PH-214 Homework 4

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Problem 1

$$k = \langle 0, 2, 0 \rangle \boxed{\text{Positive y-direction}}$$

$$\frac{1}{c_0}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} = \frac{\partial^2 \overrightarrow{E}}{\partial y^2}$$

$$\frac{\partial \overrightarrow{E}}{\partial t} = -\omega E_0 \cos(2y - \omega t) \qquad \frac{\partial^2 \overrightarrow{E}}{\partial t^2} = -\omega^2 E_0 \sin(2y - \omega t)$$

$$\frac{\partial \overrightarrow{E}}{\partial y} = 2E_0 \cos(2y - \omega t) \qquad \frac{\partial^2 \overrightarrow{E}}{\partial y^2} = -4E_0 \sin(2y - \omega t)$$

$$\frac{-\omega^2}{c_0} = -4 \quad \rightarrow \quad \omega^2 = 4c_0$$

$$\omega = ck \quad \rightarrow \quad \boxed{c^2k^2 \neq 4c}$$

$$\overrightarrow{E_0} = \langle 1, 0, -1 \rangle \qquad \overrightarrow{B_0} = \left\langle \frac{-\sqrt{2}}{2}, 0, \frac{-\sqrt{2}}{2} \right\rangle$$

$$||\overrightarrow{E_0}|| = \sqrt{2} \qquad \frac{E_0}{c} = B_0 \qquad B_0 = \frac{\sqrt{2}}{c}$$

$$\boxed{\overrightarrow{B} = \left\langle -\frac{1}{c}, 0, \frac{1}{c} \right\rangle \sin(2y - \omega t)}$$

$$\overrightarrow{s} = \frac{1}{\mu_0}\overrightarrow{E} \times \overrightarrow{B} = \boxed{\frac{1}{\mu_0}\langle 0, 6.66 * 10^{-9}, 0 \rangle}$$

Problem 2

$$\nabla \times \overrightarrow{E} = i\overrightarrow{k} \times \overrightarrow{E} =< 0, 3\pi i, 0 > \times \left\langle e^{i(3\pi y - \omega t)}, 0, 0 \right\rangle = \left\langle 0, 0, -3\pi e^{i(3\pi y - \omega t)} \right\rangle$$

$$\overrightarrow{B_0} = \left\langle 0, 0, -\frac{1}{c} \right\rangle \qquad \overrightarrow{B} = -\frac{1}{c}\,\hat{\mathbf{z}}\, e^{i(3\pi y - \omega t)}$$

$$\frac{\partial \overrightarrow{B}}{\partial t} = \frac{1}{c} * -i\omega\,\hat{\mathbf{z}}\, e^{i(3\pi y - \omega t)}$$

Problem 3

$$\sigma_{\text{Ray}} = \sigma_{\text{Th}} \left(\frac{\omega^2}{\omega^2 - \omega_0^2} \right)^2 \qquad \omega_0 >> \omega^2 = \frac{\omega^2}{\omega_0^2}$$

$$\frac{\langle P_{\text{Scat720}} \rangle}{\langle P_{\text{Scat500}} \rangle} = \left(\frac{\omega_{720}}{\omega_{500}} \right)^4 = \left(\frac{720}{500} \right)^4 = \boxed{4.3}$$

Problem 4

$$\phi_E = \frac{1}{4\pi\epsilon_0} * \frac{qdr\cos\theta}{r^3} = \frac{1}{4\pi\epsilon_0} * \frac{1.6 * 10^{-19} * 1 * 10^{-9}}{1} = \boxed{1.43 * 10^{-18}\ \text{V}}$$