

# Results: [20, 35] Crossings

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The CNN (88M params) and ViT (86M params) were fine-tuned on a new dataset  $\mathcal{D}$  comprising unknot  $\mathcal{U}$  and non-trivial knot  $\mathcal{K}$  diagrams partitioned into three disjoint data splits  $S := \{\text{train}, \text{val}, \text{test}\}$ , where:

$$\mathcal{D} = \biguplus_{s \in S} \mathcal{D}_s$$

$$|\mathcal{D}| = 560,000 \text{ diagrams,}$$

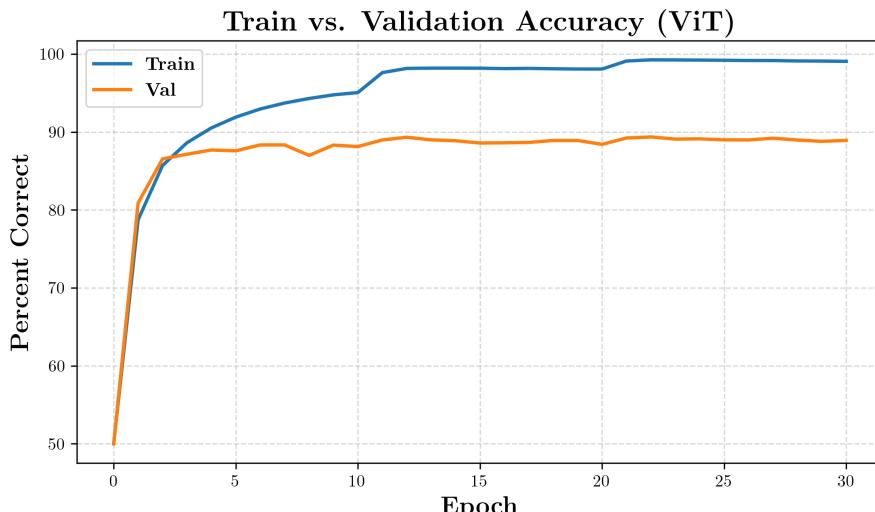
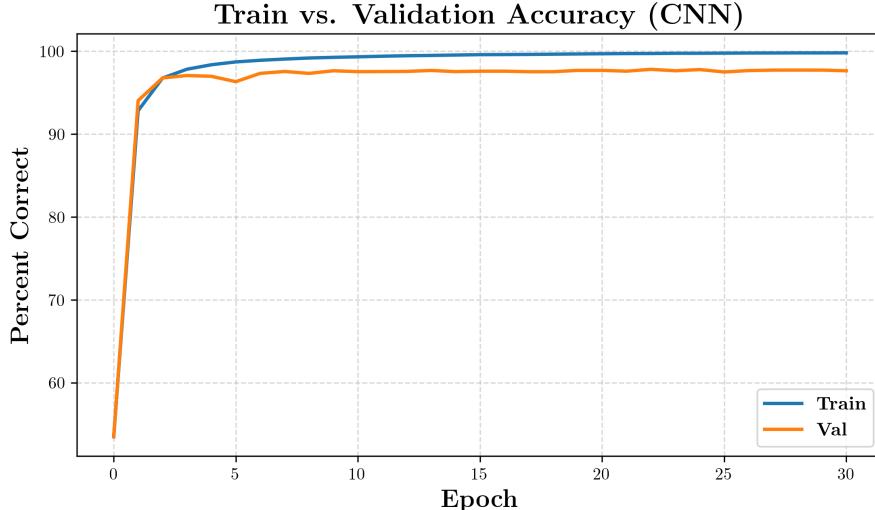
$$|\mathcal{D}_{\text{train}}| = 0.8 \cdot |\mathcal{D}| = 448,000 \text{ diagrams,}$$

$$|\mathcal{D}_{\text{val}}| = |\mathcal{D}_{\text{test}}| = 0.1 \cdot |\mathcal{D}| = 56,000 \text{ diagrams,}$$

where  $\forall s \in S, s \in \mathcal{U}_s \cup \mathcal{K}_s$  and  $|\mathcal{U}_s| = |\mathcal{K}_s|$ .

