Problem Set ~~3~~4: Demand Estimation

~~450-2 Winter 2017Due February 15th~~ 450-1

Preface

This problem set is designed to help you to understand the nuts and bolts of discrete choice demand models and imperfect competition. In this exercise, we will use simulated fake data so that you know the true distributions and then we will try alternative approaches to estimation. Please read the full problem set before starting, as it may save some heart-ache. **The problem set is quite long and you should start working on this as soon as possible!**

In addition to the lecture slides, there are several papers which can serve as good resources for this PSET. Among them are Dubé et al. (2012a), which has good guidance on implementing the MPEC algorithm (note the appendix Dubé et al. (2012b), which discusses the constraints Jacobian structure); Conlon and Gortmaker (2 , which describes in detail the popular PyBLP package (note that this is a NFP paper, but it can still guide your MPEC creation); and Nevo (2000) (again note that this is a NFP paper). There will be small differences across the papers and between the papers and the lecture notes; for instance, the lecture notes optimize δ in their MPEC while Dubé et al. (2012a) optimize ξ. **Make sure your code is internally consistent**; i.e. your objective function, constraints, gradient, and Jacobian all correspond.

This problem set can be done in multiple programming languages (the most popular historically have been Julia, Matlab, Python, and R). Some students have also used this as a chance to familiarize themselves with Knitro. As specified below, whatever language/solver you use, **you have to provide a gradient and Jacobian**.

1 Market Simulation

1.1 Model

1.1.1 Demand:

Consumers' preference for product j and market m is assumed to take the following form:

$$
\begin{aligned}
U_{ijm} &= X_{jm}\beta - \alpha_i p_{jm} + \xi_{jm} + \epsilon_{ijm} \\
\alpha_i &= \alpha + \sigma_\alpha v_{ip} \\
U_{i0} &= 0
\end{aligned} \tag{1}
$$

where the product characteristics are iid with distributions:

- $X_{jm} = (X_{1jm}, X_{2jm}, X_{3jm})$, with $X_{1jm} = 1$ (a constant).
- $X_2 \sim U[0,1]$ and $X_3 \sim N(0,1)$
- $\xi_{jm} \sim N(0,1)$.

Similarly, the consumer taste shocks are iid with distributions

- $\nu_{ip} \sim LN(0,1)$, where LN is the lognormal distribution
- ϵ_{ijm} is drawn from type I extreme value distribution.

1.1.2 Supply:

The marginal cost of producing product j in market m is given by:

$$
MC_{jm} = \gamma_0 + \gamma_1 W_j + \gamma_2 Z_{jm} + \eta_{jm} \tag{2}
$$

where $W_j \sim N(0,1)$, $Z_{jm} \sim N(0,1)$ and $\eta_{jm} \sim N(0,1)$. All products are produced by single-product firms. The markets are regional, while the firms are national. Therefore, W_j is a common cost shifter for firm j across all markets.

1.1.3 Parameters:

In the remainder of this problem set, you will estimate the demand parameters $\theta = \{\beta, \alpha, \sigma_\alpha\}$, and the supply parameters γ.

Let the true parameter values be:

- $\beta = (5, 1, 1)$
- $\alpha = 1$ and $\sigma_\alpha = 1$
- $(\gamma_0, \gamma_1, \gamma_2) = (2, 1, 1)$.

~~Provided in matlab~~

1.2 Data

Provided in CSV files are the simulated markets, prices and market shares for ~~two~~ three simulations (generated using the true parameters): 10 markets and 3 products, 100 markets and 3 products, 100 markets and 5 products. The data list observations at the product-market (jm) level. They are ordered in cycles of products within cycles of markets (product and market columns are also provided for clarity).

~~**For each of these two simulations, compare the distribution of prices, profits and consumer surplus. For consumer surplus, simulate draws of consumers from the true distribution and calculate their optimal purchasing decision and welfare**~~

1. For each of these three simulations, compare the distribution of prices, profits and consumer surplus. For consumer surplus, simulate draws of consumers from the true distribution and calculate their optimal purchasing decision and welfare.

