First Thought, Best Thought? Estimating Initial Beliefs in a Bayesian DSGE Model with Adaptive Learning

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Introduction

- Overview of Presentation
    - Explanation of Rational Expectations and how it contrasts with Adaptive Learning Framework
        - Explanation of my own Adaptive Learning algorithm
- Review Estimation of DSGE models
    - Review previous empirical work in Adaptive Learning literature
    - Introduce Sequential Monte Carlo method
- Results: Marginal likelihoods and important parameter estimates
    - Equilibrium-based initials
    - Training-sample based initials
    - Jointly estimated initials

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The Theory of Adaptive Learning

- Detailed in (Evans and Honkapohja 2001)
- Suppose there is some function that maps from agents beliefs, ϕ, to the behavior of the economy, $T(\phi)$
- The fixed point ϕ^* where $T(\phi^*) = \phi^*$, if it exists, is the rational expectations solution
  - Recall the method of undetermined coefficients

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The Theory of Adaptive Learning

- Let Z_t be some vector of forecasted variables, X_t be some vector of regressors, in our model $(1, Z_{t-1})$ or $(1, Z_{t-1}, \varepsilon_t)$ so that agents have the model $Z_t = \phi' X_t$.

- Agents update their beliefs ϕ_t according to the recursive least-squares formulae.

$$\phi_t = \phi_{t-1} + \frac{1}{t}\Sigma_t^{-1}X_t'(Z_t - \phi_{t-1}'X_t)'$$

$$\Sigma_t = \Sigma_{t-1} + \frac{1}{t}(X_tX_t' - \Sigma_{t-1})$$

- where Σ_t is $E(X_tX_t')$

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Adaptive Learning

- Empirical work, including that presented today, replaces $\frac{1}{t}$ with a constant $\bar{g}$
- This weighs more recent observations more heavily and allows beliefs to change more rapidly
- Especially useful in models of regime-switching

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Asymptotic Properties of AL algorithms

- Letting τ be notional time
- *E-stability principle*
    - A fixed-point ϕ^* of the T-map $T(\phi)$ is expectationally stable if the differential equation $\frac{d\phi}{d\tau} = T(\phi) - \phi$ is asymptotically stable
- (Marcet and Sargent 1989) show that for expectationally stable rational expectations equilibria with a suitable projection facility, agents' beliefs will converge with probability of one to the rational expectations solution
- Empirical work leans on this result for simulation and estimation of DSGE models
    - Without a projection facility, beliefs become explosive leading to undefined likelihood values, confounding estimation

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Bayesian Estimation of DSGE Models

- Fundamental object of interest is Posterior distribution of parameters, or $p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)}$
    - $p(Y|\theta)$ is the likelihood function
    - $p(\theta)$ is the prior density
    - $p(Y)$ is the marginal likelihood or model evidence, and is used to compare the evidence in favor of one model or another
    - Since $p(Y)$ does not depend on θ, one only needs to compute $p(Y|\theta)p(\theta)$ to simulate the posterior distribution since $p(Y|\theta)p(\theta) \propto \frac{p(Y|\theta)p(\theta)}{p(Y)}$

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Bayesian Estimation of DSGE Models

- Two primary concerns in present review
    - Computation of the likelihood function
    - Simulating the Posterior distribution
- Since the model is conditionally linear, I compute the likelihood function using a kalman filter
- As the posterior function is fairly irregular, I use Sequential Monte Carlo rather than Markov Chain Monte Carlo to simulate the posterior

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Bayesian Estimation of DSGE models

- The Likelihood Function is computed using a Kalman Filter
- For any parameter draw θ_i for which there is a unique solution to the DSGE model, there is a state-space representation of the following form

$$y_t = Z_t \alpha_t + d_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, H_t)$$

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t, \eta_t \sim \mathcal{N}(0, Q_t)$$

- y_t is the vector of observables
- α_t is the vector of latent variables
- $Z_t, d_t, T_t, R_t, H_t, Q_t$ are the system matrices and are all functions of θ

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Bayesian Estimation of DSGE models

- let $a_t = E(\alpha_t|y^t)$ be the optimal forecast of the state given information through time t
- $P_t = E((a_t - \alpha_t)(a_t - \alpha_t)'|y^t)$ be the variance of that optimal forecast. The Kalman filter consists of prediction and updating equations.

