

# Results: [20, 35] Crossings

Jake Weatherhead

The CNN (88M params) and ViT (86M params) were fine-tuned on a new dataset  $\mathcal{D}$  comprising unknot  $\mathcal{U}$  and non-trivial knot  $\mathcal{K}$  diagrams partitioned into three disjoint data splits  $S := \{\text{train, val, test}\}$ , where:

$$\mathcal{D} = \bigsqcup_{s \in S} \mathcal{D}_s$$

$$|\mathcal{D}| = 560,000 \text{ diagrams,}$$

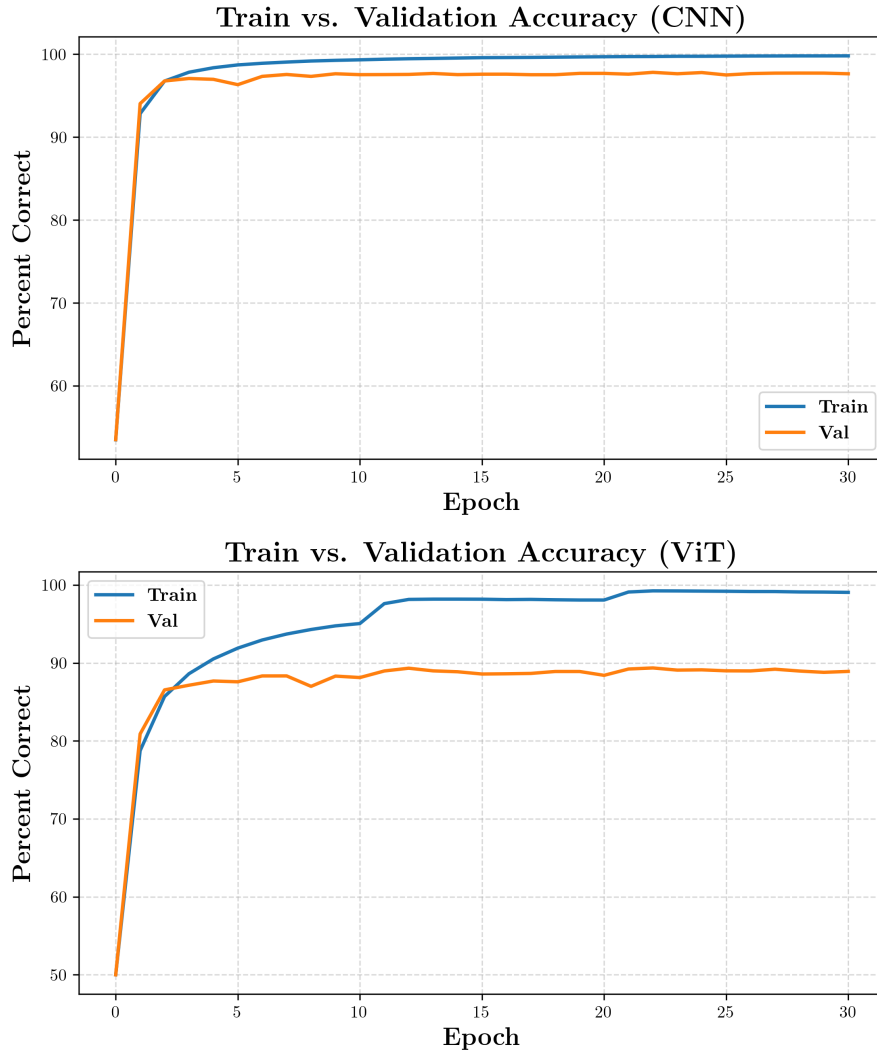
$$|\mathcal{D}_{\text{train}}| = 0.8 \cdot |\mathcal{D}| = 448,000 \text{ diagrams,}$$

$$|\mathcal{D}_{\text{val}}| = |\mathcal{D}_{\text{test}}| = 0.1 \cdot |\mathcal{D}| = 56,000 \text{ diagrams,}$$

$$\text{where } \forall s \in S, \mathcal{D}_s \in \mathcal{U}_s \cup \mathcal{K}_s \text{ and } |\mathcal{U}_s| = |\mathcal{K}_s|.$$

