equilibrium price for Firm A and Firm B increase and decrease respectively with respect to γ, $\frac{\partial p^*_{A,LM}}{\partial \gamma} > 0$ and $\frac{\partial p^*_{B,LM}}{\partial \gamma} < 0$. The comparative statics for Firm A's head-to-head equilibrium price are non-linear. When γ is low, the distribution of tourists is close to uniform, increasing γ reduces $p^*_{A,H2H}$ initially, but as γ continues to increase $p^*_{A,H2H}$ begins to increase. A more detailed discussion of why this non-linearity occurs is provided in the next section. For Firm B the head-to-head equilibrium price decreases with respect to γ, $\frac{\partial p^*_{B,H2H}}{\partial \gamma} < 0$.

The response of $p^*_{B,H2H}$ to changes in γ can be shown to be larger than the response of t_3^A. When γ is increased E_{H2H} will shift left by more than t_3^A. Therefore, if the change in the tourists distribution is large enough E_{H2H} will fall below t_3^A, and the best response functions transition from Figure 6d to Figure 6e.

Therefore $\theta > \tilde{\theta}$ increasing γ will have no effect on the Pareto optimal equilibrium. Firms will always coordinate on E_{LM}. However, when $\theta < \tilde{\theta}$ increasing γ could change the equilibrium firms play. When θ is only slightly less than $\tilde{\theta}$ increasing γ will result in the best response functions

[26]When $\theta > 2c$ all consumers buy from Firm A, and all consumers buy from both firms when $\theta > \frac{2c}{c-\beta}$.

changing from Figure 6d to Figure 6e. Thus the firms would switch from coordinating on E_{H2H} to playing E_{LM} as γ increases.

Let $\underline{\theta}$ represent the largest value of θ such that when $\theta < \underline{\theta}$ the firms will always coordinate on E_{H2H} regardless the value of γ. Additionally, let $\tilde{\theta} = \overline{\theta}$ represent the value of θ such that when $\theta > \overline{\theta}$ the firms will always coordinate on E_{LM}. Then when $\underline{\theta} < \theta < \overline{\theta}$ the Pareto optimal equilibrium will depend on the value of γ. These regions will be relevant to the welfare analysis conducted in the next section.

Welfare Analysis

Welfare will be calculated using the function below. The first three terms measure the consumer surplus of residents who buy only from Firm A, those who buy from both firms, and those residents who buy only from Firm B respectively. The last two terms measure the profits collected by Firm A and Firm B. Marginal cost is assumed to be zero so profits are just the price times demand. Tourist's consumer surplus is not included in the welfare function. Since tourists do not reside in the city policy makers are not concerned about tourists' welfare directly. Rather they are interested in the revenues generated by tourists, which are accounted for in firm profits.

$$\begin{aligned} W = &\int_0^{x_1^R} \left[V(1) - cx - p_A\right] f(x)dx + \int_{x_1^R}^{x_2^R} \left[V(2) - c + \beta D_{T,B} - p_A - p_B\right] f(x)dx \\ &+ \int_{x_2^R}^{1} \left[V(1) - c(1-x) + \beta D_{T,B} - p_B\right] f(x)dx \\ &+ p_A \left[F(x_2^T) + F(x_2^R)\right] + p_B \left[2 - F(x_1^T) - F(x_1^R)\right] \end{aligned} \tag{12}$$

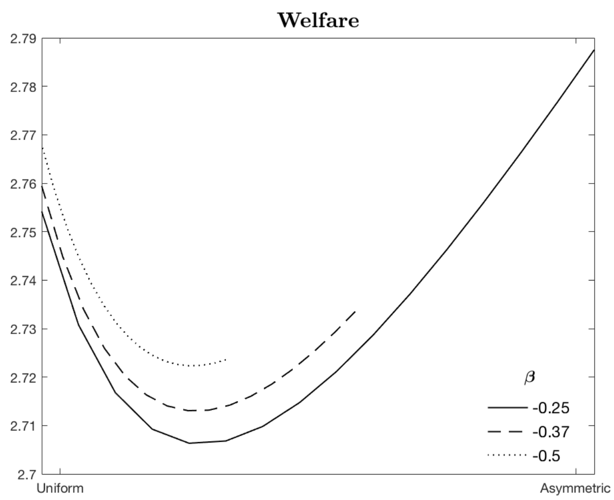
Policy makers have limited ability to control firm prices or determine the optimal location of the indifferent consumers. However, they have more control over the distribution of tourists. By loosening or restricting regulations on the P2P accommodation market policy makers can influence whether the distribution of tourists is more uniform or more asymmetric. The welfare problem then reduces to determining the distribution of tourists that maximizes welfare. Since policy makers are not setting prices, for a given tourist distribution the free market equilibrium prices will be calculated. Then these prices will be plugged into (12) to determine welfare.

The distribution of tourists will be varied by changing the value of $\gamma \in [1, \frac{c}{|\beta|}]$. The range of γ is capped at $\frac{c}{|\beta|}$ because of the condition for supermodularity to hold, which guarantees an equilibrium exists. When γ is large tourists are distributed asymmetrically and clustered near the left end point of the unit interval. As γ is lowered the tourists begin to spread out until they are uniformly distributed across the unit interval, $\gamma = 1$. The results of the welfare analysis can be divided into three cases corresponding to the three cases discussed in the previous section.

Case 1: $\theta < \underline{\theta}$

The first case occurs when $\theta < \underline{\theta}$.[27] The marginal utility of buying from both firms is relatively low, and firms will always price such that they compete head-to-head over both consumer types regardless of the tourist distribution. Figure 7 depicts welfare as a function of the tourist distribution for various levels of β when $\theta = 1$.

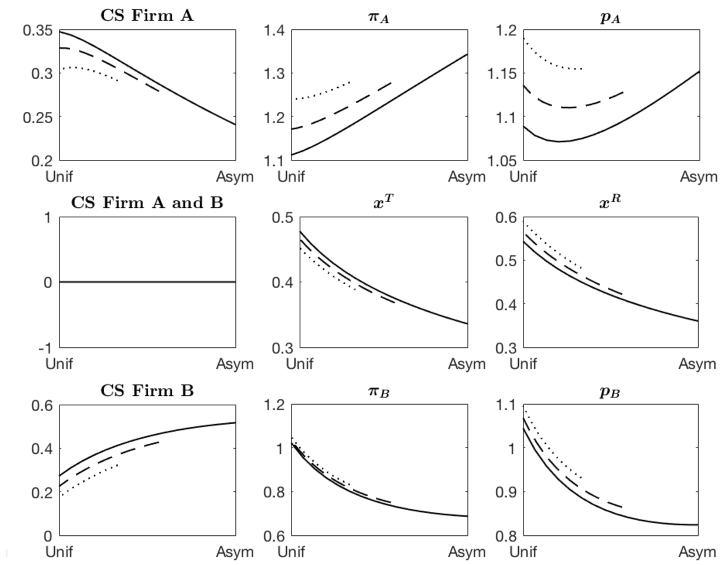
Figure 7



When β is small lowering γ initially decreases welfare, but once the tourists distribution sufficiently close to uniform welfare begins to increase. To understand why the welfare function is convex Figure 8 depicts the individual components of the welfare function as well as the location of the indifferent consumers and firm prices.

[27]The analysis shown in Figure 7 and 8 assumes $\theta = 1$.

