$$(sr, t(1-r)) \longmapsto sr + t(1-r)$$

$$\longmapsto \big(((s+t)r+t)r, ((s+t)r+t)(1+r) \big)$$

$$= \big((s+t)r + tr, (s+t)r + t + (s+t)r + tr \big)$$

$$= (sr, t(1+r)) \quad \checkmark$$

Yes!

okay....

$$R \cong Rr \times R(1-r).$$

$$\mathbb{P}(\text{win} \mid 3 \text{ bunch})\mathbb{P}(3 \text{ bunch})$$
$$+ \mathbb{P}(\text{win} \mid 2 \text{ bunch})\mathbb{P}(2 \text{ bunch})$$
$$+ \mathbb{P}(\text{win} \mid 1 \text{ bunch})\mathbb{P}(1 \text{ bunch})$$

$$= \frac{\binom{6}{4}}{\binom{7}{5}} \cdot \frac{1}{49}$$

$$+ \frac{\binom{5}{3}}{\binom{7}{5}} \cdot \frac{7\cdot 6\cdot 3}{7^3}$$

$$+ \frac{\binom{4}{2}}{\binom{7}{5}} \cdot \frac{7\cdot 6\cdot 5}{7^3}$$

players

$$7\cdot 6\cdot 3 = 7\cdot 6\cdot \binom{3}{2}$$

bach size.

$$\frac{15}{21}\cdot\frac{7}{343} + \frac{10}{21}\cdot\frac{126}{343} + \frac{6}{21}\cdot\frac{210}{343}.$$

$$= \frac{5}{343} + \frac{60}{343} + \frac{60}{343}$$

$$\frac{125}{343} \approx 36.44\%.$$

$$= \left(\frac{5}{7}\right)^3 \quad \text{If blind.}$$

If no ones behind the horse:

$$\frac{\binom{5}{4}}{\binom{6}{5}} \cdot \frac{6}{6^3} + \frac{\binom{4}{3}}{\binom{6}{5}} \cdot \frac{6 \cdot 5 \cdot 3}{6^3}$$

$$+ \frac{\binom{3}{2}}{\binom{6}{5}} \cdot \frac{6 \cdot 5 \cdot 4}{6^3}$$

$$\frac{5}{6}\cdot\frac{6}{6^3}+\frac{4}{6}\cdot\frac{90}{6^3}+\frac{3}{6}\cdot\frac{120}{6^3}$$

$$\frac{5}{6}\cdot\frac{1}{36}+\frac{4}{6}\cdot\frac{15}{36}+\frac{3}{6}\cdot\frac{20}{36}$$

$$\frac{125}{216}=\left(\frac{5}{6}\right)^3$$

$$\approx 57.87\% .$$

If one person is behind the horse.

$$\frac{\binom{5}{3}}{\binom{6}{4}}\cdot\frac{6}{6^2}+\frac{\binom{4}{2}}{\binom{6}{4}}\cdot\frac{6\cdot 5}{6^2}$$

$$\frac{10}{15}\cdot\frac{1}{6}+\frac{6}{15}\cdot\frac{5}{6}$$

$$\frac{40}{90} = \frac{4}{9}$$

$$= \left(\frac{2}{3}\right)^2 = \left(\frac{4}{6}\right)^2.$$

$$\approx 44.44\%.$$

If two players hide behind horse:

6 hide places, 1 hider, 4 guesses.

$$\frac{\binom{5}{3}}{\binom{6}{4}} = \frac{10}{15}.$$

$$= \frac{2}{3} = 66.66\%$$

$$\mathbb{P}(\text{seeker wins}) = \boxed{\left(\frac{\text{guesses}}{\text{hiding places}}\right)^{\text{hiders}}}$$

All three : 100%.