$$a_{t|t-1} = T_t a_{t-1} + c_t$$

$$P_{t|t-1} = Z_t P_{t-1} Z_t' + R_t Q_t R_t'$$

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Bayesian Estimation of DSGE models

- We get the optimal predictor of y_t based on the information set $\{y_t\}_0^{t-1}$ and the prediction error variance

$$y_{t|t-1} = Z_t a_{t|t-1} + d_t$$

$$v_t = y_t - y_{t|t-1}$$

$$E(v_t v_t') = F_t = Z_t P_{t|t-1} Z_t' + H_t$$

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Bayesian Estimation of DSGE models

- From these we get the updating equations for a_t, P_t

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} v_t$$
$$P_t = P_{t|t-1} - P_{t|t-1} Z_t' F_t^{-1} Z_t P_{t|t-1}$$

- and, because the prediction errors follow a multivariate normal distribution, the likelihood function

$$\ln(L(\theta|y)) = -\frac{NT}{2} ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \ln(|F_t(\theta)|) - \frac{1}{2} \sum_{t=1}^{T} v_t(\theta)' F_t^{-1}(\theta) v_t(\theta)$$

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Bayesian Estimation of DSGE models

- Once one has a likelihood function and a prior density one must choose a sampler algorithm in order to simulate the posterior density
- The most commonly used sampler is the Metropolis Hastings Random Walk Algorithm
- Because of time constraints and irregularity of the Posterior density of models with Adaptive Learning, I opt instead for a Sequential Monte Carlo sampler.

Sequential Monte Carlo Methods

- Sequential Monte Carlo methods are a newer class of Bayesian estimation methods of DSGE models
- Superficially similar to running thousands of MHRW chains with starting points distributed throughout the prior distribution
- Far more efficient for sampling multi-modal posterior distributions: sampler does not get "stuck" in local maxima.
- SMC Highly Parallelizable and can take advantage of HPC
- Estimates I present today were obtained using an AWS EC2 instance with 96 physical CPU cores

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Sequential Monte Carlo Algorithm

1. **Initialization:** ($\phi_0 = 0$). Draw the initial particles from the prior: $\theta_1^i \overset{\text{i.i.d.}}{\sim} p(\theta)$ and $W_1^i = 1$, $i = 1, \ldots, N$.

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Sequential Monte Carlo Algorithm (cont'd)

2. **Recursion:** For $n = 1, \ldots, N_\phi$,

- **Correction:** Reweight the particles from stage $n - 1$ by defining the incremental weights.

$$\tilde{w}_n^i = [p(Y|\theta_{n-1}^i)]^{\phi_n - \phi_{n-1}}$$
$$\tilde{W}_n^i = \frac{\tilde{w}_n^i W_{n-1}^i}{\frac{1}{N} \sum_{i=1}^N \tilde{w}_n^i W_{n-1}^i}, \quad i = 1, \ldots, N.$$

- **Selection:** If $\rho_n = 1$, resample the particles via multinomial resampling. If $\rho_n = 0$, let $\hat{\theta}_n^i = \theta_{n-1}^i$ and $W_n^i = \tilde{W}_n^i$, $i = 1, \ldots, N$.

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Sequential Monte Carlo Algorithm (cont'd)

- **Mutation:** Propagate the particles $\{\hat{\theta}^i, W_n^i\}$ via N_{MH} steps of a MH algorithm with transition density $\theta_n^i \sim K_n(\theta_n | \hat{\theta}_n^i; \zeta_n)$ and stationary distribution $\pi_n(\theta)$.

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Sequential Monte Carlo Algorithm (cont'd)

3. For $n = N_\phi(\phi_{N_\phi} = 1)$, the final importance sampling approximation of $E_\pi[h(\theta)]$ is given by:

$$\bar{h}_{N_\phi,N} = \sum_{i=1}^{N} h(\theta_{N_\phi}^i) W_{N_\phi}^i.$$

Sequential Monte Carlo

- Researcher chooses the cooling schedule, number of stages, and number of particles
- Accuracy of estimates scales linearly with number of particles
- Parallelizability of algorithm allows the use of HPC to estimate high-dimensional models.