Figure 8



As tourists are distributed more uniformly consumer surplus for those subject who buy only from Firm A (CS_A) increases, which is being driven primarily through an increase in demand. Though p_A decreases initially, which contributes to the rise in CS_A. However, p_A eventually begins to rise resulting in individual CS_A to fall, but this is not reflected in the graph of CS_A. Consumer surplus continues to increase. For this result to occur demand for Firm A by residents must be increasing, which is confirmed by the graph of x^R. Since the distribution of residents is fixed an increase (movement right) in x^R implies more residents buy from Firm A.

Conversely, the consumer surplus for residents who buy only from Firm B (CS_B) decreases as tourists are redistributed. The decrease in CS_B is being driven by a decrease in demand as well as an increase in p_B. Since the market is covered, and no consumers are buying from both firms an increase in Firm A's demand by residents means a decrease in demand for Firm B. As few residents buy from Firm B aggregate CS_B falls. The decline in consumer surplus is further exacerbated by an increase in p_B.

For p_B to increase it must be the case that demand increases or the distance between the average consumer who buys from Firm B and Firm B itself decreases.[28] The demand for Firm B

[28] This principle comes from the equation for price using a non-specificed distribution

$$p_B = \frac{2 - F_T(x) - F_R(x)}{f_T(x) + f_R(x) + \frac{\beta}{2c} f_T(x) f_R(x)}$$

. Increasing demand implies the numerator increases. Increasing the location of the average consumer occurs when by residents has already been shown to decrease. Therefore, it must be the case that the demand by tourists is increasing.

Although x^T is increasing, which implies a decrease in demand when the distribution is stationary, the rate at which tourists are being redistributed paces the rate at which the indifferent consumer is increasing. This is evident by the changes in firm profits. The profits for Firm A are decreasing which can partially be explained by the decline of p_A, but profits do not begin to increase when the price increases. Since the demand for Firm A by residents is increasing profits would only continue to decline if the demand by tourists was declining. Furthermore, the demand by tourists must be decreasing faster than the increase in demand by residents, so the net change in demand for Firm A is negative. Again, since the market is covered and no consumer buys from both firms a decrease in demand for Firm A implies an increase in demand for Firm B. Thus p_B will increase. The increase in demand by tourists will also cause CS_B to decline as a result of increased congestion.

It is clear from Figure 8 redistributing tourists has mixed effects on residents' consumer surplus and firm profits. This mixed effect is the source of welfare function's convexity. When γ is large the decrease in Firm A profits and CS_B is larger than the increase in Firm B profits and CS_A, which results in welfare decreasing as tourists are redistributed. Once γ is small enough the gain from Firm B profits and CS_A begins to outweigh the loss in Firm A profits and CS_A.

When the congestion effect is small, β is low, the resulting increase in welfare does not make up for the initial decline. Therefore, welfare is maximized when consumers are distributed asymmetrically. However, if the congestion effect is large enough then the gains in welfare from increased Firm B profits and CS_A outweighs the initial decline, and welfare is maximized when tourists are distributed uniformly.

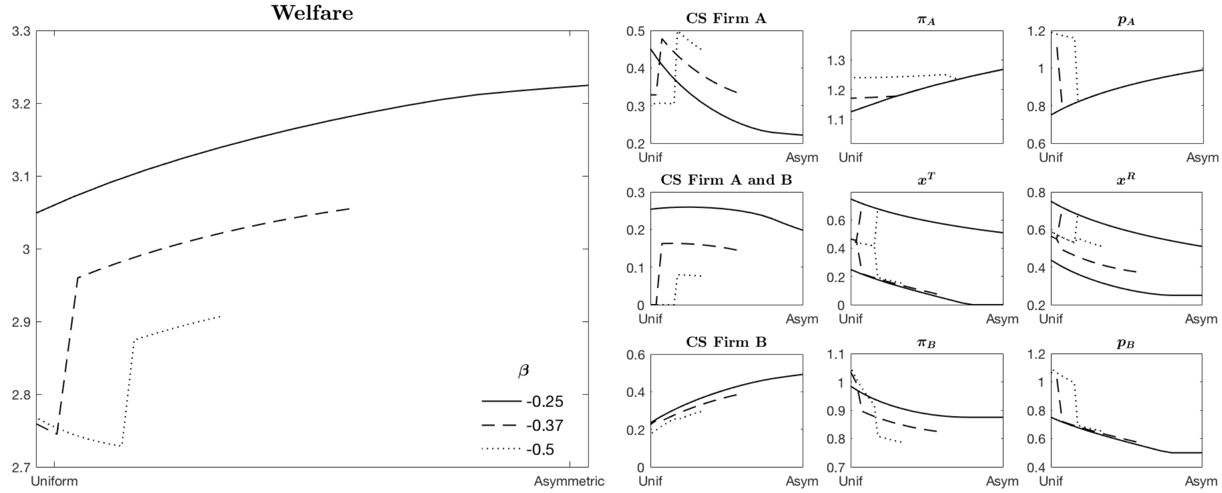
Case 2: $\underline{\theta} < \theta < \overline{\theta}$

The second case occurs when θ falls between the two threshold values.[29] In this case firms have a local monopoly when tourists are distributed asymmetrically, and will switch to competing head-to-head when the tourist distribution becomes sufficiently homogenous. Figure 9a depicts the welfare at differing levels of β. Figure 9b breaks down welfare by its principle components.

$f_T(x)$ for the indifferent consumer decreases. Holding demand constant, decreasing the mass of consumers at the indifferent location means the mass of consumers closer to the firm increases, thus decreasing the average distance between Firm B and its consumers.

[29] The analysis shown in Figure 9 assumes $\theta = 1.5$.

Figure 9



(a) (b)

Similar to the first case, when tourists are distributed asymmetrically, γ is large, the decrease in Firm A profits and CS_B outweigh the gain in welfare from increases in Firm B profits, CS_A, and consumer surplus from those residents who by from both firms (CS_{AB}). For small values of β this relation holds regardless of the tourists distribution, and welfare monotonically decreases. However, once β becomes large enough there exists a discontinuity in welfare, representing the market transitioning from a local monopoly to head-to-head competition, after which welfare increases as tourists become more uniformly distributed.

The decrease in welfare that results from the market switching to head-to-head competition is a result of both the increase in price and decrease in demand. When demand falls and fewer consumers buy from the firms aggregate consumer surplus will fall. Additionally, the increase in price lowers individual consumer surplus, which worsens the decline in consumer surplus.

A price increase in response to a decrease in demand is counterintuitive, but recall both changes in demand and the average distance between the firm and its consumers influence price. If location is thought of as brand loyalty then consumers closer to the firm will be more loyal. Therefore, decreasing the average distance between a firm and consumers is analogous to having more loyal consumers. So even though demand is decreasing the consumers who continue to buy from the firm are more loyal, which allows the firm to charge a higher price.

