

**Question 4.3-6**

The first step is to calculate the slenderness ratio using the following formula:

$$\frac{K \times L}{r}$$

Using Table 1-1,  $r$  is found to be 2.69 inches. Since one end is fixed and the other end is pinned,  $K$  is taken as 0.8. Therefore the slenderness ratio is:

$$\frac{0.8 \times 12 \text{ ft} \times 12}{2.69 \text{ in}} = 42.82$$

The buckling stress is calculated using the following formula with 29,000 ksi for  $E$ :

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{42.82^2} = 156.1 \text{ ksi}$$

The next equation that needs to be determined to decide whether to use equation E3-2 or E3-3 is:

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \times \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}} = 113.4$$

Which is greater than  $KL/r$ . Therefore, equation E3-2 is used:

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{50 \text{ ksi}}{156.1 \text{ ksi}}}\right] (50 \text{ ksi}) = 43.72 \text{ ksi}$$

The nominal strength is calculated using the formula below, with  $A_g$  being found as 35.1 in<sup>2</sup> in Table 1-1:

$$P_n = F_{cr} \times A_g = 43.72 \text{ ksi} \times 35.1 \text{ in}^2 = 1534.8 \text{ kips}$$

The design strength for LRFD is calculated by multiplying this by 0.9 (yielding equation):

$$P_u = 0.9 \times 1534.8 \text{ kips} = \boxed{1381.3 \text{ kips}}$$

Using Table 4-14, the critical stress for the member is 39.3 kips. The design strength is calculated by multiplying that number by  $A_g$ .

$$P_u = 39.3 \text{ kips} \times 35.1 \text{ in}^2 = \boxed{1379.43 \text{ kips}}$$

**Question 4.4-1**

The first step is to calculate the slenderness ratio using the following formula:

$$\frac{K \times L}{r}$$

Using Table 1-1,  $r$  is found to be 3.28 inches. Since one end is fixed-free and the other end is pinned,  $K$  is taken as 2. Therefore the slenderness ratio is:

$$\frac{2 \times 12 \text{ ft} \times 12}{3.28 \text{ in}} = 87.8$$

The buckling stress is calculated using the following formula with 29,000 for  $E$ :

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{87.8^2} = 156.1 \text{ ksi} = 37.13 \text{ ksi}$$

The next equation that needs to be determined to decide whether to use equation E3-2 or E3-3 is:

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \times \sqrt{\frac{29000 \text{ ksi}}{46 \text{ ksi}}} = 118.26$$

Which is greater than  $KL/r$ . Therefore, equation E3-2 is used:

$$F_c r = \left[0.658 \frac{F_y}{F_e}\right] F_y = \left[0.658 \frac{46 \text{ ksi}}{37.13 \text{ ksi}}\right] (46 \text{ ksi}) = 27.36 \text{ ksi}$$

The nominal strength for LRFD is calculated by multiplying this by  $A_g$ , which was found to be 6.06 in<sup>2</sup>.

$$P_n = 27.36 \text{ ksi} \times 6.06 \text{ in}^2$$

The next step is to determine  $\frac{b}{t}$  which is 43 from Table 1-11. Next is to determine the following:

$$1.4 \sqrt{\frac{E}{f}} = 1.4 \sqrt{29000 \text{ ksi}} \times 46 \text{ ksi} = 35.15$$

Where  $f$  is 46 ksi and  $E$  is 29,000 ksi. This value is less than  $\frac{b}{t}$  which means equation E7-18 can be used to determine the effective width with the  $t$  being determined at 0.174 in from Table 1-11:

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{\frac{b}{t}} \sqrt{\frac{E}{f}}\right] = 6.527 \text{ in}$$

The loss in area of each of the two base sides is calculated as follows:

$$2 \times (8 \text{ in} - 2 \times 1.5 \times 0.174 \text{ in} - 6.527 \text{ in}) \times (0.174 \text{ in}) = 0.33 \text{ in}^2$$

