The set of monomials with three variables of degree d is $\binom{2+d}{2}$. In general, it requires $\binom{2+d}{2} - 1$ general points to uniquely determine a curve of degree d.

Denote these points $P_\ell = (a_\ell, b_\ell, c_\ell)$ for $2 \leq \ell \leq \binom{2+d}{2}$. We can find the equation of this curve using a $\binom{2+d}{2} \times \binom{2+d}{2}$ matrix. The top row consists of all the monomials of the form $x^i y^j z^k$ where $0 \leq i, j, k \leq d$, and $i + j + k = d$. The ℓ^{th} row will consist of monomials of the form $a_\ell^i b_\ell^j c_\ell^k$ where i, j, and k are equal to the exponents of the x, y, and z in the corresponding column.

Call this set of points $\mathsf{P} = \left\{ P_\ell : 2 \leq \ell \leq \binom{2+d}{2} \right\}$. Then denote the aforementioned matrix M_{P}. Then $\det(M_{\mathsf{P}})$ is a homogeneous polynomial of degree d. We claim that $\det(M_{\mathsf{P}})(P_\ell) = 0$ for all $P_\ell \in \mathsf{P}$.

Note that $\det(M_{\mathsf{P}})(P_\ell)$ is equal to the determinant of the matrix M_P with $x = a_\ell$, $y = b_\ell$, and $z = c_\ell$. Since this row repeats later on in the matrix in row ℓ, the determinant must be 0. Thus $\det(M_{\mathsf{P}})(P_\ell) = 0$ for all $P_\ell \in \mathsf{P}$.

We can also do this if one of the points is infinitely near one of the P_ℓ's. Let

$$x(b_\ell v - c_\ell u) + y(c_\ell t - a_\ell v) + z(a_\ell u - b_\ell t)$$

be the line connecting P_ℓ with the point (t, u, v). Then by defining $a_{\ell,\varepsilon} = a_\ell + \varepsilon(b_\ell v - c_\ell u)$, $b_{\ell,\varepsilon} = b_\ell + \varepsilon(c_\ell t - a_\ell v)$ and $c_{\ell,\varepsilon} = c_\ell + \varepsilon(a_\ell u - b_\ell t)$ we can make the final row of the matrix consist of $a_{\ell,\varepsilon}^i b_{\ell,\varepsilon}^j c_{\ell,\varepsilon}^k$ where $i + j + k = d$. By computing the determinant of this matrix, we can factor out as many ε's as possible and set any remaining ε's equal to 0. This will give us an equation for a curve C that passes through each of the points in P and whose tangent line at P, $T_P(C)$ contains the point (t, u, v).

To see this, let $P_\ell = (0, 0, 1)$, then $ux - ty$ is the line connecting P_ℓ with $(t, u, 0)$. Then $P_{\ell,\varepsilon} = (t\varepsilon, u\varepsilon, 1)$. The bottom row of M_{P} consists of $(t\varepsilon)^i (u\varepsilon)^j (1)^k$. We want to show that $(\det(M_{\mathsf{P}})_x(0, 0, 1), \det(M_{\mathsf{P}})_y(0, 0, 1)) = (u, -t)$. We know that $\det(M_{\mathsf{P}})_z(0, 0, 1) = 0$ because any line that contains $(0, 0, 1)$ has a z-component of 0.

If we take the partial derivative with respect to x of the top row of M_{P}, we get monomials of the form $ix^{i-1} y^j z^k$. With respect to y, we get $jx^i y^{j-1} z^k$. At $P_\ell = (0, 0, 1)$, these are only nonzero when for xz^{d-1} and yz^{d-1}, respectively.

We can replace all the monomials $(t\varepsilon)^i (u\varepsilon)^j (1)^k$ with 0 except those that are 1 or 0 in the degree of ε. That is, $t\varepsilon$, $u\varepsilon$, and 1. We can do this because we are factoring out the higher degree terms in ε eventually anyway. Thus on the top row of $M_{\mathsf{P}x}$ has a z^{d-1} in the $(1, 0, d-1)$ column its bottom row has a $t\varepsilon$ in the $(1, 0, d-1)$ column, a $u\varepsilon$ in the $(0, 1, d-1)$ column and a 1 in the $(0, 0, d)$ column... Idk

The $\det(M_P)_x(0, 0, 1)$ will have a common factor of u then in all but one term?

The more analytic way of viewing this by realizing $\det(M_{\mathsf{P}})$ vanishes at (a_ℓ, b_ℓ, c_ℓ) and $(a_{\ell,\varepsilon}, b_{\ell,\varepsilon}, c_{\ell,\varepsilon})$, where $(a_{\ell,\varepsilon}, b_{\ell,\varepsilon}, c_{\ell,\varepsilon})$ is in the direction of (t, u, v) from the perspective of (a_ℓ, b_ℓ, c_ℓ). As ε approaches 0, this point $P_{\ell,\varepsilon}$ approaches P_ℓ from the direction of (t, u, v). Thus $\det(M_{\mathsf{P}})$ has vanishes at P_ℓ and the tangent line at P_ℓ contains the point (t, u, v).