After the sharp decline welfare begins to increase, which is a result of the increase in Firm

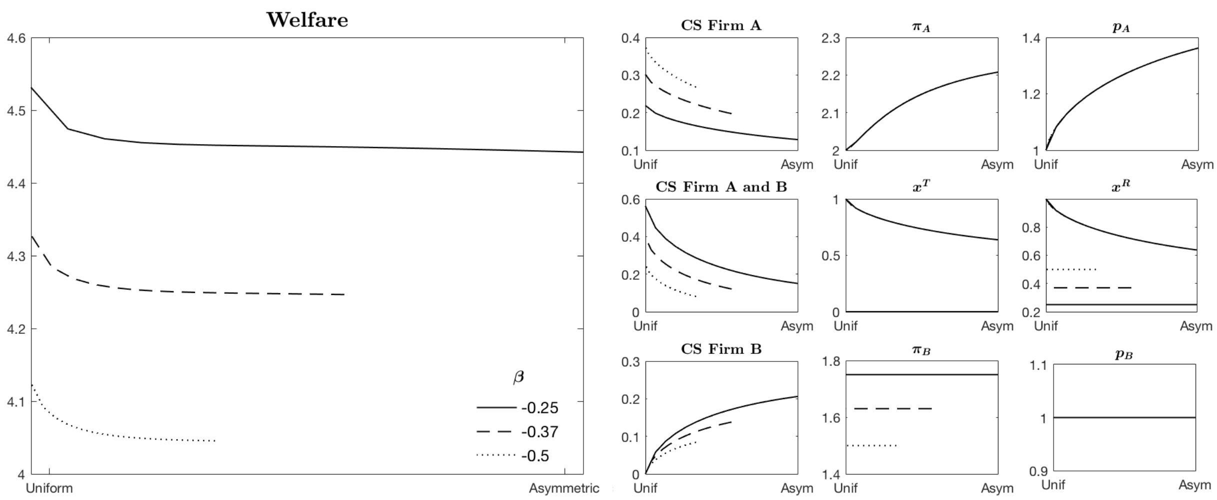
B's profits. CS_A remains relatively constant after the transition, and CS_B continues to decline.[30] So neither consumer surplus contributes to the increase in welfare. Additionally, Firm A's profits continue to decrease even though they receive a slight increase immediately after the transition. Therefore, only the increase in Firm B's profits can be contributed to the increase in welfare.

Even though redistributing the tourists increases welfare after the market transitions the gain is not enough to offset the decline in welfare that resulted from the initial redistribution. As a result welfare will be maximized when tourists are distributed asymmetrically.

Case 3: $\theta > \overline{\theta}$

The third case occurs when θ takes on large values.[31] When this case occurs firms will set prices such that they have a local monopoly over both consumer types regardless of how the tourists are distributed. Figure 10a depicts the welfare function for this case, and 10b presents the principle components of the welfare function.

Figure 10



(a) (b)

Unlike the first two cases welfare is monotonically increasing, which is driven exclusively by gains in CS_A and CS_{AB}. Profits from Firm B are constant since price and demand (x_1^R) are

[30]CS_B can receive a slight increase immediately after the transition. The gain occurs because fewer tourists are buying from Firm B after the transition. Therefore, if the congestion externality is large enough, β is sufficiently large, residents who buy from Firm B receive enough of an increase in utility from the decreased tourist demand that it can offset the decrease in residential demand and higher prices.

[31]The analysis shown in Figure 10 assumes $\theta = 2$.

constant.[32] Additionally, Firm A's profits are declining. Like before, the decline in profits is a result of the decrease in price as well as a decrease in tourist demand.

Consumers who purchase from Firm A benefit from the reduction in price. The demand for Firm A by residents who by exclusively from Firm A is constant, and so a reduction in price will increase aggregate consumer surplus. The decrease in price also raises individual CS_{AB}. Moreover, as Firm A lowers its price more consumers who were buying only from Firm B now find it optimal to buy from both firms. The increase in consumers buying from both firms further raises CS_{AB}.

Since some residents are switching from buying only from Firm B to buying from both firms CS_B will fall. It should be noted individual residents are not made worse off. Since p_B and the tourist demand for Firm B are constant individual CS_B is also constant. The fall in aggregate CS_B is a result of residents switching to buying from both firms.

By switching to buying from both firms a resident receives a net increase in their consumer surplus. This gain coupled with the increase in in consumer surplus from p_A decreasing outweighs the decrease in Firm A's profits. Thus leading to a welfare gain. Therefore, when θ is large welfare is maximized when tourists are uniformly distributed.

Discussion

The results of the analysis reveal the welfare maximizing distribution depend on the marginal benefit to consumers of buying from both firms, θ. When $\theta < \underline{\theta}$ (Case 1) the welfare maximizing distribution also depends on the weight placed on the congestion externality, β. If the congestion externality is sufficiently strong welfare is maximized when tourists are uniformly distributed. Therefore policy makers should not impose regulations on the market for P2P accommodations since a larger supply of accommodations would make it easier for tourists to spread out across the city. However, when the congestion externality is weak welfare is maximized when tourists are kept from spreading out. Therefore, policy makers should implement regulations restricting the adoption of P2P accommodations; thus restricting the growth and use of these accommodations, and slowing the movement of tourists into residential areas.

This result is counter intuitive. One would expect when the congestion externality is large, which means residents are hurt more by tourist consumption, a more asymmetric distribution of

[32]Given the assumed parameter values, Firm B would actually like to set its price lower than $p_B = 1$. However, if it were to do this then $x_1^T < 0$. Therefore, Firm B is located at a corner solution. Therefore, I follow Kim and Serfes (2006) and assume the Firm B sets the price such that $x_1^T = 0$ which implies $p_B = \theta - c$.

tourists would increase welfare not a more uniform distribution. There are two reasons why this intuition does not hold. The first is because the intuition does not take into account the benefit firms receive from redistribution. Regardless of the congestion externality Firm B receives a large increase in profits as a result of increased tourist demand.

The second reason the intuition does not hold is because of a modeling assumption. Resides not only are willing to consume both Firm A and Firm B's products, but also receive the same utility from consuming each product. This allows residents to at least partially offset a loss in consumer surplus from increased congestion by switching to Firm A. In reality, residents may not be willing to consume from both firms, or may also not receive the same utility from the two firms.

One example is a restaurant (Firm A) versus a grocery store (Firm B). Let us analyze the extreme case first where residents do not buy from the restaurant. As more tourists buy from Firm B residents are not able to switch to buying from Firm A. As a result consumer surplus would fall regardless of which product the resident is buying. If the decline in consumer surplus is large enough to outweigh the increase in Firm B's profits welfare could decrease as tourists became more uniformly distributed.

Alternatively, residents could be willing to buy from both firms, but the value the resident receives from purchasing only from Firm A is lower than the value of buying only from Firm B. Since the value of buying from Firm A has gone down the amount of congestion needed to induce a resident to switch from Firm B to Firm A has increased. Therefore, overall consumer surplus will decrease, and again, if the decline is larger than the increase in Firm B's profits welfare would decrease as tourists become more uniformly distributed.

When $\underline{\theta} < \theta < \overline{\theta}$ (Case 2) firms now find it best to price as a local monopoly when tourists are distributed asymmetrically and switch to competing head-to-head when tourists become sufficiently uniformly distributed. When the switch occurs the welfare drops significantly, which is a result of a decrease in consumer surplus and increased prices. Once firms are competing head-to-head welfare rises as tourists are redistributed, but is not able to regain the loss in welfare that occurred at the transition. Therefore, policy makers should implement policies that would restrict the supply of P2P accommodations, and thus the redistribution of tourists.

However, as θ increases along this interval the drop in welfare becomes less. Eventually, the increase in welfare from redistributing tourists when the firms compete head-to-head will outweigh the loss in welfare from the market transition, and any loss in welfare that occurred when the firms had local monopolies. In this situation the welfare maximizing policy would instead be to promote the redistribution of tourists by removing regulations on the supply of P2P accommodations.

