LogPar: Logistic PARAFAC2 Factorization for Temporal Binary Data with Missing Values

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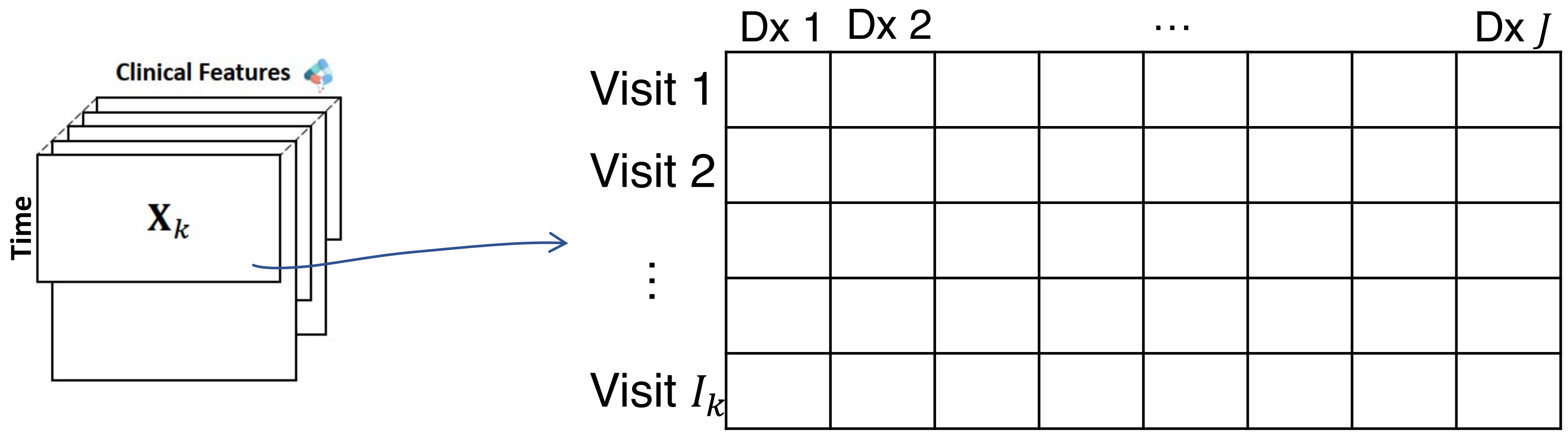
Background: Binary Irregular Tensors

- Real-world data are often collected over time.

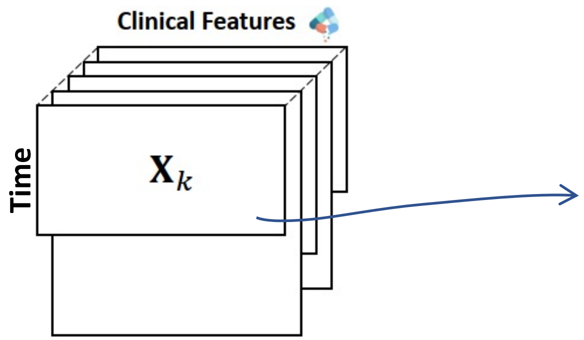
- Irregular Tensor: A collection of matrices with varying sizes in one dimension.
- Handles the temporally irregular data.

- Example: Electronic health records



- Other applications: spatio-temporal modeling, recommender systems, etc.

Background: Implicit one-class missingness



	Dx 1	Dx 2			…			Dx J
Visit 1	1	0	0	0	0	0	0	0
Visit 2	0	0	1	0	0	0	1	0
	1	0	0	0	0	0	0	0
⋮	0	0	0	1	0	0	0	0
Visit I_k	0	1	0	0	0	0	0	0

- Values of 1s: confirmed diagnosis codes assigned by doctors.
- Values of 0s?
  - The patient does have the disease.
  - The diagnosis was not performed: **missing data**.

