

**Question 6.2-2**

From Table 6-2,  $\phi M_n = 552$  kip-ft and  $\phi P_n = 619$  kips. The total load is as follows:

$$1.2D + 1.6L = 44 \text{ kips}$$

Since  $\frac{P}{P_n} < 0.2$ , the following interaction equation can be used.

$$\frac{P}{\phi P_n} + \frac{M}{\phi M_n} = 1$$

$$\frac{44}{619} + \frac{M}{552} = 1$$

$$M = 512.8 \text{ kip-ft} = \frac{wL^2}{8} = \frac{400w}{8}$$

$$w = \boxed{10.3 \text{ kip/ft}}$$

**Question 6.6-2**

Compute the buckling load:

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 1530}{(20 \times 12)^2} = 7602.67 \text{ kips}$$

Calculate the LRFD load (also established in previous question):

$$1.2D + 1.6L = 44 \text{ kips}$$

Now, calculate the moment amplification factor:

$$B_1 = \frac{C_m}{1 - \left(\alpha \frac{P_r}{P_e}\right)}$$

Using LRFD,  $\alpha = 1$  and for transversely loaded members,  $C_m = 1$ .

$$B_1 = \frac{1}{1 - \left(1 \times \frac{44}{1130}\right)} = \boxed{1.04}$$

**Question 6.6-4**

Let's find the design loading and moment by LRFD:

$$1.2 * 0.3 * P + 1.6 * 0.7 * P = 1.2 * 0.3 * 120 + 1.6 * 0.7 * 120 = 177.6 \text{ kips}$$

$$1.2 * 0.3 * M + 1.6 * 0.7 * M = 1.2 * 0.3 * 67 + 1.6 * 0.7 * 67 = 99.16 \text{ kips (bottom)}$$

$$1.2 * 0.3 * M + 1.6 * 0.7 * M = 1.2 * 0.3 * 135 + 1.6 * 0.7 * 135 = 199.8 \text{ kips (top)}$$

From the steel manual, using  $F_y$  as 50 ksi and a length of 16 feet,  $\phi P_n = 499$  kips and  $\phi M_n = 283$  kip-ft. Now calculate the modification factor.

$$C_m = 0.6 - 0.4 \left[ \frac{M_B}{M_T} \right] = 0.6 - 0.4 \left[ \frac{99.16}{199.8} \right] = 0.4$$

Compute the buckling load:

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 475}{(0.9 \times 16 \times 12)^2} = 4553 \text{ kips}$$

Compute the amplification factor:

$$B_1 = \frac{C_m}{1 - \left(\alpha \frac{P_r}{P_e}\right)} = \frac{0.4}{1 - \left(\frac{177.6}{4553}\right)} = 0.42$$

Since it's less than 1, we'll just take the amplification factor as 1. The amplified moment is the same as the moment at the top. Since  $\frac{P}{P_n} > 0.2$  the following equation is used:

$$\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

$$\frac{177.6}{499} + \frac{8}{9} \left( \frac{199.8}{\phi M_{nx}} \right) \leq 1$$

The nominal strength should be equivalent to the plastic moment since the value of  $C_b$  will make the value of the nominal strength greater than that of the plastic moment. From the manual,  $\phi M_p = 324$  kip-ft. Solving below it is seen that the equation is satisfied.

$$\frac{177.6}{499} + \frac{8}{9} \left( \frac{199.8}{324} \right) \leq 1$$

$$0.9 \leq 1$$

## Question 6.6-7

Determine design parameters according to LRFD:

$$P_1 = 1.2 * D + 1.6 * L = 356 \text{ kips}$$

$$P_2 = 1.2 * D + 1.6 * L = 37.2 \text{ kips}$$

$$w = 1.2 * D + 1.6 * L = 7.6 \text{ kips/ft}$$

The moment strength is calculated as follows (using  $L = 16$ ):

$$M = \frac{wL^2}{8} + \frac{P_2L}{4} = 385.6 \text{ kip-ft}$$

Compute the buckling load:

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 716}{(16 \times 12)^2} = 5559 \text{ kips}$$

Compute the amplification factor:

$$B_1 = \frac{C_m}{1 - \left(\alpha \frac{P_r}{P_e}\right)} = \frac{1}{1 - \left(\frac{356}{5559}\right)} = 1.068$$

Now calculate the amplified moment:

$$M_u = B_1 \times 385.6 = 411.9 \text{ kip-ft}$$

Now calculate the following two values:

$$\frac{K_x L}{r_x / r_y} = \frac{16}{1.74} = 9.2 \text{ ft}$$

$$K_y L = 1 \times 8 = 8 \text{ ft}$$

Using 9.2 ft,  $\phi P_n = 1170$  kips.

$$\frac{P_u}{\phi P_n} = \frac{356}{1170} = 0.3$$

Since this value is greater than 0.2, the following equation is used:

$$\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

$$0.3 + \frac{8}{9} \left( \frac{411.8}{511} \right) = 1.02$$

This value is *just* above 1 so it is not satisfactory.