Finally, when $\theta > \overline{\theta}$ (Case 3) firms will always choose to price such that they have a local monopoly over both consumer types, and welfare is maximized when tourists are uniformly distributed. Therefore, policy makers should place few regulates on the supply of P2P accommodations.

While the results of this paper are able to provide insight into potential policy recommendations the model should be understood as a first pass. The theory presented in this paper is a significant departure from previous work, and therefore, many simplifying assumptions were made to help gain insight into how the new complexities would affect firm behavior. Relaxing these assumptions would allow for a more complete representation the market for P2P accommodations.

For example, one such assumption is the equal value residence receive from both Firm A and Firm B discussed previously. By relaxing this assumption the model would more realistically represent the tradeoffs residents face when deciding which product to purchase. Another example is the assumption that firm locations are fixed. By relaxing this assumption the model could help provide theoretical insight into another potential problem resulting from P2P accommodations, tourism gentrification.

When locations are fixed Firm B sees profits increase a tourists are redistributed while Firm A experiences a decline in profits. Therefore, if firms were allowed to relocate the results of the fixed location model suggest Firm A would relocate closer to Firm B. The net effect on welfare is unclear since Firm A would see and increase in profits, while Firm B would see a decrease. Furthermore, if Firm A is assumed to have a lower marginal cost than Firm B the increased competition from Firm A could result in Firm B exiting the market.[33] If residents have a lower valuation for Firm A's product then Firm B shutting down could lead to a welfare loss.

The model presented in this paper also assumed no income was received from hosting and the supply of accommodations to be perfectly inelastic. Relaxing either of these assumptions would allow for the model to account for the additional income a resident could receive from listing on a P2P platform, which should increase the welfare of the residents. Furthermore, by allowing residents to decide whether or not to list their property the model provide insight into the tradeoffs residents face when making this decision.

[33]It is not uncommon in the process of tourism gentrification for resdiential firms to have to compete with national chains, which have lower marginal costs than their residential counterparts.

Conclusion

The P2P accommodation market has been shown to have an effect on the hotel industry, but also have an effect on residents of the neighborhoods were P2P accommodations are located. The increase of tourists within these residential areas can result in congestion, which makes it more difficult for residents to conduct their daily activities. This paper extends Kim and Serfes's Hotelling model with bundling to allow for two consumer types, tourists and residents, and for tourists to impose an externality on residents through the consumption of Firm B's product. There are two types of equilibria, head-to-head and local monopoly. Head-to-head is the standard Hotelling equilibrium where firms compete head-to-head, and no consumers buy from both firms. The local monopoly equilibrium occurs when a subset of residents and tourists buy from both firms. A third equilibria, the mixed market, is possible, but is never Pareto optimal. As a result, firms never coordinate on this equilibrium.

Next a welfare analysis was conducted to determine the impact redistributing tourists had on residents and firms. The results were able to be divided into there cases based on the marginal utility of buying the second product. When the marginal utility fell below the lower threshold firms would choose to coordinate on the head-to-head equilibrium regardless of how tourists were distributed. The welfare function was convex, which meant the welfare maximizing distribution would occur either when tourists were distributed uniformly or asymmetrically depending on the strength of the congestion effect.

The second case occurs when the marginal utility fell in between the two threshold values. Firms would coordinate on the local monopoly case when tourists were distributed asymmetrically, but transition to competing head-to-head once tourists were distributed sufficiently uniform. Welfare is maximized when tourists are distributed asymmetrically since there is a drop in welfare at the transition due to higher prices. The third case occurs when the marginal utility is above the upper threshold. In this case firms always coordinate on the local monopoly equilibrium, and welfare is maximized when tourists are distributed uniformly.

The policy implications that can be derived from these results are case specific. When the marginal utility is above the upper threshold few regulations should be put on the supply of P2P accommodations in order to promote the redistribution of tourists. The same policy prescription occurs when the marginal utility is below the lower threshold and the congestion effects are strong. If the congestion effects are weak or the marginal utility falls between the two thresholds the regulators should implement restrictions on the supply of P2P accommodations with the intent to restrict the redistribution of tourists.

While the results provide insight into how the potential externality P2P accommodation market may impose on residents the model does not account for all of the market's complexities. Simplify assumptions needed to be made in order to gain an initial understanding of how the congestion externality would affect firm behavior. Enriching the model by relaxing these assumptions are areas of future research that would help develop a deeper theoretical understanding of effects P2P accommodations have on residents and firms. However, the work presented in this paper is a necessary step along that path, and provides a foundation onto which these and other extensions can build.

Appendix

Proof. $x_1^i \leq b$ and $x_2^i \geq a$ where $i \in T, R$

Suppose $b \leq x_1^i$. By definition the consumer who is located at x_1^i is indifferent between buying exclusively from Firm A and both firms.

$$V(1) - c(x_1^i - a) - p_A = V(2) - c(x_1^i - a) - c(x_1^i - b) - p_A - p_B$$

$$V(1) = V(2) - c(x_1^i - b) - p_B \tag{D1}$$

Recall that a consumer who is located at $x \in (x_1^i, x_2^i)$ strictly prefers buying from both firms.

$$V(2) - c(x - a) - c(x_1^i - b) - p_A - p_B > V(1) - c(x - a) - p_A$$

$$V(1) < V(2) - c(x - b) - p_B \tag{D2}$$

Combining equations D1 and D2 and reducing gives

$$V(2) - c(x_1^i - b) - p_B < V(2) - c(x - b) - p_B$$

$$x < x_1^i$$

This contradicts the assumption $x \in (x_1^i, x_2^i)$. The same methodology used above can be used to show $x_2^i > a$. ■

Supermodulartiy

Proposition. *The game is supermodular as long as* $\gamma \leq \frac{c}{|\beta|}$.

Proof.

The game will be supermodular if the cross price derivatives of the profit functions are positive.

$$f(x_T) = \frac{(1-x_T)^{\gamma-1}}{B(1,\gamma)} = \gamma(1 - x_T)^{\gamma-1} \quad f'(x_T) = -\gamma(\gamma - 1)(1 - x_T)^{\gamma-2}$$

$$f(x_R) = 1 \qquad\qquad f'(x_R) = 0$$

<u>Firm A</u>

$$\pi_A = p_A[F(x_T) + F(x_R)]$$

$$\frac{\partial \pi_A}{\partial p_A} = F(x_T) + F(x_R) + p_A\left[f(x_T)\frac{\partial x_T}{\partial p_A} + f(x_R)\frac{\partial x_R}{\partial p_A}\right]$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = f(x_T)\frac{\partial x_T}{\partial p_B} + f(x_R)\frac{\partial x_R}{\partial p_B} + p_A\left[f'(x_T)\frac{\partial x_T}{\partial p_A}\frac{\partial x_T}{\partial p_B} + f(x_T)\frac{\partial^2 x_T}{\partial p_A \partial p_B} + f'(x_R)\frac{\partial x_R}{\partial p_A}\frac{\partial x_R}{\partial p_B} + f(x_R)\frac{\partial^2 x_R}{\partial p_A \partial p_B}\right]$$

Then the cross price derivative of the profit function will be

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = f(x_T)\frac{\partial x_T}{\partial p_B} + \frac{\partial x_R}{\partial p_B} + p_A\left[f'(x_T)\frac{\partial x_T}{\partial p_A}\frac{\partial x_T}{\partial p_B} + f(x_T)\frac{\partial^2 x_T}{\partial p_A \partial p_B} + \frac{\partial^2 x_R}{\partial p_A \partial p_B}\right]$$

