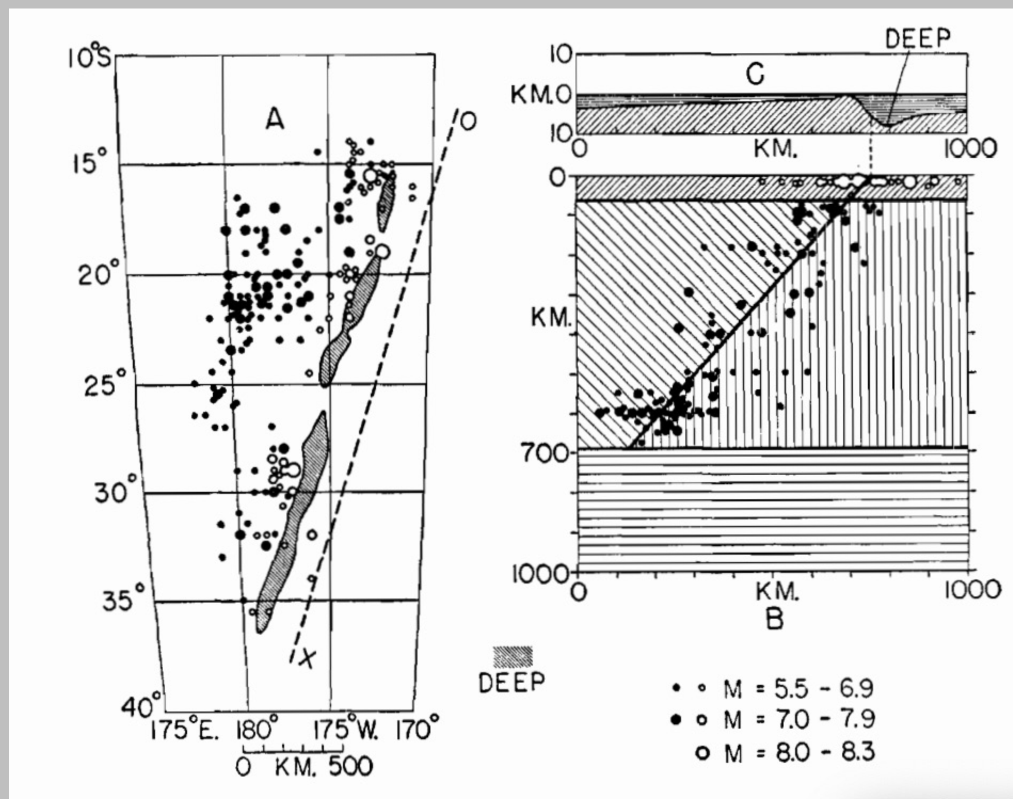
Outline

- Overview of earthquake location
- Grid-search with a known velocity model
  - Problem setup
  - Evaluation of misfit and error
  - Effect of station geometry on location
- Relative location and a few examples
- Gradient based linear inversion problem
  - Problem setup and approaches