- We do not know the missing entries (**implicit**) & we only consider missing 1s (**one-class**)

Research Questions

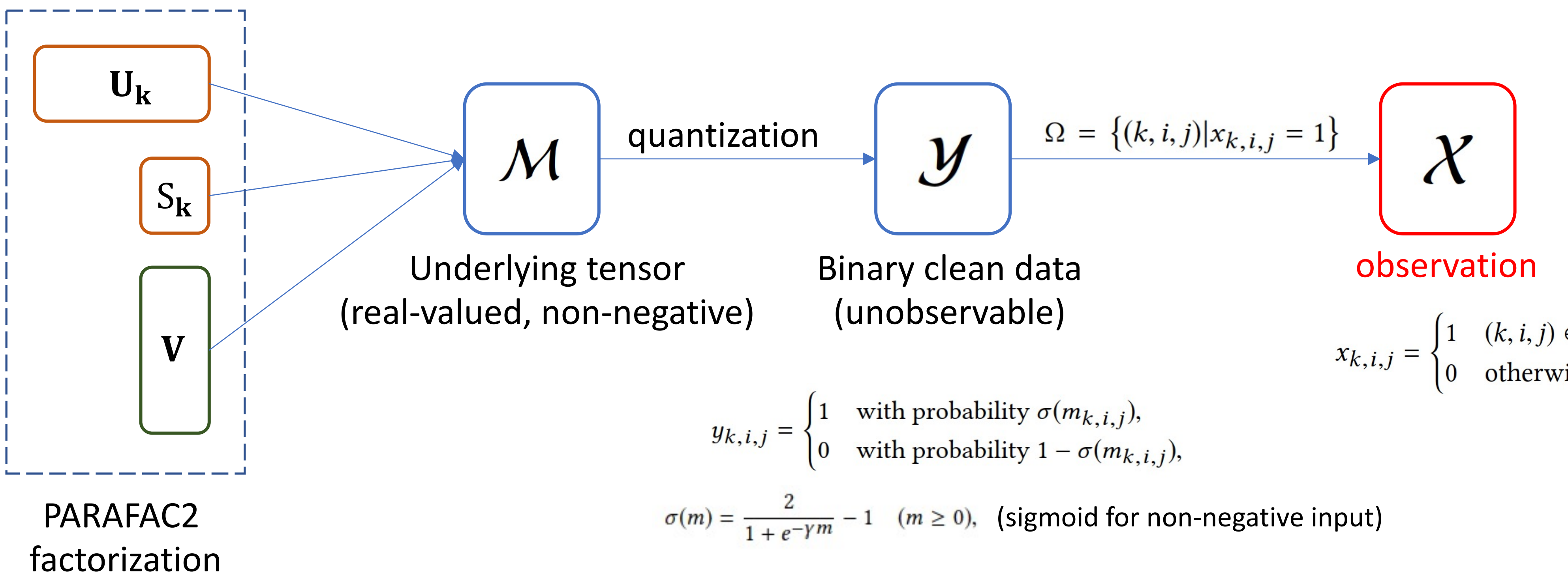
- How to complete the missing data in binary irregular tensors?
- How to discover meaningful latent factors from such data?

Our solution: Logistic PARAFAC2 (LogPar)

Problem:

Given a temporal binary irregular tensor $\mathcal{X}$, discover its latent factors and completion.

Observation model for binary data:



$$x_{k,i,j} = \begin{cases} 1 & (k,i,j) \in \Omega, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{k,i,j} = \begin{cases} 1 & \text{with probability } \sigma(m_{k,i,j}), \\ 0 & \text{with probability } 1 - \sigma(m_{k,i,j}), \end{cases}$$

$$\sigma(m) = \frac{2}{1+e^{-\gamma m}} - 1 \quad (m \geq 0),$$ (sigmoid for non-negative input)

Our solution: Logistic PARAFAC2 (LogPar)

Handling the implicit one-class missing

Extending nnPU learning to logistic PARAFAC2:

$$\widetilde{R}_{\mathrm{pu}}(g) = \pi_{\mathrm{p}} \widehat{R}_{\mathrm{p}}^{+}(g) + \max\left\{0, \widehat{R}_{\mathrm{u}}^{-}(g) - \pi_{\mathrm{p}} \widehat{R}_{\mathrm{p}}^{-}(g)\right\}$$

