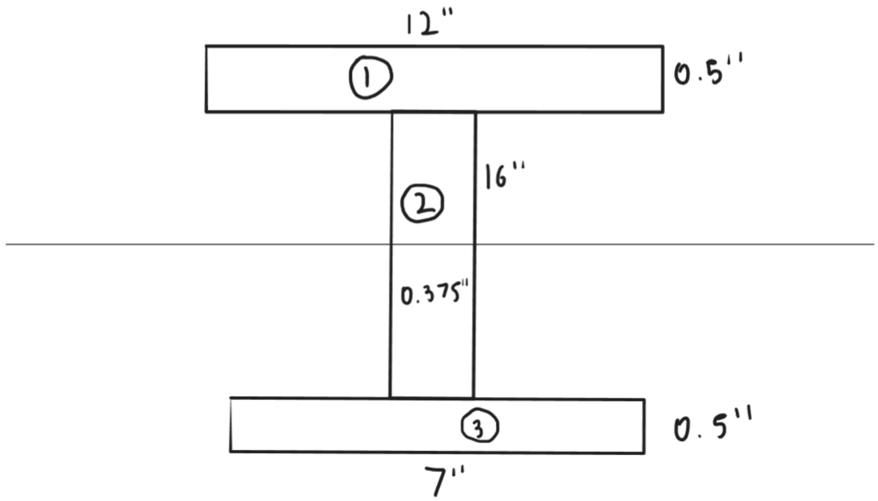
Question 5.2-2



Determine the areas of each section:

$$A_1 = 6\ \text{in}^2 \quad A_2 = 6\ \text{in}^2 \quad A_3 = 3.5\ \text{in}^2$$

Determine the centers of gravity of each section relative to the base:

$$y_1 = 16.75\ \text{in} \quad y_2 = 8.5\ \text{in} \quad y_3 = 0.25\ \text{in}$$

Determine the moments of inertia about the y-axis of each section:

$$I_1 = \frac{1}{12} \times 0.5\ \text{in} \times (7\ \text{in})^3 = 14.29\ \text{in}^4 \quad I_2 = \frac{1}{12} \times 16\ \text{in} \times (0.375\ \text{in})^3 = 0.0703\ \text{in}^4$$

$$I_3 = \frac{1}{12} \times 0.5\ \text{in} \times (12\ \text{in})^3 = 72\ \text{in}^4$$

Now determine the net center of gravity:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = 9.83\ \text{in}$$

Area of net compression zone and area of net tension zone are the same ($7.75\ \text{in}^2$). Now determine the plastic neutral axis:

$$7\ \text{in} \times 0.5\ \text{in} + 0.375\ \text{in} \times (PNA - 0.5\ \text{in}) = 7.75\text{in}^2$$

$$PNA = 11.83\ \text{in}$$

The distance from the top of the shape to the plastic neutral axis is 17 in − 11.83 in which is $\boxed{5.17\ \text{in}}$. Now determine the plastic modulus:

$$S_x = 3.5\ \text{in} \times (11.83\ \text{in} - 0.25\ \text{in}) + 6\ \text{in} \times (11.83\ \text{in} - 8.5\ \text{in}) + 6\ \text{in} \times (5.17\ \text{in} - 0.25\ \text{in}) = 90\ \text{in}^3$$

The plastic moment is determined as follows:

$$M_p = F_y \times S_x = 50\ \text{ksi} \times 90\ \text{in}^3 = \boxed{4500\ \text{kip-in}}$$

Now take the plastic modulus about the minor axis, with I being the sum of the moments of inertia and $\bar{x}$ being 6 inches.

$$Z = \frac{I}{\bar{x}} = \frac{86.4\ \text{in}^4}{6\ \text{in}} = \boxed{14.4\ \text{in}^3}$$

Question 5.5-3

Determine adjusted loads using LRFD.

$$P_y = 1.2D + 1.6L = 1.2 \times 1 \text{ kip/ft} + 1.6 \times 2 \text{ kip/ft} = 4.4 \text{ kip/ft}$$

$$P_u = 1.2D = 1.2 \times 40 \text{ kips} = 48 \text{ kips}$$

Determine the vertical reaction at the left support and use to solve for the moment.

$$\sum M_R = 0 = 48 \times 25 + 4.4 \times \frac{40^2}{2} - R_L \times 40$$

$$R_L = 118 \text{ kips}$$

$$M_u = 118 \times 15 - 4.4 \times 15 \times 7.5 = 1275 \text{ kip-ft}$$

According to Table 3-2, $\phi_b M_{px}$ is 1300 kip-ft, which is larger than the moment, therefore it is $\boxed{\text{adequate}}$.

