Results: [20, 35] Crossings

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The CNN (88M params) and ViT (86M params) were fine-tuned on a new dataset $\mathcal{D}$ comprising unknot $\mathcal{U}$ and non-trivial knot $\mathcal{K}$ diagrams partitioned into three disjoint data splits $S := \{\text{train, val, test}\}$, where:

$$\mathcal{D} = \biguplus_{s \in S} \mathcal{D}_s$$

$$|\mathcal{D}| = 560{,}000 \text{ diagrams},$$

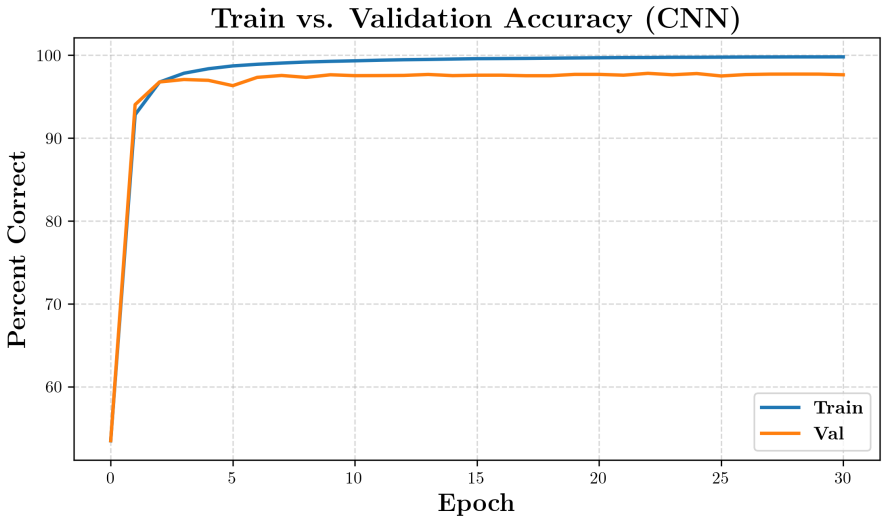
$$|\mathcal{D}_{\text{train}}| = 0.8 \cdot |\mathcal{D}| = 448{,}000 \text{ diagrams},$$

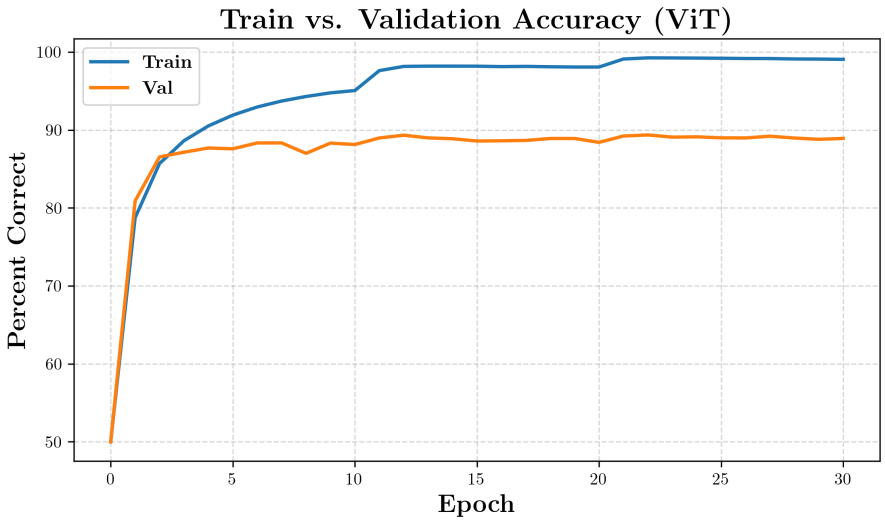
$$|\mathcal{D}_{\text{val}}| = |\mathcal{D}_{\text{test}}| = 0.1 \cdot |\mathcal{D}| = 56{,}000 \text{ diagrams},$$

$$\text{where } \forall s \in S,\ s \in \mathcal{U}_s \cup \mathcal{K}_s \text{ and } |\mathcal{U}_s| = |\mathcal{K}_s|.$$

Each split contained knot diagrams for every crossing count $n \in N := \{20, 21, \ldots, 35\}$. For all distinct $p, q \in N$, each split contained an equal number of p and q-crossing unknot and non-trivial knot diagrams.