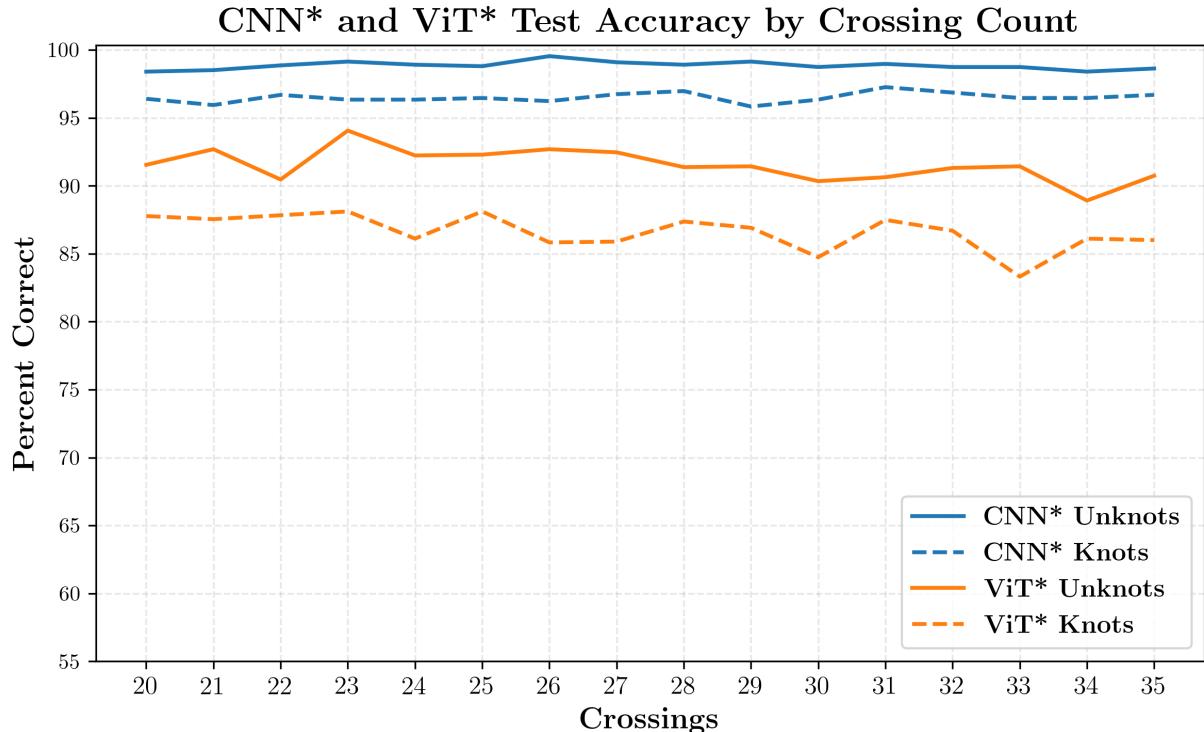
Each split contained knot diagrams for every crossing count  $n \in N := \{20, 21, \dots, 35\}$ . For all  $x, y \in N, x \neq y$ , each split contained an equal number of  $x$  and  $y$ -crossing unknot and non-trivial knot diagrams.

## Learning Curves



## Test Accuracy by Crossing Count

Let  $\text{CNN}^*$  and  $\text{ViT}^*$  denote the models with the highest test classification accuracies.



## CNN\* Saliency Maps

A sample of CNN\* saliency maps for knot diagrams in  $\mathcal{D}_{\text{test}}$ .

