Mixed integer programming

- Variables:
  - $X[i][j][k]$: Assign subject *i* to room *j* in shift *k*.

$$0 \leq i, k < N, 0 \leq j < M, D(x[i][j][k]) = \{0, 1\}$$

  - Y: total number of shifts.

$$D(Y) = \{0,1, \dots N-1\}$$

  - $conf$: pair of subjects (*c1*, *c2*) that conflict.
  - $obj = Y$
- Goal:
  - $Y \rightarrow min$

Constraint:

  - Two conflict subjects can not be assign to same shift:

$$0 \leq x[i1][j][k] + x[i2][j][k] \leq 1 \quad \forall (i1, i2) \in conf\ ;\ k = 0..N-1;\ j = 0..M-1$$

  - One subject can be assigned once:

$$1 \leq \sum_{k=0}^{N-1} \sum_{j=0}^{M-1} x[i][j][k] \leq 1 \quad \forall i = 0..N-1$$

  - In any shift, one room can contain only one subject:

$$1 \leq \sum_{i=0}^{N-1} x[i][j][k] \leq 1 \quad \forall k = 0..N-1;\ j = 0..M-1$$

  - Put subjects in the rooms with suitable capacity:

$$0 \leq \sum_{k=0}^{M-1} x[i][j][k] * d[i] \leq c[j] \quad \forall i = 0..N-1;\ j = 0..M-1$$

  - Total number of shifts:

$$-\infty \leq x[i][j][k] \times k - y \leq 0 \quad \forall\ i, k = 0..N-1;\ j = 0..M-1$$