$$\widehat{R}_{\mathrm{p}}^{+}(g) = (1/n_{\mathrm{p}}) \textstyle\sum_{i=1}^{n_{\mathrm{p}}} \boxed{\ell(g(x_i^{\mathrm{p}}), +1)}$$
$$\widehat{R}_{\mathrm{p}}^{-}(g) = (1/n_{\mathrm{p}}) \textstyle\sum_{i=1}^{n_{\mathrm{p}}} \boxed{\ell(g(x_i^{\mathrm{p}}), -1)}$$
$$\widehat{R}_{\mathrm{n}}^{-}(g) = (1/n_{\mathrm{n}}) \textstyle\sum_{i=1}^{n_{\mathrm{n}}} \boxed{\ell(g(x_i^{\mathrm{n}}), -1)}$$

Sample-wise loss

- Replace the loss function with the point-wise loss of logistic PARAFAC2

$$\ell(\widehat{x}_{k,i,j}, x_{k,i,j}) = \left(x_{k,i,j} \log \widehat{x}_{k,i,j} + (1 - x_{k,i,j}) \log(1 - \widehat{x}_{k,i,j})\right)$$

- Leading to the final objective function: $\tilde{\mathcal{L}}\left(\widehat{\mathcal{X}}\right) = \pi \frac{\langle \mathcal{X},\ \log(\widehat{\mathcal{X}})\rangle}{\|\mathcal{X}\|_1} + \max\left\{0, \frac{\langle 1-\mathcal{X},\ \log(1-\widehat{\mathcal{X}})\rangle}{\|1-\mathcal{X}\|_1} - \pi \frac{\langle \mathcal{X},\ \log(1-\widehat{\mathcal{X}})\rangle}{\|\mathcal{X}\|_1}\right\}$

Kiryo, Ryuichi, et al. "Positive-unlabeled learning with non-negative risk estimator." *Advances in Neural Information Processing Systems*. 2017.

Our solution: Logistic PARAFAC2 (LogPar)

Regularizations for Better Interpretability

- Uniqueness Regularization:

$$\mathcal{R}_1 = \sum_{k=1}^{K} \frac{\mu}{2} \left\| \mathbf{U}_k{}^{\top} \mathbf{U}_k - \boldsymbol{\Phi} \right\|_F^2$$

(promotes factor invariance)

- Time-Aware Temporal Smoothing:

$$\mathcal{R}_2 = \sum_{k=1}^{K} \sum_{i=2}^{I_k} e^{-\beta \delta_i} \left| \mathbf{u}_{k,t} - \mathbf{u}_{k,t-1} \right|$$

(discovers smoother temporal factors)

Experiments and Results

- **Datasets**
  - **Sutter**: collected from Sutter health, a large US based health provider network. We use diagnoses and medications of each clinical visit of patients.
  - **CMS**: publicly available CMS Linkable 2008-2010 Medicare Data Entrepreneurs' Synthetic Public Use File (DE-SynPUF). We use diagnoses of each clinical visit of patients.
  - **MIMIC-III**: a large-scale ICU dataset. We use medications and abnormal lab tests.

	Sutter	CMS	MIMIC-III
#Patients (K)	34,905	74,153	28,485
#Features (J)	328	319	405
Median(I_k)	26	26	22
Average(I_k)	30.5	29	28.4
#Positive entries	2.3M	4.5M	14.5M
Sparsity	0.80%	0.65%	4.43%
Single-feature visits	29.5%	37.4%	0.67%
Predictive task	Heart failure	–	Mortality
Positive label ratio	8.92%	–	8.86%

- **Baselines**
  - **COPA**: SOTA PARAFAC2 factorization model.
  - **SPARTan**: PARAFAC2 for sparse data.
  - **PU-MC**: Binary matrix completion model based on PU learning.
  - **One-class MF (OCMF)**: SOTA binary matrix completion model based on sampling.