<u>*Case 1*</u>

$$\frac{\partial x_2^T}{\partial p_A} = -\frac{1}{c} \quad \frac{\partial x_2^T}{\partial p_B} = 0 \quad \frac{\partial^2 x_2^T}{\partial p_A \partial p_B} = 0$$

$$\frac{\partial x_2^R}{\partial p_A} = -\frac{1}{c} \quad \frac{\partial x_2^R}{\partial p_B} = 0 \quad \frac{\partial^2 x_2^R}{\partial p_A \partial p_B} = 0$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = f(x_2^T)0 + 0 + p_A[f'(x_2^T)0 + f(x_2^T)0 + 0] = 0$$

Case 2

$$\frac{\partial x_2^T}{\partial p_A} = -\frac{1}{c} \qquad \frac{\partial x_2^T}{\partial p_B} = 0 \qquad \frac{\partial^2 x_2^T}{\partial p_A \partial p_B} = 0$$

$$\frac{\partial \hat{x}^R}{\partial p_A} = \frac{-1+\beta(f(x_1^T)\frac{\partial x_1^T}{\partial p_A})}{2c} \quad \frac{\partial \hat{x}^R}{\partial p_B} = \frac{1+\beta(f(x_1^T)\frac{\partial x_1^T}{\partial p_B})}{2c} \quad \frac{\partial^2 \hat{x}^R}{\partial p_A \partial p_B} = \frac{\beta(f'(x_1^T)\frac{\partial x_1^T}{\partial p_A}\frac{\partial x_1^T}{\partial p_B} + f(x_1^T)\frac{\partial^2 x_1^T}{\partial p_A \partial p_B})}{2c}$$

$$\frac{\partial x_1^T}{\partial p_A} = 0 \qquad \frac{\partial x_1^T}{\partial p_B} = \frac{1}{c} \qquad \frac{\partial^2 x_1^T}{\partial p_A \partial p_B} = 0$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = f(x_2^T)0 + \frac{1+\beta(f(x_1^T)\frac{\partial x_1^T}{\partial p_B})}{2c} + p_A\left[f'(x_2^T)0 + f(x_2^T)0 + \frac{\beta(f'(x_1^T)0 + f(x_1^T)0)}{2c}\right]$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = \frac{1 + \frac{\beta}{c}\gamma(1-x_1^T)^{\gamma-1}}{2c} \geq 0$$

$$\frac{\beta}{c}\gamma(1-x_1^T)^{\gamma-1} \geq -1$$

$$(1-x_1^T)^{\gamma-1} \leq -\frac{c}{\beta\gamma}$$

$$x_1^T \geq 1 - \left(-\frac{c}{\beta\gamma}\right)^{\frac{1}{\gamma-1}}$$

The cross price derivative will be positive if

$$1 - \left(-\frac{c}{\beta\gamma}\right)^{\frac{1}{\gamma-1}} \leq 0$$

$$1 \leq \left(-\frac{c}{\beta\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$1 \leq -\frac{c}{\beta\gamma}$$

$$\gamma \leq -\frac{c}{\beta} \Rightarrow \gamma \leq \frac{c}{|\beta|}$$

Case 3

$$\frac{\partial \hat{x}^T}{\partial p_A} = -\frac{1}{2c} \qquad \frac{\partial \hat{x}^T}{\partial p_B} = \frac{1}{2c} \qquad \frac{\partial^2 x_2^T}{\partial p_A \partial p_B} = 0$$

$$\frac{\partial \hat{x}^R}{\partial p_A} = \frac{-1+\beta(f(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_A})}{2c} \quad \frac{\partial \hat{x}^R}{\partial p_B} = \frac{1+\beta(f(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_B})}{2c} \quad \frac{\partial^2 \hat{x}^T}{\partial p_A \partial p_B} = \frac{\beta(f'(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_A}\frac{\partial \hat{x}^T}{\partial p_B}+f(\hat{x}^T)\frac{\partial^2 \hat{x}^T}{\partial p_A \partial p_B})}{2c}$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = \frac{f(\hat{x}^T)}{2c} + \frac{1+\frac{\beta}{2c}f(\hat{x}^T)}{2c} + p_A\left[-\frac{f'(\hat{x}^T)}{4c^2} + f(\hat{x}^T)0 + \frac{\beta(f'(\hat{x}^T)\frac{1}{4c^2} + f(\hat{x}^T)0)}{2c}\right]$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = \frac{f(\hat{x}^T)}{2c} + \frac{1+\frac{\beta}{2c}f(\hat{x}^T)}{2c} + p_A\left[-\frac{f'(\hat{x}^T)}{4c^2} + \frac{\beta f'(\hat{x}^T)}{8c^3}\right]$$

$$\frac{\partial^2 \pi_A}{\partial p_A \partial p_B} = \frac{f(\hat{x}^T)}{2c} + \frac{1+\frac{\beta}{2c}f(\hat{x}^T)}{2c} + p_A f'(\hat{x}^T)\left[\frac{\beta-2c}{8c^3}\right] \geq 0$$

The first term and third term on the RHS is always positive since $f(x_T) \geq 0$, $f'(x_T) \leq 0$, $c > 0$, and $\beta < 0$. The second term on the RHS will be positive if

$$\frac{1+\frac{\beta}{2c}f(\hat{x}^T)}{2c} \geq 0$$

$$\frac{1+\frac{\beta}{2c}\gamma(1-x_1^T)^{\gamma-1}}{2c} \geq 0$$

$$\frac{\beta}{2c}\gamma(1-x_1^T)^{\gamma-1} \geq -1$$

$$(1-x_1^T)^{\gamma-1} \leq -\frac{2c}{\beta\gamma}$$

$$x_1^T \geq 1 - \left(-\frac{2c}{\beta\gamma}\right)^{\frac{1}{\gamma-1}}$$

Which will always hold if

$$1 - \left(-\frac{2c}{\beta\gamma}\right)^{\frac{1}{\gamma-1}} \leq 0$$

$$1 \leq -\frac{2c}{\beta\gamma}$$

$$\gamma \leq -\frac{2c}{\beta} \Rightarrow \gamma \leq \frac{2c}{|\beta|}$$

Firm B

$$\pi_B = p_B[2 - F(x_T) - F(x_R)]$$

$$\frac{\partial \pi_B}{\partial p_B} = 2 - F(x_T) + F(x_R) - p_B\left[f(x_T)\frac{\partial x_T}{\partial p_B} + f(x_R)\frac{\partial x_R}{\partial p_B}\right]$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = -f(x_T)\frac{\partial x_T}{\partial p_A} - f(x_R)\frac{\partial x_R}{\partial p_A} - p_A\left[f'(x_T)\frac{\partial x_T}{\partial p_B}\frac{\partial x_T}{\partial p_A} + f(x_T)\frac{\partial^2 x_T}{\partial p_B \partial p_A} + f'(x_R)\frac{\partial x_R}{\partial p_B}\frac{\partial x_R}{\partial p_A} + f(x_R)\frac{\partial^2 x_R}{\partial p_B \partial p_A}\right]$$

