Homework #4

1. $C = \{w \mid w$ has an equal number of 0's & 1's$\}$

Assume C is regular and let p be the pumping length of C. Choose $s = 0^p 1^p$. So $|0^p 1^p| > p$. By pumping lemma, s can be partitioned into 3 pieces $s = xyz$ such that for all $i \geq 0$, $xy^iz \in C$.

If y contains only zeroes and $i = 2$, then $xyyz$ will have more 0's then 1's. $\therefore$ $xyyz \notin C$.

This violates condition #1. By this contradiction, C is not regular.

2. $F = \{ww \mid w$ is a string from $\{0,1\}^*\}$

Assume F is regular and let p be the pumping length of F. Choose $s = 0^p 1 0^p 1$. So $|0^p 1^p| > p$. By the PL, s can be partioned into 3 parts $s = xyz$ such that for all $i \geq 0$, $xy^iz \in F$. By condition 3, we must also have $|xy| \leq p$. Therefore, y must contain only 0's, and subsequently $xyyz = xy^2z \notin F$. Because the first part of $s \neq$ the second part of s, s cannot be pumped. This violates condition 1 of PL. By this contradiction, F is not regular.

3. $A = \{www \mid w \text{ is a string from } \{a,b\}^*\}$

Assume A is regular. Let p be the pumping length of A. Choose $s = a^p b a^p b a^p b$. So cleary $|s| > p$. By PL, s can be partioned into $s = xyz$ such that for all $i \geq 0$, $xy^iz \in A$.

$\underbrace{a^p b\, a^p}_{x}\underbrace{b}_{y}\underbrace{a^p b}_{z}$ ← for every a^p must be 1 b

If $i = 0$, $xy^0z \Rightarrow xy$ & there will only be 2 b's in the string. If there are only 2 b's in xy^0z, then $xy^0z \notin A$. By contradiction of condition 1, A is not regular.

4.

4. $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$

Assume L is regular. Let p = the pumping length of L.

Choose $s = 0^p 1 0^p$, so $|0^p 1 0^p| > p$. By PL, s can be partitioned into $s = xyz$ such that for all $i \geq 0$, $xy^iz \in L$.

$$\underbrace{0^{p-1}}_{x}\underbrace{010}_{y}\underbrace{0^{p-1}}_{z}$$

If $i = 2$ and $p = 2$, $xyyz = 0\underbrace{010}_{y}\underbrace{010}_{y}0$. In this case, the 1's dont separate the zeros correctly. There should not be zeroes stuck between the two 1's. ∴ by contradiction of condition 1 of pumping lemma, L is not regular.