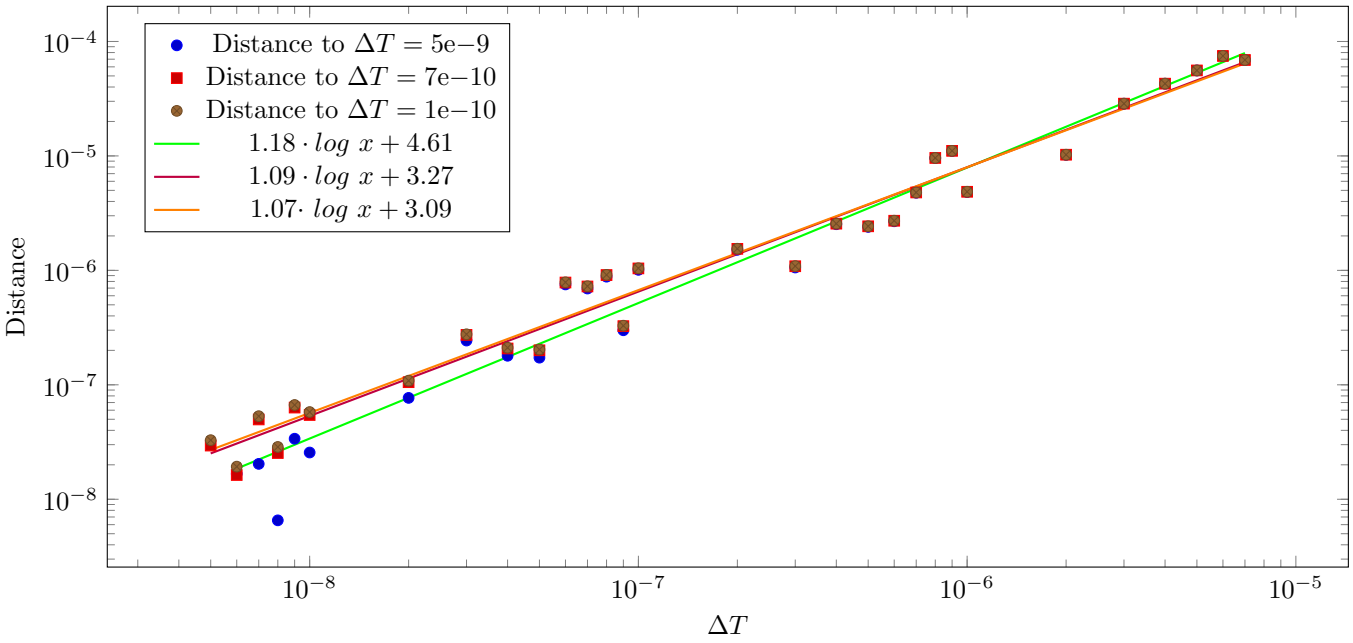
Solution Step 2 - **wtxd25** - Durham University

1 Results



2 How does the position of points depend on the time step chosen?

If the points were to become more accurate as we decrease the time step size then this would mean that the smallest time step size that we are testing, which is $\Delta T = 5e{-}9$, would be the most accurate value. As such on the above graph the x-axis represents the time step sizes, and the y-axis represents the distance from the collision point in the assumed most accurate case; when $\Delta T = 5e{-}9$. As we can see in the graph above there is a linear relationship in the Log-Log graph. The linear regression line we get on the data is $log\ y(x) = 1.18 \cdot log\ x + 4.61$. In monomial form this results in a function $y(x) = e^{4.61} \cdot x^{1.18}$. Therefore, we have almost a linear relationship between the time step size and the distance from the point chosen. This suggests that a smaller time step size will result in a smaller distance from the point chosen; as the time step size approaches 0, so will the distance from the point. The variation from the trend line in the Log-Log plot will stem from machine precision in the calculations performed, as such the relationship is not perfectly linear in the monomial achieved. Another issue is that the point we have used as absolute truth is not that because it also has errors; this also leads to imperfections in the results. I have also included $\Delta T = 1e{-}10$ because as we have seen from the previous result, the smaller time step size means more accurate results. On the plot, this value of ΔT can be seen to backup this conclusion.

3 What is the convergence order of the explicit Euler method?

The convergence order of an algorithm is p if $|y_h(t) - y(t)| \leq C \cdot h^p$. Given a logarithmic equation $1.07 \cdot log\ x + 3.09$ we convert it to a monomial equation in the following way; $|y_h(t) - y_g(t)| = e^{3.09} \cdot x^{1.07}$ where $g = 1e{-}10$ is the . This would mean that the data we have gathered gives a convergence order of 1.07, because as we have established from the other data points, the value of p decreases as we decrease g. This conforms to what we would expect because the convergence order of explicit Euler is 1.