Learning Curves





CNN* Saliency Maps

Saliency maps for knot diagrams in $\mathcal{D}_{\text{test}}$ from the CNN with the highest test accuracy, CNN*.

True Positives: Unknots Predicted as Unknots

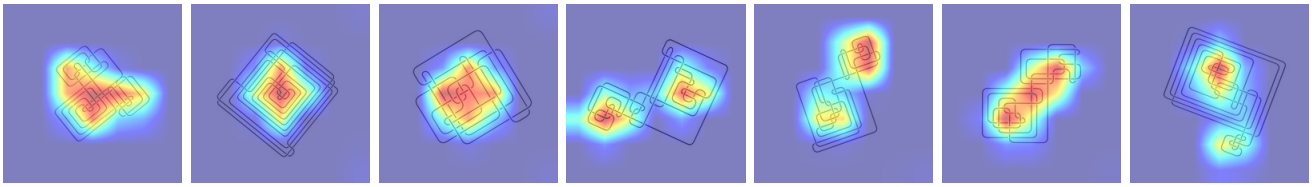


Figure 1: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, CNN* produced 27,677 true positives.

False Negatives: Unknots Predicted as Non-Trivial

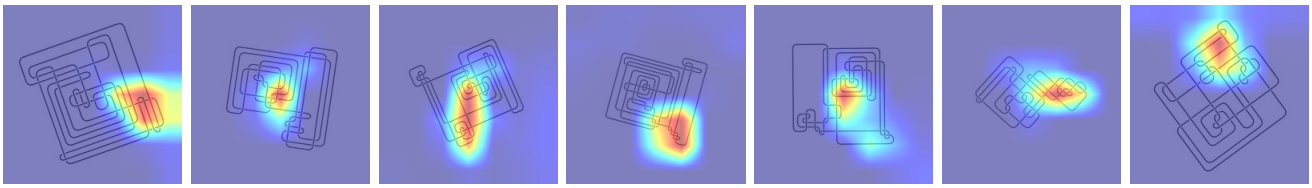


Figure 2: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, CNN* produced 323 false negatives.

True Negatives: Non-Trivials Predicted as Non-Trivial

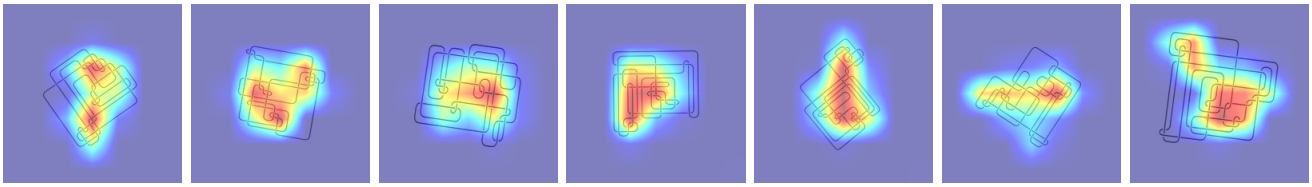


Figure 3: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, CNN* produced 27,020 true negatives.

False Positives: Non-Trivials Predicted as Unknots

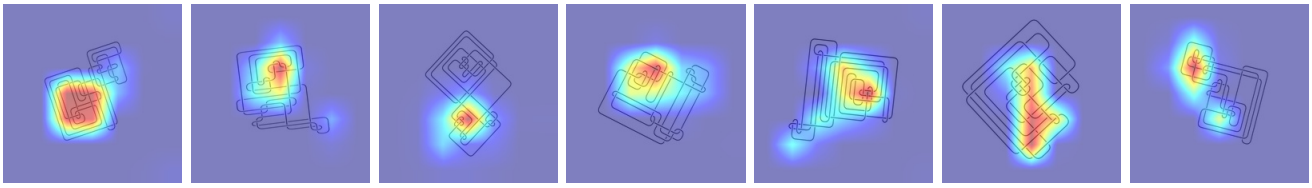


Figure 4: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, CNN* produced 980 false positives.

ViT* Saliency Maps

Saliency maps for knot diagrams in $\mathcal{D}_{\text{test}}$ from the ViT with the highest test accuracy, ViT*.

ViT TP: True Unknots Predicted as Unknots

ViT FN: True Unknots Predicted as Knots

ViT TN: True Knots Predicted as Knots

ViT FP: True Knots Predicted as Unknots