## Question 6.7-1

The first step is to calculate the following two values:

$$\frac{K_{\text{sway}} L}{r_x / r_y} = \frac{1.7 \times 14}{6.05 / 2.48} = 9.75 \text{ ft}$$

$$K_{\text{non-sway}} L = 1 \times 14 = 14 \text{ ft}$$

Using a  $KL$  of 14 feet,  $\phi P_n = 701$  kips. Now calculate the following:

$$\frac{P_u}{\phi P_n} = \frac{400}{701} = 0.5706$$

Since this value is greater than 0.2,  $C_m$  is calculated as follows:

$$C_m = 0.6 - 0.2 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.2 \left( \frac{24}{45} \right) = 0.39$$

Now calculate buckling load:

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 881}{(14 \times 12)^2} = 8934 \text{ kips}$$

Compute the amplification factors:

$$B_{1x} = \frac{C_m}{1 - \left( \alpha \frac{P_u}{P_e} \right)} = \frac{0.39}{1 - \left( \frac{400}{8934} \right)} = 0.405$$

$$B_2 = \frac{C_m}{1 - \left( \alpha \frac{P_u}{P_e} \right)} = \frac{1}{1 - \left( \frac{6000}{40000} \right)}$$

Now calculate the moments at the top and bottom:

$$M_T = 1 \times 45 + 1.176 \times 40 = 92 \text{ kip-ft}$$

$$M_B = 1 \times 24 + 1.176 \times 95 = 135.7 \text{ kip-ft}$$

We know that  $\phi M_p = 479$  kip-ft, and since  $\frac{P_u}{\phi P_n}$  is greater than 0.2, use the following equation:

$$\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

$$0.5706 + \frac{8}{9} \left( \frac{135.7}{479} \right) = 0.82$$

Since the value is less than one, this member satisfies the provisions.

## Question 6.7-2

The first step is to calculate the following two values:

$$\frac{K_x L}{r_x / r_y} = \frac{1.2 \times 16}{2.44} = 7.87 \text{ ft}$$

$$K_y L = 1 \times 16 = 16 \text{ ft}$$

Using a  $KL$  of 16 feet,  $\phi P_n = 698$  kips. Now calculate the following:

$$P_n = 1.2D + 1.6L = 1.2 \times 128 + 1.6 \times 240 = 528 \text{ kips}$$

$$M_T = 1.2D + 1.6L = 1.2 \times 15 + 1.6 \times 40 = 82 \text{ kip-ft}$$

$$M_B = 1.2D + 1.6L = 1.2 \times 18 + 1.6 \times 48 = 98.4 \text{ kip-ft}$$

Now calculate  $C_m$  and the critical buckling load:

$$C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.4 \left( \frac{82}{98.4} \right) = 0.27$$

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 881}{(9.6)^2} = 9467 \text{ kips}$$

Compute the amplification factor:

$$B_1 = \frac{C_m}{1 - \left( \alpha \frac{P_r}{P_c} \right)} = \frac{0.27}{1 - \left( \frac{528}{9467} \right)} = 0.29$$

Since it's less than 1, we'll just take the amplification factor as 1. The amplified moment is the same as the moment at the bottom. Now calculate the following:

$$\frac{P_u}{\phi P_n} = \frac{528}{697} = 0.8$$

The nominal strength should be equivalent to the plastic moment since the value of  $C_b$  will make the value of the nominal strength greater than that of the plastic moment. From the manual,  $\phi M_p = 473$  kip-ft. This is greater than 0.2, so the following is used:

$$\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

$$0.8 + \frac{8}{9} \left( \frac{98.4}{473} \right) = 0.985$$

Now check the braced condition:

$$P_n = 1.2D + 0.5L = 1.2 \times 120 + 0.5 \times 240 = 264 \text{ kips}$$

$$M_T = 1.2D + 0.5L = 1.2 \times 15 + 0.5 \times 40 = 38 \text{ kip-ft}$$

$$M_B = 1.2D + 0.5L = 1.2 \times 18 + 0.5 \times 48 = 45.6 \text{ kip-ft}$$

Now calculate  $C_m$  and the critical buckling load:

$$C_m = 0.6 - 0.4 \left( \frac{M_1}{M_2} \right) = 0.6 - 0.4 \left( \frac{38}{45.6} \right) = 0.27$$

$$P = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 881}{(1.2 \times 16 \times 12)^2} = 4750 \text{ kips}$$