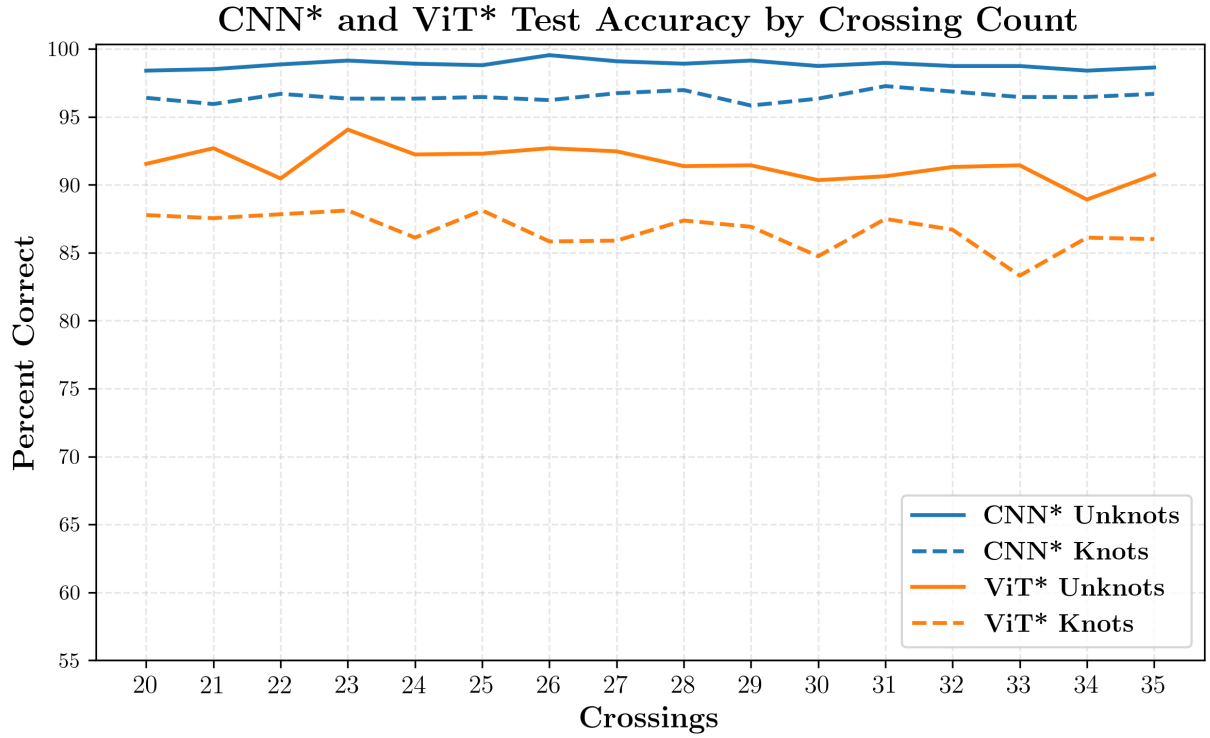
Each split contained knot diagrams for every crossing count  $n \in N := \{20, 21, \dots, 35\}$ . For all  $x, y \in N$ ,  $x \neq y$ , each split in  $S$  contained an equal number of  $x$  and  $y$ -crossing unknot and non-trivial knot diagrams.

## Learning Curves



## Test Accuracy by Crossing Count

Let CNN\* and ViT\* denote the models with the highest test classification accuracies. The test accuracy of CNN\* is 97.673%; the test accuracy of ViT\* is 89.075%.



## CNN\* Saliency Maps

A sample of CNN\* saliency maps for knot diagrams in  $\mathcal{D}_{\text{test}}$ .