Sequential Monte Carlo

- Sequential Monte Carlo also allows for sequential computation of marginal data density

$$\widetilde{w}_n^i = \left[p(Y|\theta_{n-1}^i) \right]^{(\phi_n - \phi_{n-1})}$$

$$\frac{1}{N} \sum_{i=1}^{N} \widetilde{w}_n^i W_{n-1}^i \approx \int [p(Y|\theta)]^{(\phi_n - \phi_{n-1})} \frac{p^{\phi_{n-1}}(Y|\theta)p(\theta)}{\int p^{\phi_{n-1}}(Y|\theta)p(\theta)\,d\theta}\,d\theta$$

$$= \frac{\int p(Y|\theta)^{\phi_n} p(\theta)\,d\theta}{\int p(Y|\theta)^{\phi_{n-1}} p(\theta)\,d\theta}$$

Which implies

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Sequential Monte Carlo

$$\prod_{n=1}^{N_\phi} \left(\frac{1}{N} \sum_{i=1}^{N} \widetilde{w}_n^i W_{n-1}^i \right) \approx \int p(Y|\theta)p(\theta)\, d\theta$$

The Model

- I estimate a small scale New Keynesian Model with two sources of mechanical persistence, developed in detail by (Woodford 2003) and first estimated under AL by (Milani 2005)
    - Habit persistence in consumption by households
    - Inflation indexation by price-setting firms
- In the end we will have five equations:
    - A New Keynesian Phillips Curve
    - A New Keynesian IS curve
    - Taylor Rule for Monetary Policy
    - Productivity Shock
    - Aggregate Demand shock

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The Model

$$\tilde{\pi}_t = \xi_p[\omega\tilde{x}_t + [(1-\eta\beta)\sigma]^{-1}\tilde{x}_t] + \beta\hat{\mathbb{E}}_t\tilde{\pi}_{t+1} + u_t \quad \text{(NK Phillips Curve)}$$

$$\tilde{x}_t = \hat{\mathbb{E}}_t\tilde{x}_{t+1} - (1 - \beta\eta)\sigma[i_t - \hat{\mathbb{E}}_t\pi_{t+1} - r_t^n] \quad \text{(NK IS Curve)}$$

$$i_t = \rho i_{t-1} + (1-\rho)(\psi_\pi\tilde{\pi}_t + \psi_x\tilde{x}_t) + \varepsilon_t \quad \text{(Taylor Rule for Monetary Policy)}$$

$$r_t^n = \phi^r r_{t-1}^n + v_t^r \quad \text{(Natural Interest Rate process)}$$

$$u_t = \phi^u u_{t-1} + v_t^u \quad \text{(Productivity shock process)}$$

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Integrating Learning within the Model

- Solving the model under bounded rationality is mechanical and straightforward, no solution by undetermined coefficients or eigenvalue or schur decompositions needed!
- Agents have a "Perceived Law of Motion", a belief about how the economy operates
    - I model this as a VAR(1) process $\alpha_{t+1} = c + \Phi\alpha_t + \Psi\varepsilon_t$
- One can solve easily for the period t expectations of period $t + 1$ variables: $E_t\alpha_{t+1} = c + \Phi\alpha_t$
- If agents only have information up to time $t - 1$ then $E_{t-1}\alpha_{t+1} = c + \Phi c + \Phi^2\alpha_{t-1}$
- If simulating the model, simply substitute these $E_{t-n}\alpha_{t+1}$ terms into the agents' Euler equations and one has a solution.

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Integrating Learning within the model

- For estimation, recall the state-space representation of a model

$$y_t = Z_t \alpha_t + d_t + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, H_t)$$

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t, \eta_t \sim \mathcal{N}(0, Q_t)$$

- One can cast any DSGE model into the following form:
$$s_t = P + Q s_{t+1}^e + R s_{t-1} + S \varepsilon_t$$

- To make the model amenable to estimation, then, I take the coefficients from $E_{t-1}\alpha_{t+1} = c + \Phi c + \Phi^2 \alpha_{t-1}$ and simply substitute Φ^2 for Q and this gives me the VAR(1) representation of the model

- This gives time-varying coefficients as well in the system matrices.