Question 5.5-12

From Table 3-2, L_p is 6.36 feet, L_r is 17.3 feet. L_b is given as 10 ft. This corresponds with the second case in section F2, therefore equation F2-2 is used.

$$M_n = C_b \left[M_p - (M_p - 0.7F_y \times S_x) \times \left(\frac{L_b - L_p}{L_r - L_p} \right) \right]$$

Since there will be five supports laterally, evenly spaced out, from Table 3-1, C_b is 1.12. From Table 3-2, Z_x is 160 in^3. Therefore, the plastic moment is:

$$M_p = 50 \text{ ksi} \times 160 \text{ in}^3 = 8000 \text{ kip-in}$$

From Table 1-1, S_x is 140 in^3. Now plug everything into equation F2-2 above to obtain 7804.78 kip-in, which is 650.39 kip-ft. The design moment is:

$$M_d = 0.9 \times M_n = 0.9 \times 650.39 \text{ kip-ft} = 585.35 \text{ kip-ft}$$

We know that this moment is equivalent to one eighth of the distributed load multiplied by the length squared.

$$585.35 \text{ kip-ft} = \frac{wL^2}{8} = \frac{50^2 \times w}{8}$$
$$w = 1.87 \text{ kips/ft} = 1.2D + 1.6L$$

We know that the dead load is 68 lbs/ft from Table 1-1. Therefore:

$$1.87 \text{ kips/ft} - 1.2 \times \frac{68 \text{ lbs/ft}}{1000} = 1.6L$$

$$L = \boxed{1.11 \text{ kips/ft}}$$

Question 5.6-4

The first step is to check for compactness:

$$\lambda_p = 0.38 \times \sqrt{\frac{E}{F_y}} = 0.38 \times \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{16 \text{ in}}{2 \times \frac{3}{4} \text{ in}} = 10.67$$

$$\lambda_r = 0.95 \times \sqrt{\frac{k_c \times E}{F_L}}$$

$$k_c = \frac{4}{\sqrt{\frac{h}{t_w}}} = \frac{4}{\sqrt{\frac{40 \text{ in}}{0.5 \text{ in}}}} = 0.4472 \qquad F_L = 0.7F_y = 35 \text{ ksi}$$

$$\lambda_r = 0.95 \times \sqrt{\frac{0.4472 \times 29000 \text{ ksi}}{35 \text{ ksi}}} = 18.29$$

The flange is not compact since $\lambda_p < \lambda < \lambda_r$. The width to thickness ratio of the web is 80. Now check if the web is compact:

$$\lambda_p = 3.76 \times \sqrt{\frac{E}{F_y}} = 3.76 \times \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}} = 90.55$$

Since the width to thickness ratio is less, the web is compact. Now calculate half the area of the entire section:

$$A = 0.75 \text{ in} \times 16 \text{ in} + \frac{1}{2} \times 0.5 \text{ in} \times 40 \text{ in} = 22 \text{ in}^2$$

Now calculate the centroid:

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{12 \text{ in} \times (20 \text{ in} + 3/8 \text{ in}) + 10 \text{ in} \times 10 \text{ in}}{22 \text{ in}^2} = 15.66 \text{ in}$$

$$Z = \frac{A}{2} \times a = \frac{A}{2} \times 2\bar{y} = 344.5 \text{ in}^3$$

The plastic moment is this value multiplied by 50 ksi.

$$M_p = 50 \text{ ksi} \times 344.5 \text{ in}^3 = 17225 \text{ kip-in}$$

Now calculate the moment of inertia about the x-axis.

$$I_x = \frac{1}{12} b \times h^3 + 2\left[\frac{1}{12} b \times h^3 + b \times d^2\right]$$

$$I_x = \frac{1}{12}(0.5 \text{ in}) \times (40 \text{ in})^3 + 2\left[\frac{1}{12}(16 \text{ in}) \times (0.75 \text{ in})^3 + (12 \text{ in})\left(20 \text{ in} + \frac{3}{8} \text{ in}\right)^2\right] = 12631.17 \text{ in}^4$$

Now use the moment of inertia to calculate the section modulus.

$$S_x = \frac{I_x}{c} = \frac{I_x}{\frac{h_w}{2} + t_f} = \frac{12630 \text{ in}^4}{20 \text{ in} + 0.75 \text{ in}} = 608.7 \text{ in}$$

This corresponds with the second case in section F2, therefore equation F2-2 is used. Substituting the calculated plastic moment for M_p, 1 for C_b, the calculated section modulus for S_x, 50 ksi for F_y, 10.67 for λ, 9.15 for λ_p and 18.29 for λ_r.

$$M_n = C_b\left[M_p - (M_p - 0.7F_y \times S_x) \times \left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right] = 18038.76 \text{ kip-in} = \boxed{1503.23 \text{ kip-ft}}$$