Hwk 7

1. Proof. Assume A is CFL and let p be the Pumping length. Choose $s = 0^p \# 0^{2p} \# 0^{3p}$ so $|s| > p$. By PL for CFL we partition $s = uvxyz$ so for any $i \geq 0$, $s' = uv^ixy^iz \in A$. Let's consider this:

   - Consider $s = 0\#00\#000$. In this case; $u = 0\#$, $v = 0$, $x = 0\#$, $y = 0$, $z = 00$. Therefore when pumped up ($i = 2$), $s' = 0\#000\#0000$. This violates condition 1 of PL; therefore, A is not context-free.

2. Proof. Assume C is CFL and let p be the pumping length. Choose $s = 1^p 3^p 2^p 4^p$ so $|s| > p$. By PL for CFL, we can partition $s = uvxyz$ so for any $i \geq 0$, $s' = uv^ixy^iz \in C$. Let's consider this:

   - Let $i = 2$ so $s = 11332244$ where $u = 1$, $v = 1$, $x = 3322$, $y = 4$, $z = 4$. When $p = 2$, then $s' = 1^3 3^2 2^2 4^3$. Number of 1's $\neq$ Number of 2's. Therefore s' is not in the language. This violated condition 1 of PL; hence, A is not context free.

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$B = \{w\#t \mid w \text{ is a substring of } t, t \in \{a,b\}^*\}$

3. Proof. Assume B is a CFL & let p be the pumping length. Choose the string $s = a^p b^p \# a^p b^p$ so $|s| > p$. By PL for CFL, we can partition $s = uvxyz$, so that for $i \geq 0$, $s' = uv^i xy^i z \in B$. Let's consider these cases:

1) The string $s = ab\#ab$ where $p = 1$. Then, $s' = uv^0xy^0z$ removes b from the left & a from the right. By pumping down, $s' \notin B$ and condition 1 of PL violated. This is because a is not 'a' substring of 'b'. Therefore, this contradicts our assumption that B is context-free.

2) The string aabb#aabb where $p = 2$. Then, $s' = uv^2xy^2z = uvvxyyz$ so that when pumped up $s' = \underbrace{aa}_{u}\underbrace{bbbb}_{v}\underbrace{\#}_{x}\underbrace{aaa}_{y}\underbrace{abb}_{z}$. Because aabbbb is not a substring of aaaabb, $s' \notin B$ and condition 1 is violated. Like case 1, this contradicts our assumption that B is context-free.

Therefore, by these contradictions, C is not context-free.