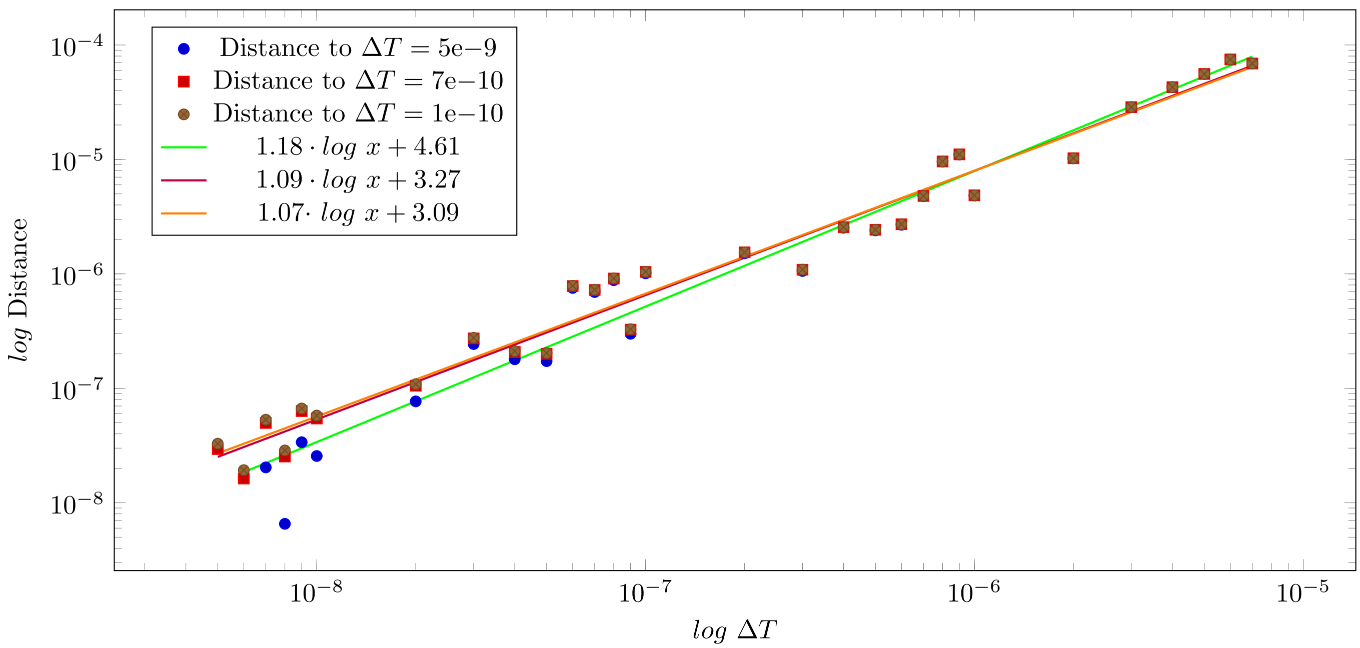
Solution Step 2 - **wtxd25** - Durham University

1 Results



2 How does the position of points depend on the time step chosen?

Let us assume that the points become more accurate as we decrease the time step size. Then this would mean that the one of the smallest time step sizes that we have calculated is more accurate than all larger time steps, we will use $\Delta T = 5\text{e}{-9}$ as the "absolute truth" value. As such on the above graph the x-axis represents the logarithm of time step sizes, and the y-axis represents the logarithm of the distance from the collision point in the assumed "absolute truth" case; when $\Delta T = 5\text{e}{-9}$. As we can see in the graph above there is a linear relationship in the Log-Log graph. The linear regression we get on the data is $log\ y(x) = 1.18 \cdot log\ x + 4.61$. In monomial form this results in a function $y(x) = e^{4.61} \cdot x^{1.18}$. Therefore, we have almost a linear relationship between the time step size and the distance from the point chosen. This suggests that a smaller time step size will result in a smaller distance from the "absolute truth" point; as the time step size approaches $\Delta T = 5\text{e}{-9}$ so will the distance. The variation from the trend line in the Log-Log plot will stem from machine precision in the calculations performed, as such the relationship is not perfectly linear in the monomial achieved. Another issue is that the point we have used is not the absolute truth because it also has errors; this also leads to imperfections in the results. I have also included $\Delta T = 7\text{e}{-10}$ and $\Delta T = 1\text{e}{-10}$ because as we have seen from the previous result, the smaller time step size means more accurate results. On the plot, these values of ΔT can be seen to backup this hypothesis.

3 What is the convergence order of the explicit Euler method?

The convergence order of an algorithm is p if $|y_h(t) - y(t)| \leq C \cdot h^p$. Given a logarithmic equation $1.07 \cdot log\ x + 3.09$ we convert it to a monomial equation in the following way; $|y_h(t) - y_g(t)| = e^{3.09} \cdot x^{1.07}$ where $g = 1\text{e}{-10}$. This would mean that the data we have gathered gives a convergence order of 1.07, because as we have established from the other data points, the value of p decreases as we decrease g. This conforms to what we would expect because the convergence order of explicit Euler is 1 if we use $y_g(t) = y(t)$.