23.01

A Note About This Chapter

- Last chapter was fairly brutal
  - Physics is really about making hard problems easy with abstraction
    - This chapter will be less labor-intensive
- Through *symmetry*, we can skip parts of problems
  - Like how we only summed the vertical components of $d\vec{E}$ in 22.04

Gauss' Law

- **Gauss' law** = a law that relates net charge of a volume enclosed by a closed surface and the $\vec{E}$ field about that closed surface
  - Discovered by Carl Friedrich Gauss
    - Lived from 1777 until 1855
- Imagine a particle of positive charge q
  - Now superimpose a sphere centered at the particle
    - The surface of the sphere is called a **Gaussian surface**
    - The $\vec{E}$ vectors around the surface point radially outwards
      - Because the particle is *positive*
    - Those same vectors are said to **pierce** the surface of the sphere
- The essential utility of **Gauss' Law** is that we can infer things about the net charge of an object by examining the $\vec{E}$ field about its outer surface
  - Or, equivalently, we can use the net charge to infer information about the $\vec{E}$ about the object's outer surface

Electric Flux

- **Electric flux** = a metric of *how much* the $\vec{E}$ field *pierces* the Gaussian surface
  - The symbol for **electric flux** is ϕ
- The best way to learn about this is to just do a bunch of examples
- The ϕ is
  - Positive if $\vec{E}$ pierces outward
  - Zero if $\vec{E}$ is parallel to the differential area
  - Negative if $\vec{E}$ pierces inward

Electric Flux On a Flat Surface in a Uniform $\vec{E}$ Field

- Imagine we had a uniform $\vec{E}$ field
  - Now superimpose a flat surface of area A
    - Orient it along with yz-plane with its center point at the origin
  - Denote the angle that the uniform $\vec{E}$ vectors make with the x-axis as θ
  - Then, we can imagine splitting the $\vec{E}$ vectors into two components
    - One that *directly* pierces the surface
      - Directly perpendicular to the surface
      - This vector is the **electric flux** for any given differential area
    - One that doesn't pierce the surface at all
      - Directly parallel to the surface
- We can define the magnitude of the electric flux in a subarea of A as

$$d\phi = |\vec{E}|cos(\theta)$$

- This is valid, but there is a more elegant solution
  - This value can calculated with a **dot product**

$$d\phi = \vec{E} \cdot d\vec{A}$$

- where $d\vec{A}$ is a vector perpendicular to the surface with a magnitude equal to the area of the surface
- At some points, the $\vec{E}$ field may pierce *into* the surface and in other points, it may pierce outwards
  - In order to find the **net electric flux**, we use integration

$$\phi = \int \vec{E} \cdot d\vec{A}$$

Electric Flux On a Closed Surface in a Non-Uniform $\vec{E}$ Field