$$\mathbb{P}(\text{no one picks horse}) = \frac{6^3}{7^3}.$$

$$\mathbb{P}(1 \text{ person picks horse}) = \frac{6^2 \cdot 3}{7^3}$$

$$\mathbb{P}(2 \text{ people pick horse}) = \frac{6 \cdot 3}{7^3}$$

$$\mathbb{P}(3 \text{ people pick horse}) = \frac{1}{7^3}$$

$$\frac{6^3}{7^3} \cdot \left(\frac{5}{6}\right)^3 + \left(\frac{6^2 \cdot 3}{7^3}\right)\left(\frac{4}{6}\right)^2$$

$$+ \frac{6 \cdot 3}{7^3}\left(\frac{4}{6}\right) + \frac{1}{7^3}(1).$$

$$\frac{\cancel{6^3} \cdot 5^3}{7^3 \cdot \cancel{6^3}} + \frac{\cancel{6^2} \cdot 3 \cdot 4^2}{7^3 \cdot \cancel{6^2}} + \frac{\cancel{6} \cdot 3 \cdot 4}{7^3 \cdot \cancel{6}} + \frac{1}{7^3}.$$

$$\frac{5^3}{7^3} + \frac{3 \cdot 4^2}{7^3} + \frac{3 \cdot 4}{7^3} + \frac{1}{7^3}$$

$$\frac{186}{343}$$

$\approx 54.23\%$ total.

If: seeker knows about rocking horse & hiders choose uniformly & independently.

If seeker is ignorant @ beginning of game.

$$\frac{6^3}{7^3}\left(\frac{5}{7}\right)^3 + \left(\frac{6^2 \cdot 3}{7^3}\right)\left(\frac{4}{6}\right)^2$$

$$+ \frac{6 \cdot 3}{7^3}\left(\frac{4}{6}\right) + \frac{1}{7^3}(1).$$

$$\frac{6^3 \cdot 5^3}{7^6} + \frac{3 \cdot 4^2}{7^3} + \frac{3 \cdot 4}{7^3} + \frac{1}{7^3}$$

$$\frac{47923}{117649}$$

$$\approx 40.73\%.$$

and $\frac{127}{343} \approx 37.03\%$ chance of leaving the rule.

$$\frac{14^{10}}{15^{10}} \approx \frac{1}{2}.$$

$\mathbb{P}(\text{hider picks horse}) = p.$

$\mathbb{P}(\text{hider picks other}) = \frac{1-p}{6}.$

$\mathbb{P}(\text{no one picks horse}) = (1-p)^3$

$\mathbb{P}(\text{one person picks horse}) = p(1-p)^2 \cdot 3$

$\mathbb{P}(\text{two people pick horse}) = p^2(1-p) \cdot 3.$

$\mathbb{P}(\text{all 3 pick horse}) = p^3.$

$\mathbb{P}(\text{seeker wins}) = S(p)$

$$(1-p)^3 \cdot \left(\frac{5}{6}\right)^3 + 3p(1-p)^2 \left(\frac{4}{6}\right)^2$$

$$+3p^2(1-p) \left(\frac{4}{6}\right) + p^3 (1)$$

minimize $S(p)$ for

$p \in [0, 1]$.

$$\frac{125}{216}(1-p)^3 + \frac{12}{9}p(1-p)^2$$

$$+2p^2(1-p) + p^3.$$

$S(p)$ minimal at

$$p \approx 0.2024.$$
$$= 20.24\%.$$

$$\frac{125}{216}\left(1-3p+3p^2-p^3\right)+\frac{4}{3}\left(p-2p^2+p^3\right)$$
$$+2p^2-2p^3+p^3$$

$$\frac{125}{216}-\frac{125}{72}p+\frac{125}{72}p^2-\frac{125}{216}p^3$$

$$+\frac{4}{3}p-\frac{8}{3}p^2+\frac{4}{3}p^3+2p^2-p^3$$

$$\frac{125}{216}+\left(\frac{4}{3}-\frac{125}{72}\right)p+\left(\frac{125}{72}-\frac{8}{3}+2\right)p^2$$
$$+\left(\frac{4}{3}-1-\frac{125}{216}\right)p^3$$

$$\frac{125}{216} - \frac{29}{72}p + \frac{77}{72}p^2 - \frac{53}{216}p^3$$

$$\mapsto -87 + 462p - 159p^2$$

roots

$$\frac{-462 \pm \sqrt{213444 - 55332}}{-318}$$

$$\boxed{\frac{462 - \sqrt{158112}}{318}}$$

optimum probability

$$158112 = 2^5 \times 3^4 \times 61$$

$$= 2^4 \times 3^4 \times 122$$

$$462 = 2 \times 3 \times 7 \times 11$$

$$318 = 2 \times 3 \times 53$$

$$\frac{2 \times 3 \times 7 \times 11 - 2^2 \times 3^2 \times \sqrt{122}}{2 \times 3 \times 53}$$

$$= \boxed{\frac{77 - 6\sqrt{122}}{53}}$$