Let $V = \mathrm{Mat}_k(3 \times 3)$ be the vector space of 3×3 matrices over a field k and let $\mathbb{P}V \cong \mathbb{P}^8_k$ be its projectivization. Then we can say a few things about interesting subvarieties of $\mathbb{P}V$.

Let $D = \{m \in \mathbb{P}V : \det m = 0\}$. Then D is a seven-dimensional hypersurface of degree three, given by $\mathfrak{V}(\det(x_0, \ldots, x_8))$.

Within D, we can characterize the subvariety of matrices whose rank is one, called D^2. Note that rank $m = 1$ if and only if all 2×2 minors of m are 0. A 3×3 matrix has 9 such minors, but a computation in Macaulay2 reveals that the dimension of D^2 is four, and has degree six.

Furthermore, we can characterize the left and right annihilators of matrices in D as subvarieties. We will define $\mathrm{Lann}\ d = \{a \in D : ad = 0\}$ and $\mathrm{Rann}\ d = \{a \in D : da = 0\}$, and $\mathrm{ann}\ d = \mathrm{Lann}\ d \cap \mathrm{Rann}\ d$.

The properites of Lann d, Rann d, and ann d will differ based and rank d. First, we will look at the case rank $d = 2$. Note that $da = 0$ if and only if im $a \subseteq \ker d$. When rank $d = 2$, this kernel is generated by one vector (k_1, k_2, k_3). Then for im $a \subseteq \ker d$, we must have

$$a = \begin{pmatrix} \lambda_1 k_1 & \lambda_2 k_1 & \lambda_3 k_1 \\ \lambda_1 k_2 & \lambda_2 k_2 & \lambda_3 k_2 \\ \lambda_1 k_3 & \lambda_2 k_3 & \lambda_3 k_3 \end{pmatrix} \in \mathbb{P}^2_k,$$

so Rann $d \cong \mathbb{P}^2_k$.

The left annihilator is easier to examine with an explicit example, so let us look at $d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then $ad = 0$ if and only if im $d \subseteq \ker a$, with im $d = \langle (1,0,0), (0,1,0) \rangle$. Thus we wish a to have $\langle (1,0,0), (0,1,0) \rangle \subseteq \ker a$, meaning

$$a = \begin{pmatrix} 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 \\ 0 & 0 & \lambda_3 \end{pmatrix} \in \mathbb{P}^2_k.$$

And so we again have Lann $d \cong \mathbb{P}^2_k$. Note that in the above case $d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, we have Rann $d = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} \right\}$, and so the intersection is ann $d = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{pmatrix} \right\}$, a single point.

Now on to rank $d = 1$. To find Rann d, we need im $a \subseteq \ker d = \langle (k_1, k_2, k_3), (\ell_1, \ell_2, \ell_3) \rangle$. So

$$a = \begin{pmatrix} \lambda_1 k_1 + \mu_1 \ell_1 & \lambda_2 k_1 + \mu_2 \ell_1 & \lambda_3 k_1 + \mu_3 \ell_1 \\ \lambda_1 k_2 + \mu_1 \ell_2 & \lambda_2 k_2 + \mu_2 \ell_2 & \lambda_3 k_2 + \mu_3 \ell_2 \\ \lambda_1 k_3 + \mu_1 \ell_3 & \lambda_2 k_3 + \mu_2 \ell_3 & \lambda_3 k_3 + \mu_3 \ell_3 \end{pmatrix} \in \mathbb{P}^5_k$$

and so Rann $d \cong \mathbb{P}^5_k$.

Again, Lann d is easier to examine with an explicit example, say $d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Then we need $\operatorname{im} d \subseteq \ker a$, where $\operatorname{im} d = \langle (1,0,0) \rangle$. Then we have

$$a = \begin{pmatrix} 0 & * & * \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \in \mathbb{P}^5_k$$

and so $\text{Lann } d \cong \mathbb{P}^5_k$. Note in the example $d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, we have $\text{Rann } d = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ * & * & * \end{pmatrix} \right\}$, and so

$$\text{ann } d = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \right\} \cong \mathbb{P}^3_k.$$