### True Positives: Unknots Predicted as Unknots

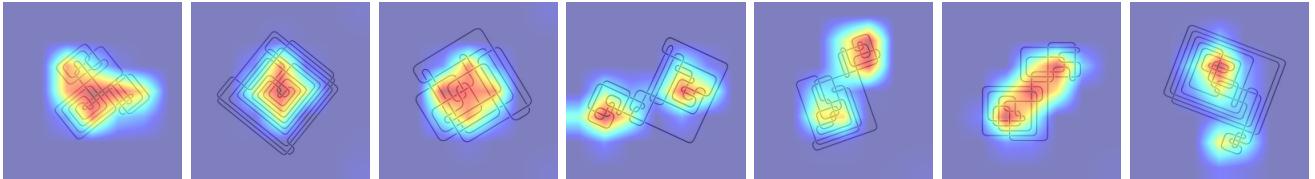


Figure 1: Of the 28,000 unknot diagrams in  $\mathcal{D}_{\text{test}}$ , CNN\* produced 27,677 true positives.

### False Negatives: Unknots Predicted as Non-Trivial

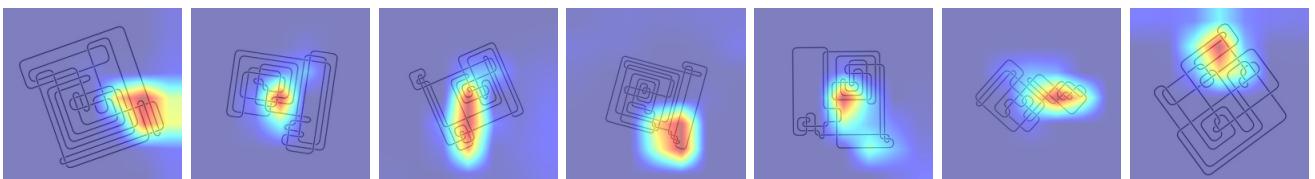


Figure 2: Of the 28,000 unknot diagrams in  $\mathcal{D}_{\text{test}}$ , CNN\* produced 323 false negatives.

### True Negatives: Non-Trivials Predicted as Non-Trivial

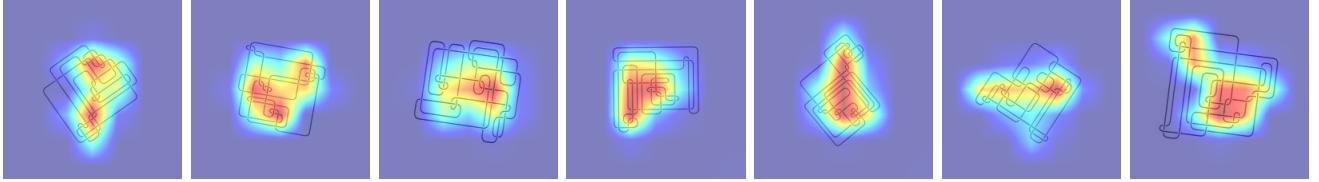


Figure 3: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , CNN\* produced 27,020 true negatives.

### False Positives: Non-Trivials Predicted as Unknobs

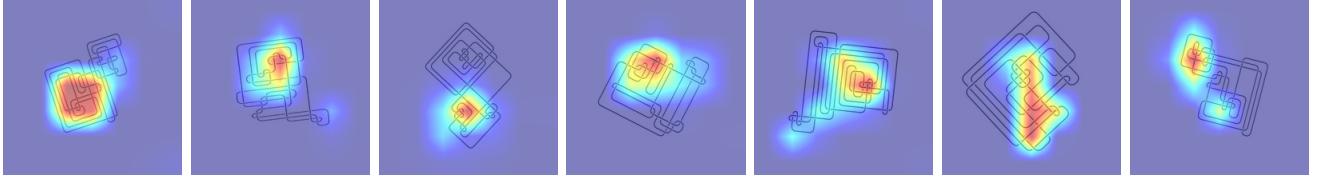


Figure 4: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , CNN\* produced 980 false positives.

### ViT\* Saliency Maps

A sample of ViT\* saliency maps for knot diagrams in  $\mathcal{D}_{test}$ .