Next, the effective width is calculated for the height, with  $\frac{h}{t}$  being determined to be 54.5 from Table 1-11:

$$b_e = 1.92t \sqrt{\frac{E}{f}} \left[1 - \frac{0.38}{\frac{h}{t}} \sqrt{\frac{E}{f}}\right] = 6.527 \text{ in} = 6.92 \text{ in}$$

The loss in area of each of the two base sides is calculated as follows:

$$2 \times (10 \text{ in} - 2 \times 1.5 \times 0.174 \text{ in} - 6.92 \text{ in}) \times (0.174 \text{ in}) = 0.89 \text{ in}^2$$

The gross area was found to be 6.06 in<sup>2</sup> from Table 1-11. The effective area is calculated as follows:

$$A_e = 6.06 \text{ in}^2 - 0.33 \text{ in}^2 - 0.89 \text{ in}^2 = 4.84 \text{ in}^2$$

$Q$  is taken as the ratio of net effective area to area:

$$Q = \frac{A_e}{A} = \frac{4.84 \text{ in}^2}{6.06 \text{ in}^2} = 0.81$$

Now, it must be determined whether to use equation E7-2 or E7-3. The following quantity is calculated:

$$4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \times \sqrt{\frac{29000 \text{ ksi}}{0.81 \times 46 \text{ ksi}}} = 131.6$$

Which is still greater than the slenderness ratio, therefore, equation E7-2 is used:

$$F_{cr} = Q \left[ 0.658^{\frac{QF_y}{F_e}} \right] F_y = 0.81 \left[ 0.658^{\frac{0.81 \times 46 \text{ ksi}}{37.13 \text{ ksi}}} \right] (46 \text{ ksi}) = 30.22 \text{ ksi}$$

The nominal strength for LRFD is calculated by multiplying this by  $A_g$ .

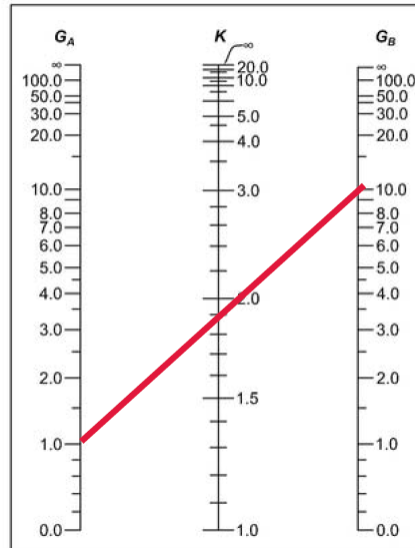
$$P_n = 30.22 \text{ ksi} \times 6.06 \text{ in}^2 = \boxed{183 \text{ kips}}$$

### Question 4.7-9

The moment of inertia for W18X50 is  $800 \text{ in}^4$ . The moment of inertia for W18X97 is  $1750 \text{ in}^4$ . The moment of inertia for W18X130 is  $2460 \text{ in}^4$ . Since joint A is fixed, the stiffness ratio is 1. The stiffness ratio for joint B is calculated using the following equation:

$$G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{L_g}} = \frac{\frac{1750 \text{ in}^4}{13 \text{ in}} + \frac{2460 \text{ in}^4}{13 \text{ in}}}{\frac{800 \text{ in}^4}{25 \text{ in}}} = 10.1$$

Using the alignment chart as depicted below, the effective length factor is 1.9.



Next, the factored load is calculated using the following equation for LRFD.

$$P_u = 1.2D + 1.6L = 1.2 \times 204 \text{ kips} + 1.6 \times 408 \text{ kips} = 897.6 \text{ kips}$$

Stress is taken as force over area ( $38.2 \text{ in}^2$  from Table 1-1):

$$\sigma = \frac{P_u}{A_g} = \frac{897.6 \text{ kips}}{38.2 \text{ in}^2} = 23.5 \text{ ksi}$$

Using Table 4-21, the stiffness reduction factor is as follows:

$$\frac{0.934 + 0.913}{2} = 0.924$$

Multiplying this by the original calculated stiffness for B the following is obtained:

$$0.924 \times 10.1 = 9.3$$

Using the alignment chart above, the effective length is approximately 1.85.