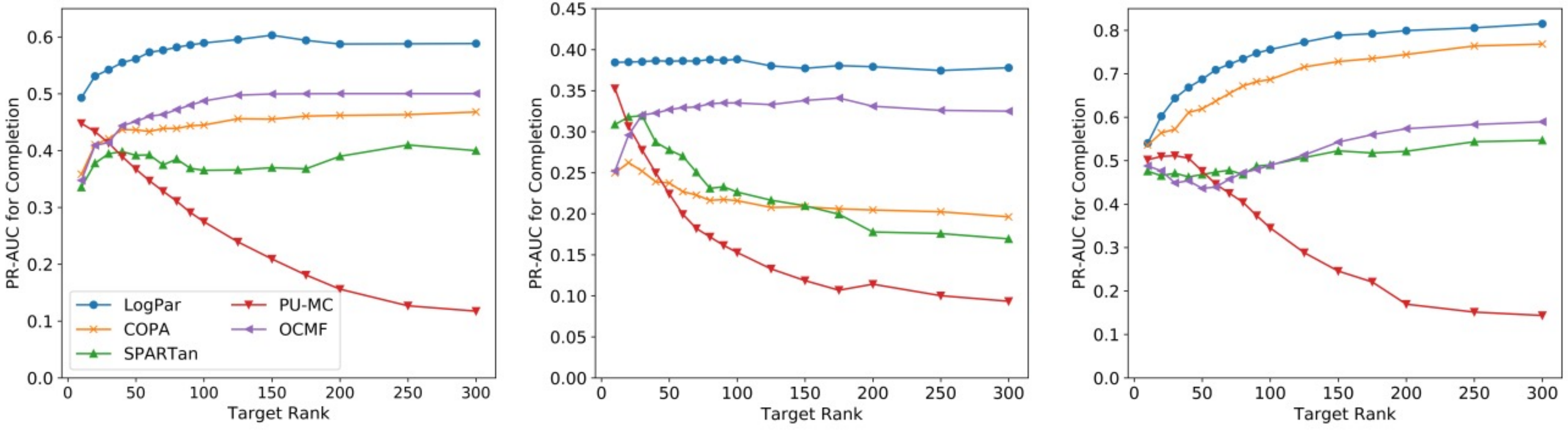
Experiments and Results

- **Experiment Setting**
  - Extract 10% positive entries for validation and parameter tuning.
  - Hold out 20% positive entries for test.
  - Randomly match 10 negative entries (value 0s) for each positive test entry as test set.
  - Use the remaining 70% positive entries as training set.

- **Evaluation Metric**
  - **PR-AUC**: binary & imbalanced

Experiments and Results

- **Completion performance with varying ranks**

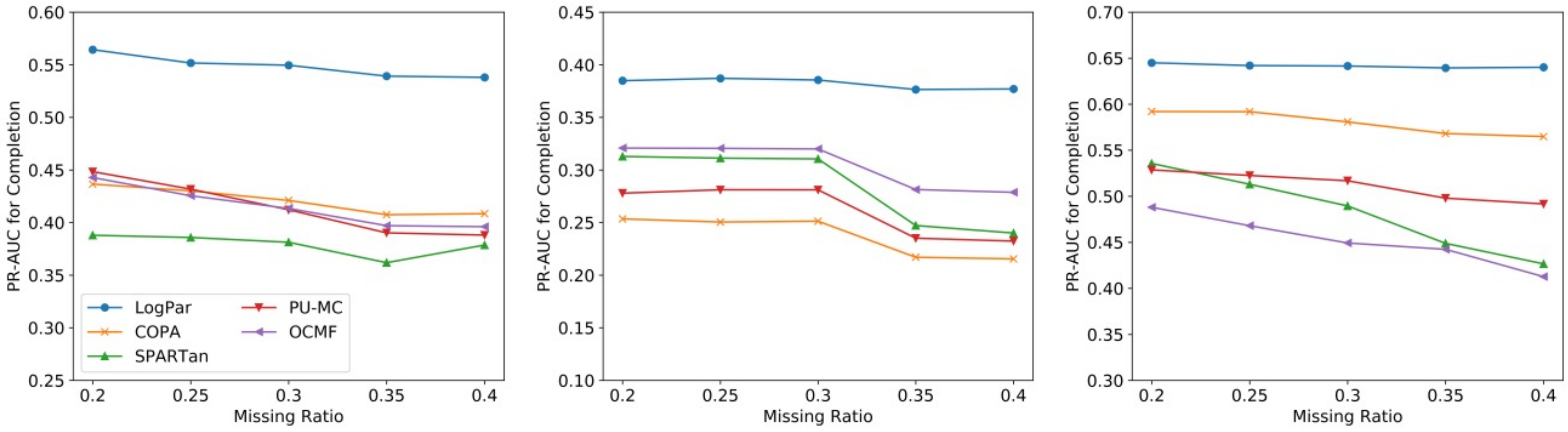


(a) Sutter Dataset (b) CMS Dataset (c) MIMIC-III Dataset

- LogPar consistently outperforms all baselines for all target ranks
- Performance of PU-MC decrease with increasing rank: severe overfitting.

