

Let $a := \frac{1 + \sqrt{3}}{2}$ and $b := \frac{1 - \sqrt{3}}{2}$. Then the 19 triple points of the B_{12} configuration are as follows.

1. $(0,0)$
2. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
3. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
4. $(1,0)$
5. $(2b, 0)$
6. $(2, 0)$
7. $(2a, 0)$
8. $(-1, 1)$
9. $(-1, -1)$
10. $(-1, b - a)$
11. $(-a, a(b - a))$
12. $(-1, a - b)$
13. $(-a, a(a - b))$
14. $(a, a - 1)$
15. $(a, -a + 1)$
16. $(1 - a, a)$
17. $(1 - a, -a)$
18. $(-b, 1 + b)$
19. $(-b, -1 - b)$

Call this set of points Z_{12} . Then $H^0(Z_{12}, \mathcal{O}(5))$ has three generators. We can generate each of them as products of three lines and a smooth conic.

First let us start with the product of lines $L_1 = (y)(x + y(1 + 2a) - 2a)(x - y(1 + 2a) - 2a)$. Note that L_1 vanishes on all the points except 10, 11, 12, 13, 16, and 17. There exists a conic vanishing on these six remaining points: in particular:

$$Q_1 = (a^2 - 3b^2) \left(x + \frac{1}{2} (1 + \sqrt{3}) \right) (x + 1) - \left(b + \frac{1}{2} (1 + \sqrt{3}) \right) (b + 1) (y^2 - 3x^2).$$

Then $U_1 = L_1 Q_1$ is a quintic polynomial vanishing on the nineteen points.

Similarly, we can construct another quintic using the product of lines

$$\begin{aligned} L_2 = & (x(a^2 - ab) + ay) \\ & \times (x(a^2 - ab - 1) + y(a - 1) + a^2 - ab - a) \\ & \times (x(a^2 - ab) + y(a + 1) - a^2 + ab). \end{aligned}$$

We can define a conic by first defining the four lines

$$\begin{aligned} M_1 &= y - \frac{1}{3}x + \frac{2}{3} \\ M_2 &= (b - a) + y(1 + 2a) - 2a(b - a) \\ M_3 &= -x + y(1 + 2a) + 2a \\ M_4 &= x(b - a) + 3y + 2(a - b) \end{aligned}$$

$$Q_2 = M_1(a, a - 1)M_2(a, a - 1)M_3(x, y)M_4(x, y) - M_1(x, y)M_2(x, y)M_3(a, a - 1)M_4(a, a - 1).$$

Then $U_2 = L_2 Q_2$ is cuts out a quintic curve containing the nineteen points of Z_{12} .

Finally, we can create one more quintic curve by taking

$$U_3 = y(3x^2 - y^2)(x^2 + y^2 - 2).$$

Then $H^0(Z_{12}, \mathcal{O}(5)) = (U_1, U_2, U_3)$. We can observe two elements $f, g \in (U_1, U_2, U_3)$ meeting in 25 distinct points by taking

$$\begin{aligned} f &= 15U_1 + 12U_2 + 26U_3 \\ g &= U_1 + U_2 - 3U_3. \end{aligned}$$