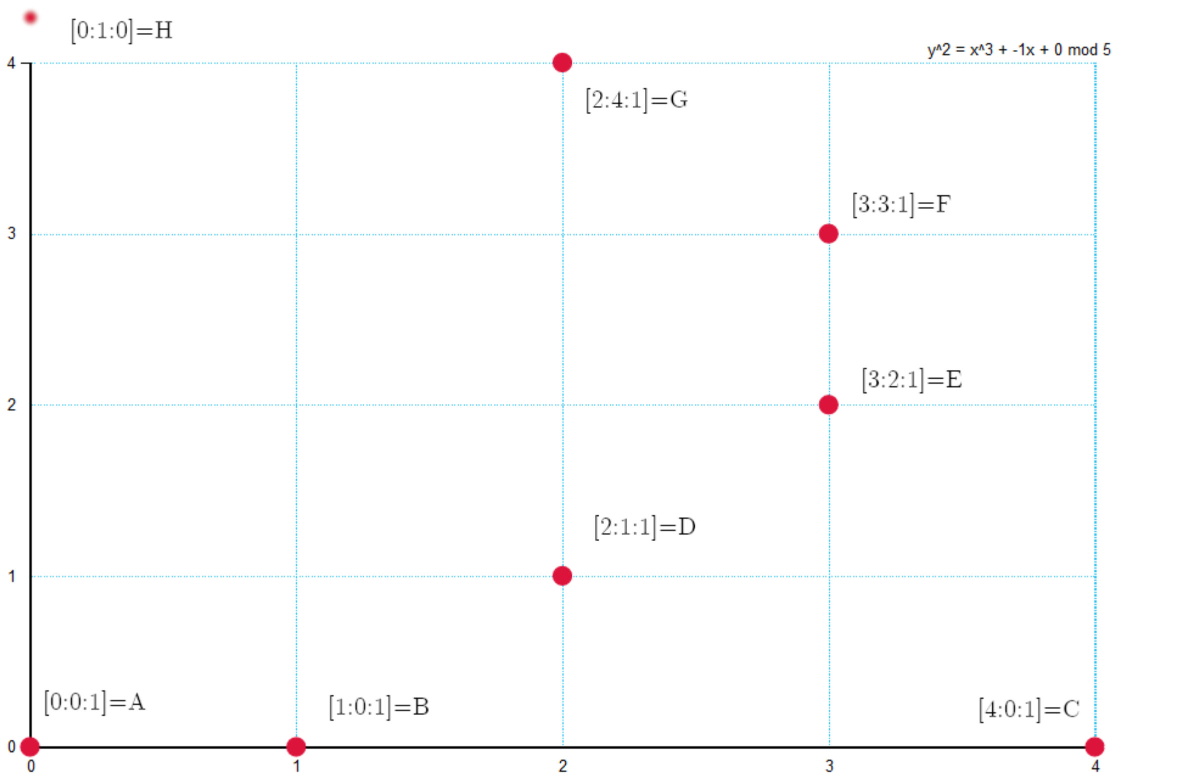
Consider the scheme $X = \text{Proj}(\mathbb{Z}/5\mathbb{Z}[x, y, z]/(x^3 - y^2z - xz^2))$. We have the following graph of X :



Note that $3H = (z)$ and so $H = 0 \in \text{Cl}^\circ X$. Note that when $x = 0$, we have $-y^2z = 0$, which intersects A with multiplicity 2 and H with multiplicity 1. Therefore $0 = (x) = 2A+H = 2A$. Similarly, we have $2B = 2C = 0$.

Now let's look at the principal divisor $(x-y)$. We get the polynomial $x^3 - x^2z - xz^2 = 0$. This has a solution at $A = [0 : 0 : 1]$ (note this does not have a solution at H, since the x-coordinate of H does not equal the y-coordinate of H). This polynomial factors into $x(x^2 - xz - z^2) = 0$. Then we have $(x^2 - xz - z^2) = (x - 3z)^2$, which has a solution at F with multiplicity 2. Thus $0 = A + 2F$. Therefore $0 = 2A + 4F = 4F$ and so $3F = -F = E$. Since $E = -F$, we also have $4E = 0$. Furthermore, $2E = 2(3F) = 6F = 4F + 2F = 2F$. Thus $2E = 2F = -A = A$.

We also have $F + G + B = (3x - 2y - 3z) = 0$, so $F + G = B$ and $F + B = D$ and $G + B = E$. Thus $2(F + G) = 0$ and so $A + 2G = 0$, thus $2G = A$, so $4G = 0$. Similarly as before, this means $2D = A$ and $4D = 0$, since $D = -G$.

Therefore $\text{Cl}^\circ X$ has one element of order 1 (H), three elements of order 2 ($A, B,$ and C), and four elements of order 4 ($D, E, F,$ and G). Therefore $\text{Cl}^\circ X \cong C_2 \times C_4$. We can draw its Cayley graph as follows (generated by B and F).

$$\begin{tikzcd}
D \arrow[rrr, "+F"] \arrow[dr, bend left, "+B"] & & & C \arrow[dl, bend right, "+B"'] \arrow[ddd, "+F"] \\
& F \arrow[r, "+F"] \arrow[ul, bend left, "+B"] & A \arrow[d, "+F"] \arrow[ur, bend right, "+B"'] & \\
& H \arrow[u, "+F"] \arrow[dl, bend right, "+B"'] & E \arrow[l, "+F"] \arrow[dr, bend left, "+B"] & \\
B \arrow[uuu, "+F"] \arrow[ur, bend right, "+B"'] & & & G \arrow[lll, "+F"] \arrow[ul, bend left, "+B"]
\end{tikzcd}$$