**Question 4.8-2**

Using an  $r$  of 0.762 in from Table 1-5 and a  $K$  of 0.65 from Table C-A-7.1, the slenderness ratio is:

$$\frac{KL}{r} = \frac{0.65 \times 12 \text{ in} \times 12}{0.762 \text{ in}} = 122.8$$

The buckling stress is:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{122.8^2} = 18.98 \text{ ksi}$$

The next equation that needs to be determined to decide whether to use equation E3-2 or E3-3 is:

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \times \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}} = 113.4$$

Which is less than the slenderness ratio, so equation E3-3 is used:

$$F_{cr} = 0.877 \times F_e = 0.877 \times 18.98 \text{ ksi} = 16.65 \text{ ksi}$$

The nominal compressive strength is as follows, using  $8.81 \text{ in}^2$  for  $A_g$ , as found in Table 1-5.

$$P_n = 16.65 \text{ ksi} \times 8.81 \text{ in}^2 = 147 \text{ kips}$$

Repeat all the above calculations for the  $y$  axis.

$$\frac{KL}{r} = \frac{0.65 \times 12 \text{ in} \times 12}{4.29 \text{ in}} = 21.82$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{21.82^2} = 601.2 \text{ ksi}$$

Now buckling must be calculated for the  $z$  axis. However,, since the shape is singly symmetrical, Equation E4-7 is used.

$$F_e = \left[ \frac{\pi^2 EC_w}{(L_{cz})^2} + GJ \right] \frac{1}{A_g \bar{r}_0^2}$$

Well, we know  $E$  is 29000 ksi,  $L_{cz}$  is the effective length which is  $0.65 \times 12 \text{ in} \times 12$ ,  $C_w$  is in Table 1-5 as  $151 \text{ in}^6$ ,  $J$  is in the table as  $0.861 \text{ in}^4$ ,  $\bar{r}_0$  is 4.54 inches.  $G$  was found in the textbook to be 11200 ksi for structural steel.

$$F_e = \left[ \frac{\pi^2 \times 29000 \text{ ksi} \times 151 \text{ in}^6}{(0.65 \times 12 \text{ in} \times 12)^2} + 11200 \text{ ksi} \times 0.861 \text{ in}^4 \right] \frac{1}{8.81 \text{ in}^2 \times (4.54 \text{ in})^2} = 80.27 \text{ ksi}$$

From here, add this to the buckling stress I found for the  $y$  axis.

$$80.27 \text{ ksi} + 601.2 \text{ ksi} = 681.47 \text{ ksi}$$

Now, use equation E4-3 to determine the flexural-torsional buckling strength:

$$F_e = \frac{F_{ey} + F_{ez}}{2H} \left[ 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right]$$

$H$  is found to be 0.919 in from Table 1-5. The rest we should know:

$$F_e = \frac{681.47 \text{ ksi}}{2 \times 0.919 \text{ in}} \left[ 1 - \sqrt{1 - \frac{4 \times 601.2 \text{ ksi} \times 80.27 \text{ ksi} \times 0.919 \text{ in}}{(681.47 \text{ ksi})^2}} \right] = 79.29 \text{ ksi}$$

Now we compare  $F_e$  to  $F_y$ :

$$0.44 \times F_y = 22 \text{ ksi} < 79.29 \text{ ksi}$$

Therefore, Equation E3-2 is used:

$$F_{cr} = \left[ 0.658^{\frac{F_y}{F_e}} \right] F_y = \left[ 0.658^{\frac{50 \text{ ksi}}{79.29 \text{ ksi}}} \right] (50 \text{ ksi}) = 38.4 \text{ ksi}$$

The nominal strength is calculated as follows:

$$P_n = F_{cr} \times A_g = 38.4 \text{ ksi} \times 8.81 \text{ in}^2 = \boxed{338.3 \text{ kips}}$$

## Question 4.9-10

Equations 1-3 show various required calculations, with all needed values determined from Table 1-7:

$$\frac{L_c}{r_x} = \frac{KL}{r_x} = \frac{18 \text{ ft} \times 12}{1.89 \text{ in}} = 114.3 \quad (1)$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{114.3^2} = 21.91 \text{ ksi} \quad (2)$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000 \text{ ksi}}{50 \text{ kips}}} = 113.4 \quad (3)$$