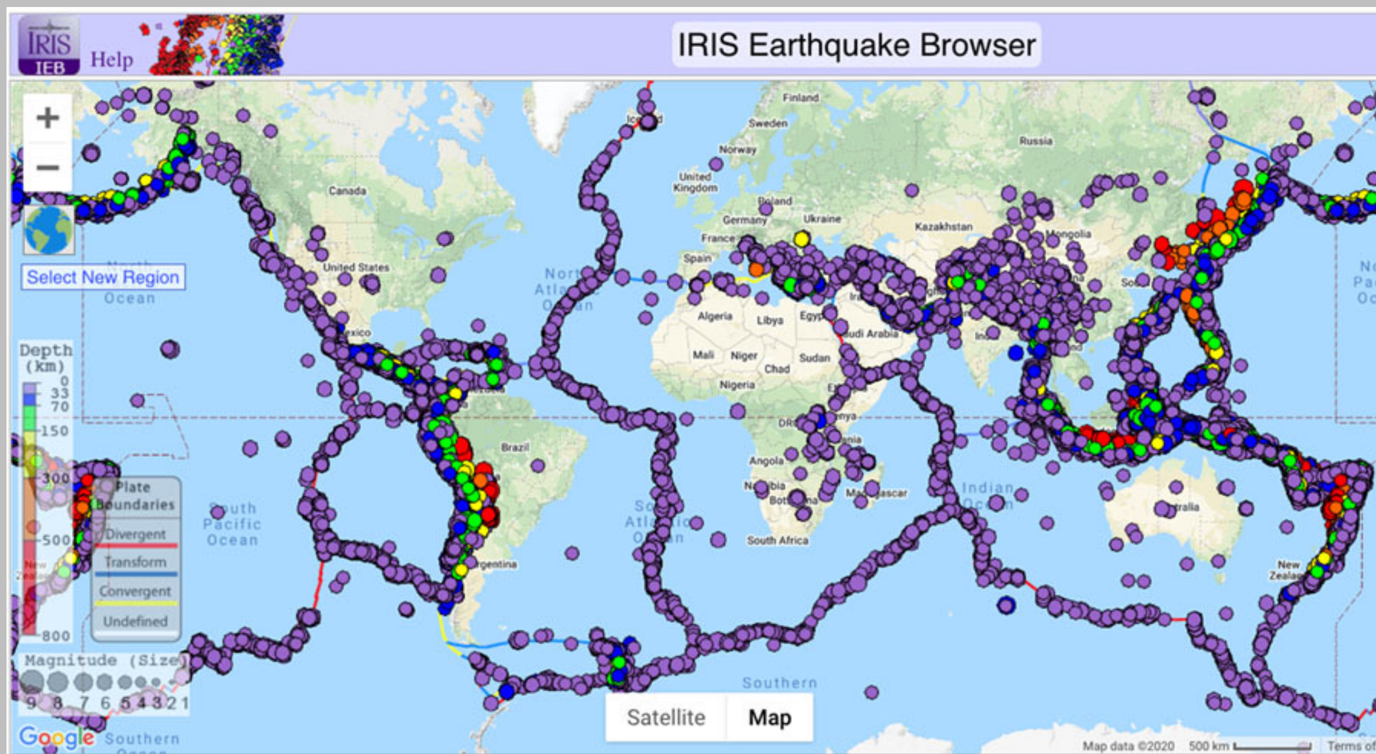
Part 1

Overview of earthquake location


N
50 kilometers



Benioff, 1949


IRIS Earthquake Browser
Help
Select New Region
Depth (km)
Plate Boundaries
Divergent
Transform
Convergent
Undefined
Magnitude (Size)
Satellite
Map
Map data ©2020
500 km

Earthquake location is:

- The most fundamental problem in earthquake seismology.
- Important for hazard assessment, early warning, seismicity, and find faults!
- Broadly speaking, the earthquake location problem has two parts:
  - Absolute location for individual earthquakes
  - Relative location for a group of earthquakes

Part 2

Grid-search for earthquake location.
Uncertainties and complications

Problem set up

Model vector (to be determined):

$$\mathbf{m} = (m_1, m_2, m_3, m_4) = (T, x, y, z).$$

Observations: Arrival time at different stations.

$$t_i^p = F_i(\mathbf{m}),$$

Goal: find a model vector that minimize the residual between prediction and observation

Predicted arrival time = Original time (T) + travel time (t_i^p).

$$r_i = t_i - t_i^p = t_i - F_i(\mathbf{m}),$$

Travel-time is non-linear!

- For a 2D problem:

$$t_i = \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{v},$$

- Therefore, it is not easy to use linear inversion to perform earthquake location problems (but there are ways to do this!)

Grid-search for best model parameter.

Least-square: square of model residual, L2 norm

$$\epsilon = \sum_{i=1}^{n} \left[t_i - t_i^p\right]^2,$$

variance: averaged residual = ε/n

“Variance reduction”: a common term in research papers.
If Parameter set m1 has a variance of 10;
Parameter set m2 has a variance of 5;
We would say that “we got a variance reduction of 50% with parameter set m2.

Sometimes, we use L1-norm, this handles outliers better than L2-norm, but the uncertainty assessment is not straightforward.

$$\epsilon = \sum_{i=1}^{n} \left|t_i - t_i^p\right|.$$

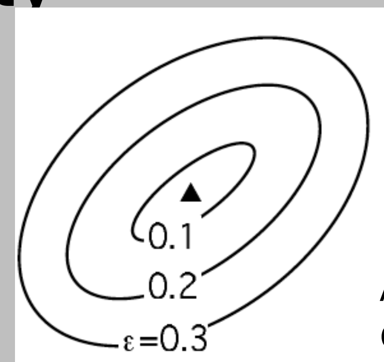
Estimate model uncertainty

Contours of variance for a set of different models will provide some idea of

$$\sigma^2(\mathbf{m}_{\text{best}}) = \frac{\sum_{i=1}^{n} \left[t_i - t_i^p(\mathbf{m}_{\text{best}})\right]^2}{n_{\text{df}}}$$

$$\chi^2 = \sum_{i=1}^{n} \frac{\left[t_i - t_i^p\right]^2}{\sigma_i^2},$$

The expected value of χ2 is the number of degrees of freedom:
Ndf = N – Np ,, where the Np is number of model parameters. In this case, Np = 4, because we have X, Y, Z and T0 in the model parameter.



A typical table of χ2 distribution.
Can be computed from Python

Table 5.1: Percentage points of the χ^2 distribution.

n_{df}	$\chi^2(95\%)$	$\chi^2(50\%)$	$\chi^2(5\%)$
5	1.15	4.35	11.07
10	3.94	9.34	18.31
20	10.85	19.34	31.41
50	34.76	49.33	67.50
100	77.93	99.33	124.34

standard deviation is critical in chi-square test..

- In practice, we typically use the observations at the best fitting location to estimate the chi-square value.

$$\sigma^2(\mathbf{m}_{\text{best}}) = \frac{\sum_{i=1}^{n} \left[t_i - t_i^p(\mathbf{m}_{\text{best}})\right]^2}{n_{df}}$$

$$\chi^2(\mathbf{m}) = \frac{\sum_{i=1}^{n} \left[t_i - t_i^p(\mathbf{m})\right]^2}{\sigma^2}$$