Results: [20, 35] Crossings

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The CNN (88M params) and ViT (86M params) were fine-tuned on a new dataset $\mathcal{D}$ comprising unknot $\mathcal{U}$ and non-trivial knot $\mathcal{K}$ diagrams partitioned into three mutually disjoint data splits, where:

$$S := \{\text{train, val, test}\},$$

$$\mathcal{D} \;=\; \biguplus_{s \in S} \mathcal{D}_s,$$

$$|\mathcal{D}| \;=\; 560{,}000 \;\text{ diagrams},$$

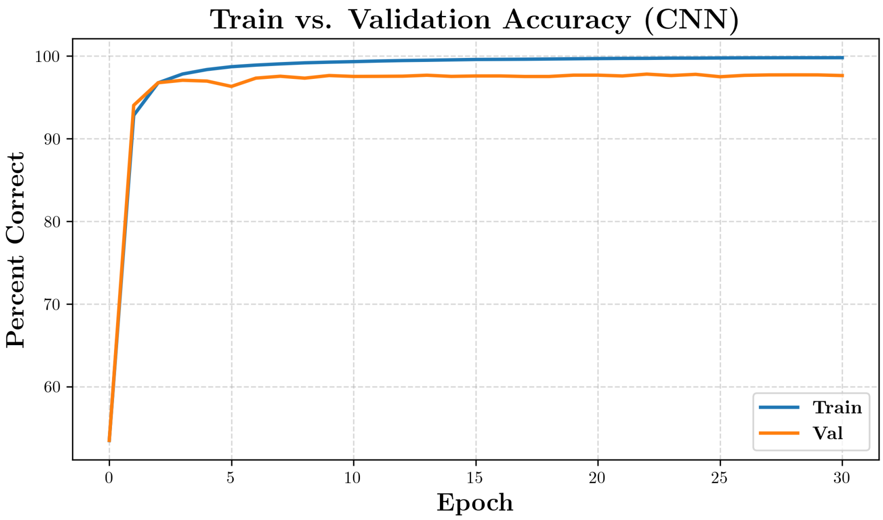
$$|\mathcal{D}_{\text{train}}| \;=\; 0.8 \cdot |\mathcal{D}| \;=\; 448{,}000 \text{ diagrams},$$

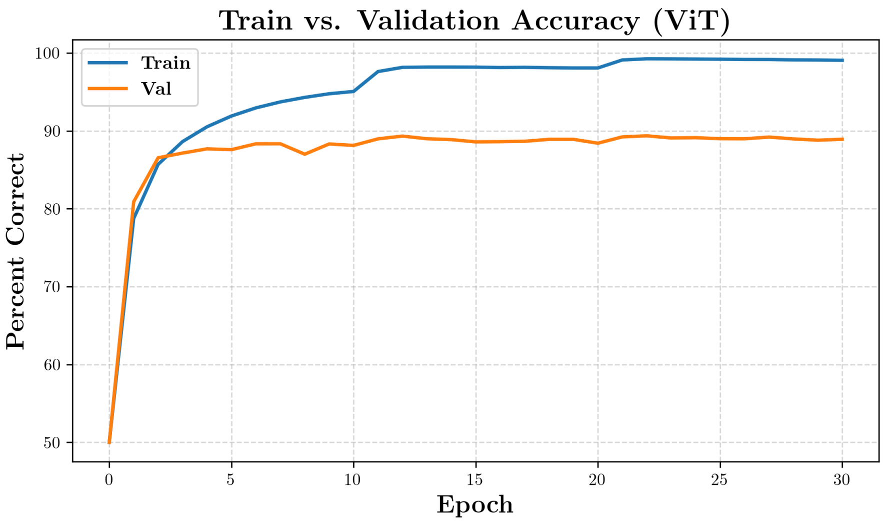
$$|\mathcal{D}_{\text{val}}| \;=\; |\mathcal{D}_{\text{test}}| \;=\; 0.1 \cdot |\mathcal{D}| \;=\; 56{,}000 \text{ diagrams},$$

$$\text{where } \;\forall s \in S,\;\; \mathcal{D}_s \in \;\mathcal{U}_s \cup \mathcal{K}_s \;\;\text{ and }\;\; |\mathcal{U}_s| = |\mathcal{K}_s|.$$

Each split comprised knot diagrams for all crossing counts $n \in N := \{n \in \mathbb{Z} \mid 20 \leq n \leq 35\}$. For all $x, y \in N$, $x \neq y$, each split in S contained an equal number of x and y-crossing unknot and non-trivial knot diagrams.