Then the cross price derivative of the profit function will be

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = -f(x_T)\frac{\partial x_T}{\partial p_A} - \frac{\partial x_R}{\partial p_A} - p_A\left[f'(x_T)\frac{\partial x_T}{\partial p_B}\frac{\partial x_T}{\partial p_A} + f(x_T)\frac{\partial^2 x_T}{\partial p_B \partial p_A} + \frac{\partial^2 x_R}{\partial p_B \partial p_A}\right]$$

<u>*Case 1*</u>

$$\frac{\partial x_1^T}{\partial p_A} = 0 \qquad \frac{\partial x_1^T}{\partial p_B} = \frac{1}{c} \qquad \frac{\partial^2 x_1^T}{\partial p_B \partial p_A} = 0$$

$$\frac{\partial x_1^R}{\partial p_A} = \frac{\beta f(x_1^T)\frac{\partial x_1^T}{\partial p_A}}{c} \quad \frac{\partial \hat{x}^R}{\partial p_B} = \frac{1+\beta f(x_1^T)\frac{\partial x_1^T}{\partial p_B}}{c} \quad \frac{\partial^2 x_1^R}{\partial p_B \partial p_A} = \frac{\beta\left(f'(x_1^T)\frac{\partial x_1^T}{\partial p_A}\frac{\partial x_1^T}{\partial p_B} + f(x_1^T)\frac{\partial^2 x_1^T}{\partial p_B \partial p_A}\right)}{c}$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = -f(x_1^T)0 - \frac{\beta f(x_1^T)0}{c} - p_A\left[f'(x_1^T)0 + f(x_1^T)0 + \frac{\beta\left(f'(x_1^T)0 + f(x_1^T)0\right)}{c}\right]$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = -0 - 0 - p_A\left[0 + 0 + 0\right] = 0$$

<u>*Case 2*</u>

$$\frac{\partial x_1^T}{\partial p_A} = 0 \qquad \frac{\partial x_1^T}{\partial p_B} = \frac{1}{c} \qquad \frac{\partial^2 x_1^T}{\partial p_B \partial p_A} = 0$$

$$\frac{\partial \hat{x}^R}{\partial p_A} = \frac{-1+\beta f(x_1^T)\frac{\partial x_1^T}{\partial p_A}}{2c} \quad \frac{\partial \hat{x}^R}{\partial p_B} = \frac{1+\beta f(x_1^T)\frac{\partial x_1^T}{\partial p_B}}{2c} \quad \frac{\partial^2 \hat{x}^R}{\partial p_B \partial p_A} = \frac{\beta\left(f'(x_1^T)\frac{\partial x_1^T}{\partial p_A}\frac{\partial x_1^T}{\partial p_B} + f(x_1^T)\frac{\partial^2 x_1^T}{\partial p_B \partial p_A}\right)}{2c}$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = -f(x_T)0 - \frac{-1+\beta f(x_1^T)0}{2c} - p_A\left[f'(x_T)0 + f(x_T)0 + \frac{\beta\left(f'(x_1^T)0 + f(x_1^T)0\right)}{2c}\right]$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = \frac{1}{2c} \geq 0$$

<u>*Case 3*</u>

$$\frac{\partial x_1^T}{\partial p_A} = -\frac{1}{2c} \qquad \frac{\partial x_1^T}{\partial p_B} = \frac{1}{2c} \qquad \frac{\partial^2 x_1^T}{\partial p_B \partial p_A} = 0$$

$$\frac{\partial \hat{x}^R}{\partial p_A} = \frac{-1+\beta f(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_A}}{2c} \quad \frac{\partial \hat{x}^R}{\partial p_B} = \frac{1+\beta f(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_B}}{2c} \quad \frac{\partial^2 \hat{x}^R}{\partial p_B \partial p_A} = \frac{\beta\left(f'(\hat{x}^T)\frac{\partial \hat{x}^T}{\partial p_A}\frac{\partial \hat{x}^T}{\partial p_B} + f(\hat{x}^T)\frac{\partial^2 \hat{x}^T}{\partial p_B \partial p_A}\right)}{2c}$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = \frac{f(\hat{x}^T)}{2c} + \frac{1+\frac{\beta}{2c}f(\hat{x}^T)}{2c} - p_A\left[\frac{f'(\hat{x}^T)}{4c^2} + f(\hat{x}^T)0 + \frac{\beta\left(-\frac{f'(\hat{x}^T)}{4c^2} + f(\hat{x}^T)0\right)}{2c}\right]$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = \frac{f(\hat{x}^T)}{2c} + \frac{1 + \frac{\beta}{2c} f(\hat{x}^T)}{2c} + p_A \left[-\frac{f'(\hat{x}^T)}{4c^2} + \frac{\beta f'(\hat{x}^T)}{8c^3} \right]$$

$$\frac{\partial^2 \pi_B}{\partial p_B \partial p_A} = \frac{f(\hat{x}^T)}{2c} + \frac{1 + \frac{\beta}{2c} f(\hat{x}^T)}{2c} + p_A \left[f'(\hat{x}^T) \left(\frac{\beta - 2c}{8c^3} \right) \right] \geq 0$$

The first term and third term on the RHS is always positive since $f(x_T) \geq 0$, $f'(x_T) \leq 0$, $c > 0$, and $\beta < 0$. The second term on the RHS will be positive if The first term and third term on the RHS is always positive since $f(x_T) \geq 0$, $f'(x_T) \leq 0$, $c > 0$, and $\beta < 0$. The second term on the RHS will be positive if

$$\frac{1 + \frac{\beta}{2c} f(\hat{x}^T)}{2c} \geq 0$$

$$\frac{1 + \frac{\beta}{2c} \gamma (1 - x_1^T)^{\gamma - 1}}{2c} \geq 0$$

$$\frac{\beta}{2c} \gamma (1 - x_1^T)^{\gamma - 1} \geq -1$$

$$(1 - x_1^T)^{\gamma - 1} \leq -\frac{2c}{\beta \gamma}$$

$$x_1^T \geq 1 - \left(-\frac{2c}{\beta \gamma} \right)^{\frac{1}{\gamma - 1}}$$

Which will always hold if

$$1 - \left(-\frac{2c}{\beta \gamma} \right)^{\frac{1}{\gamma - 1}} \leq 0$$

$$1 \leq -\frac{2c}{\beta \gamma}$$

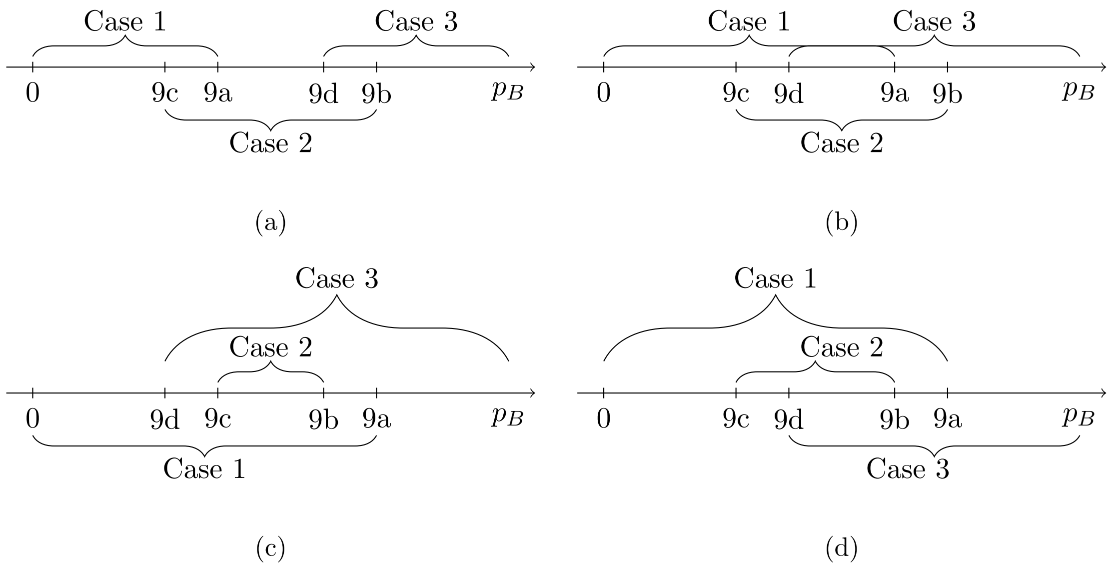
$$\gamma \leq -\frac{2c}{\beta} \Rightarrow \gamma \leq \frac{2c}{|\beta|}$$

