

PH-214 Cheat Sheet

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Constants

$$\begin{aligned}\mu_0 &= 12.57 * 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}} & \varepsilon_0 &= 8.85 * 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} & m_{\text{Proton}} &= 1.67 * 10^{-27} \text{ kg} \\ m_{\text{Electron}} &= 9.11 * 10^{-31} \text{ kg} & q &= 1.60 * 10^{-19} \text{ C} & \sigma &= 5.67 * 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}\end{aligned}$$

Vector Derivatives

$$\begin{aligned}\text{Gradient: } \nabla t &= \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}} & \text{Divergence: } \nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} & \text{Laplacian: } \nabla^2 t &= \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \\ \text{Curl: } \nabla \times \mathbf{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}}\end{aligned}$$

Vector Identities

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Maxwell's Equations

Integral Form	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$	$\oint \vec{B} \cdot d\vec{A} = 0$	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt}$	$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} + \mu_0 i_{\text{enc}}$
Differential Form	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	$\nabla \cdot \vec{B} = 0$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j}$

Wave Equations for \vec{E} and \vec{B}

$$\begin{aligned}\vec{E} &= \vec{E}_0 e^{i(k \cdot \mathbf{r} - \omega t)} = \vec{E}_0 \cos(\omega t - kx) & \vec{B} &= \vec{B}_0 e^{i(k \cdot \mathbf{r} - \omega t)} = \vec{B}_0 \cos(\omega t - kx) \\ \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} &= \frac{\partial^2 \vec{E}}{\partial x^2} & \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} &= \frac{\partial^2 \vec{B}}{\partial x^2} & \omega &= ck & \vec{B} &= \frac{1}{c} \hat{\mathbf{k}} \times \vec{E} \\ \nabla \cdot [\] &= i \vec{k} \cdot [\] & \nabla \times [\] &= i \vec{k} \times [\] & \frac{\partial}{\partial t} [\] &= i \omega [\] & \frac{\partial^2}{\partial t^2} [\] &= -\omega^2 [\]\end{aligned}$$

Energy Density

$$u_E = \frac{1}{2} \varepsilon_0 |\vec{E}|^2 \quad u_B = \frac{1}{2 \mu_0} \varepsilon_0 |\vec{B}|^2 \quad u_T = \varepsilon_0 |\vec{E}|^2 = \frac{1}{\mu_0} |\vec{B}|^2$$

Radiation

$$\begin{aligned}\vec{s} &= \frac{1}{\mu_0} \vec{E} \times \vec{B} & \frac{E}{B} &= c & P &= \frac{q^2 a^2}{6 \pi \varepsilon_0 c^3} & \sigma_{\text{Th}} &= \frac{8 \pi}{3} \left(\frac{q^2}{4 \pi \varepsilon_0 m_e c^2} \right)^2 & \langle P \rangle &= \frac{q^4 E_0^2}{12 \pi \varepsilon_0 m^2 c^3} \\ E_\theta &= \frac{a \sin \theta q}{4 \pi c^2 \varepsilon_0 R} & E_R &= \frac{a T \sin \theta q}{4 \pi c \varepsilon_0 R^2} & \sigma_{\text{Ray}} &= \sigma_{\text{Th}} \left(\frac{\omega^2}{\omega^2 - \omega_0^2} \right)^2\end{aligned}$$

Potential for a Dipole

$$\phi_E = \frac{1}{4 \pi \varepsilon_0} * \frac{q dr \cos \theta}{r^3}$$

Electric and Magnetic Fields in Materials

$$\begin{aligned}\vec{D} &= \varepsilon_0(1 + \chi_e)\vec{E} = \varepsilon\vec{E} & \varepsilon &= \varepsilon_0(1 + \chi_e) = \varepsilon_0\varepsilon_r & \varepsilon_r &= \frac{\varepsilon_m}{\varepsilon_0} \\ \vec{B} &= \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H} & \mu &= \mu_0(1 + \chi_m) = \mu_0\mu_r & \mu_r &= \frac{\mu_m}{\mu_0}\end{aligned}$$

Maxwell's Equations in Materials

$$\boxed{\nabla \cdot \vec{D} = 0 \quad \nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}}$$

Boundary Conditions

$$E_1^{\parallel} = E_2^{\parallel} \quad \varepsilon_1 \quad E_1^{\perp} = \varepsilon_2 E_2^{\perp} \quad B_1^{\perp} = B_2^{\perp} \quad \frac{B_1^{\parallel}}{\mu_1} = \frac{B_2^{\parallel}}{\mu_2}$$

Wave Impedance

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \quad E = ZH \quad n = \frac{c}{v} \quad c^2 = \frac{1}{\mu_0\varepsilon_0} \quad v^2 = \frac{1}{\varepsilon\mu}$$

Separations Between Media

$$\text{Snell's Law: } n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

P Polarization	S Polarization
\vec{E} is in the incidence plane.	\vec{E} is perpendicular to the incidence plane.
$\frac{E_{0r}}{E_{0i}} = \frac{\left(\frac{n_2}{n_1}\right)^2 \cos(\theta_i) - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}{\left(\frac{n_2}{n_1}\right)^2 \cos(\theta_i) + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}$	$\frac{E_{0r}}{E_{0i}} = -\frac{\cos(\theta_i) - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}{\cos(\theta_i) + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}$
$\frac{E_{0t}}{E_{0i}} = \frac{n_1}{n_2} \left(1 + \frac{E_{0r}}{E_{0i}}\right)$	$\frac{E_{0t}}{E_{0i}} = 1 - \frac{E_{0r}}{E_{0i}}$

$$\text{Reflection Coefficient: } R = \frac{|E_{0r}|^2}{|E_{0i}|^2} \quad \text{Transmission Coefficient: } T = \frac{|E_{0t}|^2}{|E_{0i}|^2} \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)}$$

$$\text{Brewster Angle (P waves with no reflection): } \theta_B = \arctan\left(\frac{n_2}{n_1}\right)$$

$$\text{Total internal reflection: } k = k_R + ik_I \quad \theta_{\text{crit}} = \arcsin\left(\frac{n_2}{n_1}\right) \quad \text{Factor changing the amplitude: } e^{\frac{z}{z_0}} \quad \text{e-folding length: } z_0$$

Normal Incidence

$$\frac{E_{0t}}{E_{0i}} = \frac{n_2 - n_1}{n_2 + n_1} \quad \frac{E_{0r}}{E_{0i}} = \frac{2n_1}{n_2 + n_1}$$

Conductors

$$\boxed{\nabla \cdot \varepsilon \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \frac{\vec{B}}{\mu} = \frac{\partial(\varepsilon \vec{E})}{\partial t} + \sigma_c \vec{E}}$$

Quantum Mechanics

$$I = \sigma T^4 \quad \lambda_{\text{max}} = \frac{2.898 * 10^{-3}}{T}$$