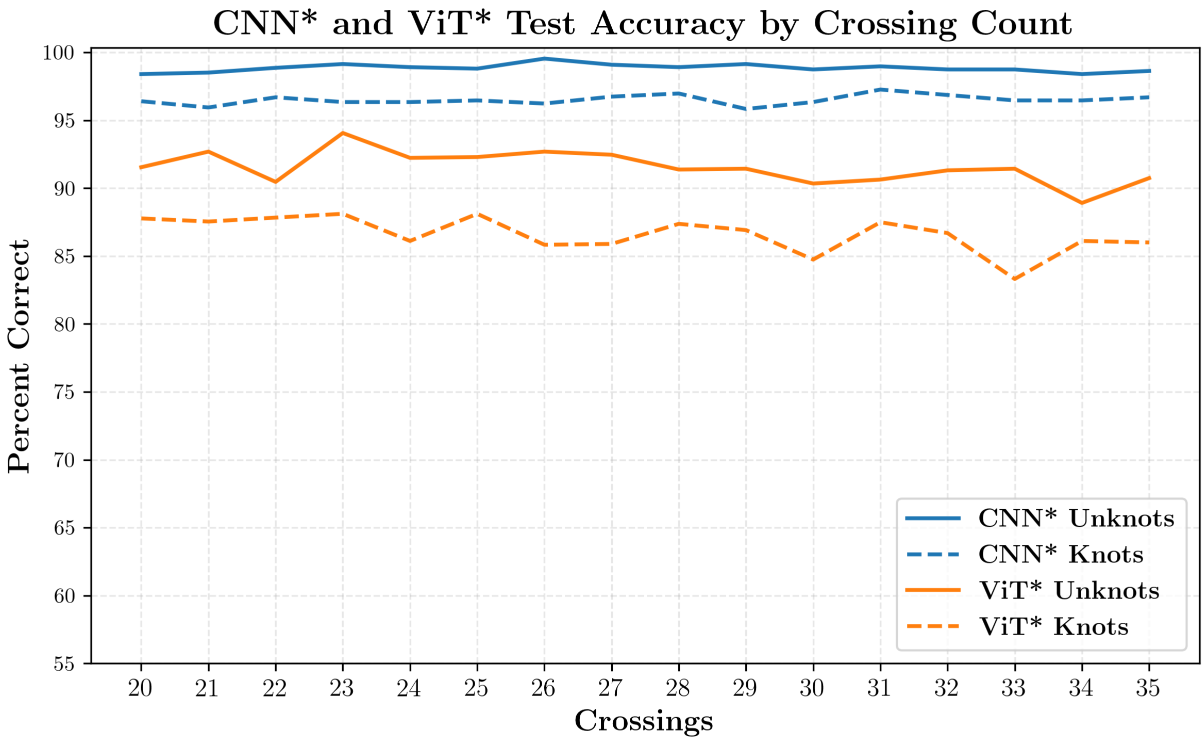
Learning Curves





Test Accuracy by Crossing Count

Let CNN* and ViT* denote the models with the highest test classification accuracies. The test accuracy of CNN* is 97.673% ($\mathcal{U}$=98.846%, $\mathcal{K}$=96.5%); the test accuracy of ViT* is 89.075% ($\mathcal{U}$=91.536%, $\mathcal{K}$=86.614%).



CNN* Saliency Maps

A sample of CNN* saliency maps for knot diagrams in $\mathcal{D}_{\text{test}}$.