Experiments and Results

- **Completion performance with varying missing ratios**
  - Fix rank of all models to 30

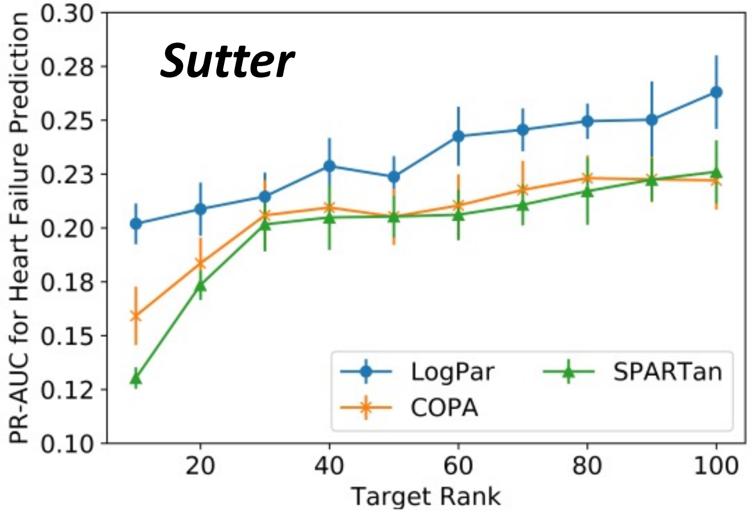


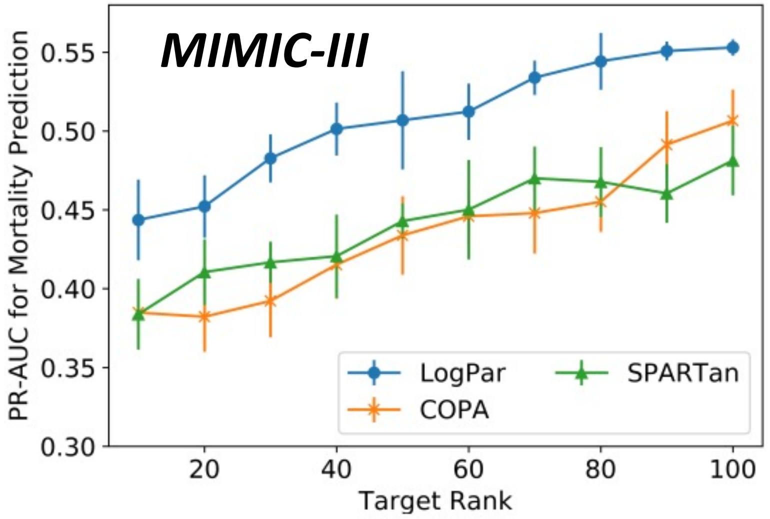
(a) Sutter Dataset (b) CMS Dataset (c) MIMIC-III Dataset

  - LogPar consistently outperforms all baselines for all level of missingness.

Experiments and Results

- **Downstream prediction tasks**
  - Fix missing ratio to 30% and split patients into training set and test set
  - Train LogPar with training set to learn the latent factor $\mathbf{V}$
  - Project test set onto the learned factor $\mathbf{V}$
  - Use $\boldsymbol{S}_k$ as features and train a logistic regression for classification. We use five-fold cross validation.





- **LogPar outperforms all baselines**

Conclusion

- We propose LogPar for binary irregular tensor factorization with explicit consideration of one-class missingness.
- LogPar is the **first PARAFAC2 model considering missingness**.
- We **incorporated the PU learning**, which greatly helps the completion.
- We **propose the uniqueness and the temporal smoothness regularization**.
- Empirical evaluation validates the effectiveness of LogPar and its components.

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