The value of equation 3 is less than the value of equation 1, equation E3-3 is used:

$$F_{cr} = 0.877 \times F_e = 0.877 \times 21.91 \text{ ksi} = 19.22 \text{ ksi}$$

The nominal compressive strength is calculated as follows:

$$P_n = F_{cr} \times A_g = 19.22 \text{ ksi} \times 11.7 \text{ in}^2 = 224.9 \text{ kips}$$

Now we use table 1-15 and calculate 3/4 of the slenderness ratio:

$$\frac{3}{4} \frac{L_c}{r_y} = \frac{3}{4} \frac{18 \text{ ft} \times 12}{1.66 \text{ in}} = 97.59$$

Two fully tightened bolts mean that there are three spaces. The spacing of the connectors is:

$$a = \frac{L_c}{3} = \frac{18 \text{ ft} \times 12}{3} = 72 \text{ in}$$

The modified slenderness ratio is:

$$\frac{a}{r_z} = \frac{72 \text{ in}}{0.859 \text{ in}}$$

The next step is to calculate the separation ratio according to section E6 of the manual:

$$\alpha = \frac{h}{2r_y} = \frac{2 \times \bar{x} + 3/8 \text{ in}}{2 \times r_y} = \frac{2 \times 1.03 \text{ in} + 3/8 \text{ in}}{2 \times 1.13 \text{ in}} = 1.08$$

The modified slenderness ratio is calculated in equation 4 according to equation E6-2:

$$\sqrt{\left(\frac{KL}{r_y}\right)^2 + 0.82 \frac{\alpha^2}{1 + \alpha^2} \times \left(\frac{a}{r_y}\right)^2} = \sqrt{\left(\frac{18 \text{ ft} \times 12}{1.66 \text{ in}}\right)^2 + 0.82 \frac{1.08^2}{1 + 1.08^2} \times \left(\frac{72 \text{ in}}{1.13 \text{ in}}\right)^2} = 136.8 \quad (4)$$

The remaining calculations are done in equations 5 and 6:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000 \text{ ksi}}{136.8^2} = 15.29 \text{ ksi} \quad (5)$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000 \text{ ksi}}{50 \text{ kips}}} = 113.4 \quad (6)$$

The value of equation 6 is less than the value of equation 4, equation E3-3 is used:

$$F_{cr} = 0.877 \times F_e = 0.877 \times 15.29 \text{ ksi} = 13.41 \text{ ksi}$$

Now we look at equation E4-2:

$$F_{cr} = \left( \frac{F_{cry} + F_{crz}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}} \right]$$

$F_{crz}$  is calculated using equation E4-3, using all values obtained from Table 1-15 along with 11200 ksi for  $G$ :

$$F_{crz} = \frac{GJ}{A_g \bar{r}_0^2} = \frac{11200 \text{ ksi} \times (0.775 \text{ in} \times 2)}{11.7 \text{ in}^2 \times (3.05 \text{ in})^2} = 159.5 \text{ ksi}$$

Now, using 13.41 ksi for  $F_{cry}$  and 0.684 in for  $H$  (from Table 1-15), we can solve equation E4-2:

$$F_{cr} = \left( \frac{13.41 \text{ ksi} + 159.5 \text{ ksi}}{2 \times (0.684 \text{ in})} \right) \left[ 1 - \sqrt{1 - \frac{4(13.41 \text{ ksi}) \times (159.5 \text{ ksi}) \times (0.684 \text{ in})}{(13.41 \text{ ksi} + 159.5 \text{ ksi})^2}} \right] = 13.04 \text{ ksi}$$

Nominal strength is calculated as follows:

$$P_n = 13.04 \text{ ksi} \times 11.7 \text{ in}^2 = 152.6 \text{ kips}$$

Multiply this by 0.9 to get the LRFD design strength:

$$0.9 \times 152.6 \text{ kips} = \boxed{137.3 \text{ kips}}$$