True Positives: Unknots Predicted as Unknots

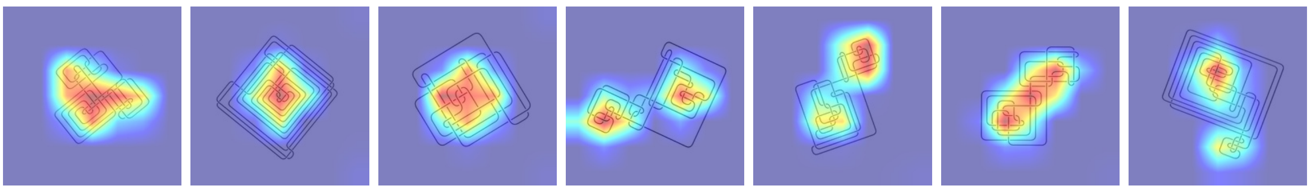


Figure 1: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, CNN* produced 27,677 true positives.

False Negatives: Unknots Predicted as Non-Trivial

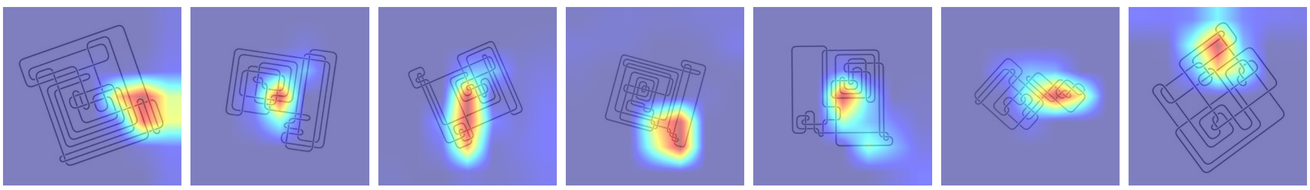


Figure 2: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, CNN* produced 323 false negatives.

True Negatives: Non-Trivials Predicted as Non-Trivial

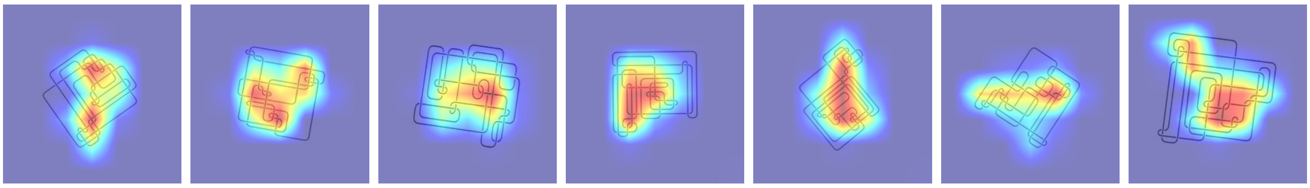


Figure 3: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, CNN* produced 27,020 true negatives.

False Positives: Non-Trivials Predicted as Unknots

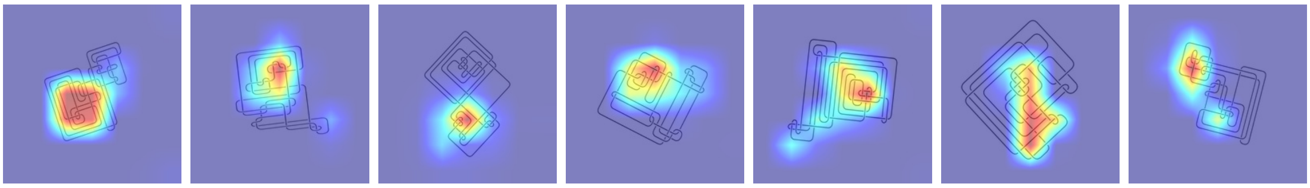


Figure 4: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, CNN* produced 980 false positives.

ViT* Saliency Maps

A sample of ViT* saliency maps for knot diagrams in $\mathcal{D}_{\text{test}}$.