2 BLP and Hausman Instruments

Unless specified, use the dataset with $J = 3$ and $M = 100$ for the following exercises:

1. Consider the following set of moment conditions: $E[\xi|X] = 0$ and $E[\xi|p] = 0$.
   (a) Using the $(J, M) = (3, 200)$ dataset, compute the values of $E[\xi_{jm}X_{jm}]$, $E[\xi_{jm}p_{jm}]$ and $E[\xi_{jm}\bar{p}_{jm}]$, where $\bar{p}_{jm}$ is the average price of products in the other markets.
   (b) Which of these moment conditions is valid? Which of them are relevant? Why?
   (c) Can you use both BLP and Hausman instruments in this setting? Why? Why not?
2. Estimate θ a.la. BLP, but using *demand-side moments only* i.e. $E[\xi|X] = 0$ and writing the problem as an MPEC
   (a) Write down the BLP moments.
   (b) Construct your objective function.

(c) Construct the constraints function.

(d) Construct the gradient and ~~Hessian~~Jacobian.

(e) Try to estimate θ and the standard errors. Report the estimates, bias and standard errors for each parameter.

Note: You should start the optimization routine at several different starting values, and ensure that you are confident about your results. Comment on which parameters appear to be most stable across runs?

(f) Compute the price elasticity of demand at equilibrium prices, profits and consumer surplus at the estimated parameters. Compare with the true values.

(g) Repeat the estimation for $M = 10$. How do the estimates, standard errors and stability of the optimization routine change?

3. Estimate θ a.la. BLP, but assuming incorrectly that $E[\xi|p] = 0$ within each market. Compare the parameter estimates to the true values and the ones obtained using BLP instruments. Comment.

3 ~~Bonus:~~ Adding Supply-side~~Instruments~~

~~If you are able to solve the previous problem, you can move on to the following two sections for extra credit. Unless specified, use the dataset with $J = 3$ and $M = 100$ for the following exercises:~~

1. Estimate θ assuming $E[\xi|X, W] = 0$.

   (a) Write down the BLP moments, as well as a moment with the cost shifter W.

   (b) Estimate θ and the standard errors. Report the estimates, bias and standard errors for each parameter. Compute elasticity of demand at equilibrium prices, profits and consumer surplus at the estimated parameters. Compare with the true values.

   (c) Repeat the estimation for $M = 10$. How do the estimates, standard errors and stability of the optimization routine change?

   (d) Compare the answers obtained here with the true values and the estimates using the BLP instruments alone.

2. ~~Estimate θ and γ jointly, assuming that $E[\xi, \eta|X, W] = 0$~~Estimating marginal costs.

(a) Write down marginal costs under the three pricing assumptions: 1) perfect competition, 2) perfect collusion, and 3) oligopoly (correct model).

(b) Using your most preferred estimates so far, compute the marginal costs. Comment on why you prefer these estimates. Compare the marginal costs to the true marginal costs in the data.

4 Bonus: Joint Estimation

Estimate θ and γ jointly, assuming that $E[\xi, \eta | X, W] = 0$

1. Estimate the demand and supply parameters under ~~these three assumptions~~1) perfect competition, 2) perfect collusion, and 3) oligopoly (correct model). Comment on the estimates, standard errors, and demand elasticities at observed prices comparing them to previous estimates.

5 Bonus: Merger exercise

1. Pick a set of parameter estimates that you trust the most for the following exercises. Why do you prefer these over the others?

2. Suppose firm 1 and firm 2 plan to merge.

   (a) Write down the merged firm's pricing problem.

   (b) Predict the new set of prices using *estimated* parameters. How does markup change?

   (c) Compare consumer surplus, prices and profits.

References

Conlon, C. and J. Gortmaker (2020, December). Best practices for differentiated products demand estimation with PyBLP. *The RAND Journal of Economics 51*(4), 1108–1161.

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