The cross price derivatives for π_A and π_B are positive for all cases when $\gamma \leq \frac{c}{|\beta|}$. ■

Alternative Overlapping Bounds

The possible ways in which the regions each case is valid can overlap are depicted in Figure 11. Figure 11a was discussed in the main body of the paper, $thresh_1 \in [9c, 9a]$ and $thresh_2 \in [9d, 9b]$. In the remaining figures Case 2 overlaps entirely with on or more of the other cases. For Figure 11b, like in Figure 11b $thresh_1 \in [9c, 9a]$ and $thresh_2 \in [9d, 9b]$, but now $thresh_1$ maybe greater than $thresh_2$. If $thresh_1 > thresh_2$ then the conditions for Case 2 to be valid will never be met. The best response function will then transition directly from Case 1 to Case 2, and the break will occur at $thresh_3 \in [9d, 9a]$.

Figure 11



For Figure 11c Case 2 falls entirely inside Case 1 and Case 2, and $thresh_1 \in [9c, 9b]$ and $thresh_2 \in [9c, 9b]$. When $p_B \in [9c, thresh_1]$ profits for Case 1 are larger than profits for Case 2 which are larger than profits for Case 3, $\pi_1 > \pi_2 > \pi_3$, assuming $thresh_1 < thresh_2$. Since profits are increasing with respect to p_B profits for Case 1 will be greater than profits for Case 3 when $p_B \in [9d, 9c]$. An analogous argument can be made to show profits for Case 3 will be greater than profits for Case 1 when $p_B \in [9b, 9a]$. Therefore, the bet response function for Figure 11c will have two breaks at $thresh_1$ and $thresh_2$ when $thresh_1 < thresh_2$.

Lastly, in Figure 11d Case 2 overlaps with Case 3 and falls entirely within the region for Case 1, $thresh_1 \in [9c, 9b]$ and $thresh_2 \in [d, 9b]$. If $thresh_1 < thresh_2$ Case 1 will be the best response when $p_B < thresh_1$, Case 2 will be the best response when $thresh_1 < p_B < thresh_2$, and Case 3 will be the best response when $p_B > thresh_2$. Case 3 is the best response when $p_B \in [9b, 9a]$ because profits are increasing with respect to p_B and the profits for Case 3 are larger than Case 2 which are larger than Case 1, $\pi_3 > \pi_2 > \pi_1$, when $p_B \in [thresh_2, 9b]$.

Profit function increases with respect to other firm's price.

Firm A

Case 1

$$\pi_A = p_A\left(1 - \left(1 - \frac{\theta - p_A}{c}\right)^{\gamma} + \frac{\theta - p_A}{c}\right)$$

$$\frac{\partial \pi_A}{\partial p_B} = 0$$

Case 2

$$\pi_A = p_A\left(1 - \left(1 - \frac{\theta - p_A}{c}\right)^{\gamma} + \frac{p_B - p_A + c - \beta D_{T,B}}{2c}\right)$$

$$\frac{\partial \pi_A}{\partial p_B} = p_A\left(\frac{1}{2c}\right) > 0$$

Case 3

$$\pi_A = p_A\left(1 - \left(1 - \frac{p_B - p_A + c}{2c}\right)^{\gamma} + \frac{p_B - p_A + c - \beta D_{T,B}}{2c}\right)$$

$$\frac{\partial \pi_A}{\partial p_B} = p_A\left(\frac{\gamma}{2c}\left(1 - \frac{p_B - p_A + c}{2c}\right)^{\gamma-1} + \frac{1 + \beta(\frac{\gamma}{2c}\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1})}{2c}\right)$$

Assuming $\gamma \geq 1$, the derivative will be positive when the condition for supermodularity holds.

Proof.

$$p_A\left(\frac{\gamma}{2c}\left(1 - \frac{p_B - p_A + c}{2c}\right)^{\gamma-1} + \frac{1 + \beta(\frac{\gamma}{2c}\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1})}{2c}\right) > 0$$

$$\frac{1 + \beta(\frac{\gamma}{2c}\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1})}{2c} > -\frac{\gamma}{2c}\left(1 - \frac{p_B - p_A + c}{2c}\right)^{\gamma-1}$$

$$-\frac{1}{\gamma\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1}} - \frac{\beta}{2c} < 1$$

$$-\beta < 2c\left(1 + \frac{1}{\gamma\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1}}\right)$$

$$-\beta < 2c < 2c\left(1 + \frac{1}{\gamma\left(1 - \frac{p_B-p_A+c}{2c}\right)^{\gamma-1}}\right)$$

For supermodularity to hold $\gamma \leq \frac{c}{|\beta|}$. Additionally, the tourist distribution is assumed to be bounded by the uniform distribution, $\gamma \geq 1$. This assumption in conjunction with the condition for supermodularity implies $|\beta| \leq c$. Therefore when the game is supermodular the derivative will be positive. ■

Firm B

Case 1

$$\pi_B = p_B\left(\left(\frac{\theta - p_B}{c}\right)^{\gamma} + \frac{\theta - p_B + \beta D_{T,B}}{c}\right)$$

$$\frac{\partial \pi_B}{\partial p_A} = 0$$

Case 2

$$\pi_B = p_B\left(\left(\frac{\theta - p_B}{c}\right)^{\gamma} + \frac{p_A - p_B + \beta D_{T,B} + c}{2c}\right)$$

$$\frac{\partial \pi_B}{\partial p_A} = p_B\left(\frac{1}{2c}\right) > 0$$

Case 3

$$\pi_B = p_B\left(\left(\frac{p_A - p_B + c}{2c}\right)^{\gamma} + \frac{p_A - p_B + \beta D_{T,B} + c}{2c}\right)$$

$$\frac{\partial \pi_B}{\partial p_A} = p_B\left(\frac{\gamma}{2c}\left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1} + \frac{1 + \beta(\frac{\gamma}{2c}\left(\frac{p_A-p_B+c}{2c}\right)^{\gamma-1})}{2c}\right)$$

Assuming $\gamma \geq 1$, the derivative will be positive when the condition for supermodularity holds.