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Choice of Initial beliefs: Equilibrium-Based initials

- For every θ_i that has a unique Rational Expectations solution, there is a state-space representation

$$y_t = Z\alpha_t + d + \varepsilon_t, \varepsilon_t \sim \mathcal{N}(0, H_t)$$

$$\alpha_t = T\alpha_{t-1} + c + R\eta_t, \eta_t \sim \mathcal{N}(0, Q_t)$$

- When choosing Equilibrium-based initials, then, for every θ_i drawn from the posterior distribution, I substitute the corresponding element of $T(\theta_i)$ into b_0 in agents PLM $\alpha_t = a_0 + b_0\alpha_{t-1} + \epsilon_t$

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Choice of Initial Beliefs: Training Sample

- This involves estimating a reduced-form state-space model based on pre-sample data
- The state-space model in my case has three observable variables and five latent variables $y_t = (\pi_t, x_t, i_t)$ or inflation, the output gap, and the federal funds rate and the latent variables are the inflation process, output process, interest rate process, natural interest rate process, and productivity shock process

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Choice of Initial Beliefs: Training Sample

- I also exploit the restrictions on the effect of the autoregressive shocks

$$
\begin{bmatrix} \pi_t \\ x_t \\ i_t \\ r_t^n \\ g_t \end{bmatrix} = \mathbf{a} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{15} \\ b_{21} & b_{22} & b_{23} & \dots & b_{25} \\ b_{31} & b_{32} & b_{33} & \dots & b_{35} \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ i_{t-1} \\ r_{t-1}^n \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \varepsilon_t
$$

- After finding the likelihood maximum, I substitute the coefficients into agents' PLM

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Choice of Initial Beliefs: Joint Estimation

- One could estimate initial beliefs along with the rest of the model parameters
- This requires researcher to choose an explicit prior distribution
- It also increases substantially the number of estimated parameters, which can confound Bayesian samplers.

Joint Estimation, How to compute the prior density

- I take the simulated Posterior distribution of θ under Rational Expectations
- For each θ_i I then back out the VAR-1 representation of the model

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \\ r_t^n \\ g_t \end{bmatrix} = \mathbf{a} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & \ldots & b_{15} \\ b_{21} & b_{22} & b_{23} & \ldots & b_{25} \\ b_{31} & b_{32} & b_{33} & \ldots & b_{35} \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ i_{t-1} \\ r_{t-1}^n \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \varepsilon_t$$

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Joint Estimation

- I then gather elements $(b_{11}, b_{12}, ...)'$ into a single vector for each θ_i
- I then fit a multivariate normal distribution over the vectors from the simulated posterior distribution from the model under rational expectations
- As the posterior density can be made arbitrarily large by shrinking the prior sample space, I set each diagonal element of the covariance matrix equal to the maximum of the sample covariance and one

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Priors on Parameters

- I seek to understand just the effects of initial beliefs, and so I seek priors that match the usual conventions in the DSGE literature

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The Data

- The estimates I report are based on 1982-2002 data, but some robustness checks are performed with 1960-2008 data
- Data are the time series including inflation, output gap, and the federal funds rate
- Because the latent variables are zero-mean processes but the observables are not zero-mean processes due to the Fed's inflation rate, I use de-meaned values for the federal funds rate and the inflation rate
- Another solution used by (An and Schorfheide 2007) treats those means as parameters to be estimated along with structural parameters

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Results: Rational Expectations Baseline

- Parameter estimates under Rational Expectations match somewhat closely those found in (Milani 2005)

Table 1: SMC Estimates, 5000 particles with 100 stages, Rational Expectations, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.67	0.52	0.78
β	0.99	0.97	1.00
σ	0.26	0.16	0.34
γ	0.97	0.91	1.00
ξ_p	0.00	0.00	0.00
ω	0.75	0.19	1.25
ρ	0.95	0.92	0.97
ξ_π	1.56	1.30	1.84
ξ_x	0.43	0.23	0.61
ϕ_r	0.94	0.91	0.96
ϕ_u	0.04	0.00	0.12
σ_e	0.23	0.20	0.26
σ_r	1.15	0.84	1.49
σ_u	0.40	0.36	0.44

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Equilibrium-based Initial Beliefs

Table 2: SMC Estimates, 5000 particles with 300 stages, Equilibrium Initials, Full Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.54	0.33	0.72
β	0.99	0.97	1.00
σ	0.19	0.09	0.33
γ	0.96	0.88	1.00
ξ_p	0.00	0.00	0.00
ω	0.86	0.25	1.47
ρ	0.94	0.91	0.97
ξ_π	1.58	1.24	1.94
ξ_x	0.35	0.07	0.68
ϕ_r	0.95	0.90	0.97
ϕ_u	0.04	0.00	0.12
σ_e	0.23	0.20	0.26
σ_r	1.10	0.63	1.80
σ_u	0.42	0.36	0.48
gain	0.0218	0.0049	0.0484

