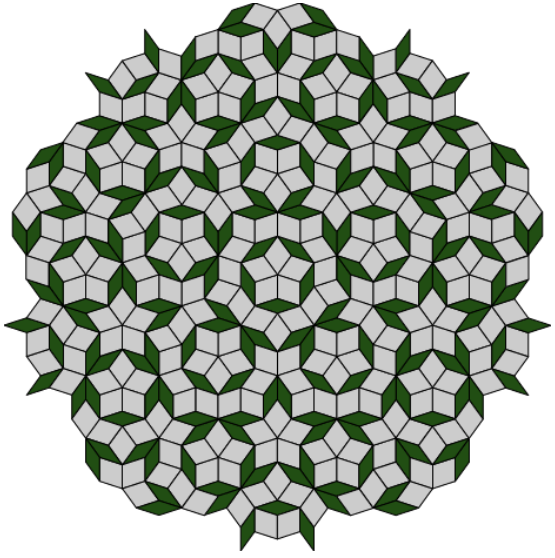
School of Mathematics and Statistics

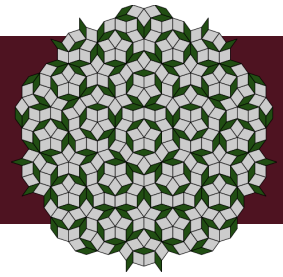
Divisibility



Dr Adam Day

Math 161 – Discrete Mathematics and Logic

The Divisibility Relation



- $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \ldots\}$
- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Definition

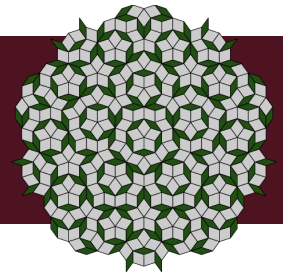
Let a and b be integers. We say that a *divides* b and write $a \mid b$ if for some integer e we have that $a \cdot e = b$.

- if a does not divide b we write $a \nmid b$.
- e.g. $4 \mid 12$, $3 \nmid 100$.
- For any n, $n \mid 0$.

Lemma

The divisibility relation on $\mathbb{Z}$ is reflexive and transitive.

More Lemmas



Definition

Let a and b be integers. We say that a *divides* b and write $a \mid b$ if for some integer e we have that $a \cdot e = b$.

Lemma

If a and b are integers such that $a \mid b$ and $b \mid a$ then $a = \pm b$.

Lemma

Let $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$ then $a \mid (b + c)$ and $a \mid (b - c)$.