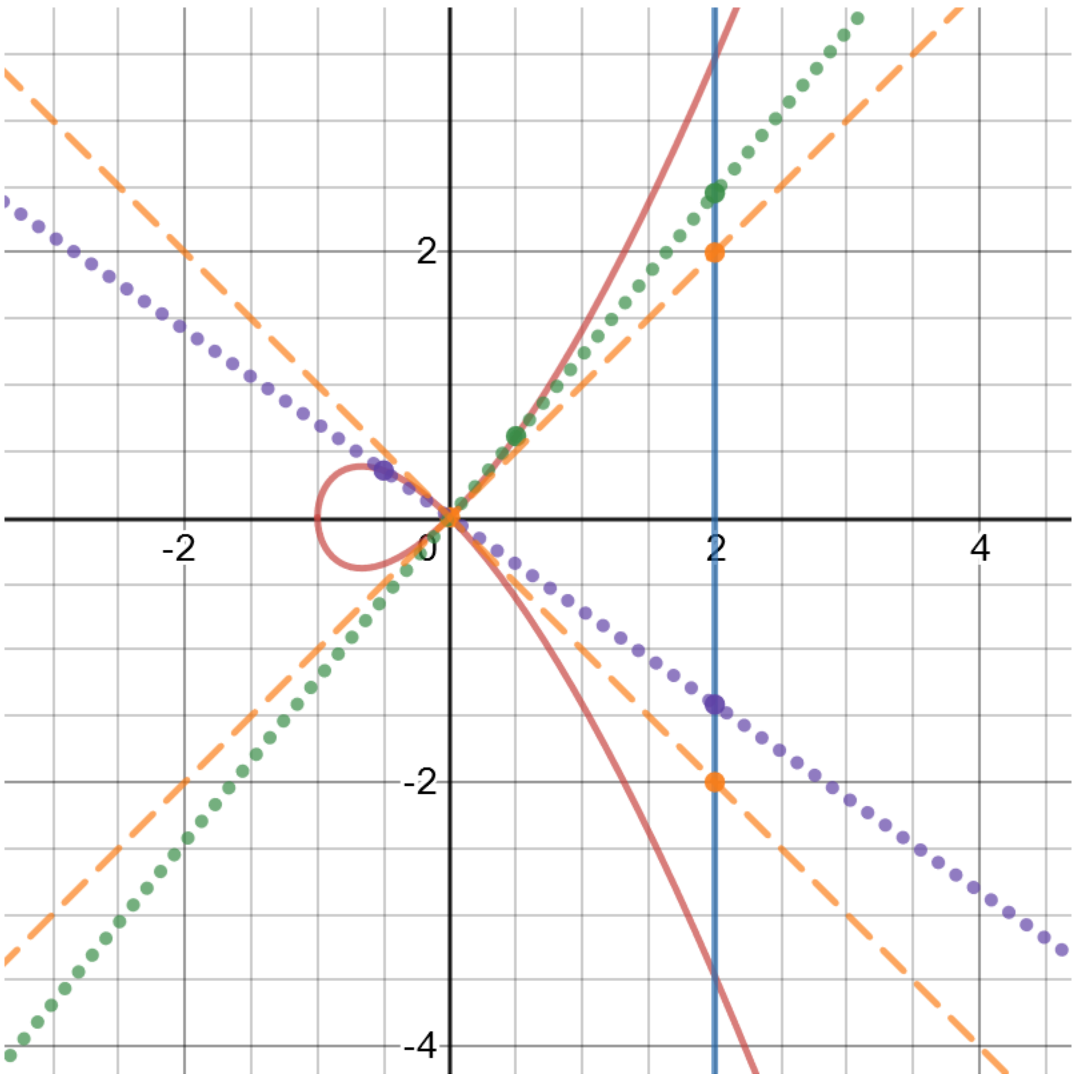
We shall show that the cubic given by the equation $C = y^2z - x^3 - x^2z$ is birationally equivalent to the projective line $L = x - 2z$. The rational map f will be given by projection from the point $(0, 0, 1)$ on C and will be defined on $C\setminus\{(0, 0, 1)\}$ and $L\setminus\{(2, 2, 1), (2, -2, 1)\}$.



We shall show that this map, which is explicitly given by $(a, b, c) \mapsto (2a, 2b, a)$, induces an isomorphism on the rings $f^* : \mathcal{O}_L(\mathfrak{D}(y^2 - 4z^2)) \to \mathcal{O}_C(\mathfrak{D}(y^2 - x^2))$.

Note that $\mathcal{O}_L(\mathfrak{D}(y^2-4z^2)) = (k[x, y, z]/(x-2z))_{\mathfrak{D}(y^2-4z^2),0}$ (i.e., you are allowed to $\mathfrak{D}$ivide by the element $y^2 - 4z^2$) and $\mathcal{O}_C(\mathfrak{D}(y^2 - x^2)) = (k[x, y, z]/(y^2z - x^3 - x^2z))_{\mathfrak{D}(y^2-x^2),0}$. Also note that in $\mathcal{O}_L$, we have $2z = x$ and so $y^2 - 4z^2 = y^2 - x^2$. Then note $f^*(x) = 2x$, $f^*(y) = 2y$ and $f^*(z) = x$. Then note that f^* is surjective since $z = x^3/(y^2 - x^2)$ in $\mathcal{O}_C$ and so $f^*\left(\frac{x^3}{2y^2-2x^2}\right) = z$.

The inverse map is given by $(f^*)^{-1}(x) = x/2$, $(f^*)^{-1}(y) = y/2$, and $(f^*)^{-1}(z) = x^3/(2y^2-2x^2) = (x^3)/(2(y^2 - 4z^2))$.

Technically, we should only be looking at degree-0 stuff but showing that this stuff works for degree-1 stuff gets us that.