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Equilibrium-based Initial Beliefs

Table 3: SMC Estimates, 5000 particles with 300 stages, Equilibrium Initials, Limited Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.26	0.10	0.45
β	0.99	0.97	1.00
σ	0.29	0.16	0.45
γ	0.59	0.10	0.97
ξ_p	0.00	0.00	0.00
ω	0.81	0.19	1.48
ρ	0.93	0.88	0.97
ξ_π	1.65	1.27	2.00
ξ_x	0.28	0.04	0.62
ϕ_r	0.61	0.52	0.71
ϕ_u	0.12	0.01	0.36
σ_e	0.23	0.21	0.27
σ_r	3.01	1.75	4.94
σ_u	0.42	0.36	0.48
gain	0.0176	0.0088	0.0313

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Training Sample Initial Beliefs

- Extremely low estimated gain value due to projection facility

Table 4: SMC Estimates, 5000 particles with 300 stages, Training Sample Initials, Full Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.40	0.14	0.61
β	0.98	0.96	1.00
σ	0.34	0.20	0.52
γ	0.23	0.02	0.53
ξ_p	0.00	0.00	0.01
ω	0.82	0.22	1.45
ρ	0.97	0.96	0.97
ξ_π	1.43	1.03	1.83
ξ_x	0.13	0.01	0.35
ϕ_r	0.82	0.66	0.95
ϕ_u	0.37	0.03	0.83
σ_e	0.23	0.21	0.27
σ_r	0.37	0.33	0.43
σ_u	0.32	0.27	0.37
gain	0.0061	0.0012	0.0153

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Training Sample

Table 5: SMC Estimates, 5000 particles with 300 stages, Training Sample Initials, Limited Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.24	0.09	0.42
β	0.98	0.96	1.00
σ	0.24	0.18	0.33
γ	0.15	0.01	0.38
ξ_p	0.00	0.00	0.00
ω	0.80	0.23	1.38
ρ	0.97	0.96	0.97
ξ_π	1.42	1.05	1.81
ξ_x	0.15	0.02	0.34
ϕ_r	0.27	0.06	0.51
ϕ_u	0.22	0.03	0.45
σ_e	0.24	0.21	0.27
σ_r	0.62	0.50	0.75
σ_u	0.39	0.34	0.44
gain	0.0106	0.0025	0.0212

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Joint Estimation Results

Table 6: SMC Estimates, 10000 particles with 500 stages, Jointly Estimated Initials, Full Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.28	0.13	0.42
β	1.00	0.99	1.00
σ	0.27	0.18	0.38
γ	0.93	0.77	1.00
ξ_p	0.00	0.00	0.01
ω	0.67	0.27	1.05
ρ	0.93	0.90	0.96
ξ_π	1.81	1.44	2.08
ξ_x	0.31	0.11	0.59
ϕ_r	0.87	0.76	0.94
ϕ_u	0.12	0.01	0.36
σ_e	0.24	0.21	0.28
σ_r	1.12	0.83	1.44
σ_u	0.53	0.41	0.74
gain	0.0145	0.0079	0.0256

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Joint Estimation Results

Table 7: SMC Estimates, 10000 particles with 500 stages, Jointly Estimated Initials, Limited Information, 5 runs

Parameter	Mean	5% Interval	95% Interval
η	0.39	0.12	0.66
β	0.99	0.98	1.00
σ	0.60	0.37	0.87
γ	0.37	0.02	0.94
ξ_p	0.01	0.00	0.02
ω	0.80	0.28	1.35
ρ	0.93	0.89	0.96
ξ_π	1.65	1.29	2.01
ξ_x	0.31	0.07	0.61
ϕ_r	0.79	0.60	0.94
ϕ_u	0.67	0.17	0.90
σ_e	0.23	0.20	0.27
σ_r	1.48	0.84	2.35
σ_u	0.38	0.33	0.44
gain	0.0062	0.0011	0.0138

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Summary of Important Parameter Estimates

- Somewhat consistent reductions in Inflation Indexation compared to RE
- Less so with Habit persistence
- Unable to match the near-elimination of mechanical persistence in (Milani 2005)
- Fairly close to those estimated in (Cole and Milani 2019)
- Unusually low gain value compared to (Milani 2005)
- Likely driven by data, as post-1982 estimated model reports extremely low constant gain of .0058

