PH-214 Cheat Sheet

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Constants

$$\mu_0 = 12.57 * 10^7 \, \frac{\mathrm{T \cdot m}}{\mathrm{A}} \qquad \varepsilon_0 = 8.85 * 10^{-12} \frac{\mathrm{C}^2}{\mathrm{N \cdot m^2}} \qquad m_{\text{Proton}} = 1.67 * 10^{-27} \text{ kg}$$

$$m_{\text{Electron}} = 9.11 * 10^{-}31 \text{ kg} \qquad q = 1.60 * 10^{-19} \text{ C} \qquad \sigma = 5.67 * 10^{-8} \mathrm{Wm^{-2}K^{-4}}$$

Vector Derivatives

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$ Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 z}{\partial z^2}$

$$\text{Curl: } \nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$$

Vector Identities

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}) \qquad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Maxwell's Equations

Integral Form	$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$	$\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0$	$\oint \overrightarrow{E} \cdot d\overrightarrow{s} = -\frac{d\phi_B}{dt}$	$\oint \overrightarrow{B} \cdot d\overrightarrow{s} = \mu_0\varepsilon_0 \frac{d\phi_B}{dt} + \mu_0 i_{\text{enc}}$
Differential Form	$\nabla \cdot \overrightarrow{E} = 0$	$\nabla \cdot \overrightarrow{B} = 0$	$\nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$	$\nabla \times \overrightarrow{B} = \mu_0\varepsilon_0 \frac{\partial \overrightarrow{E}}{\partial t}$

Wave Equations for $\overrightarrow{E}$ and $\overrightarrow{B}$

$$\overrightarrow{E} = \overrightarrow{E_0} e^{i(k\cdot r - \omega t)} = \overrightarrow{E_0}\cos(\omega t - kx) \qquad \overrightarrow{B} = \overrightarrow{B_0} e^{i(k\cdot r - \omega t)} = \overrightarrow{B_0}\cos(\omega t - kx)$$

$$\frac{1}{c_0^2}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} = \frac{\partial^2 \overrightarrow{E}}{\partial x^2} \qquad \frac{1}{c_0^2}\frac{\partial^2 \overrightarrow{B}}{\partial t^2} = \frac{\partial^2 \overrightarrow{B}}{\partial x^2} \qquad \omega = ck \qquad \overrightarrow{B} = \frac{1}{c}\hat{k} \times \overrightarrow{E}$$

$$\nabla \cdot [\,] = i\overrightarrow{k} \cdot [\,] \qquad \nabla \times = i\overrightarrow{k} \times [\,] \qquad \frac{\partial}{\partial t}[\,] = i\omega * [\,] \qquad \frac{\partial^2}{\partial t^2}[\,] = -\omega^2 * [\,]$$

Energy Density

$$u_E = \frac{1}{2}\varepsilon_0 \left|\overrightarrow{E}\right|^2 \qquad u_B = \frac{1}{2\mu_0}\varepsilon_0 \left|\overrightarrow{B}\right|^2 \qquad u_T = \varepsilon_0 \left|\overrightarrow{E}\right|^2 = \frac{1}{\mu_0}\left|\overrightarrow{B}\right|^2$$

Radiation

$$\overrightarrow{s} = \frac{1}{\mu_0}\overrightarrow{E} \times \overrightarrow{B} \qquad \frac{E}{B} = c \qquad P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} \qquad \sigma_{\text{Th}} = \frac{8\pi}{3}\left(\frac{q^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 \qquad \langle P \rangle = \frac{q^4 E_0^2}{12\pi\varepsilon_0 m^2 c^3}$$

$$E_\theta = \frac{a \sin\theta \, q}{4\pi \, c^2 \varepsilon_0 R} \qquad E_R = \frac{a\,T \sin\theta \, q}{4\pi c\,\varepsilon_0 R^2} \qquad \sigma_{\text{Ray}} = \sigma_{\text{Th}}\left(\frac{\omega^2}{\omega^2 - \omega_0^2}\right)^2$$

