Let $Y = \mathfrak{V}(f_1, \ldots, f_m)$ be closed in the Zariski space $X$. Then $Y(\mathbb{C})$ is closed in $X(\mathbb{C})$ under the complex topology, as

$$X(\mathbb{C}) \setminus Y(\mathbb{C}) = \mathfrak{V}(X; \varepsilon + f_1, \ldots, \varepsilon + f_m; \varepsilon) \cup \mathfrak{V}(X; \varepsilon - f_1, \ldots, \varepsilon - f_m; \varepsilon).$$

That's not right but I'll keep thinking about it.

Maybe it's $\mathfrak{V}(X; 1/f_1, \ldots, 1/f_m; \varepsilon)$? Or the union of these for all $\varepsilon > 0$?

I know $X(\mathbb{C}) \setminus Y(\mathbb{C}) = \bigcap_{\varepsilon>0} \mathfrak{V}(X; f_1, \ldots, f_m; \varepsilon)$, but this is not guaranteed to be open or closed. Show every limit point is a point in here? I think it relies on continuity, barf on my life.

Let $x$ be a limit point. Then for all $V = \mathfrak{V}(U; g_1, \ldots, g_n; \varepsilon)$ containing $x$, $(V \setminus \{x\}) \cap Y(\mathbb{C}) \neq \emptyset$. We wish to show this implies $f_i(x) = 0$ for $1 \leq i \leq m$.

Want to show that if for all $g_i$ such that $|g_i(x)| = 0$, there is a $y \in Y$ such that $|g_i(y)| < \varepsilon$ for all $\varepsilon > 0$, then $x \in Y$. In this case, $|g_i(y)| = 0$.

If $x \notin Y$, then $\inf_{y \in Y} \{|y - x|\} > 0$ if $Y$ is compact.

If the $f_i : X \subseteq \mathbb{C}^n \to \mathbb{C}$ are continuous in the complex topology, and if $\{0\} \subseteq \mathbb{C}$ is closed in the complex topology, then $Y = \bigcap_{i=1}^{m} f_i^{-1}(\{0\})$ is closed in the complex topology.