Now compute both amplification factors, accounting for the wind load as 30 kips:

$$B_{1x} = \frac{C_m}{1 - \left( \alpha \frac{P_r + P_w}{P_e} \right)} = \frac{0.27}{1 - \left( \frac{264 + 30}{9467} \right)} = 0.275$$

$$B_2 = \frac{1}{1 - \left( \alpha \frac{P_r}{P_e} \right)} = \frac{1}{1 - \left( \frac{264}{4750} \right)} = 1.059$$

$B_1$  is used as 1. The amplified moment is as follows (accounting for the wind moment as 130 kip-ft):

$$M_u = B_1 M_B + B_2 M_W = 45.6 + 1.059 \times 130 = 183.3 \text{ kip-ft}$$

Since  $\frac{P_u}{\phi P_n}$  is greater than 0.2, use the following equation:

$$\frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

$$0.424 + \frac{8}{9} \left( \frac{183.3}{473} \right) = 0.77$$

The member is adequate.

## Question 6.8-2

Let's start by calculating  $C_b$ . Below is the equation:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_a + 4 M_b + 3 M_c}$$

$M_a$  is the moment at 1/4 of the unbraced length. Since the moment diagram is a straight line, linearization can be used:

$$M_a = M_T - \frac{M_T - M_B}{L} \times L_a = 182 - \frac{182 - 42}{12} \times 9 = 150.5 \text{ kip-ft}$$

Now repeat with  $M_b$  (1/2 unbraced length) and  $M_c$  (3/4 unbraced length).

$$M_b = M_T - \frac{M_T - M_B}{L} \times L_b = 182 - \frac{182 - 42}{12} \times 6 = 161 \text{ kip-ft}$$

$$M_c = M_T - \frac{M_T - M_B}{L} \times L_c = 182 - \frac{182 - 42}{12} \times 3 = 171.5 \text{ kip-ft}$$

We know that the maximum moment is 182 kip-ft, so  $C_b$  can now be calculated:

$$C_b = \frac{12.5 \times 182}{2.5 \times 182 + 3 \times 150.5 + 4 \times 161 + 3 \times 171.5} = 1.1$$

Here are the details we're looking for before I pick my test shape:

$$P_u = 400 \text{ kips}$$

$$M_u = 182 \text{ kip-ft}$$

$$KL = 12 \text{ ft}$$

It seems that the most comfort is with a W10X77. Let's modify this with our  $C_b$ .

$$\phi_b M_{nx} \times C_b = 355 \times 1.1 = 390.5 \text{ kip-ft}$$

Now calculate  $\frac{P_u}{\phi P_n}$  for the interaction equation:

$$\begin{aligned}\frac{P_u}{\phi P_n} &= \frac{400}{816} \\ \frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) &\leq 1 \\ 0.49 + \frac{8}{9} \left( \frac{182}{390.5} \right) &= 0.9\end{aligned}$$

I'm happy with this, but I will check the lighter shape (W10X68) just in case.

$$\begin{aligned}\phi_b M_{nx} \times C_b &= 309 \times 1.1 = 339.9 \text{ kip-ft} \\ \frac{P_u}{\phi P_n} &= \frac{400}{564} \\ \frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) &\leq 1 \\ 0.71 + \frac{8}{9} \left( \frac{182}{339.9} \right) &= 1.18\end{aligned}$$

Well I'm satisfied, W10X77 is my section of choice.

## Question 6.8-8

The ultimate moment is 300 kip-ft and the ultimate load is 75 kips. Looking at Table 6-2, W12X65 makes the most sense. Let's calculate the critical buckling load.

$$\begin{aligned}P_{sway} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 553}{(2 \times 16 \times 12)^2} = 1035.7 \text{ kips} \\ P_{brace} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 29000 \times 553}{(1 \times 16 \times 12)^2} = 4136.3 \text{ kips}\end{aligned}$$

Now calculate the amplification factors:

$$\begin{aligned}B_{1x} &= \frac{C_m}{1 - \left( \alpha \frac{P_r + P_w}{P_e} \right)} = \frac{0.6}{1 - \left( \frac{75}{4136.3} \right)} = 0.611 \\ B_2 &= \frac{1}{1 - \left( \alpha \frac{P_r}{P_e} \right)} = \frac{1}{1 - \left( \frac{75}{1035.7} \right)} = 1.078\end{aligned}$$

Now calculate the amplified moment:

$$M_u = 1 \times 270 + 1.078 \times 30 = 302.34 \text{ kip-ft}$$

Now calculate  $\frac{P_u}{\phi P_n}$  for the interaction equation:

$$\begin{aligned}\frac{P_u}{\phi P_n} &= \frac{300}{640} \\ \frac{P}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi M_{nx} \times C_b} + \frac{M_{uy}}{\phi M_{ny}} \right) &\leq 1 \\ 0.47 + \frac{8}{9} \left( \frac{302.34}{334 \times 1.67} \right) &= 0.95\end{aligned}$$

The section W12X65 is my section of choice.