Potential for a Dipole

$$\phi_E = \frac{1}{4\pi\varepsilon_0} * \frac{qdr\cos\theta}{r^3}$$

Electric and Magnetic Fields in Materials

$$\vec{D} = \varepsilon_0(1+\chi_e)\vec{E} = \varepsilon\vec{E} \qquad \varepsilon = \varepsilon_0(1+\chi_e) = \varepsilon_0\varepsilon_r \qquad \varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0}$$

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu\vec{H} \qquad \mu = \mu_0(1+\chi_m) = \mu_0\mu_r \qquad \mu_r = \frac{\mu_m}{\mu_0}$$

Maxwell's Equations in Materials

$$\boxed{\nabla\cdot\vec{D} = 0 \quad \nabla\cdot\vec{B} = 0 \quad \nabla\times\vec{E} = -\tfrac{\partial\vec{B}}{\partial t} \quad \nabla\times\vec{H} = \tfrac{\partial\vec{D}}{\partial t}}$$

Boundary Conditions

$$E_1^{||} = E_2^{||} \qquad \varepsilon_1 \qquad E_1^{\perp} = \varepsilon_2 E_2^{\perp} \qquad B_1^{\perp} = B_2^{\perp} \qquad \frac{B_1^{||}}{\mu_1} = \frac{B_2^{||}}{\mu_2}$$

Wave Impedance

$$Z = \sqrt{\frac{\mu}{\varepsilon}} \qquad E = ZH \qquad n = \frac{c}{v} \qquad c^2 = \frac{1}{\mu_0\varepsilon_0} \qquad v^2 = \frac{1}{\varepsilon\mu}$$

Separations Between Media

Snell's Law: $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

P Polarization	**S Polarization**
$\vec{E}$ is in the incidence plane.	$\vec{E}$ is perpendicular to the incidence plane.
$\frac{E_{0r}}{E_{0i}} = \frac{\left(\frac{n_2}{n_1}\right)^2\cos(\theta_i) - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}{\left(\frac{n_2}{n_1}\right)^2\cos(\theta_i) + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}$	$\frac{E_{0r}}{E_{0i}} = -\frac{\cos(\theta_i) - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}{\cos(\theta_i) + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2(\theta_i)}}$
$\frac{E_{0t}}{E_{0i}} = \frac{n_1}{n_2}\left(1 + \frac{E_{0r}}{E_{0i}}\right)$	$\frac{E_{0t}}{E_{0i}} = 1 - \frac{E_{0r}}{E_{0i}}$

Reflection Coefficient: $R = \dfrac{|E_{0r}|^2}{|E_{0i}|^2}$ Transmission Coefficient: $T = \dfrac{|E_{0t}|^2}{|E_{0i}|^2}\dfrac{n_2\cos(\theta_2)}{n_1\cos(\theta_1)}$

Brewster Angle (P waves with no reflection): $\theta_B = \arctan\left(\dfrac{n_2}{n_1}\right)$

Total internal reflection: $k = k_R + ik_I$ $\theta_{\text{crit}} = \arcsin\left(\dfrac{n_2}{n_1}\right)$ Factor changing the amplitude: $e^{\frac{z}{z_0}}$

e-folding length: z_0

Normal Incidence

$$\frac{E_{0t}}{E_{0i}} = \frac{n_2 - n_1}{n_2 + n_1} \qquad \frac{E_{0r}}{E_{0i}} = \frac{2n_1}{n_2 + n_1}$$

Conductors

$$\boxed{\nabla\cdot\varepsilon\vec{E} = 0 \quad \nabla\cdot\vec{B} = 0 \quad \nabla\times\vec{E} = -\tfrac{\partial\vec{B}}{\partial t} \quad \nabla\times\tfrac{\vec{B}}{\mu} = \tfrac{\partial(\varepsilon\vec{E})}{\partial t} + \sigma_c\vec{E}}$$

Quantum Mechanics

$$I = \sigma T^4 \qquad \lambda_{\max} = \frac{2.898 * 10^{-3}}{T}$$