### True Positives: Unknots Predicted as Unknots

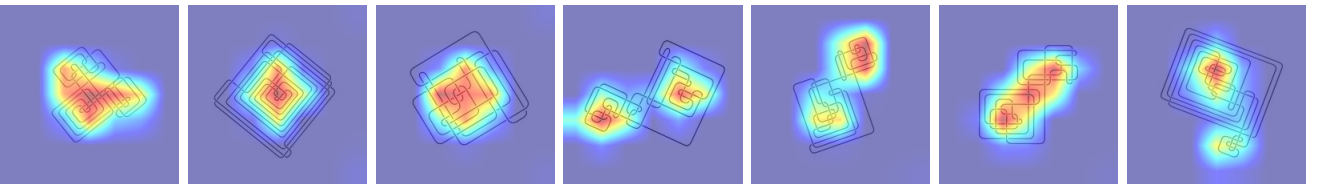


Figure 1: Of the 28,000 unknot diagrams in  $\mathcal{D}_{\text{test}}$ , CNN\* produced 27,677 true positives.

### False Negatives: Unknots Predicted as Non-Trivial

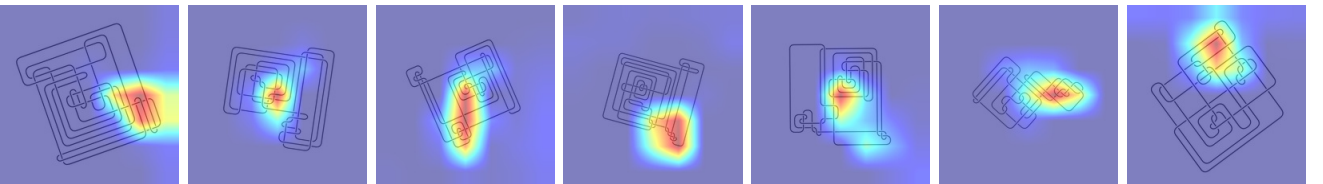


Figure 2: Of the 28,000 unknot diagrams in  $\mathcal{D}_{\text{test}}$ , CNN\* produced 323 false negatives.

### True Negatives: Non-Trivials Predicted as Non-Trivial

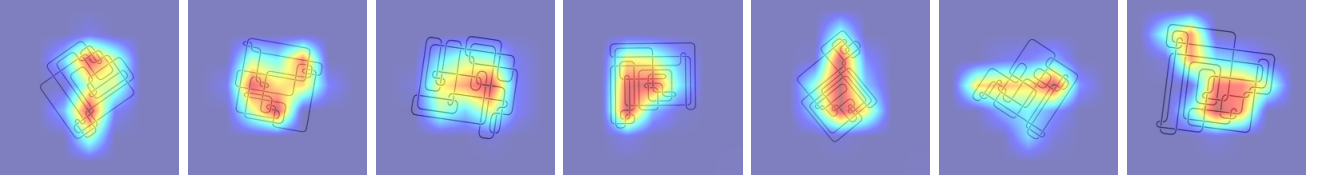


Figure 3: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , CNN\* produced 27,020 true negatives.

### False Positives: Non-Trivials Predicted as Unknots

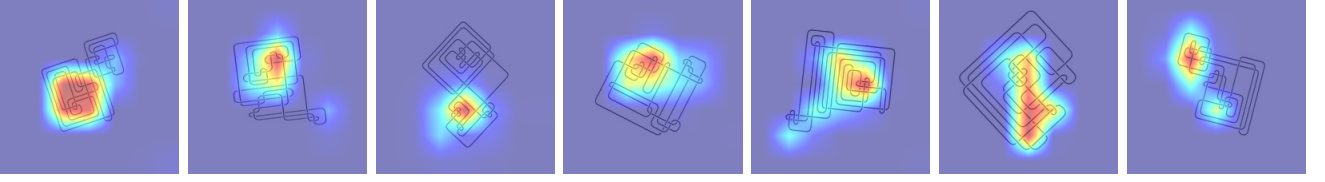


Figure 4: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , CNN\* produced 980 false positives.

## ViT\* Saliency Maps

A sample of ViT\* saliency maps for knot diagrams in  $\mathcal{D}_{test}$ .

