Configurations with Geogebra!

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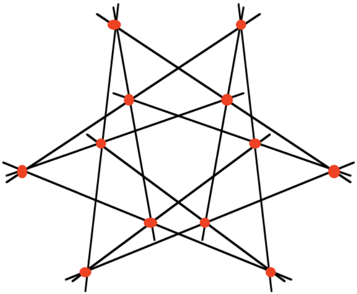
What is a configuration?

Definition

A set of points and lines in the plane is a **configuration** if every point is on the same number of lines and every line contains the same number of points.



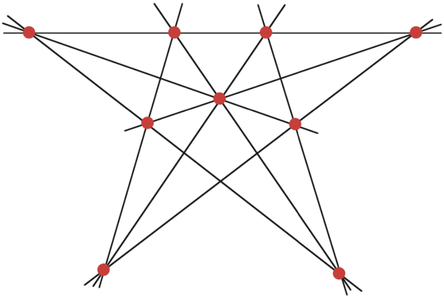


Figure: The left is a configuration, but the right is not.

Notation

We use the notation (a_b, c_d) to refer to configurations comprising a points with b lines per point, and c lines with d points per line. If $a = c$ and $b = d$, we can just call it an (a_b)-configuration.

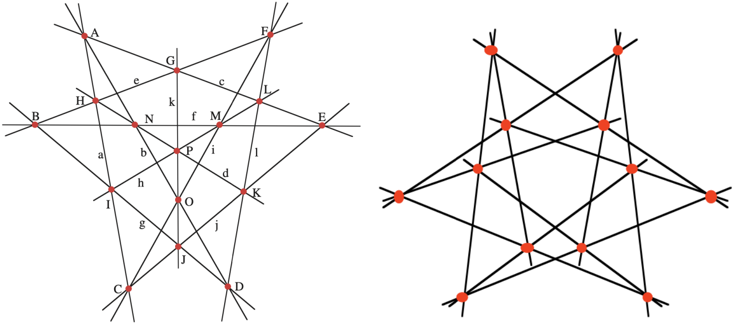
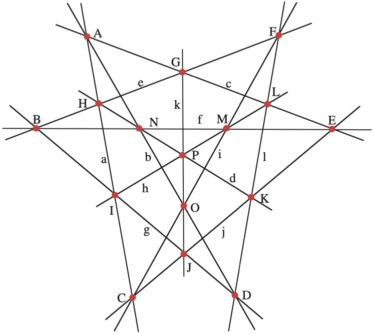


Figure: The left is a $(16_3, 12_4)$-configuration, the right is (12_3).

Geometric and combinatorial configurations

We can label the points and lines of a configuration like so and make a table.



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
a	e	a	b	c	e	c	a	a	g	d	c	f	b	b	d
b	f	i	g	f	i	e	d	g	j	j	h	h	d	i	h
c	g	j	l	j	l	k	e	h	k	l	l	i	f	k	k

This is a **combinatorial configuration**, as opposed to a **geometric configuration**.

Can we go the other way?

Let's take a look at the table

A	B	C	D	E	F	G
a	a	a	b	b	c	c
b	d	f	d	e	d	e
c	e	g	f	g	g	f

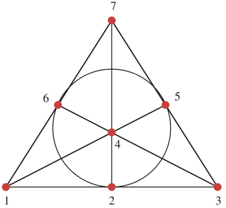
This is a combinatorial (7_3)-configuration. But is it geometrically realizable?

Can we go the other way?

Let's take a look at the table

A	B	C	D	E	F	G
a	a	a	b	b	c	c
b	d	f	d	e	d	e
c	e	g	f	g	g	f

This is a combinatorial (7_3)-configuration. But is it geometrically realizable? No! This is a special configuration called the **Fano plane**, and it is only realizable in special geometric spaces, not in the regular Euclidean plane.



Cyclic Configurations

Given any number $n \geq 7$ and a starting seed of $(0, 1, 3)$, you can make a combinatorial (n_3) configuration that places point p_1 at the intersection of lines 0, 1, and 3, and point p_i at the intersection of lines $i \mod n$, $1 + i \mod n$, and $3 + i \mod n$. Like so:

p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	0
3	4	5	6	7	8	0	1	2

This is called a **cyclic** configuration, denoted $C_3(n)$.

Pappus

The smallest geometric (n_3)-configurations are (9_3). One of them is $C_3(9)$. Another is known as the Pappus configuration.

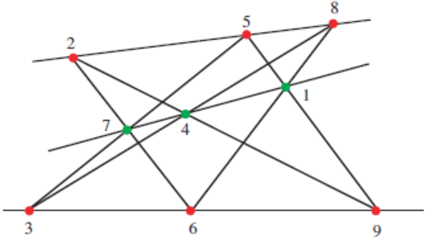


Figure: Pappus' configuration

Augmenting an (n_3)

Below is a table for the Pappus configuration we saw:

1	2	3	4	5	6	7	8	9
A	D	G	A	B	C	A	C	B
B	E	H	D	E	F	F	E	D
C	F	I	G	H	I	H	G	I

We can add a new point and line and reconfigure this to get a new (10_3)-configuration:

1	2	3	4	5	6	7	8	9	10
A	D	G	A	B	C	A	C	B	E'
B	E'	H	D	I'	F	F	J	D	I'
C	F	E'	G	H	I'	H	G	J	J

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A	D	G	A	B	C	A	C	B
B	E	H	D	E	F	F	E	D
C	F	I	G	H	I	H	G	I

We can add a new point and line and reconfigure this to get a new (10_3)-configuration:

1	2	3	4	5	6	7	8	9	10
A	D	G	A	B	C	A	C	B	E'
B	E'	H	D	I'	F	F	J	D	I'
C	F	E'	G	H	I'	H	G	J	J

Undoing an augmentation is **reducing**. Some (n_3) configurations are **irreducible.**

Configurations in 3D!

We can also make configurations in 3D! Two of the best known are the Reye configuration and the Schläfli double six.

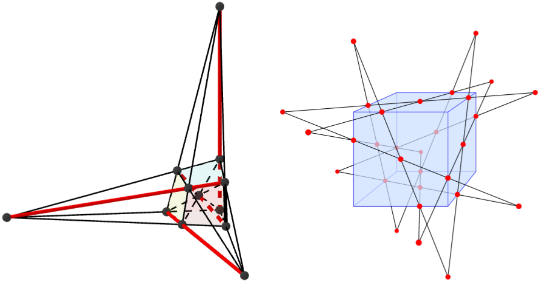

Figure: The $(12_4, 16_3)$ Reye configuration (left) and the $(30_2, 12_5)$ Schläfli double six (right)

Thanks for coming!

Happy Math Day!

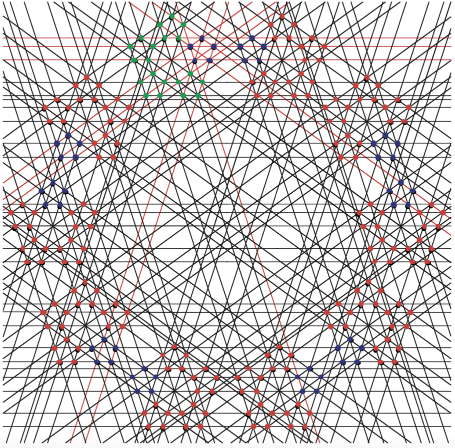

Figure: A floral $(120_5, 150_4)$-configuration