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Comparison of Model Fit

- Marginal Data Density allows one to rank models according to in-sample forecasting performance
- They tell researchers, given the prior density, how likely are the data to appear
- Driven by priors, so want to use "reasonable" priors
- Use different priors, or even use different data sets

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Comparison of Model Fit

Table 8: Mean and Standard Deviation of Natural Logarithms of the Marginal Likelihoods, 1982-2002 data

	Full Information	Limited Information
Rational Expectations	-331.8948 (0.9613)	N/A
Equilibrium Initials	-329.5719 (0.9909)	-332.9946 (0.4705)
Training Sample Initials	-650.1220 (112.3248)	-351.2298 (7.3816)
Jointly Estimated Initials	-328.9922 (2.6250)	-326.1411 (0.9756)

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Comparison of Model Fit

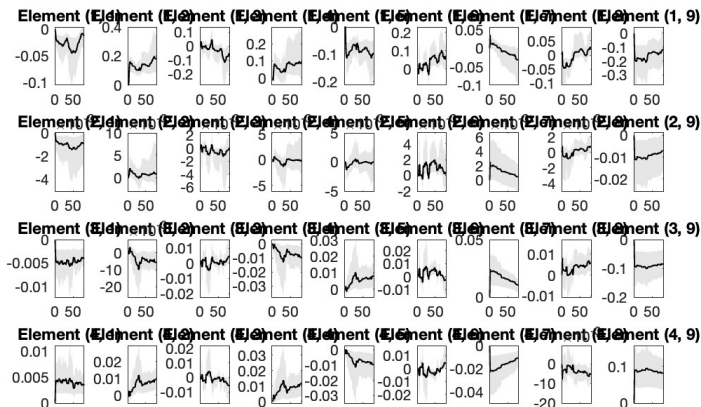Table 9: Mean and Standard Deviation of Natural Logarithms of the Marginal Likelihoods, 1961-2006 data

	Full Information	Limited Information
Rational Expectations	-839.2973 (3.4430)	N/A
Equilibrium Initials	-833.9175 (0.8594)	-838.5927 (0.7877)
Training Sample Initials	-2764.7828 (0.6802)	-859.9934 (0.3554)
Jointly Estimated Initials	-885.0740 (12.0577)	-833.8648 (1.7775)

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Evolution of Beliefs

- Want to see if results are driven by the choice of projection facility
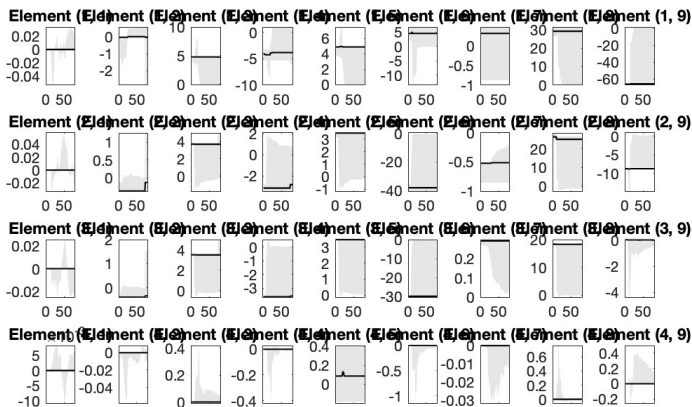- If beliefs are driven by projection facility then results are trivial

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Evolution of Beliefs

- For most models beliefs are moving, and thus not hitting projection facility

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Evolution of Beliefs

- For training sample inits with full information, beliefs (nearly) always hitting projection facility and thus not moving at all

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Evolution of Beliefs

- Impact of projection facility unsurprising for training sample beliefs
- Projection facility forces beliefs to not update when entering explosivity
- Training sample forces initial beliefs for all θ_i to be a single ϕ_i, Σ_i

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Summary

- SMC results unable to eliminate mechanical persistence
- Equilibrium Initials generally at least as good or better than Rational Expectations Baseline in terms of marginal likelihood
- Joint Estimation performs the best
- Training sample performs poorly
- I propose this rule-of-thumb for choosing initial beliefs
    - For models wherein agents use very small forecasting models, jointly estimate initial beliefs
    - For models wherein agents use richer forecasting models a la (Smets and Wouters 2007), consider using equilibrium-based initials

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