### True Positives: Unknobs Predicted as Unknobs

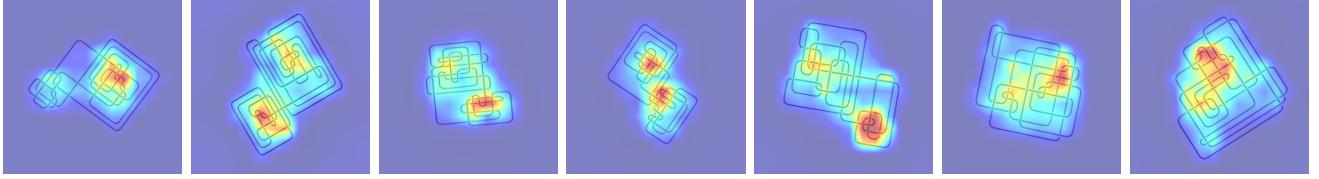


Figure 5: Of the 28,000 unknot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 25,630 true positives.

### False Negatives: Unknobs Predicted as Non-Trivial

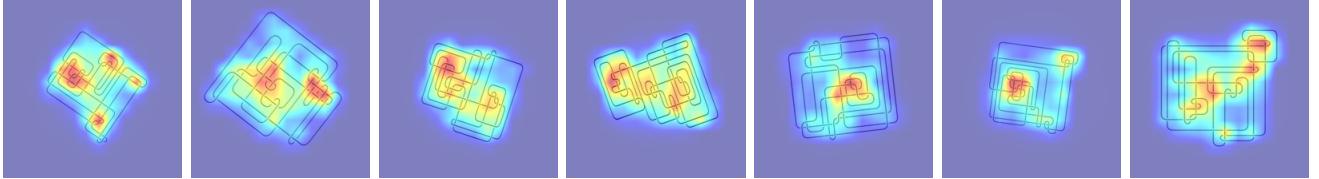


Figure 6: Of the 28,000 unknot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 2,370 false negatives.

### True Negatives: Non-Trivials Predicted as Non-Trivial

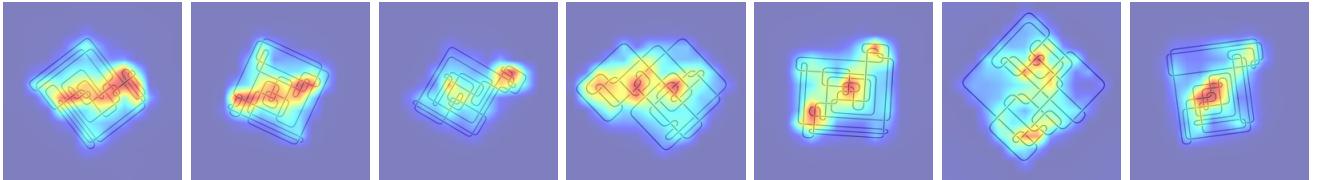


Figure 7: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 24,252 true negatives.

## False Positives: Non-Trivials Predicted as Unknots

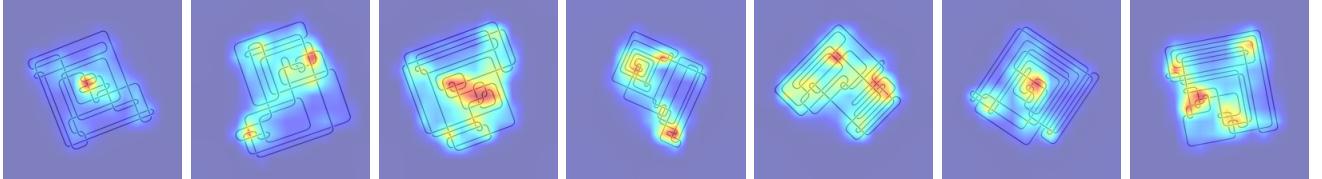


Figure 8: Of the 28,000 non-trivial knot diagrams in  $\mathcal{D}_{test}$ , ViT\* produced 3,748 false positives.

## Further Evidence of Learning

Let  $m_v$  denote checkpoint model  $m \in \{\text{CNN}, \text{ViT}\}$  with validation accuracy  $v$ . The figures below are examples of a model  $m$  initially misclassifying a knot diagram  $d \in \mathcal{D}_{test}$ , later learning to correctly classify  $d$ . Let  $p$  and  $t$  respectively denote the predicted and true triviality of  $d$  as given by model  $m_v$ .

