Denote by P_n^i the point $\left(\cos\left(\frac{2\pi}{n}i\right), \sin\left(\frac{2\pi}{n}i\right)\right)$, and denote by $L_n(i,j)$ either

- the line containing the points P_n^i and P_n^j if $P_n^i \neq P_n^j$ (that is, if $i \not\equiv j \mod n$), or
- the line containing $P_n^i = P_n^j$ and is tangent to the circle $x^2+y^2=1$.

Either way, we can define

$$L_n(i,j) = \lim_{\varepsilon\to 0} V\left(\begin{vmatrix} x & y & 1 \\ x(P_n^i) & y(P_n^i) & 1 \\ x(P_n^j)+y(P_n^j)\varepsilon & y(P_n^j)-x(P_n^j)\varepsilon & 1 \end{vmatrix}\right)$$

in affine coordinates.

Definition 1. For $n \in \mathbb{N}$, the **Böröczky configuration** B_n is the union of lines of the form $\ell_n(i) := L_n(i, n/2-2i)$ for $0 \le i < n-1$, together with the points of triple intersection.

The B_n configuration has $\frac{n(n-3)}{6}+1$ triple points when $n \equiv 0 \mod 3$. The set of triple points is acted upon by the D_6 group. There is one orbit of size 1: the point $(0,0)$. The orbits of size 3 comprise the points on the lines $y=0$, $\sqrt{3}x+y=0$, and $\sqrt{3}x-y=0$. Each of these lines has $\lfloor\frac{n-1}{2}\rfloor$ triple points (including $(0,0)$). This is because every line $\ell_n(i)$ meets $\ell_n(n-i)$ on the line $\ell_n(0)=V(y)$, except the line $\ell_n(n/2)$ when n is even. And so there are

$$\frac{n(n-3)}{6} - 3\left\lfloor\frac{n-1}{2}-1\right\rfloor$$

points whose orbits are size 6. Thus there are

$$\frac{\frac{n(n-3)}{6} - 3\left\lfloor\frac{n-1}{2}-1\right\rfloor}{6}$$

orbits of size 6.

Let $h(n) = \frac{n(n-3)}{6}+1$. Recall that the number of general points required to determine a unique curve of degree d is $\binom{d+2}{2}-1 = \frac{d^2-3d}{2}$. Therefore the degree of the minimal generators of the curves that contain all $h(n)$ points of the B_n configuration is bounded above by setting $\frac{d^2-3d}{2} = h(n)$ and solving for d. We get

$$d \ge \left\lceil \sqrt{2h(n)+\frac{9}{4}} - \frac{3}{2}\right\rceil.$$

We will begin with the B_{12} configuration. This configuration has