23.01

A Note About This Chapter

- Last chapter was fairly brutal
  - Physics is really about making hard problems easy with abstraction
    - This chapter will be less labor-intensive
- Through *symmetry*, we can skip parts of problems
  - Like how we only summed the vertical components of $d\vec{E}$ in 22.04

Gauss' Law

- **Gauss' law** = a law that relates net charge of a volume enclosed by a closed surface and the $\vec{E}$ field about that closed surface
  - Discovered by Carl Friedrich Gauss
    - Lived from 1777 until 1855
- Imagine a particle of positive charge q
  - Now superimpose a sphere centered at the particle
    - The surface of the sphere is called a **Gaussian surface**
    - The $\vec{E}$ vectors around the surface point radially outwards
      - Because the particle is *positive*
    - Those same vectors are said to **pierce** the surface of the sphere
- The essential utility of **Gauss' Law** is that we can infer things about the net charge of an object by examining the $\vec{E}$ field about its outer surface
  - Or, equivalently, we can use the net charge to infer information about the $\vec{E}$ about the object's outer surface

Electric Flux

- **Electric flux** = a metric of *how much* the $\vec{E}$ field *pierces* the Gaussian surface
  - The symbol for **electric flux** is ϕ
- The best way to learn about this is to just do a bunch of examples
- The ϕ is
  - Positive if $\vec{E}$ pierces outward
  - Zero if $\vec{E}$ is parallel to the differential area
  - Negative if $\vec{E}$ pierces inward

Electric Flux On a Flat Surface in a Uniform $\vec{E}$ Field

- Imagine we had a uniform $\vec{E}$ field
  - Now superimpose a flat surface of area A
    - Orient it along with yz-plane with its center point at the origin
  - Denote the angle that the uniform $\vec{E}$ vectors make with the x-axis as θ
  - Then, we can imagine splitting the $\vec{E}$ vectors into two components
    - One that *directly* pierces the surface
      - Directly perpendicular to the surface
      - This vector is the **electric flux** for any given differential area
    - One that doesn't pierce the surface at all
      - Directly parallel to the surface
- We can define the magnitude of the electric flux in a subarea of A as

$$d\phi = |\vec{E}|cos(\theta)$$

- This is valid, but there is a more elegant solution
  - This value can calculated with a **dot product**

$$d\phi = \vec{E} \cdot d\vec{A}$$

- where $d\vec{A}$ is a vector perpendicular to the surface with a magnitude equal to the area of the surface
- At some points, the $\vec{E}$ field may pierce *into* the surface and in other points, it may pierce outwards
  - In order to find the **net electric flux**, we use integration

$$\phi = \oint \vec{E} \cdot d\vec{A}$$

23.02

Gauss' Law

- **Gauss' law** = a mathematical model that relates **net flux(ϕ)** and enclosed charge
- Mathematically it looks like this

$$\epsilon_0 \Sigma\phi = \Sigma q$$

- Or, substituting the definition of net flux, we get

$$\epsilon_0 \oint \left(\vec{E} \cdot d\vec{A} \right) = \Sigma q$$

- The charge of Σq determines whether the flux is *inwards* or *outwards*
  - If Σq is *positive*, $\Sigma\phi$ points outward
  - If Σq is *negative*, $\Sigma\phi$ points inward
  - If Σq is zero, $\Sigma\phi$ is a zero
- The interesting thing about Gauss' law is that charges external to the enclosed volume do not affect the net flux
  - Think about that: *if you put a charged particle right up against the barrier, the field lines would change but the net flux wouldn't*

Deriving Coulomb's Law with Gauss' Law

- Recall **Coulomb's law**

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

- This can actually be proven using Gauss' law
- Imagine we had a particle of point q
  - Now superimpose a gaussian sphere that envelops that particle
    - We can use the integral form of Gauss' law to set up an equation

$$\epsilon_0 \oint \left(\vec{E} \cdot d\vec{A} \right) = \Sigma q$$

- A property of dot products is that

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|cos(\theta)$$

- where θ is the angle between $\vec{a}$ and $\vec{b}$
- Using that fact, we can rewrite our equation as

$$\epsilon_0 \oint \left(|\vec{E}||d\vec{A}|cos(\theta) \right) = \Sigma q$$

- Since our problem is basically a one-particle problem, we know that $\vec{E}$ will radiate outwards perpendicular to conceсntric spheres
  - As such, $\vec{E}$ and $d\vec{A}$ actually point in the same direction
    - So, θ is zero

$$\epsilon_0 \oint \left(|\vec{E}||d\vec{A}|cos(0) \right) = \Sigma q$$

$$\epsilon_0 \oint \left(|\vec{E}||d\vec{A}|(1) \right) = \Sigma q$$

$$\epsilon_0 \oint \left(|\vec{E}||d\vec{A}| \right) = \Sigma q$$

- At this point, we can rewrite Σq as just q, since our gaussian sphere only contains that one particle

$$\epsilon_0 \oint \left(|\vec{E}||d\vec{A}| \right) = q$$

- Now, the direction of $\vec{E}$ clearly changes from point to point on the gaussian sphere
  - However, the *magnitude* does not change
    * So, we can pull it out of the integral

$$\epsilon_0 |\vec{E}| \oint |d\vec{A}| = q$$

- Now, surface integrals, which is what that $\oint$ symbol denotes, aren't in the scope of this course
  - Really, all you need to know is that they integrate a function over every point on a surface(in this case, the surface area of the sphere)
  - If the function your surface integrates is just 1, then the surface integral returns the surface area
    * So, really, our surface integral just returns the surface area of our sphere
      · Which, if you remember from geometry is

$$SA = 4\pi r^2$$

- Substituting that, we get

$$\epsilon_0 |\vec{E}| \left(4\pi r^2\right) = q$$

- Then, some simple algebra gets us to

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

23.03

Gauss' Law and the Behavior of Conductors

- Gauss' law can actually be used to explain phenomenon regarding conductors
  - For example, in a conductor, the free electrons will disperse themselves amongst the outer surface of the object
    * This kind of makes intuitive sense, as like charges repel, and that permutation ensures maximum distance between particles