Let us use the basis $\langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, e_7\rangle = K^{\perp} \oplus \langle e_7\rangle = \mathrm{Pic}X$. I will single out e_7 because in the quasielliptic fibration I'm thinking of, p_7 is the only point that is blown up once instead of twice. For our change of basis matrix between $\{r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, e_7\}$ and $\{\ell, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$, we get

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}^{-1}$$

which makes use of only integers.

Thus we can rewrite

$$\begin{aligned} \ell &= r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 3r_5 + 3r_6 + 3e_7 \\ e_1 &= r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + e_7 \\ e_2 &= r_2 + r_3 + r_4 + r_5 + r_6 + e_7 \\ e_3 &= r_3 + r_4 + r_5 + r_6 + e_7 \\ e_4 &= r_4 + r_5 + r_6 + e_7 \\ e_5 &= r_5 + r_6 + e_7 \\ e_6 &= r_6 + e_7 \\ e_7 &= e_7 \\ e_8 &= -r_7 + e_7 \\ e_9 &= -r_7 - r_8 + e_7 \end{aligned}$$

The sixteen (-2)-curves we get are as follows:

$$\begin{aligned}
\ell - e_1 - e_2 - e_7\\
2\ell - e_3 - e_4 - e_5 - e_6 - e_8 - e_9\\
\ell - e_3 - e_4 - e_7\\
2\ell - e_1 - e_2 - e_5 - e_6 - e_8 - e_9\\
\ell - e_5 - e_6 - e_7\\
2\ell - e_1 - e_2 - e_3 - e_4 - e_8 - e_9\\
\ell - e_7 - e_8 - e_9\\
2\ell - e_1 - e_2 - e_3 - e_4 - e_5 - e_6\\
e_1 - e_2\\
3\ell - 2e_1 - e_3 - e_4 - e_5 - e_6 - e_7 - e_8 - e_9\\
e_3 - e_4\\
3\ell - e_1 - e_2 - 2e_3 - e_5 - e_6 - e_7 - e_8 - e_9\\
e_5 - e_6\\
3\ell - e_1 - e_2 - e_3 - e_4 - 2e_5 - e_7 - e_8 - e_9\\
e_8 - e_9\\
3\ell - e_1 - e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - 2e_8.
\end{aligned}$$

Writing these in our new basis $\langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8, e_7\rangle$ yields

$$r_0 + r_3 + r_4 + r_5 + r_6 \tag{1}$$
$$2r_0 + 2r_1 + 4r_2 + 5r_3 + 4r_4 + 3r_5 + 2r_6 + 2r_7 + r_8 \tag{2}$$
$$r_0 + r_1 + 2r_2 + 2r_3 + r_4 + r_5 + r_6 \tag{3}$$
$$2r_0 + r_1 + 2r_2 + 4r_3 + 4r_4 + 3r_5 + 2r_6 + 2r_7 + r_8 \tag{4}$$
$$r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 2r_5 + r_6 \tag{5}$$
$$2r_0 + r_1 + 2r_2 + 3r_3 + 2r_4 + 2r_5 + 2r_6 + 2r_7 + r_8 \tag{6}$$
$$r_0 + r_1 + 2r_2 + 3r_3 + 3r_4 + 3r_5 + 3r_6 + 2r_7 + r_8 \tag{7}$$
$$2r_0 + r_1 + 2r_2 + 3r_3 + 2r_4 + r_5 \tag{8}$$
$$r_1 \tag{9}$$
$$3r_0 + r_1 + 4r_2 + 6r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 + r_8 \tag{10}$$
$$r_3 \tag{11}$$
$$3r_0 + 2r_1 + 4r_2 + 5r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 + r_8 \tag{12}$$
$$r_5 \tag{13}$$
$$3r_0 + 2r_1 + 4r_2 + 6r_3 + 5r_4 + 3r_5 + 3r_6 + 2r_7 + r_8 \tag{14}$$
$$r_8 \tag{15}$$
$$3r_0 + 2r_1 + 4r_2 + 6r_3 + 5r_4 + 4r_5 + 3r_6 + 2r_7 \tag{16}$$

Notice that there is no e_7 vector present because all of these elements are in $K^\perp$. Now we want to take the free $\mathbb{Z}$-module $K^\perp = \langle r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8\rangle$ and mod out by the space generated by these sixteen (-2)-curves. Modding out by $\langle r_1, r_3, r_5, r_8\rangle$ yields $\langle r_0, r_2, r_4, r_6, r_7\rangle$. We then get from (1) and (3): $r_0 + r_4 + r_6 = r_0 + 2r_2 + r_4 + r_6 = 0$, and so $2r_2 = 0$ in the quotient.