Proof.

$$p_B \left(\frac{\gamma}{2c} \left(\frac{p_A - p_B + c}{2c} \right)^{\gamma-1} + \frac{1 + \beta(\frac{\gamma}{2c} \left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1})}{2c} \right) > 0$$

$$\frac{1 + \beta(\frac{\gamma}{2c} \left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1})}{2c} > -\frac{\gamma}{2c} \left(\frac{p_A - p_B + c}{2c} \right)^{\gamma-1}$$

$$-\frac{1}{\gamma \left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1}} - \frac{\beta}{2c} < 1$$

$$-\beta < 2c \left(1 + \frac{1}{\gamma \left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1}} \right)$$

$$-\beta < 2c < 2c \left(1 + \frac{1}{\gamma \left(\frac{p_A - p_B + c}{2c}\right)^{\gamma-1}} \right)$$

For supermodularity to hold $\gamma \leq \frac{c}{|\beta|}$. Additionally, the tourist distribution is assumed to be bounded by the uniform distribution, $\gamma \geq 1$. This assumption in conjunction with the condition for supermodularity implies $|\beta| \leq c$. Therefore when the game is supermodular the derivative will be positive. ■

$$\pi_B = \begin{cases} p_B \left(\left(\frac{\theta - p_B}{c} \right)^{\gamma} + \frac{\theta - p_B + \beta D_{T,B}}{c} \right), & \text{if } p_A + p_B \leq 2\theta - c + \beta D_{T,B} \\ p_B \left(\left(\frac{\theta - p_B}{c} \right)^{\gamma} + \frac{p_A - p_B + \beta D_{T,B} + c}{2c} \right), & \text{if } p_A + p_B \leq 2\theta - c \ \& \\ & \quad p_A + p_B > 2\theta - c + \beta D_{T,B} \\ p_B \left(\left(\frac{p_A - p_B + c}{2c} \right)^{\gamma} + \frac{p_A - p_B + \beta D_{T,B} + c}{2c} \right), & \text{if } p_A + p_B > 2\theta - c. \end{cases} \tag{13}$$

References

(1926). *Village of Euclid v Ambler Reality Co, 272 US 365.*

Anderson, S. P. and Goeree, J. K. (1997). Location, location, location. *Journal of Economic Theory*, 77:102–127.

Arben, A. and Pizam, A. (1977). Some determinants of urban hotel location: The tourists' inclinations. *Journal of Travel Research*, 15(3).

Burtenshaw, D., Bateman, M., and Ashworth, G. (1991). *The European City: A Western Perspective.* Halsted Press.

Choi, K.-H., Jung, J., Ryu, S., Kim, S.-D., and Yoon, S.-M. (2015). The relationship between airbnb and the hotel revenue: In the case of korea. *Indian Journal of Science and Technology*, 8(26).

Cócola-Grant, A. (2015). Tourism and commercial gentrification. In *The ideal city. Between myth and reality*, RC21 Conference, pages 1–25, Urbino.

d'Aspermont, C., Gabszewicz, J. J., and Thisse, J.-F. (1979). On hotelling's "stability in competition". *Econometrica*, 47(5):1145–1150.

Economides, N. (1986). Nash equilibrium in duopoly with products defined by two charateristics. *Rand Journal of Economics*, 17(3):431–439.

Egan, D. J. and Nield, K. (2000). Towards a theory of intraurban hotel location. *Urban Studies*, 37(3).

Elizalde, J. (2013). Competition in multiple characteristics: An empirical test of location equilibrium. *Regional Science and Urban Economics*, 43:938–950.

Grant, J. (2002). Mixed use in theory and practice: Canadian experience with implementing a planning principle. *Journal of the American Planning Association*, 68(1).

Grilo, I., Shy, O., and Thisse, J.-F. (2001). Price competiton when consumer behavior is characterized by conformity or vanity. *Journal of Public Economics*, 80(3).

Gutiérrez, J., García-Palomares, J. C., Romanillos, G., and Salas-Olmedo, M. H. (2017). The eruption of airbnb in tourists cities: Comparing spatial patterns of hotels and peer-to-peer accommodation in barcelona. *Tourism Management*, 62:278–291.

Guttentag, D. (2015). Airbnb: disrupive innovation and the rise of an informal touristm acomodation sector. *Current Issues in Tourism*, 18(12):1192–1217.

Hirt, S. (2012). Mixed use by default: How the europeans (don't) zone. *Journal of Planning Literature*, 27(4).

Hoppenbrouwer, E. and Louw, E. (2005). Mixed-use developmet: Theory and practice in amsterdam's eastern docklands. *European Planning Studies*, 12(7).

Hotelling, H. (1929). Stability in competition. *The Economic Journal*, 39(15):41–57.

Irmen, A. (1998). Competition in multi-characteristics spaces: Hotelling was almost right. *Journal of Economic Theory*, 78:76–102.

Kim, H. and Serfes, K. (2006). A location model with preference for variety. *The Journal of Industrial Economics*, 54(4):569–595.

Lamertini, L. and Orsini, R. (2005). The existence of equilibrium in a differentiated duopoly with network externalities. *The Japanese Economic Review*, 56(1).

Larralde, H., Stehlé, J., and Jensen, P. (2009). Analytical solution of a multi-dimensional hotelling model with quadratic transportation costs. *Regional Science and Urban Economics*, 39:343–349.

Laussel, D., de Montmarin, M., and Long, N. V. (2004). Dynamic duopoly with congestion effects. *International Journal of Industrial Organization*, 22(5).

Milgrom, P. and Roberts, J. (1990). Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica*, 58(6):1255–1277.

Neuts, B. and Nijkamp, P. (2012). Tourist crowding perception and acceptability in cities: An applied modelling study on bruges. *Annals of Tourism Research*, 39(4):2133–2153.

Neven, D. J. (1986). On hotelling's competition with non-uniform customer distributions. *Economic Letters*, 21:121–126.

Russo, A. P. (2002). The "vicious circle" of tourism development in heritage cities. *Annals of Tourism Research*, 29(1):165–182.

Russo, A. P. and Domínguez, A. Q. (2014). La lógica espacial del intercambio de casas: una aproximación a las nuevas geografías de lo cotidiano en el turismo contemporáneo. *Scripta Nova. Revista Electrónica de Geografía y Ciencias Sociales*, 18.

Sans, A. A. and Domínguez, A. Q. (2016). Unravelling airbnb: Urban perspectives from barcelona. In Russo, A. P. and Richards, G., editors, *Reinventing the Local in Tourism*, chapter 13, pages 209–228. Channel View.

Shlay, A. B. and Rossi, P. H. (1981). Keeping up the neighborhood: Estimating net effects of zoning. *American Sociological Review*, 46(6):703–719.

Shoval, N., McKercher, B., Ng, E., and Birenboim, A. (2011). Hotel location and tourist activity in cities. *Annals of Tourism Research*, 38(4):1594–1612.

Stors, N. and Kagermeier, A. (2015). Share economy in metropolitan tourism. the role of authenticity-seeking. In *RSA-Tourism Research Network*.

Tabuchi, T. and Thisse, J.-F. (1995). Asymmetric equilibria in spatial competition. *International Journal of Industrial Organization*, 13(2):213–227.

Topkis, D. M. (1968). Optimal ordering and rationing policies in a nonstationary dynamic inventoy model with n demand classes. *Management Science*, 15(3):160–176.

Walker, H. (1997). Mixed use development as an agent of sustainability. In Coupland, A., editor, *Reclaiming the City: Mixed Use Development*, chapter 3. E. & F.N. Spon.

Zervas, G., Proserpio, D., and Byers, J. W. (2017). The rise of the sharing economy: Esimating the impact of airbnb on the hotel industry. *Journal of Marketing Research*, 54(5):687–705.