### True Positives: Unknots Predicted as Unknots

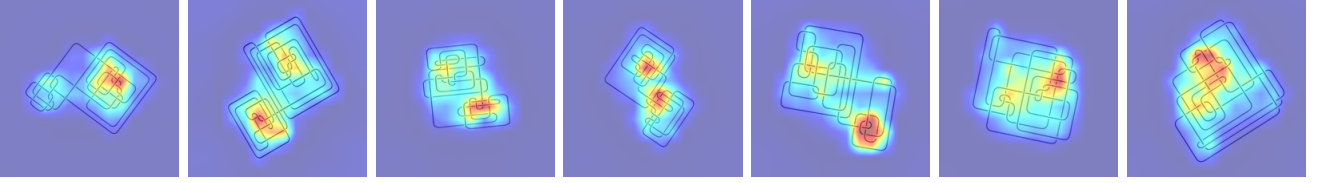


Figure 5: Of the 28,000 unknot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 25,630 true positives.

### False Negatives: Unknots Predicted as Non-Trivial

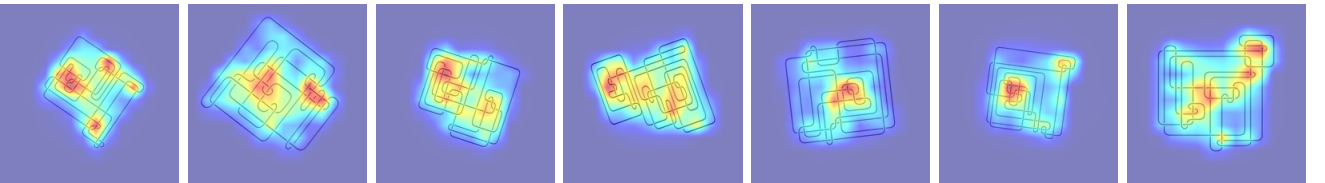


Figure 6: Of the 28,000 unknot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 2,370 false negatives.

### True Negatives: Non-Trivials Predicted as Non-Trivial

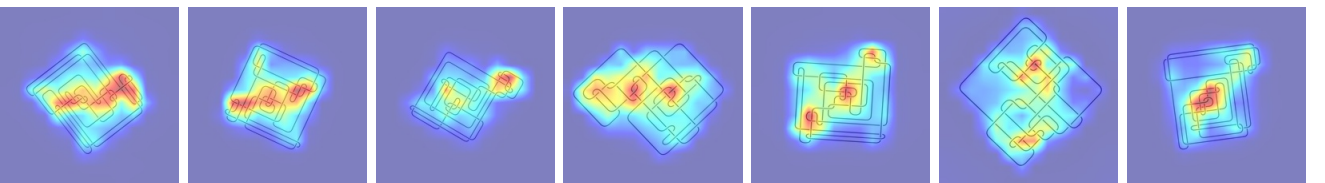


Figure 7: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 24,252 true negatives.

## False Positives: Non-Trivials Predicted as Unknots

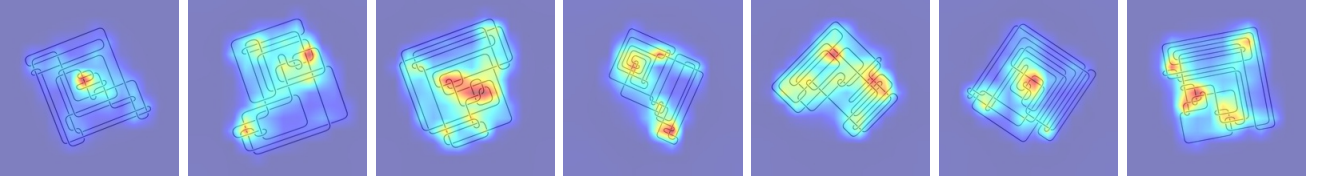


Figure 8: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 3,748 false positives.

## Further Evidence of Learning

Let  $m_v$  denote checkpoint model  $m \in \{\text{CNN}, \text{ViT}\}$  with validation accuracy  $v$ . The figures below are examples of a model  $m$  initially misclassifying a knot diagram  $d \in \mathcal{D}_{test}$ , later learning to correctly classify  $d$ . Let  $p$  and  $t$  respectively denote the predicted and true triviality of  $d$  as given by model  $m_v$ .