From line (5) we also get $r_0 + 3r_4 + r_6 = 0$, and since we know $r_0 + r_4 + r_6 = 0$ from before, we get $2r_4 = 0$.

From line (4) we know that $2r_0+2r_6+2r_7 = 0$. From line (7) we get $r_0+r_4+3r_6+2r_7 = 0$, but since $r_0 + r_4 + r_6 = 0$, we get $2r_6 + 2r_7 = 0$. Since $2r_0 + 2r_6 + 2r_7 = 2r_6 + 2r_7 = 0$, we know $2r_0 = 0$.

From line (2), we know $2r_0+4r_4+2r_6+2r_7 = 0 = 2(r_0+r_4+r_6)+2r_4+2r_7 = 0+0+2r_7$. Thus $2r_7 = 0$ and since $2r_7 + 2r_6 = 0$, we know $2r_6 = 0$.

Thus our group is $\langle r_0, r_2, r_4, r_6, r_7 | 2r_{0\le i\le 7}, r_0 + r_4 + r_6\rangle \cong (\mathbb{Z}/2\mathbb{Z})^{\oplus 4}$. We can choose representative elements:

$$
\begin{aligned}
&0\\
r_0 &= \ell - e_1 - e_2 - e_3\\
r_2 &= e_2 - e_3\\
r_4 &= e_4 - e_5\\
r_6 &= e_6 - e_7\\
r_7 &= e_7 - e_8\\
r_0 + r_2 &= \ell - e_1 - 2e_3\\
r_0 + r_7 &= \ell - e_1 - e_2 - e_3 + e_7 - e_8\\
r_2 + r_4 &= e_2 - e_3 + e_4 - e_5\\
r_2 + r_6 &= e_2 - e_3 + e_6 - e_7\\
r_2 + r_7 &= e_2 - e_3 + e_7 - e_8\\
r_4 + r_7 &= e_4 - e_5 + e_7 - e_8\\
r_6 + r_7 &= e_6 - e_8\\
r_0 + r_2 + r_7 &= \ell - e_1 - 2e_3 + e_7 - e_8\\
r_2 + r_4 + r_7 &= e_2 - e_3 + e_4 - e_5 + e_7 - e_8\\
r_2 + r_6 + r_7 &= e_2 - e_3 + e_6 - e_8
\end{aligned}
$$

The sixteen (-1)-curves I found are:

$$\begin{aligned}
e_2\\
e_4\\
e_6\\
e_7\\
e_9\\
\ell - e_1 - e_3\\
\ell - e_1 - e_5\\
\ell - e_3 - e_5\\
\ell - e_1 - e_8\\
\ell - e_3 - e_8\\
\ell - e_5 - e_8\\
2\ell - e_1 - e_3 - e_5 - e_7 - e_8\\
2\ell - e_1 - e_3 - e_5 - e_8 - e_9\\
2\ell - e_1 - e_2 - e_3 - e_5 - e_8\\
2\ell - e_1 - e_3 - e_4 - e_5 - e_8\\
2\ell - e_1 - e_3 - e_5 - e_6 - e_8
\end{aligned}$$

Note: I didn't find these using the $E + v + mK$ method, I just guessed and showed that they each meet every (-2)-curve nonnegatively. The other potential candidates for (-1)-curves are imposters because they meet some (-2)-curves negatively. For example, $(2\ell - e_1 - e_2 - e_5 - e_6 - e_8).(2\ell - e_1 - e_2 - e_5 - e_6 - e_8 - e_9) = -1$.