Summary: $L.C + C[2]$ is a $(12_4, 16_3)$-configuration.

Let C be a smooth cubic curve and let L be a line in $\mathbb{P}^2$ such that $L \cap C = \{A, B, C\}$, three distinct points. Let $O \in C$ be flex and the zero of the group law of C. Then let $C[2] = \langle \gamma_1, \gamma_2 \rangle \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

Then the set of twelve points given by $\{A, B, C\} + C[2]$ is the image of some projection of the $D_4 \subseteq \mathbb{P}^3$ into the plane. One can check easily that it satisfies all the required incidence relations (line three points on every line, four lines through every point). This also makes it a $(12_4, 16_3)$-configuration.

Question: How can this construction be modified to produce new interesting configurations? Is $L.C + C[3]$ interesting? What about $Q.C + C[2]$? Or perhaps replacing C with a higher-degree curve and finding an interesting subgroup of its Picard group??? Idk...