True Positives: Unknots Predicted as Unknots

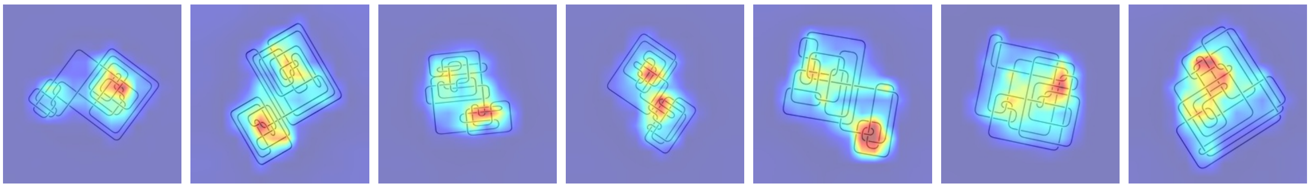


Figure 5: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, ViT* produced 25,630 true positives.

False Negatives: Unknots Predicted as Non-Trivial

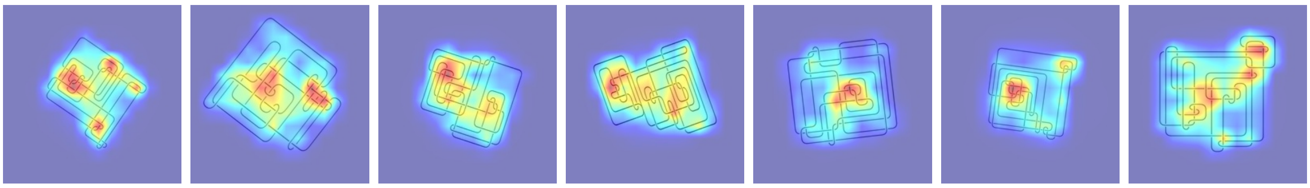


Figure 6: Of the 28,000 unknot diagrams in $\mathcal{D}_{test}$, ViT* produced 2,370 false negatives.

True Negatives: Non-Trivials Predicted as Non-Trivial

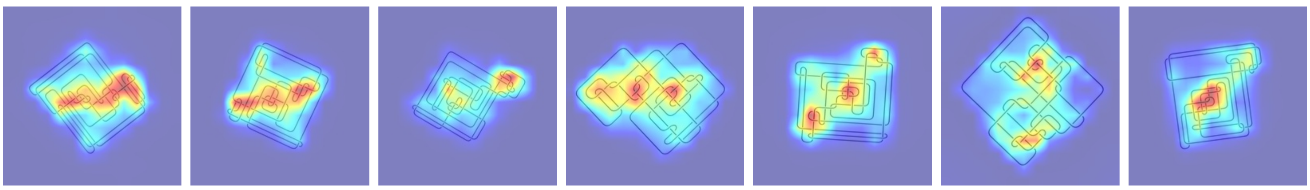


Figure 7: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, ViT* produced 24,252 true negatives.

False Positives: Non-Trivials Predicted as Unknots

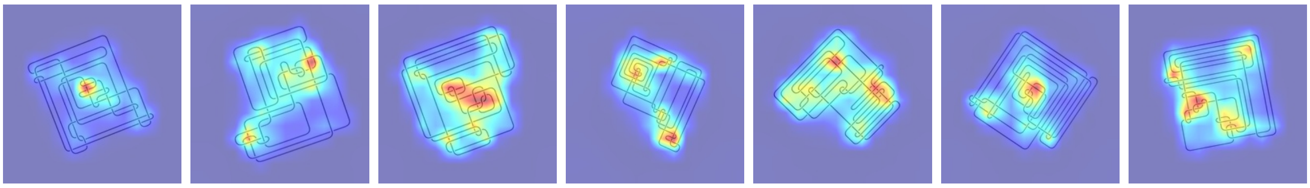


Figure 8: Of the 28,000 non-trivial knot diagrams in $\mathcal{D}_{test}$, ViT* produced 3,748 false positives.

Further Evidence of Learning

Let m_v denote checkpoint model $m \in \{\text{CNN}, \text{ViT}\}$ with validation accuracy v. The figures below are examples of a model m initially misclassifying a knot diagram $d \in \mathcal{D}_{test}$, later learning to correctly classify d. Let p and t respectively denote the predicted and true triviality of d as given by model m_v.