Let $P_1, \ldots, P_7 \in \mathbb{P}^2_k$. First note that $\binom{8+2}{2} = 45$, and so 44 conditions will determine a unique octic. Now consider the six triple points $P_1, \ldots, P_6$. Six triple points impose $6 * \binom{3+1}{2} = 6 * 6 = 36$ conditions, and choosing P_7 as a double point imposes an extra $\binom{2+1}{2} = 3$ conditions for a total of 39. Thus there is a $45 - 39 = 6$-dimensional vector space of octics that have triple points at $P_1, \ldots, P_6$ and a double point at P_7. The six generating octics are simple to construct:

$$\begin{aligned}
O_1 &= C\,(1,2,3,4,5)\,C\,(2,3,4,5,6)\,C\,(1,6,3,4,7)\,C\,(1,6,2,5,7)\\
O_2 &= C\,(1,2,3,4,6)\,C\,(1,3,4,5,6)\,C\,(2,5,7,1,3)\,C\,(2,5,7,4,6)\\
O_3 &= C\,(1,2,3,5,6)\,C\,(1,2,4,5,6)\,C\,(3,4,1,2,7)\,C\,(3,4,7,5,6)\\
O_4 &= C\,(1,2,3,4,6)\,C\,(2,3,4,5,6)\,C\,(1,5,7,2,3)\,C\,(1,6,7,4,5)\\
O_5 &= C\,(1,3,4,5,6)\,C\,(1,2,3,4,5)\,C\,(2,6,7,1,5)\,C\,(2,6,7,3,4)\\
O_6 &= C\,(1,2,3,5,6)\,C\,(2,3,4,5,6)\,C\,(1,4,7,2,3)\,C\,(1,4,7,5,6)
\end{aligned}$$

where each octic is a product of four conics determined by the five specified points.

In order to upgrade P_7 from a double point to a triple point, we need to choose coefficients $c_1, \ldots, c_6$ that produces an adequate linear combination of $O_1, \ldots, O_6$. Since seven triple points induces 42 conditions, we need only use the first four octics $O_1, \ldots, O_4$.

We want a linear combination

$$O = c_1 O_1 + c_2 O_2 + c_3 O_3 + c_4 O_4$$

where $O_{xy}(P_7) = O_{xz}(P_7) = O_{yz}(P_7) = 0$. This is sufficient since P_7 is already a double point. That is, we are already given that $O_x(P_7) = O_y(P_7) = O_z(P_7) = 0$, and so if the desired conditions are fulfilled, then $O_x(P_7) = xO_{xx}(P_7) + yO_{xy}(P_7) + zO_{xz}(P_7) = 0$ and we must have $O_{xx}(P_7) = 0$ as well. Same goes for $O_{yy}(P_7)$ and $O_{zz}(P_7)$.

We want

$$\begin{aligned}
c_1 O_{1xy}(P_7) + c_2 O_{2xy}(P_7) + c_3 O_{3xy}(P_7) + c_4 O_{4xy}(P_7) &= 0\\
c_1 O_{1xz}(P_7) + c_2 O_{2xz}(P_7) + c_3 O_{3xz}(P_7) + c_4 O_{4xz}(P_7) &= 0\\
c_1 O_{1yz}(P_7) + c_2 O_{2yz}(P_7) + c_3 O_{3yz}(P_7) + c_4 O_{4yz}(P_7) &= 0
\end{aligned}$$

which can be turned into the following matrix equation:

$$\begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} (P_7) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

So

$$\vec{c} \in \ker \begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} (P_7)$$

and so we can take

$$\vec{c} = \begin{pmatrix} \det \begin{pmatrix} O_{2xy} & O_{3xy} & O_{4xy} \\ O_{2xz} & O_{3xz} & O_{4xz} \\ O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} \\ -\det \begin{pmatrix} O_{1xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{3yz} & O_{4yz} \end{pmatrix} \\ \det \begin{pmatrix} O_{1xy} & O_{2xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{4yz} \end{pmatrix} \\ -\det \begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} \\ O_{1xz} & O_{2xz} & O_{3xz} \\ O_{1yz} & O_{2yz} & O_{3yz} \end{pmatrix} \end{pmatrix} (P_7).$$

Then $\vec{c} \in \ker \begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} (P_7)$ because for example

$$\begin{aligned} O_{1xy} \det \begin{pmatrix} O_{2xy} & O_{3xy} & O_{4xy} \\ O_{2xz} & O_{3xz} & O_{4xz} \\ O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} &- O_{2xy} \det \begin{pmatrix} O_{1xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{3yz} & O_{4yz} \end{pmatrix} \\ +O_{3xy} \det \begin{pmatrix} O_{1xy} & O_{2xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{4yz} \end{pmatrix} &- O_{4xy} \det \begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} \\ O_{1xz} & O_{2xz} & O_{3xz} \\ O_{1yz} & O_{2yz} & O_{3yz} \end{pmatrix} \\ &= \det \begin{pmatrix} O_{1xy} & O_{2xy} & O_{3xy} & O_{4xy} \\ O_{1xy} & O_{2xy} & O_{3xy} & O_{4xy} \\ O_{1xz} & O_{2xz} & O_{3xz} & O_{4xz} \\ O_{1yz} & O_{2yz} & O_{3yz} & O_{4yz} \end{pmatrix} = 0. \end{aligned}$$

The same applies to the other two rows of $\vec{c}$.

So finally

$$\begin{pmatrix} O_1 & O_2 & O_3 & O_4 \end{pmatrix} \vec{c}$$

is an octic polynomial that vanishes with multiplicity 3 at $P_1, \ldots, P_7$.

You can find a Desmos gadget illustrating this octic here, but be warned that it is very slow to load.